# Lecture 9

Graphs, BFS and DFS

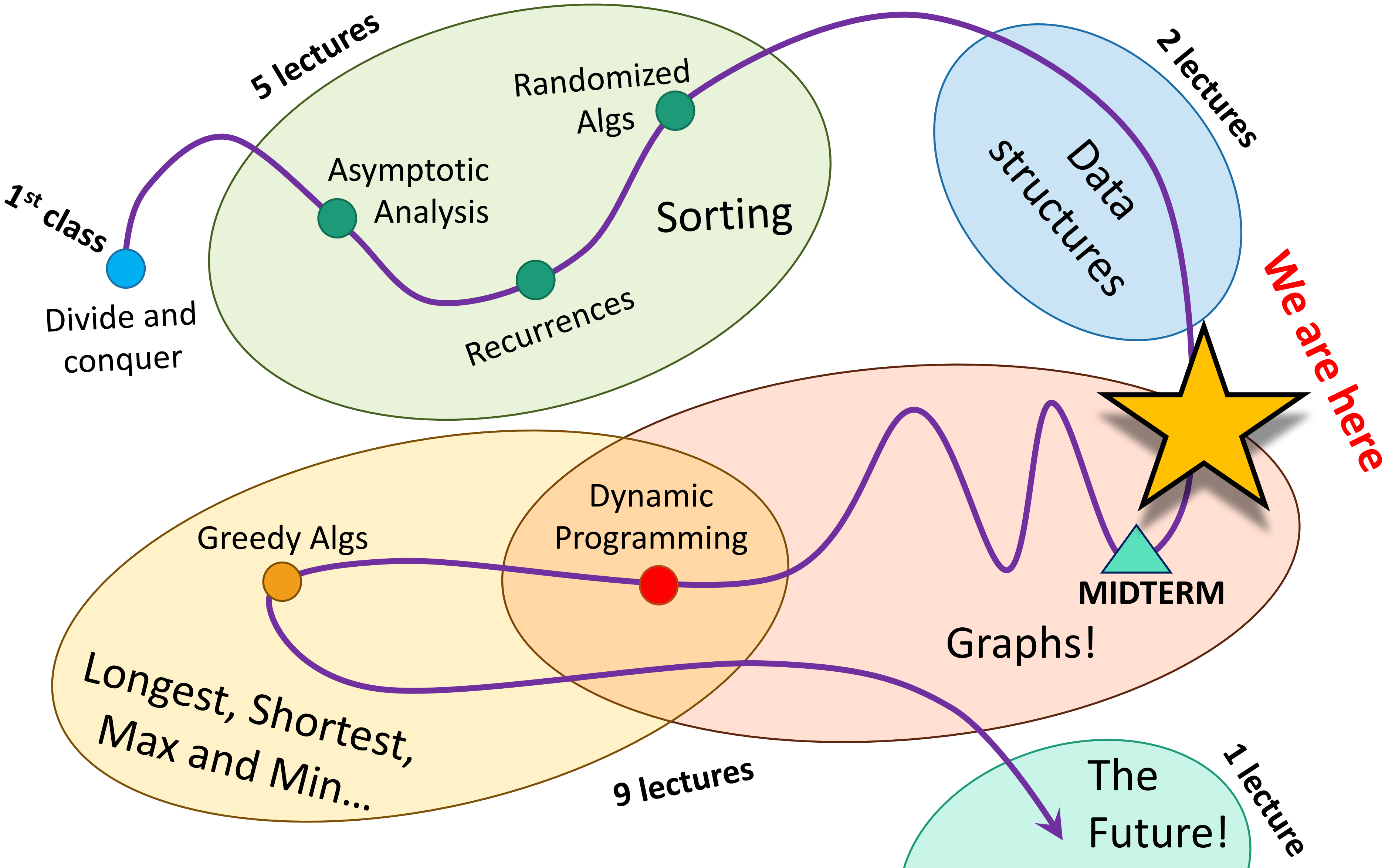
# Announcements!

- HW4 due Friday
- **MIDTERM** in class, Monday 10/30.
  - That's 1 week from today. **Please show up.**
  - During class, 1:30-2:50
    - If your last name is A-M: 370-370 (here)
    - If your last name is N-V: 160-124
    - If your last name is W-Z: 160-323
  - You may bring one double-sided letter-size page of notes, that ***you have prepared yourself***.

  - Any material through Hashing (Lecture 8) is fair game.
  - Practice exams on the website
  - Review Session tomorrow in Section

More detailed schedule on the website!

# Roadmap

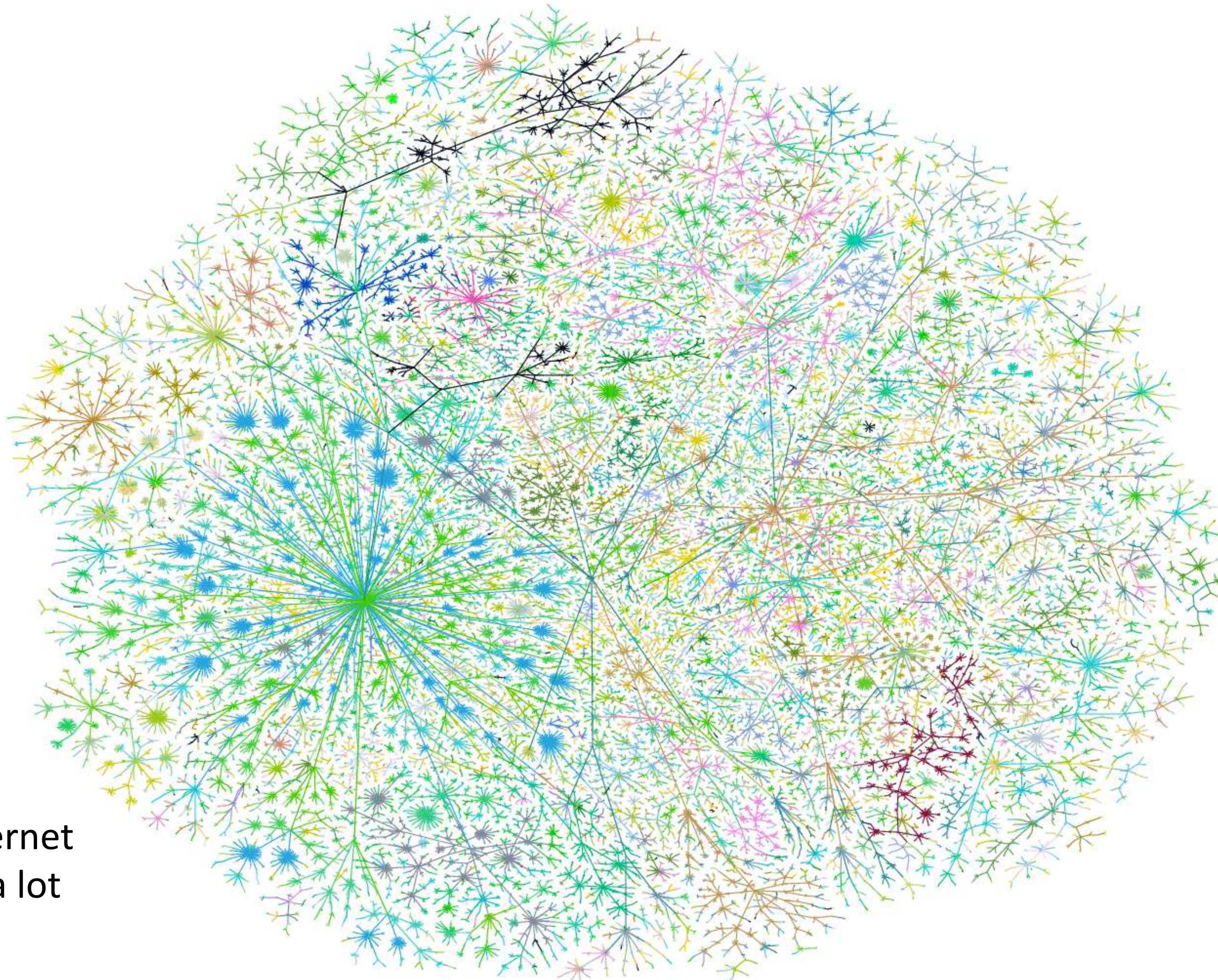


# Outline

- Part 0: Graphs and terminology
- Part 1: Depth-first search
  - Application: topological sorting
  - Application: in-order traversal of BSTs
- Part 2: Breadth-first search
  - Application: shortest paths
  - Application (if time): is a graph bipartite?

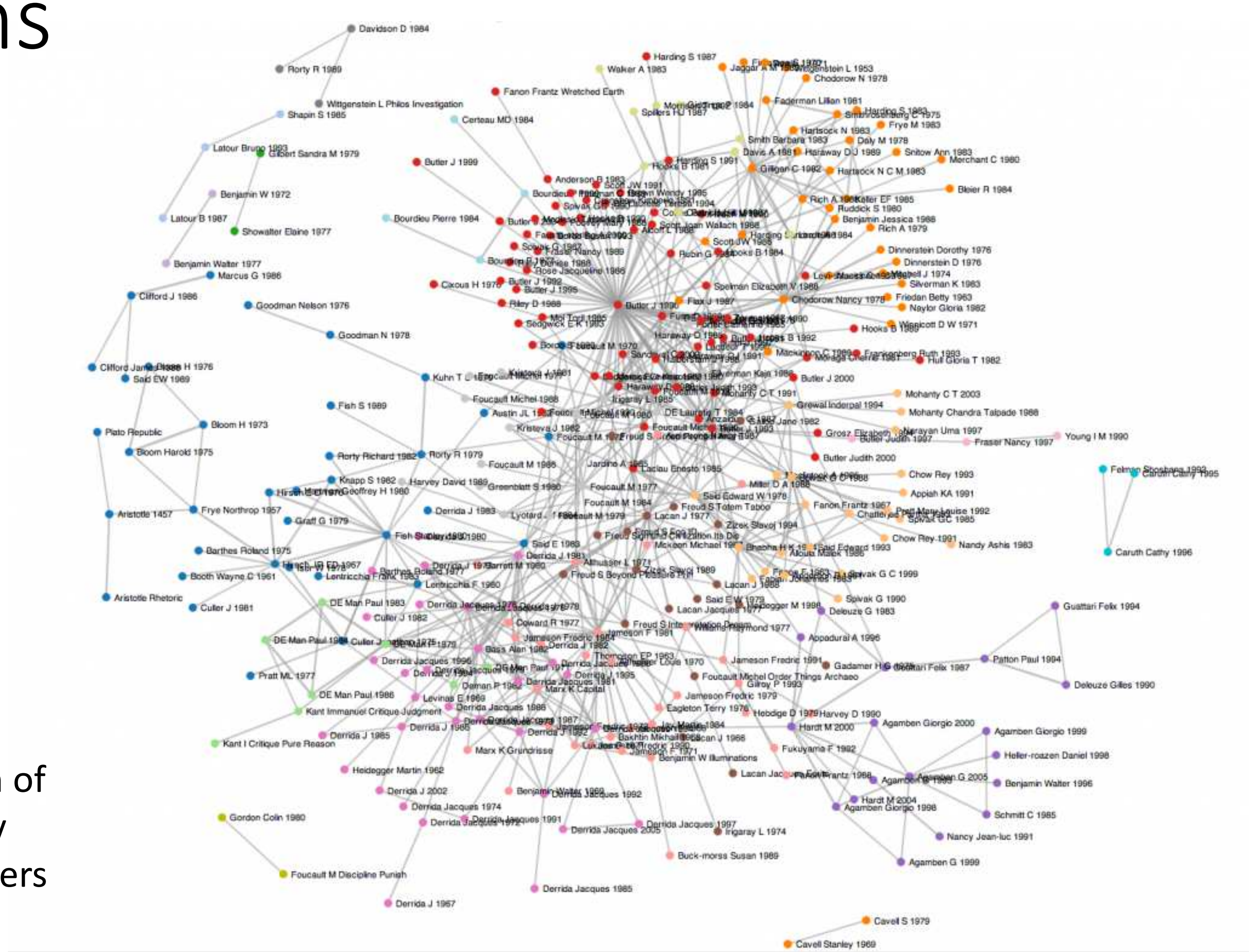
# Part 0: Graphs

# Graphs



Graph of the internet (circa 1999…it's a lot bigger now…)

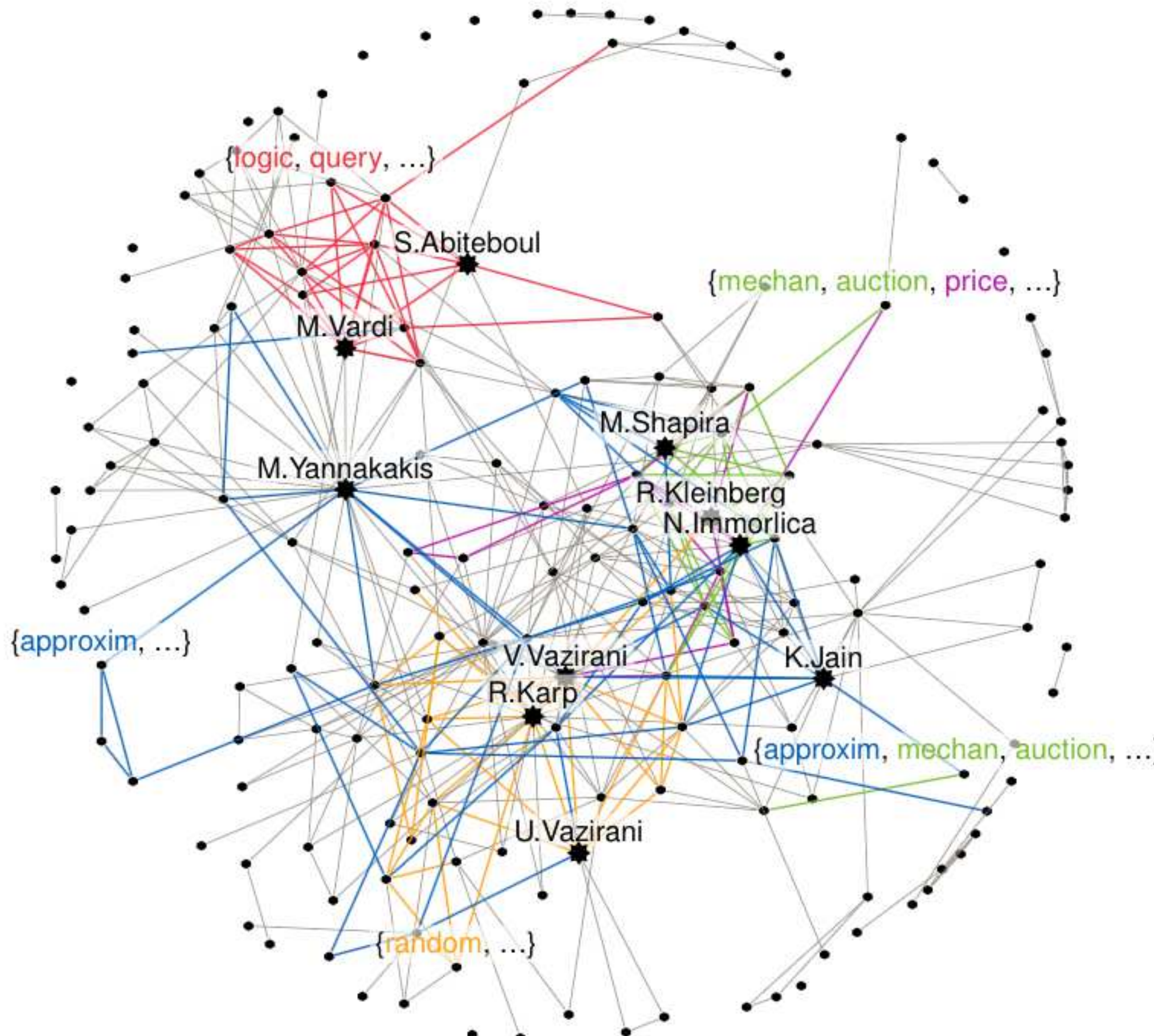
# Graphs




Citation graph of literary theory academic papers

# Graphs

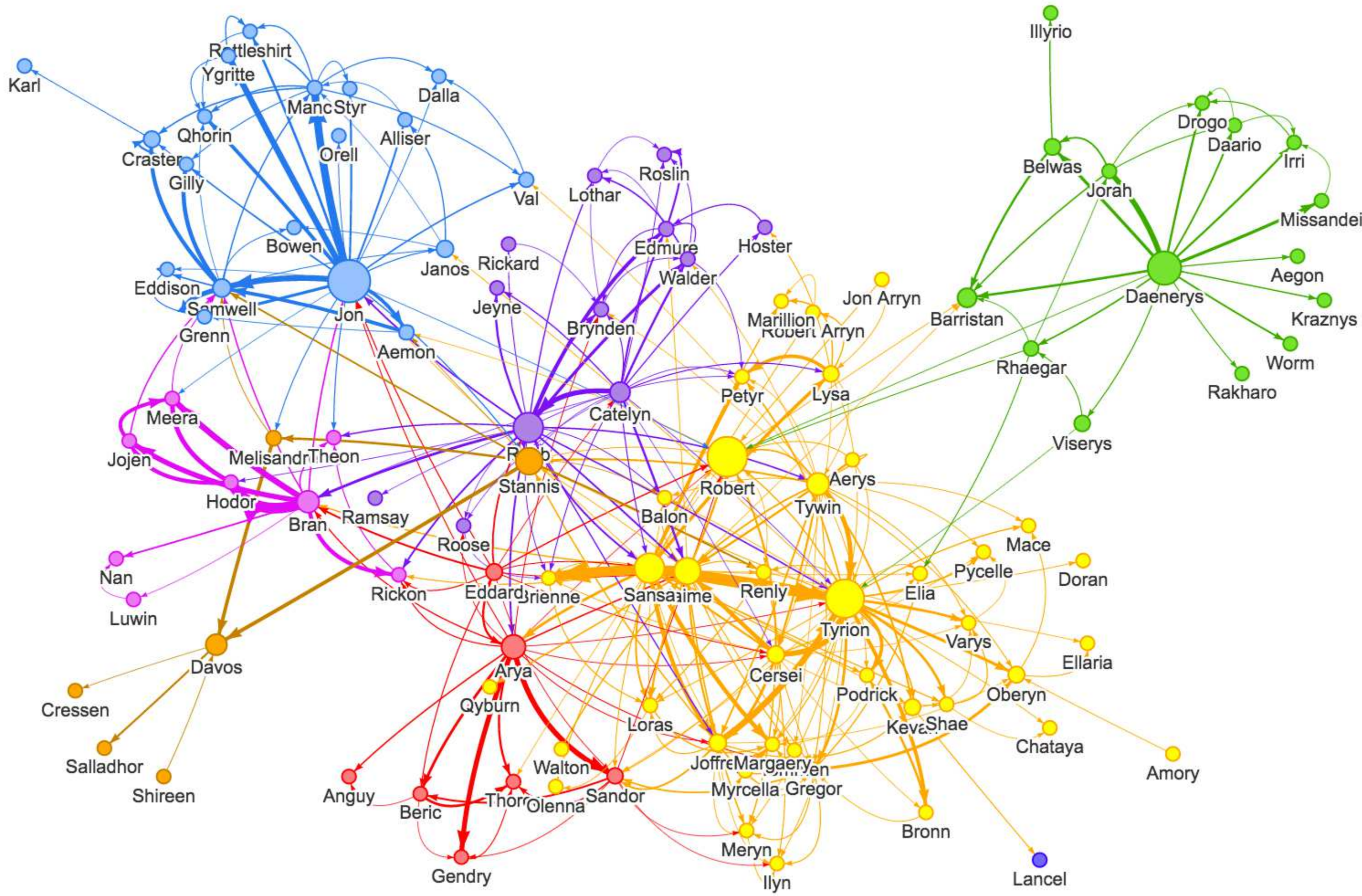
Theoretical Computer Science academic communities




*Example from DBLP:*
Communities within the co-authors of Christos H. Papadimitriou

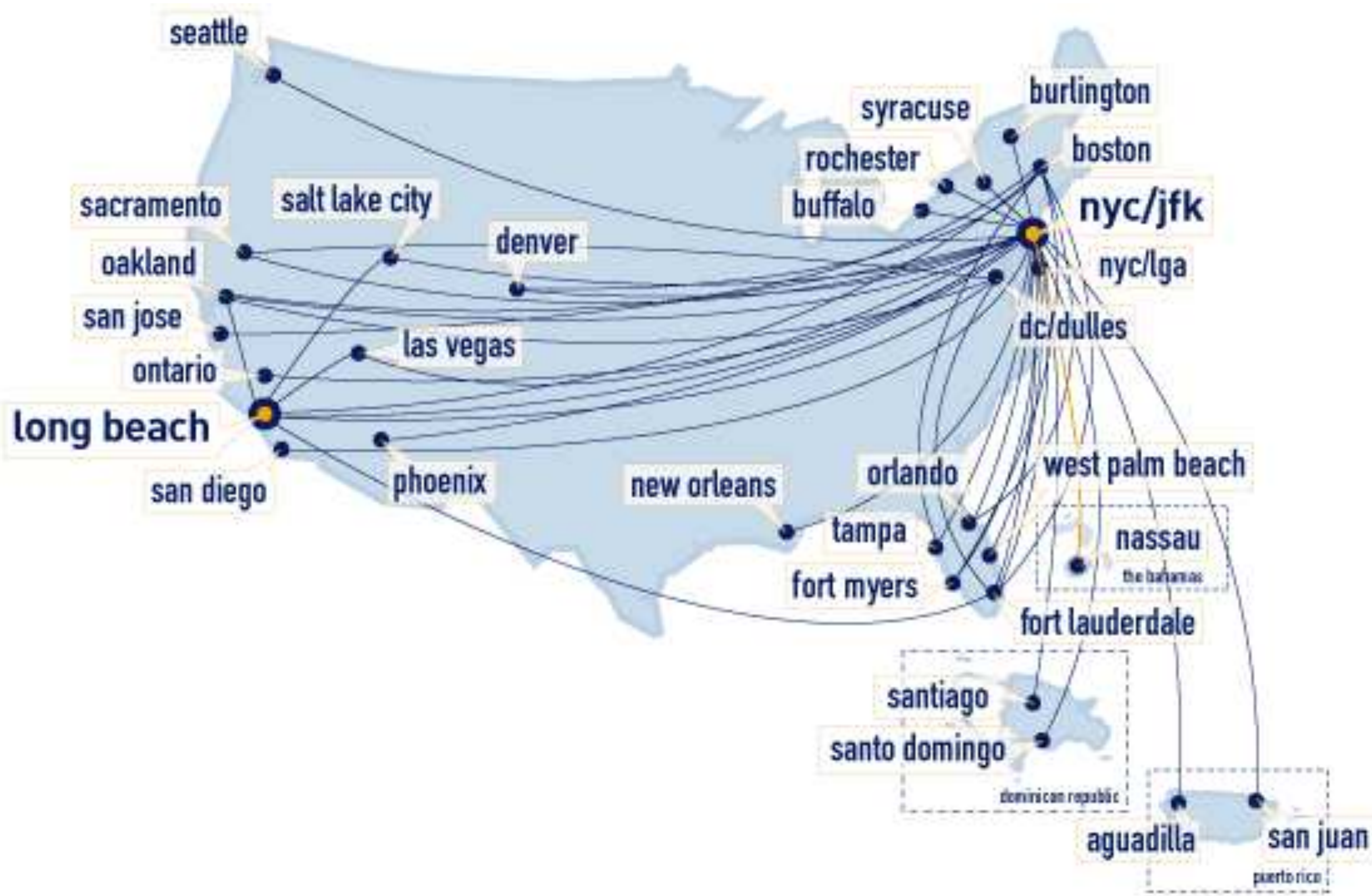
# Graphs

Game of Thrones Character Interaction Network



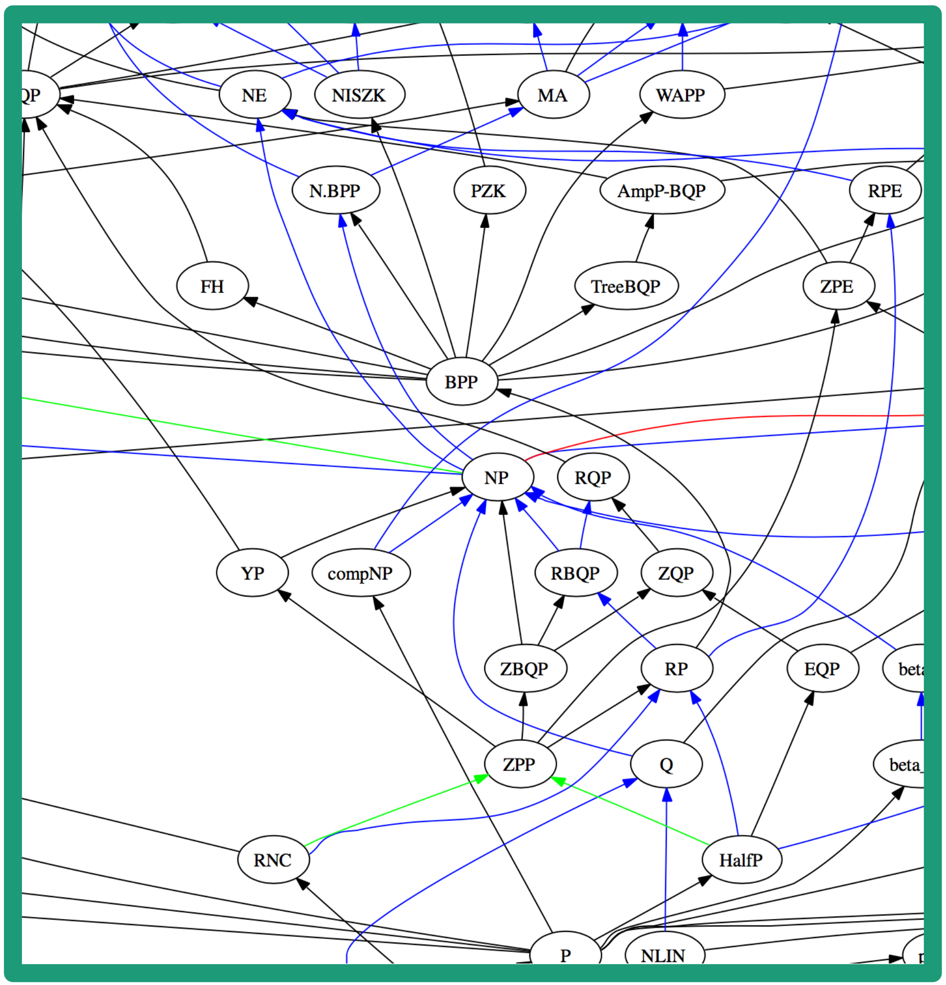
# Graphs

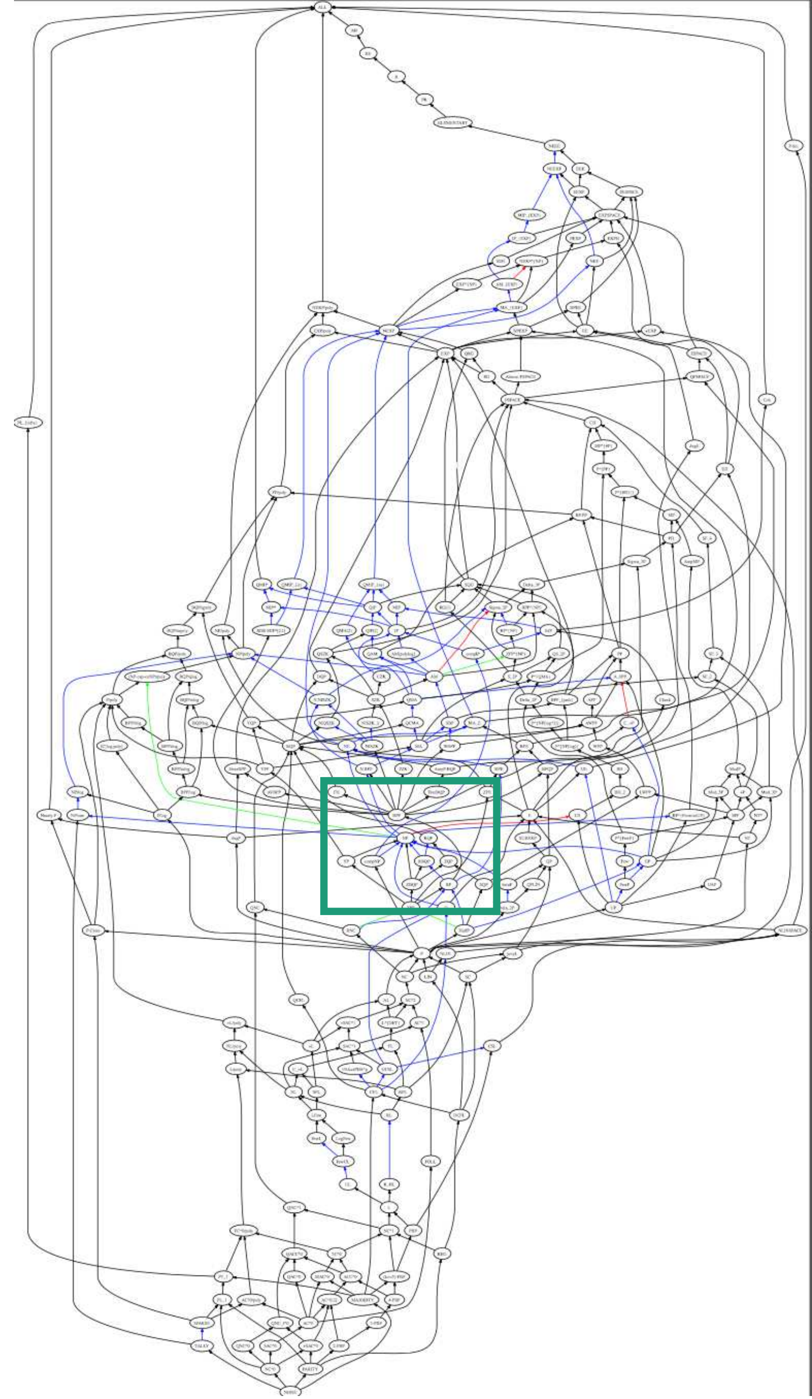
jetblue flights



# Graphs

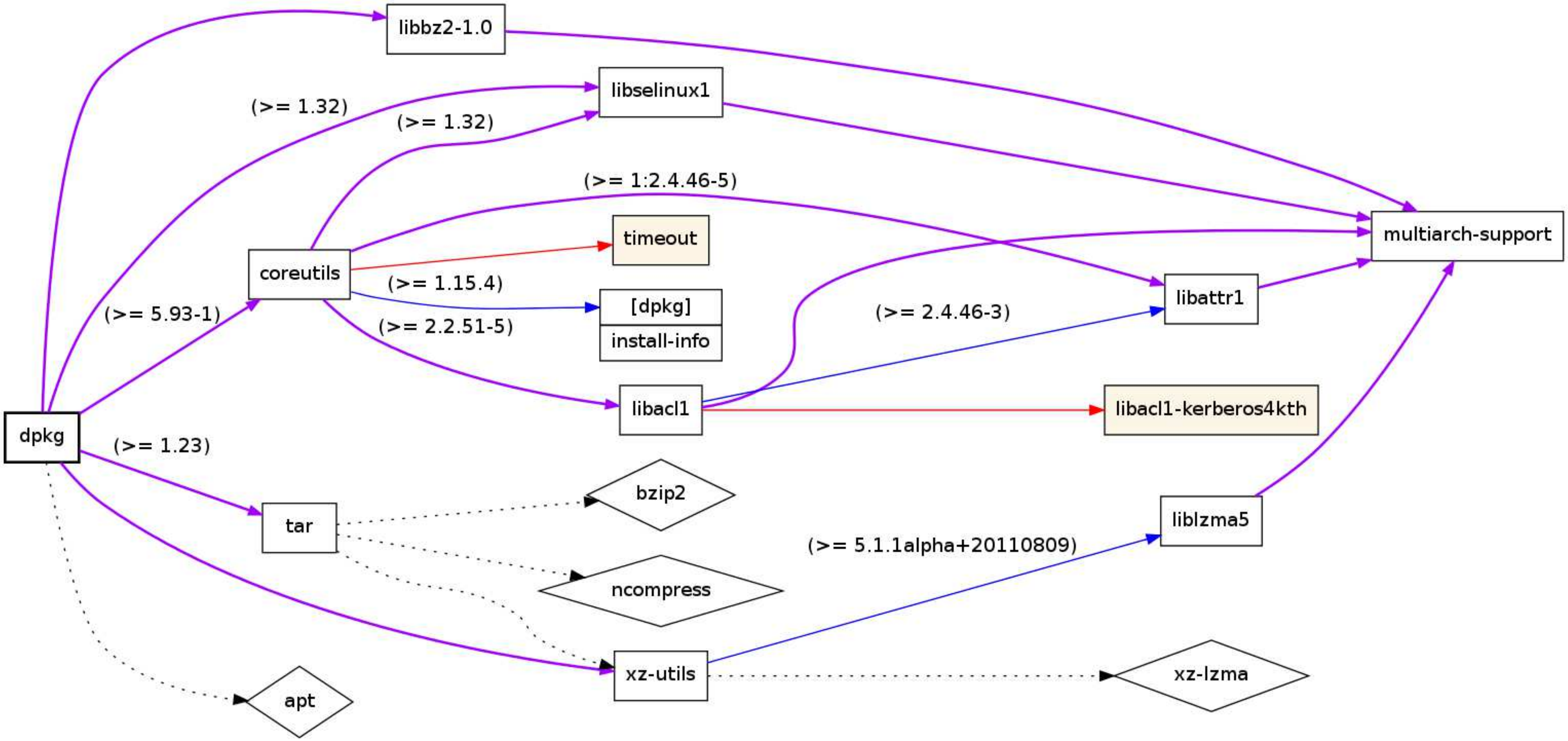
Complexity Zoo
containment graph



# Graphs

debian dependency (sub)graph



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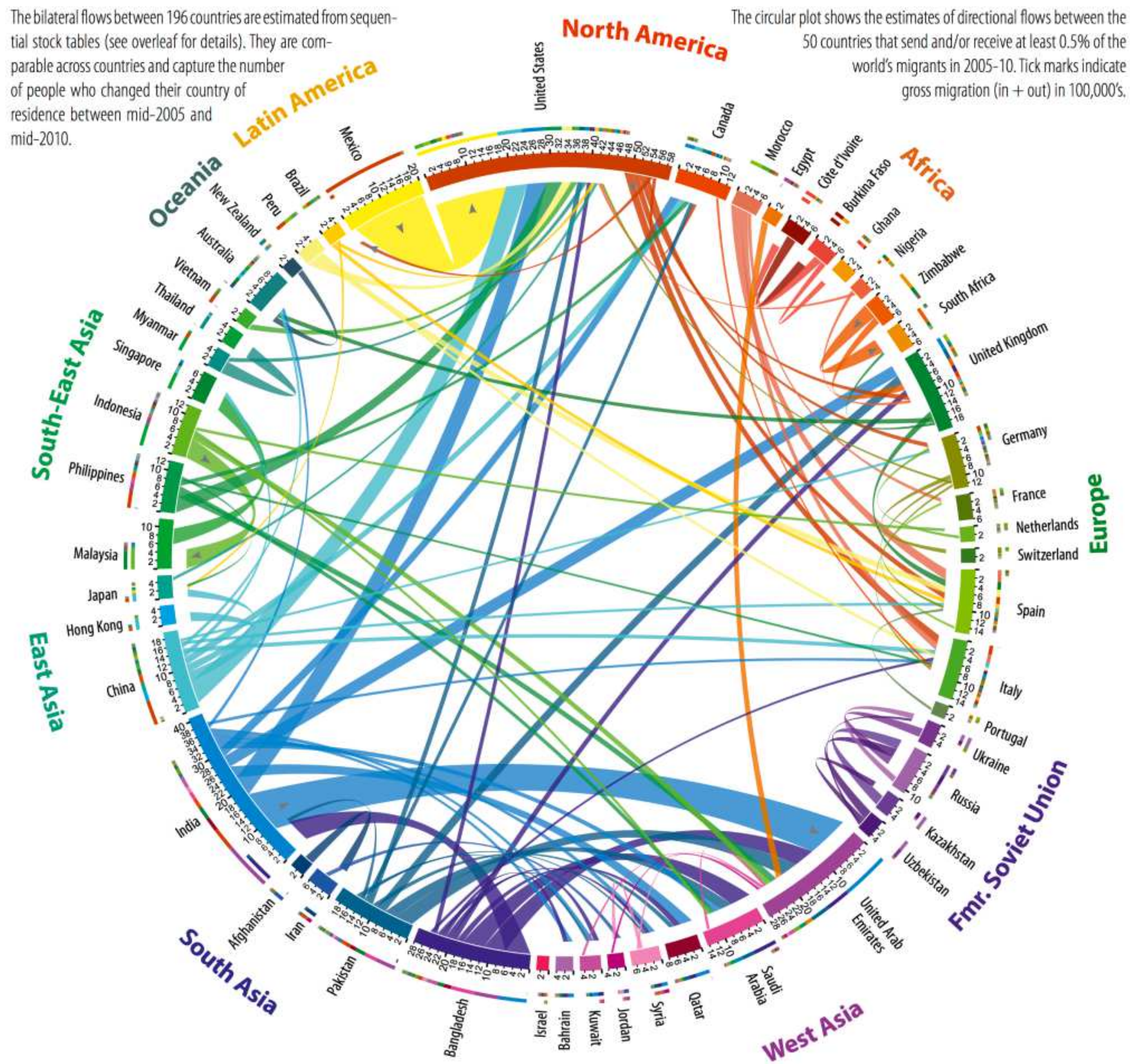
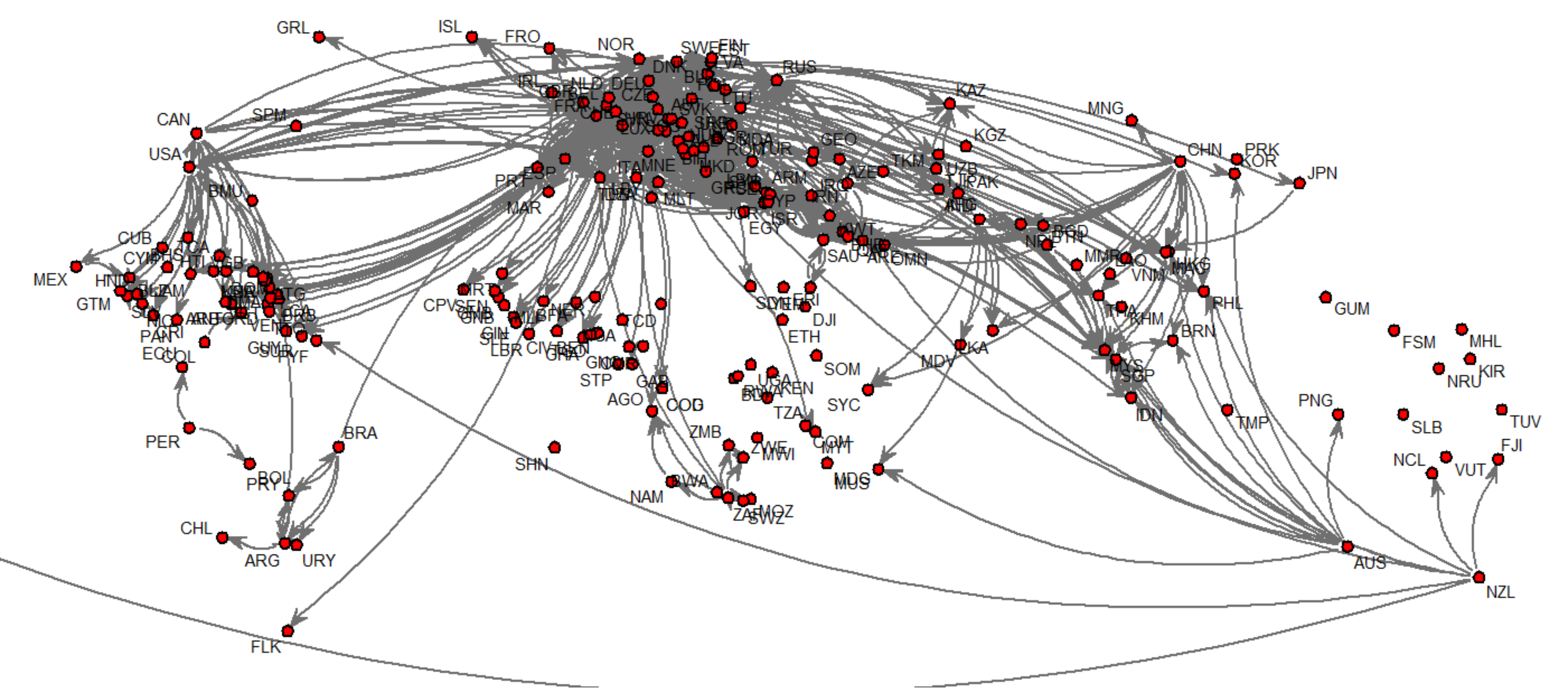



Immigration flows

# Graphs

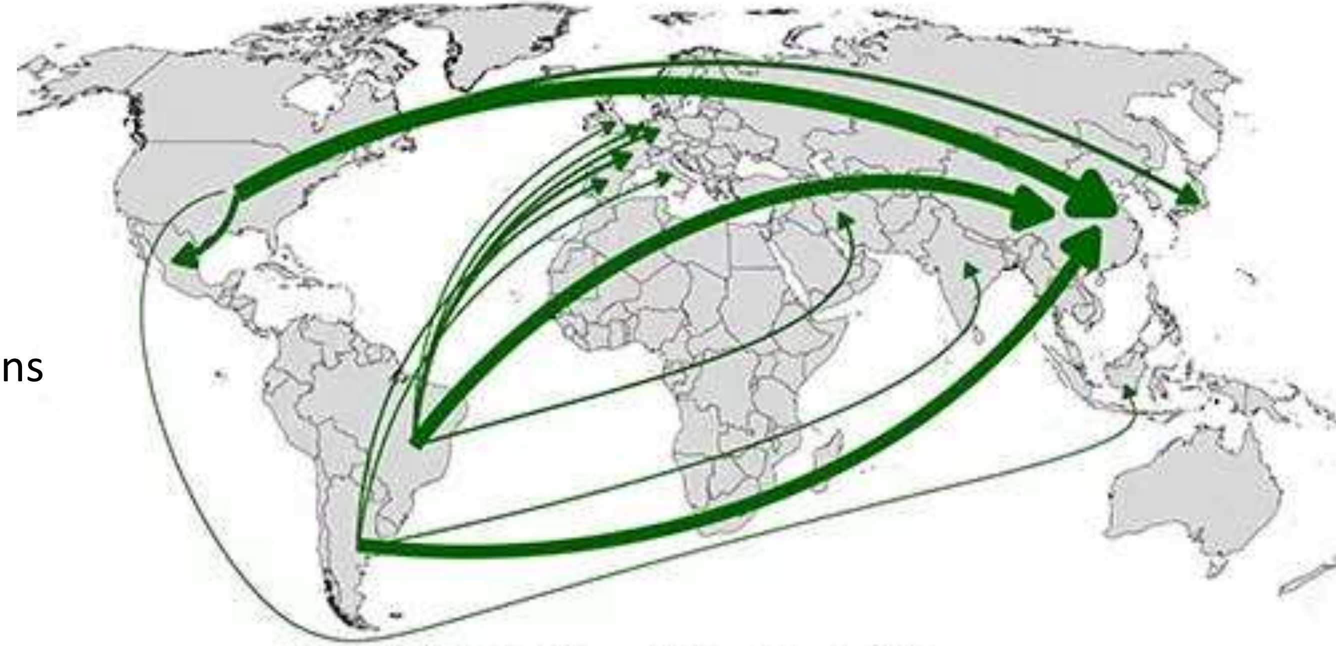
Potato trade

World trade in fresh potatoes, flows over 0.1 m US$ average 2005-2009

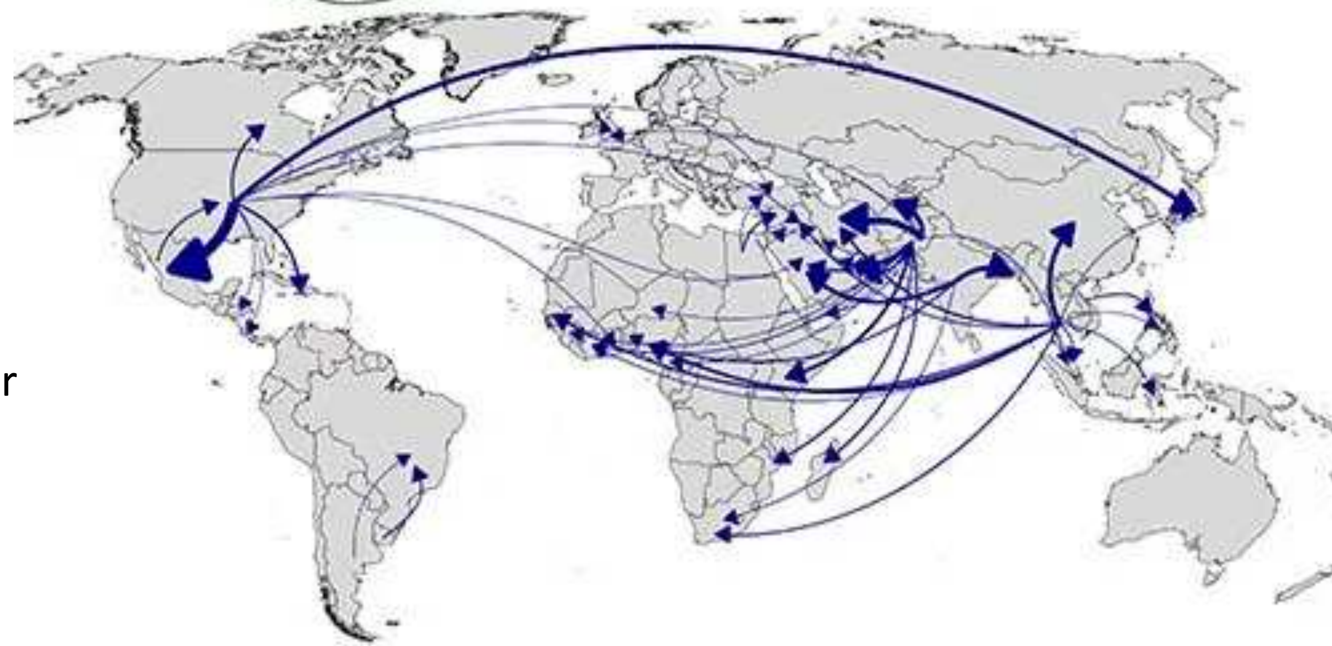


# Graphs

Soybeans

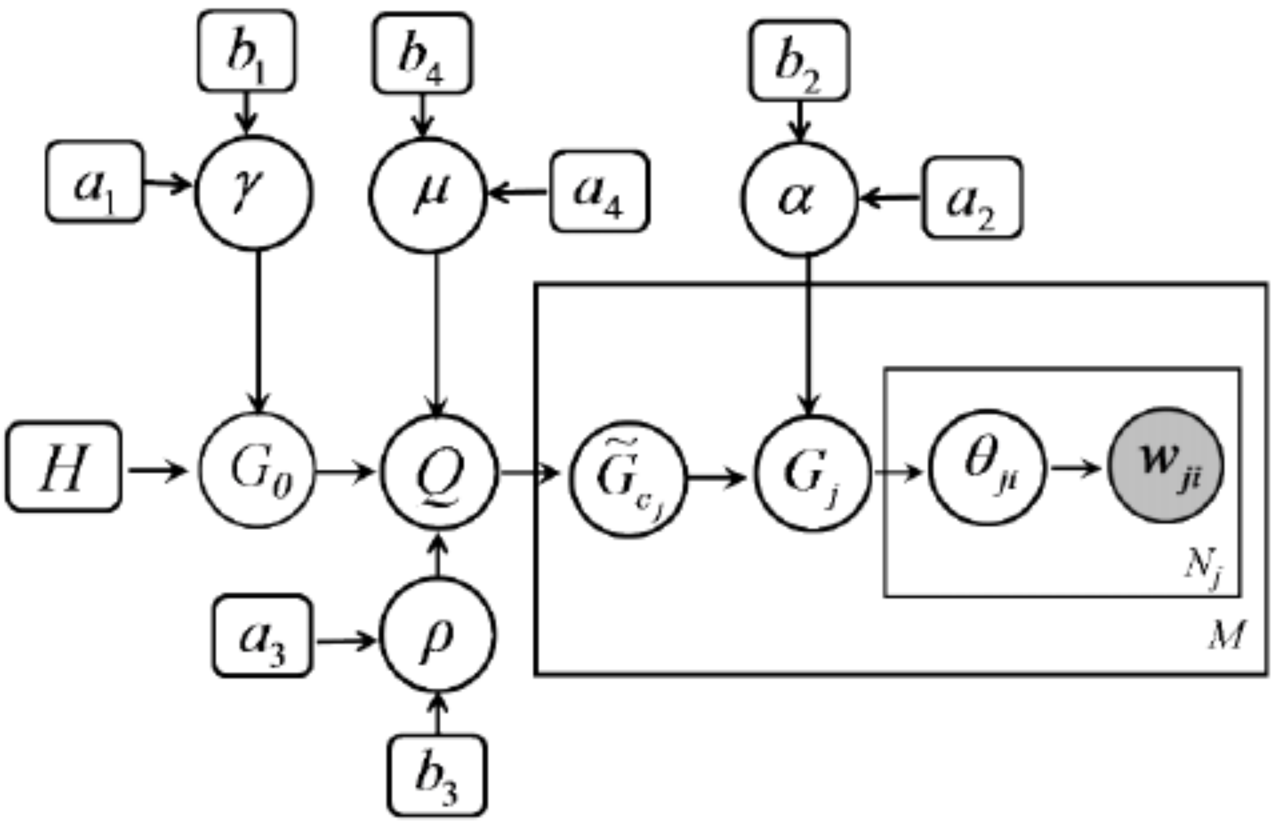


Water

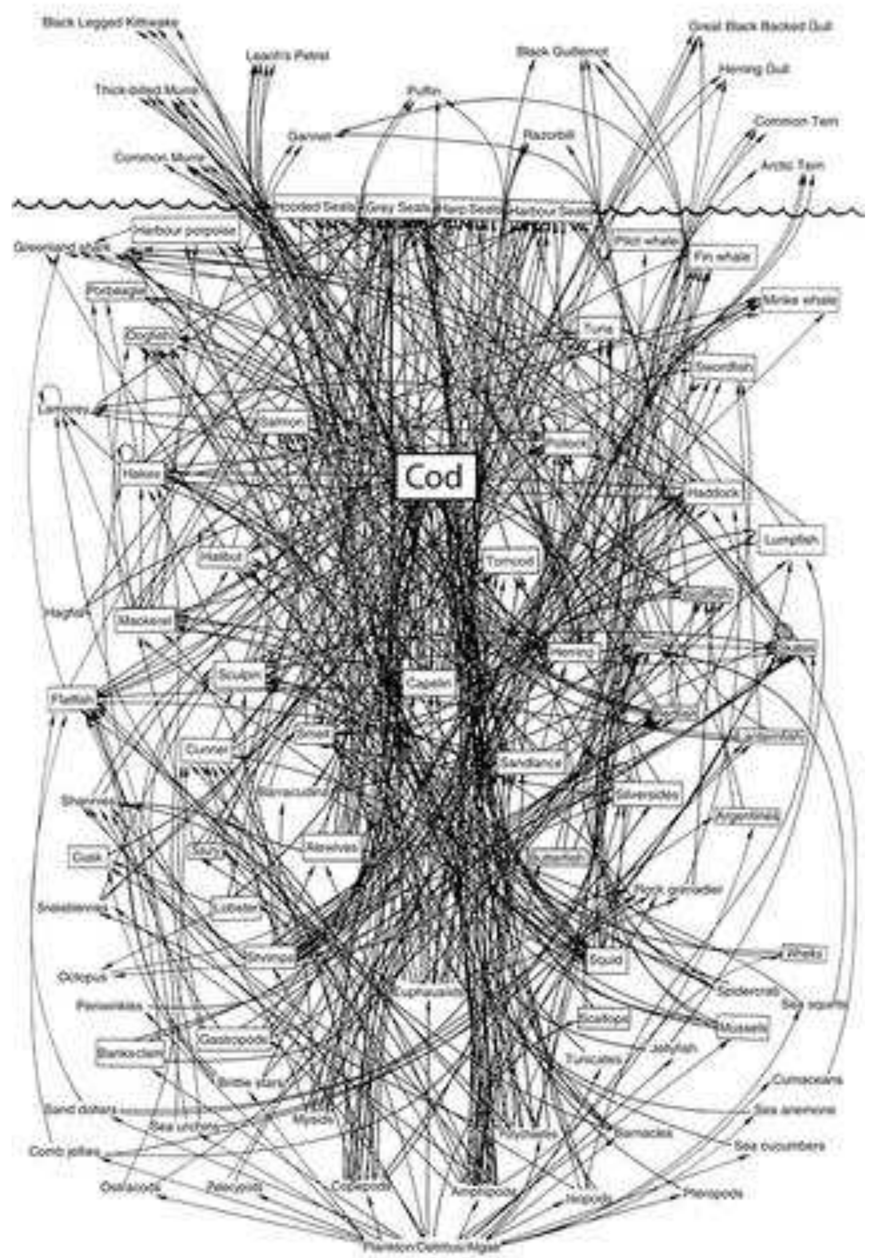


# Graphs

Graphical models



# Graphs

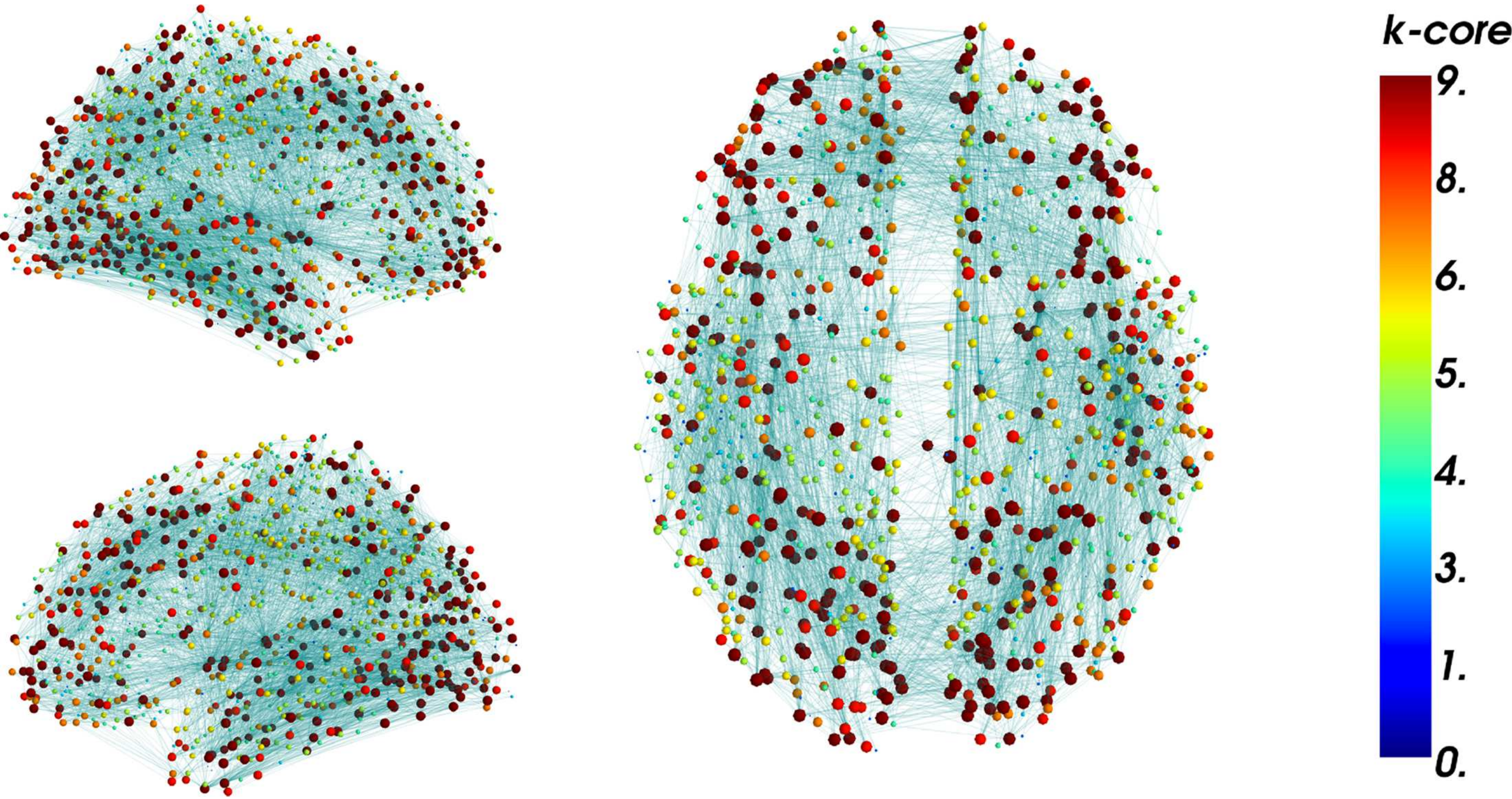



A simplified food web for the Northwest Atlantic. © IMMA

What eats what in the Atlantic ocean?

# Graphs

Neural connections
in the brain



# Graphs

- **There are a lot of graphs.**

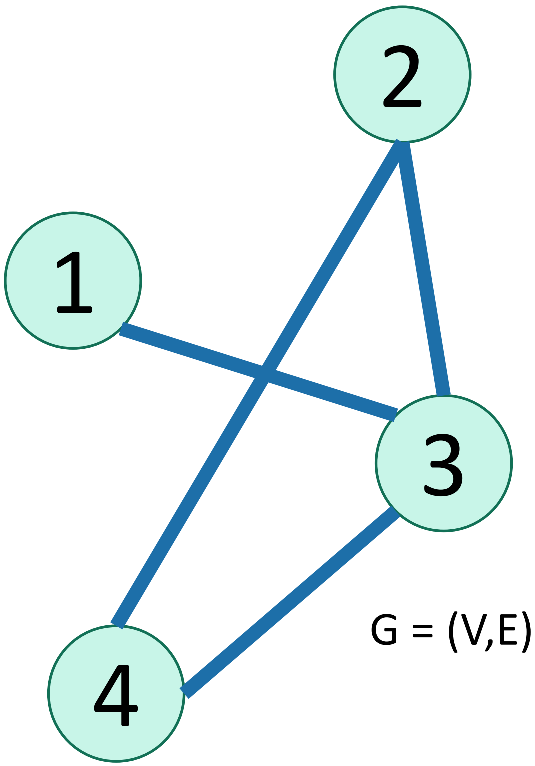
- We want to answer questions about them.
  - Efficient routing?
  - Community detection/clustering?
  - From pre-lecture exercise:
    - Computing Bacon numbers
    - Signing up for classes without violating pre-req constraints
    - How to distribute fish in tanks so that none of them will fight.

- This is what we'll do for the next several lectures.

# Undirected Graphs

- Has vertices and edges
  - V is the set of vertices
  - E is the set of edges
  - Formally, a graph is G = (V,E)

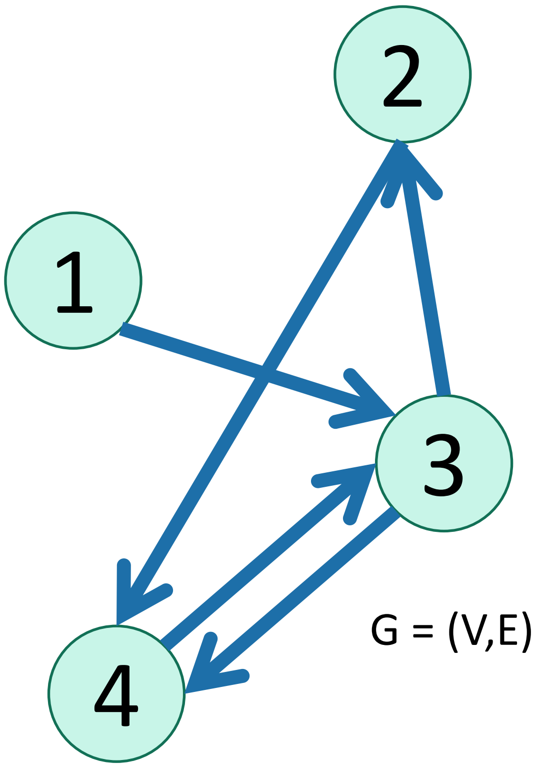
- Example
  - V = {1,2,3,4}
  - E = { {1,3}, {2,4}, {3,4}, {2,3} }




- The **degree** of vertex 4 is 2.
  - There are 2 edges coming out
- Vertex 4's **neighbors** are 2 and 3

# Directed Graphs

- Has vertices and edges
  - V is the set of vertices
  - E is the set of **DIRECTED** edges
  - Formally, a graph is G = (V,E)



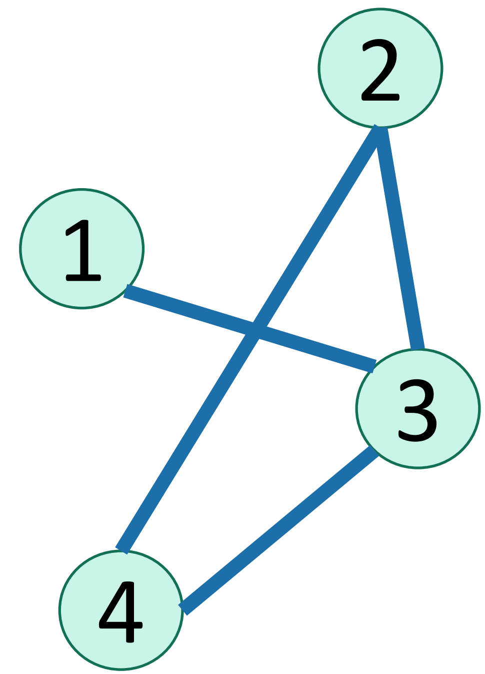

- Example
  - V = {1,2,3,4}
  - E = { (1,3), (2,4), (3,4), (4,3), (3,2) }

- The **in-degree** of vertex 4 is 2.
- The **out-degree** of vertex 4 is 1.
- Vertex 4's **incoming neighbors** are 2,3
- Vertex 4's **outgoing neighbor** is 3.

# How do we represent graphs?

- Option 1: adjacency matrix

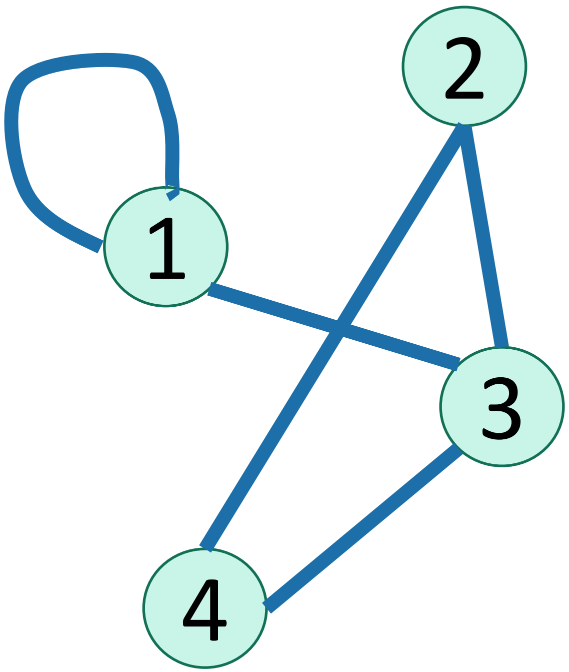
| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 1 |
| 3 | 1 | 1 | 0 | 1 |
| 4 | 0 | 1 | 1 | 0 |



# How do we represent graphs?

- Option 1: adjacency matrix

$$
\begin{array}{c c}
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}
\end{array}
$$

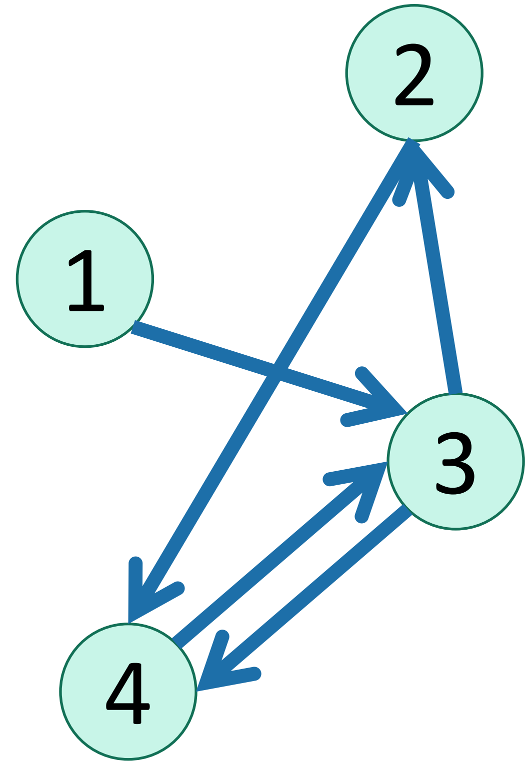


# How do we represent graphs?

- Option 1: adjacency matrix

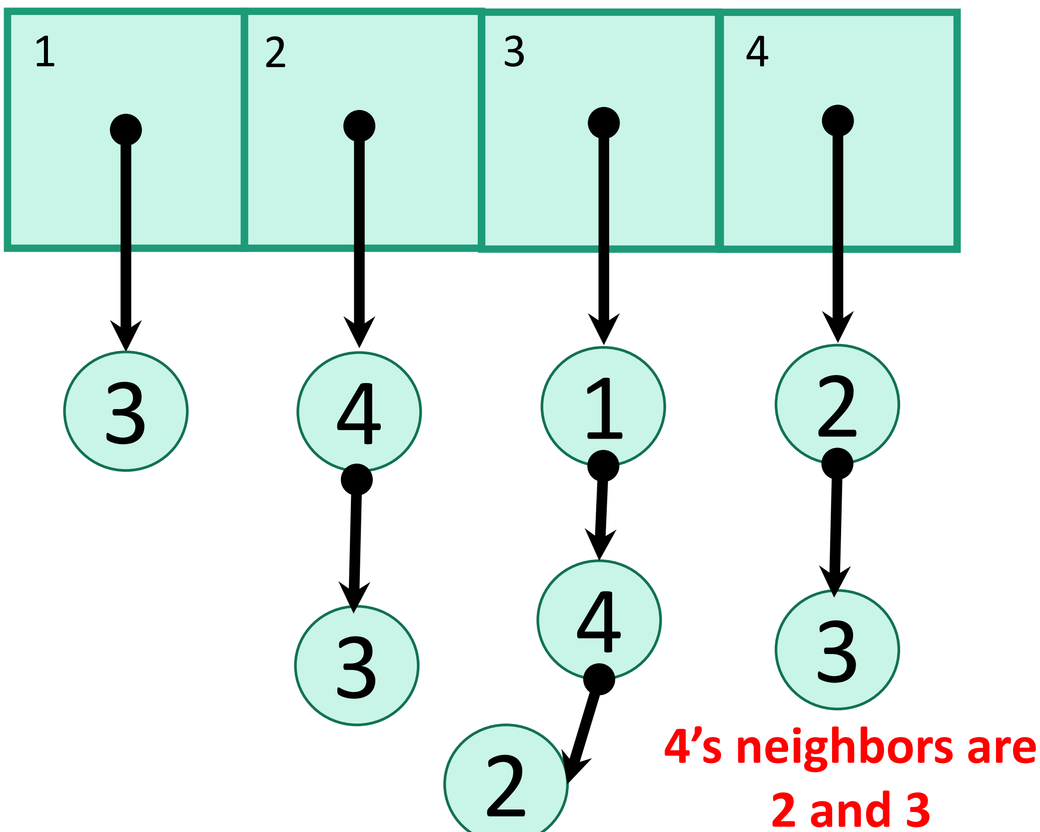
Destination

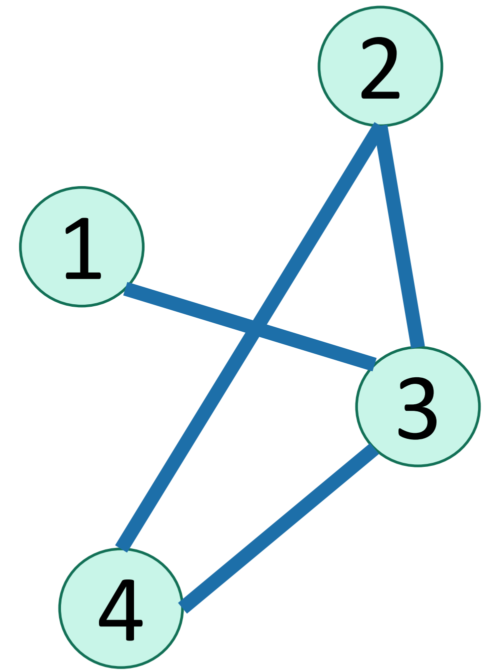
| Source | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 0 | 1 |
| 3 | 0 | 1 | 0 | 1 |
| 4 | 0 | 0 | 1 | 0 |



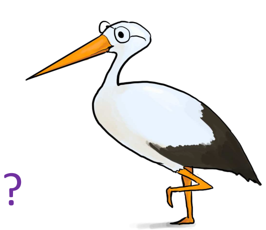
# How do we represent graphs?

- Option 2: linked lists.




How would you modify this for directed graphs?

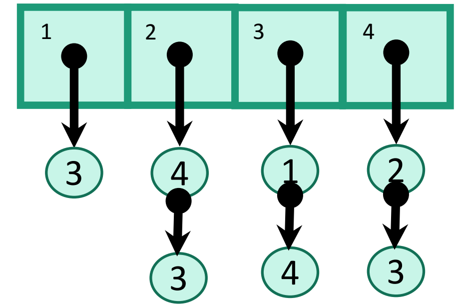


# In either case

- Vertices can store other information
  - Attributes (name, IP address, …)
  - helper info for algorithms that we will perform on the graph
- Want to be able to do the following operations:
  - **Edge Membership**: Is edge e in E?
  - **Neighbor Query**: What are the neighbors of vertex v?

# Trade-offs

Generally better for **sparse** graphs

| Say there are n vertices and m edges. | $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ |  |
|---|---|---|
| Edge membership<br>Is e = {v,w} in E? | O(1) | O(deg(v)) or O(deg(w)) |
| Neighbor query<br>Give me v's neighbors. | O(n) | O(deg(v)) |
| Space requirements | $O(n^2)$ | O(n + m) |

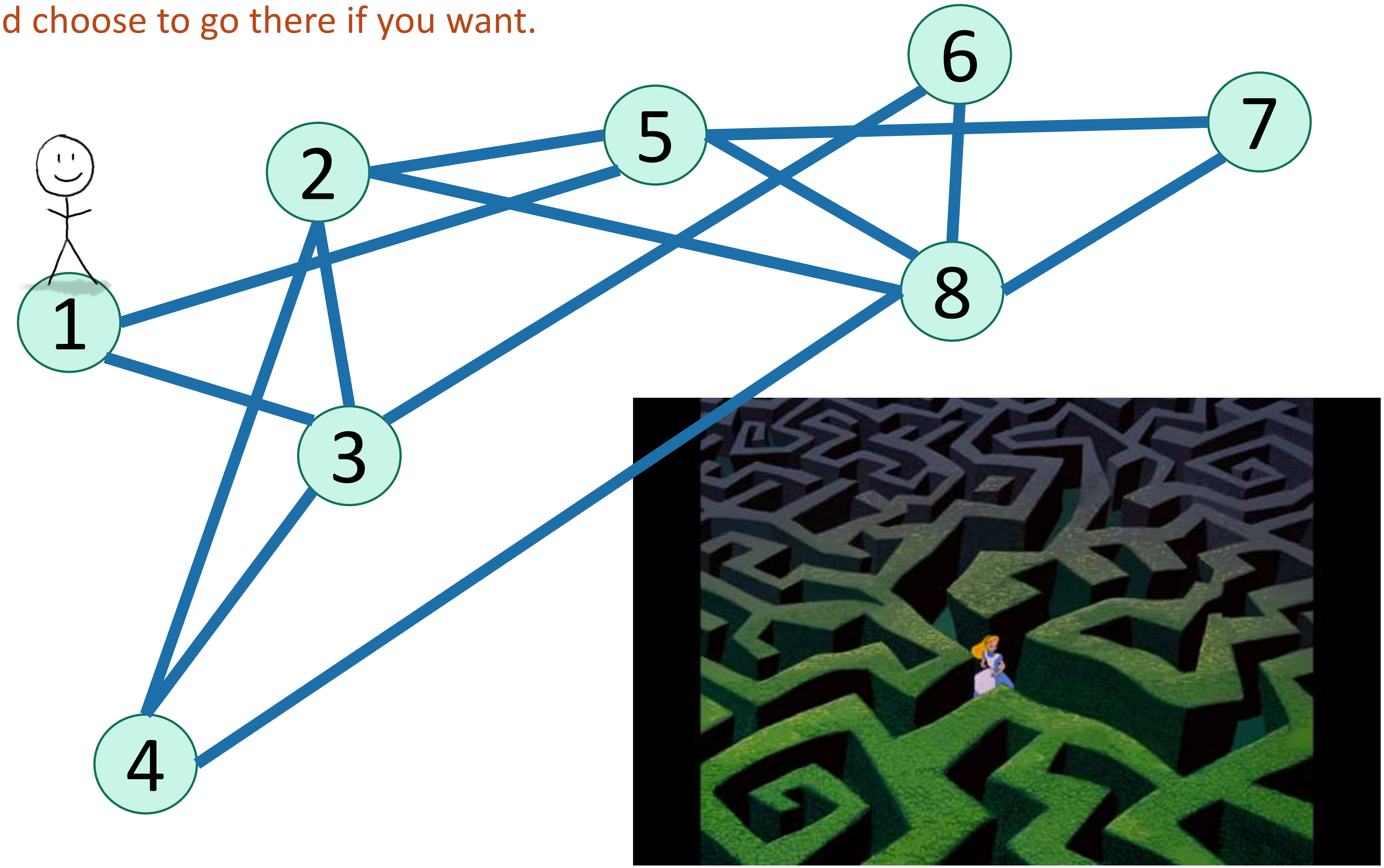
**See Lecture 9 IPython notebook for the actual data structure that we will be using!**

**We'll assume this representation for the rest of the class**

# Part 1: Depth-first search

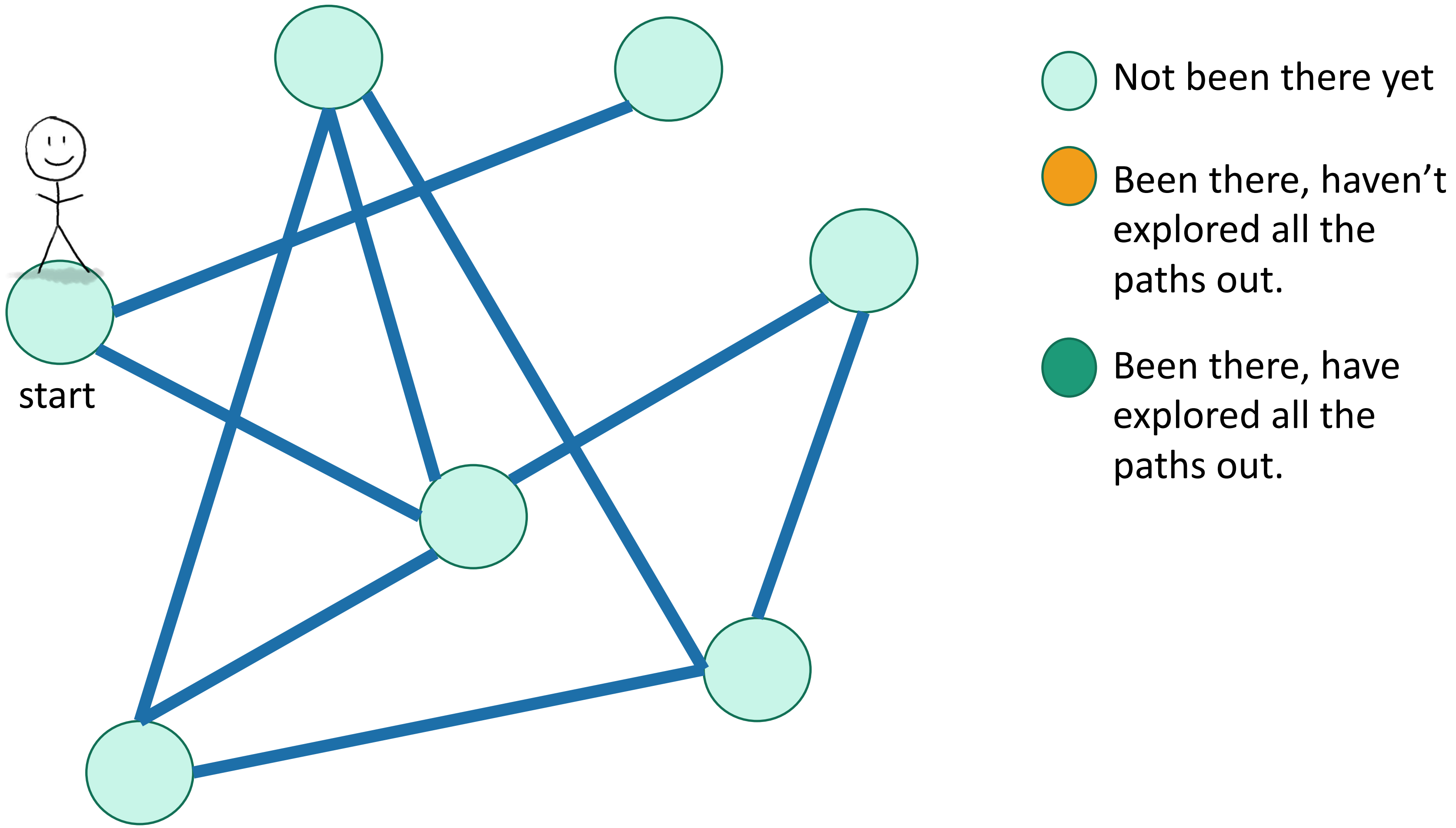
# How do we explore a graph?

At each node, you can get a list of neighbors,
and choose to go there if you want.



# Depth First Search

## Exploring a labyrinth with chalk and a piece of string



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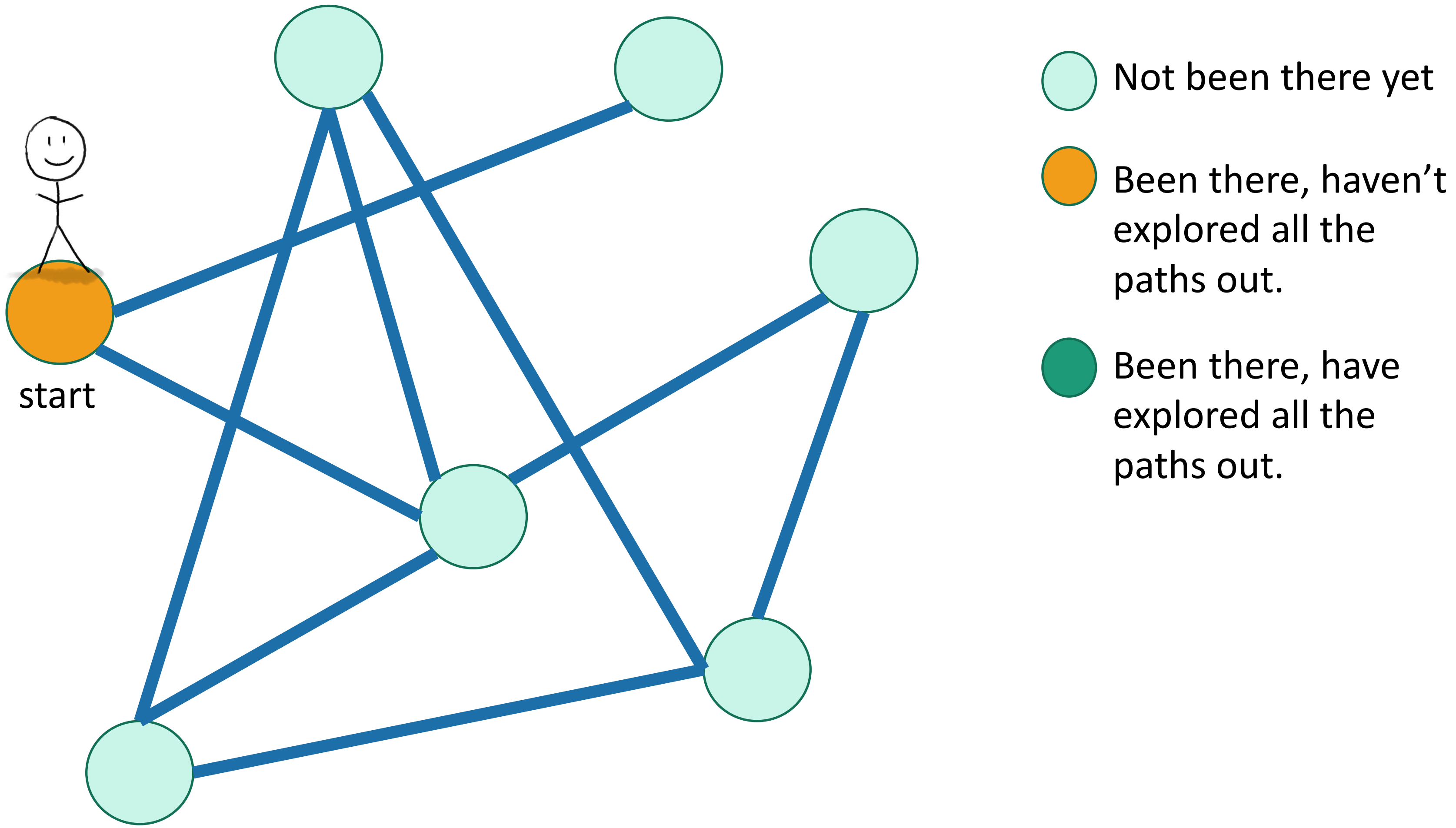

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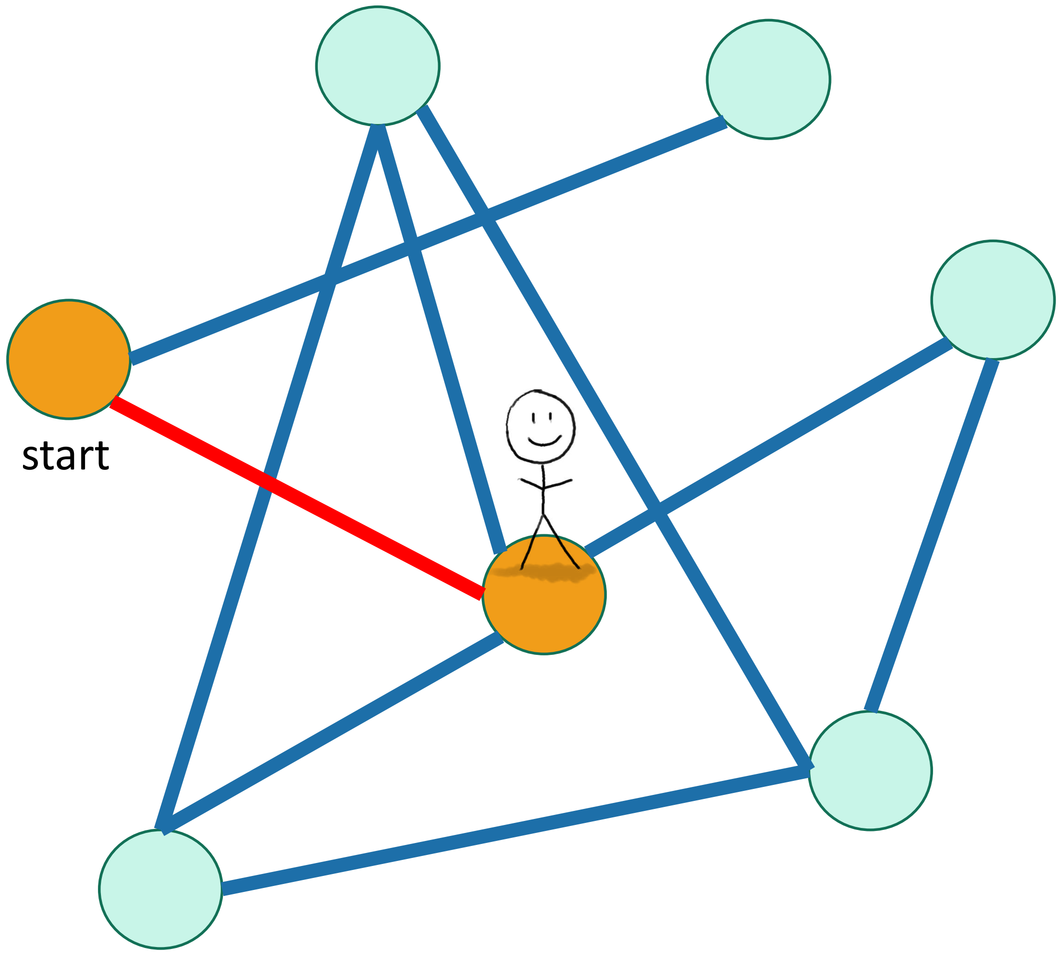


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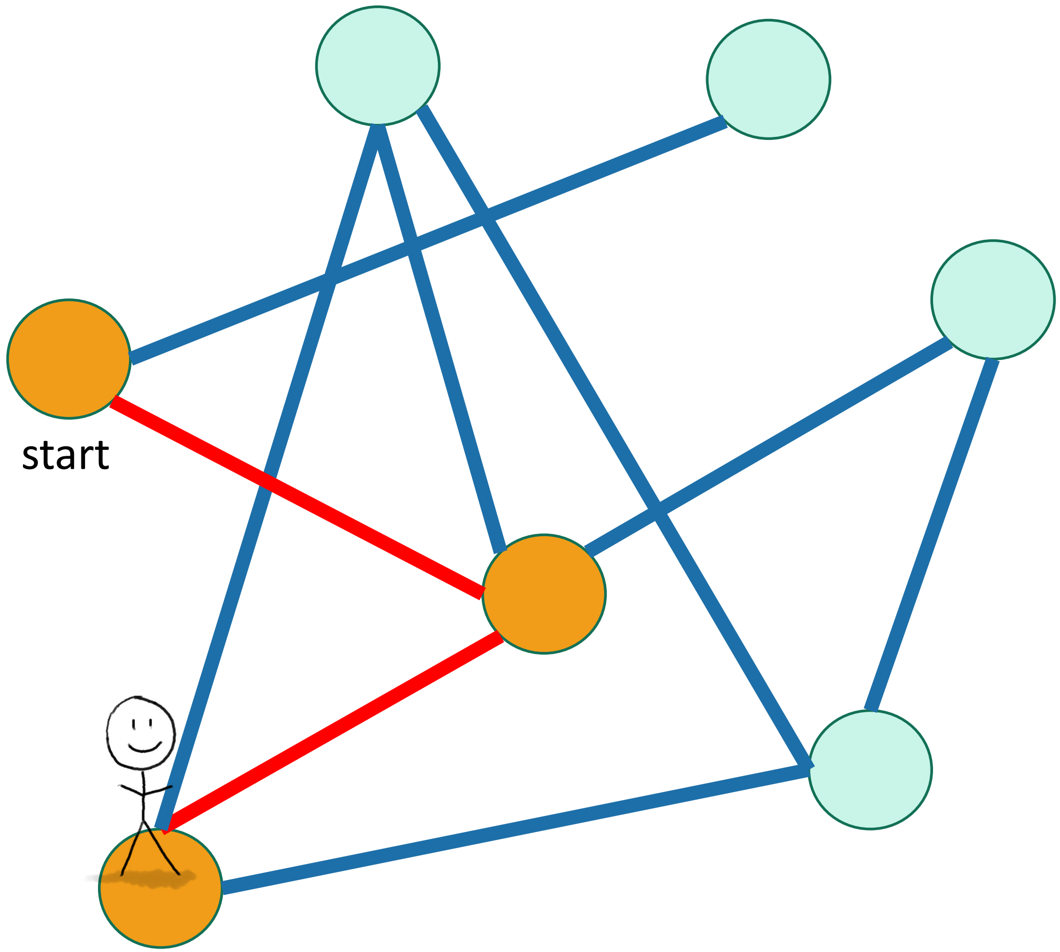


- Not been there yet
- Been there, haven't explored all the paths out.
- Been there, have explored all the paths out.

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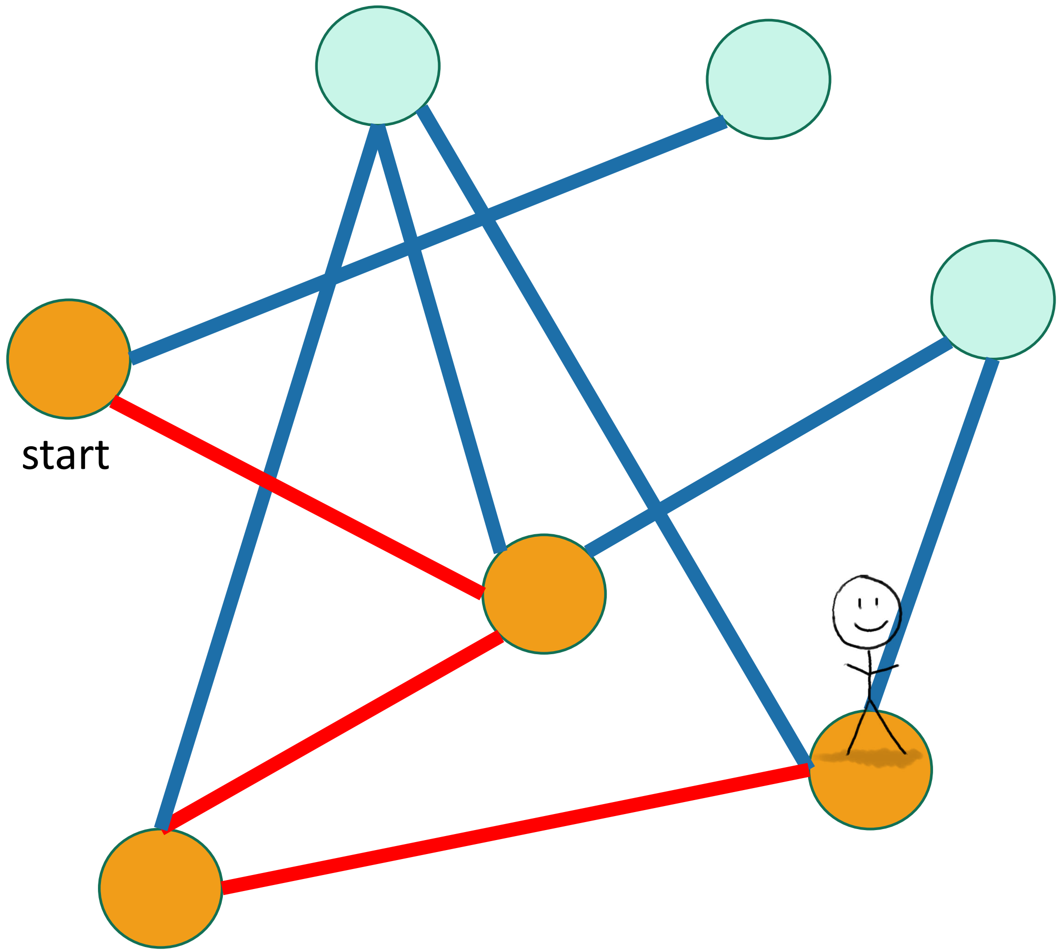


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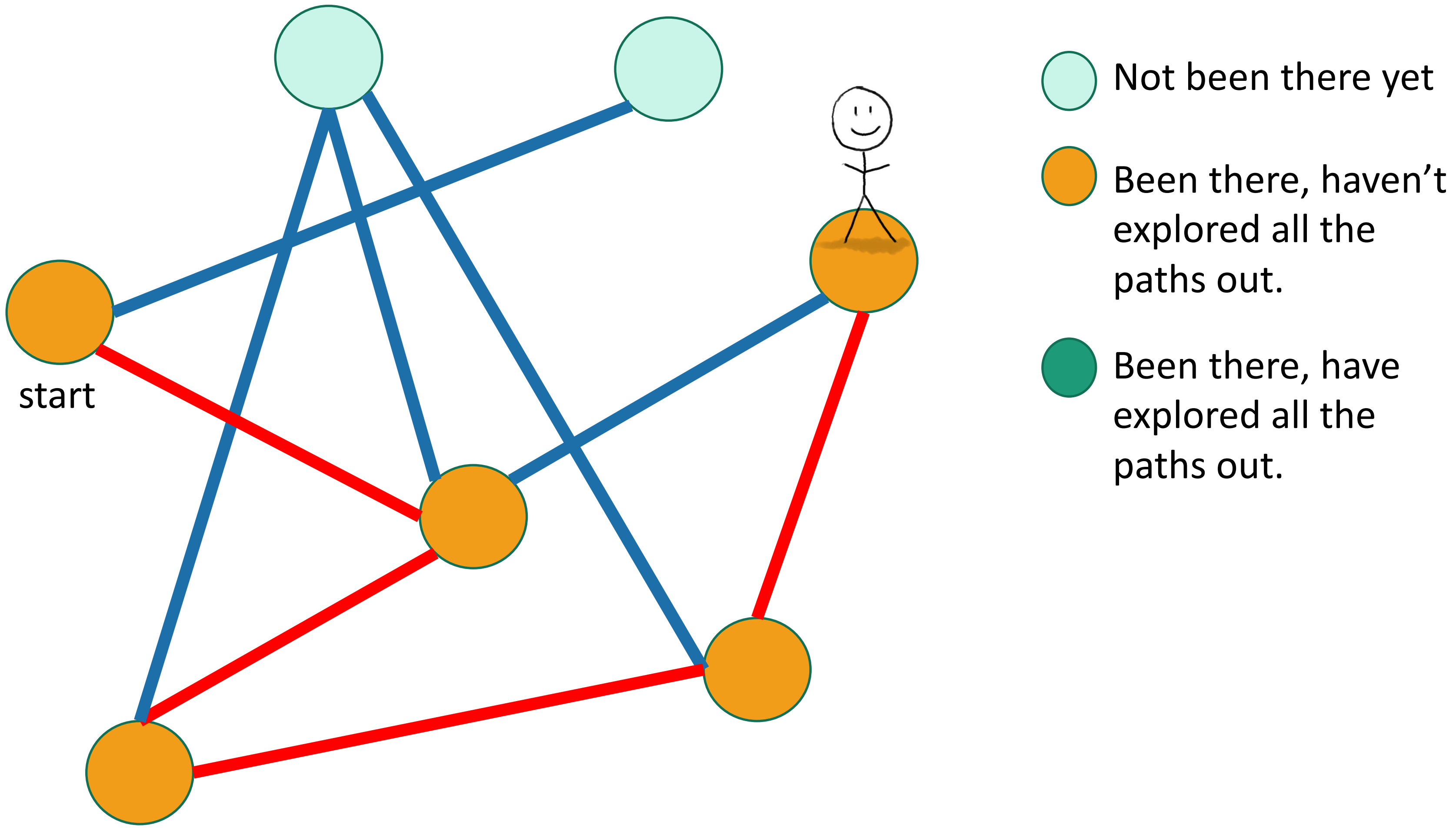

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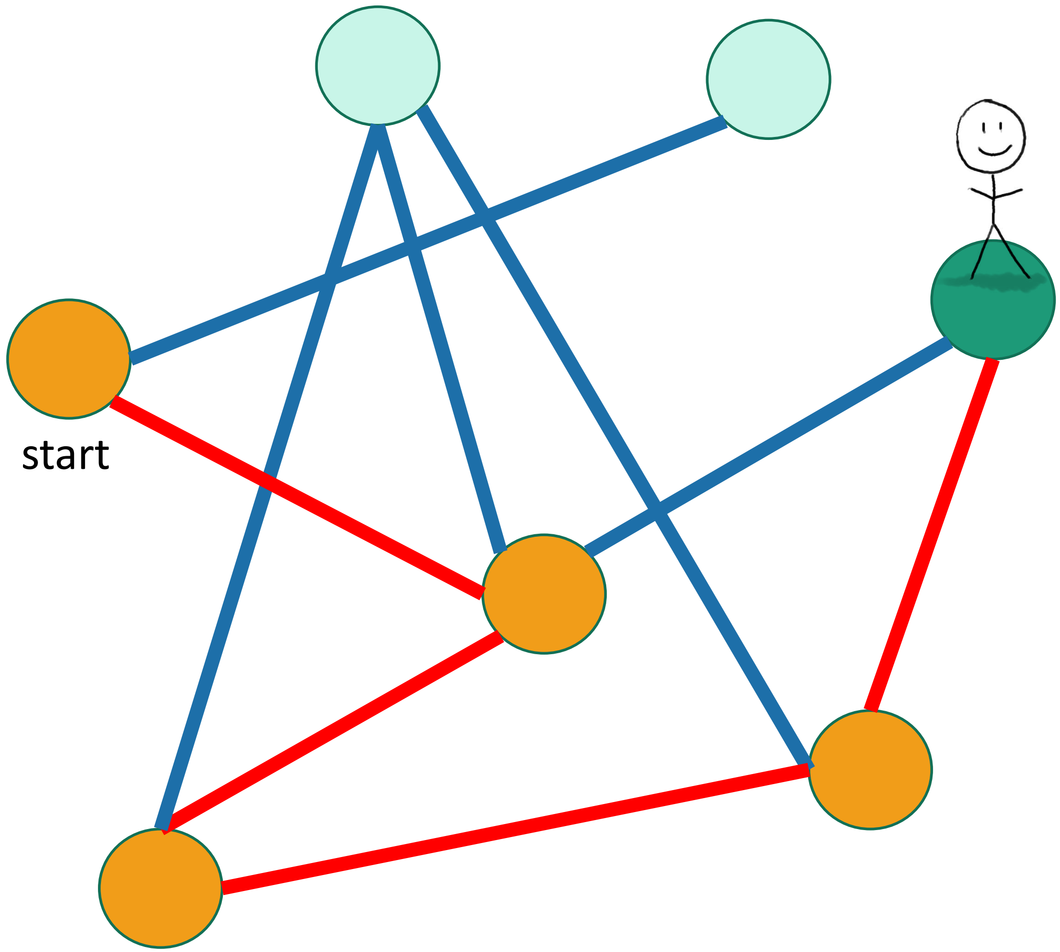


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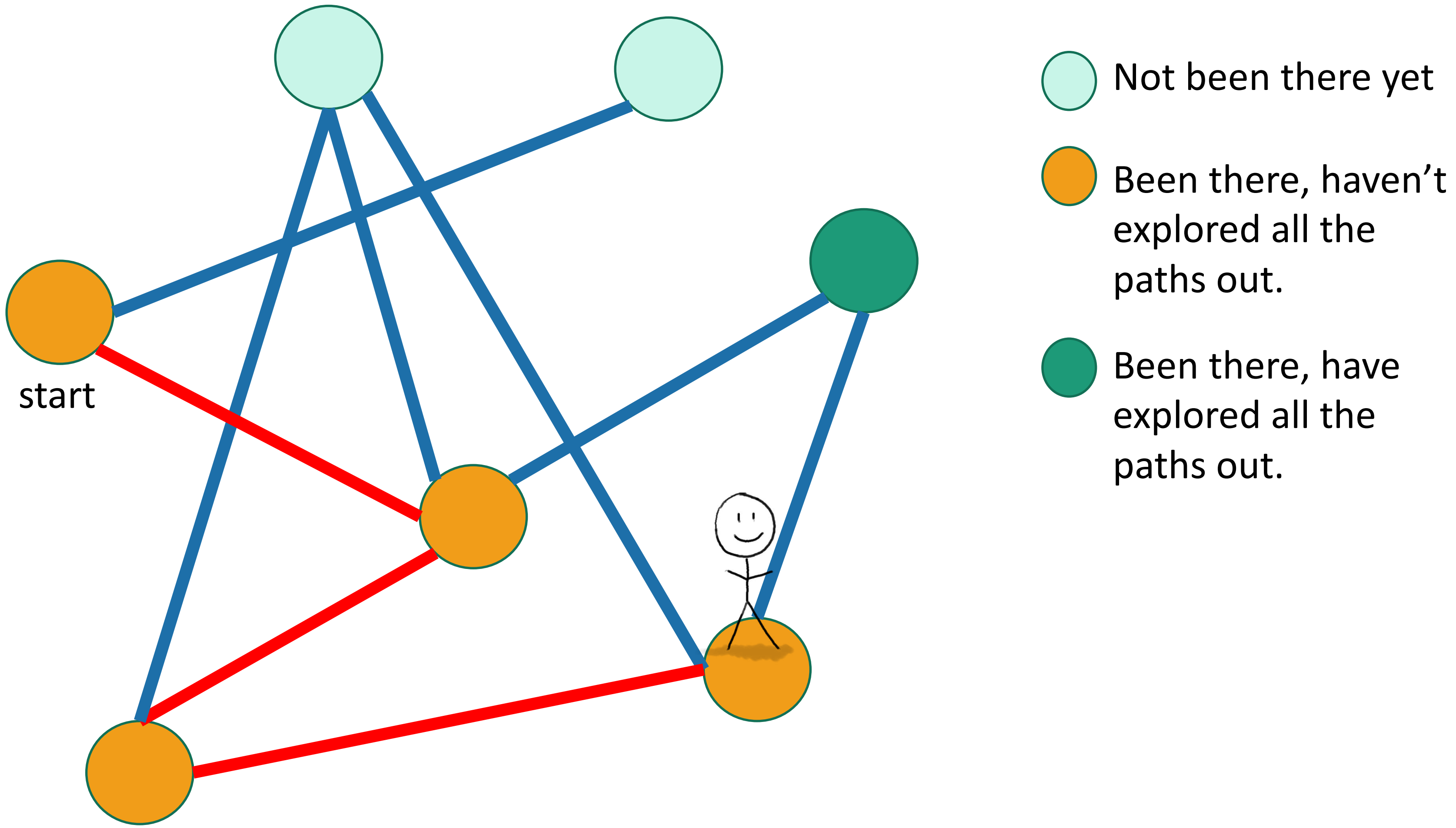

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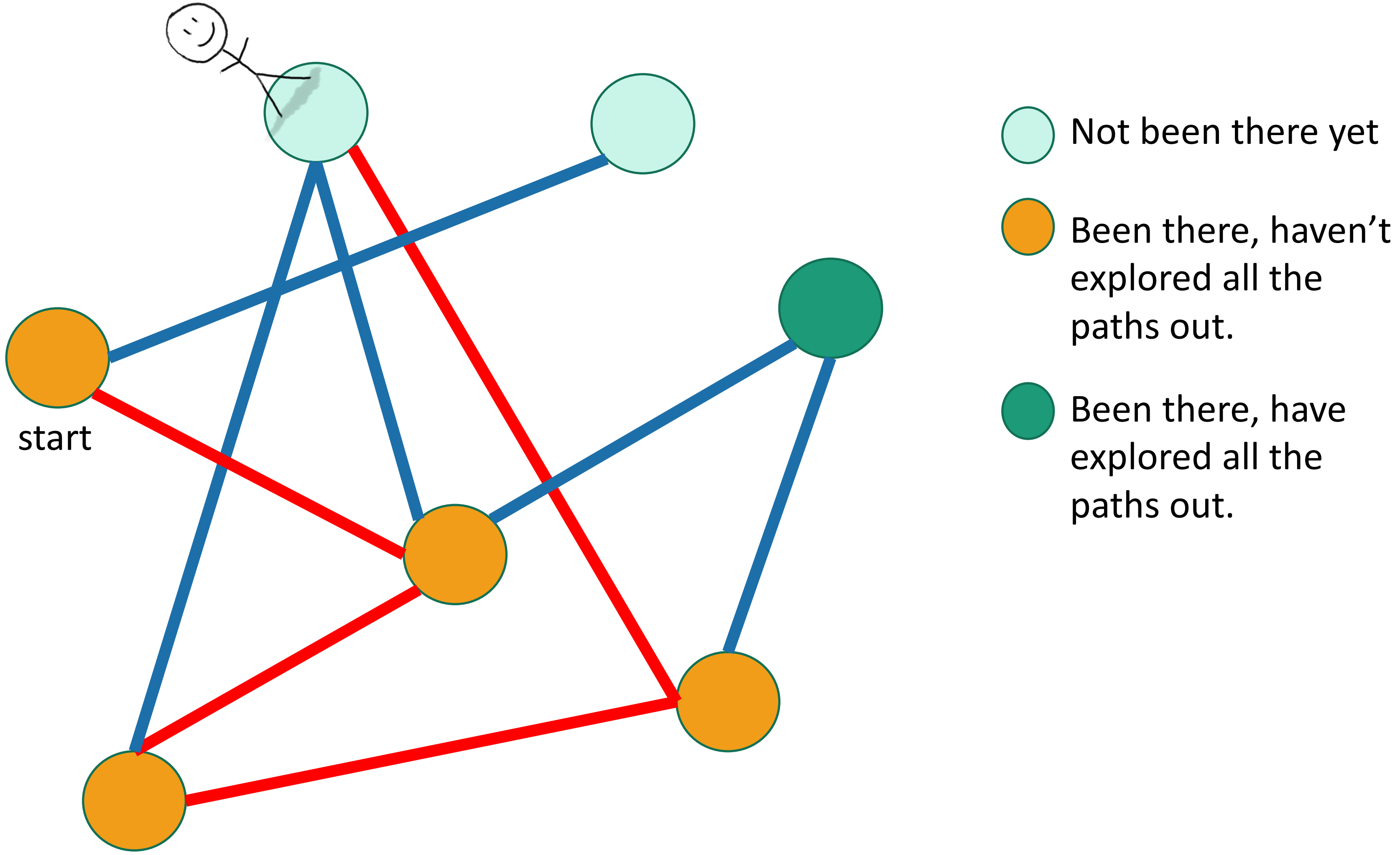

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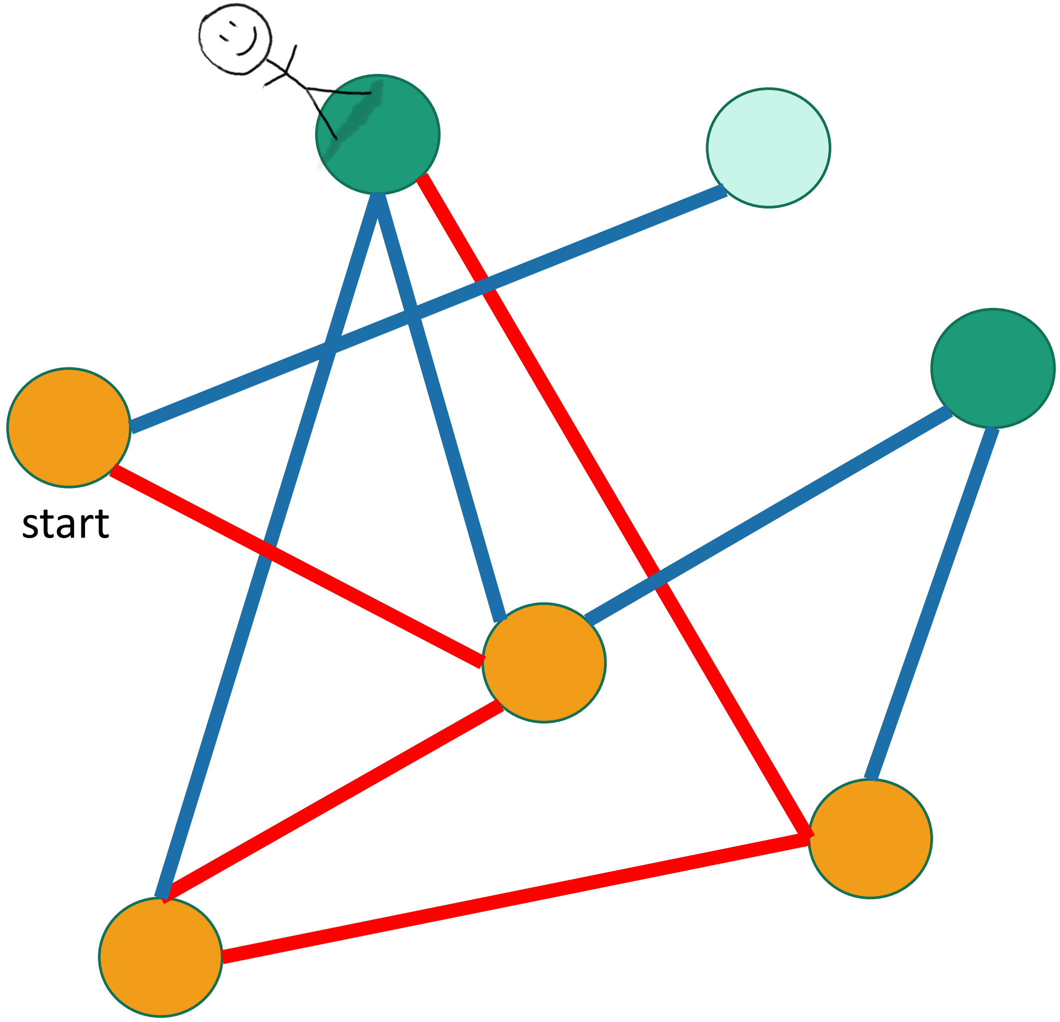


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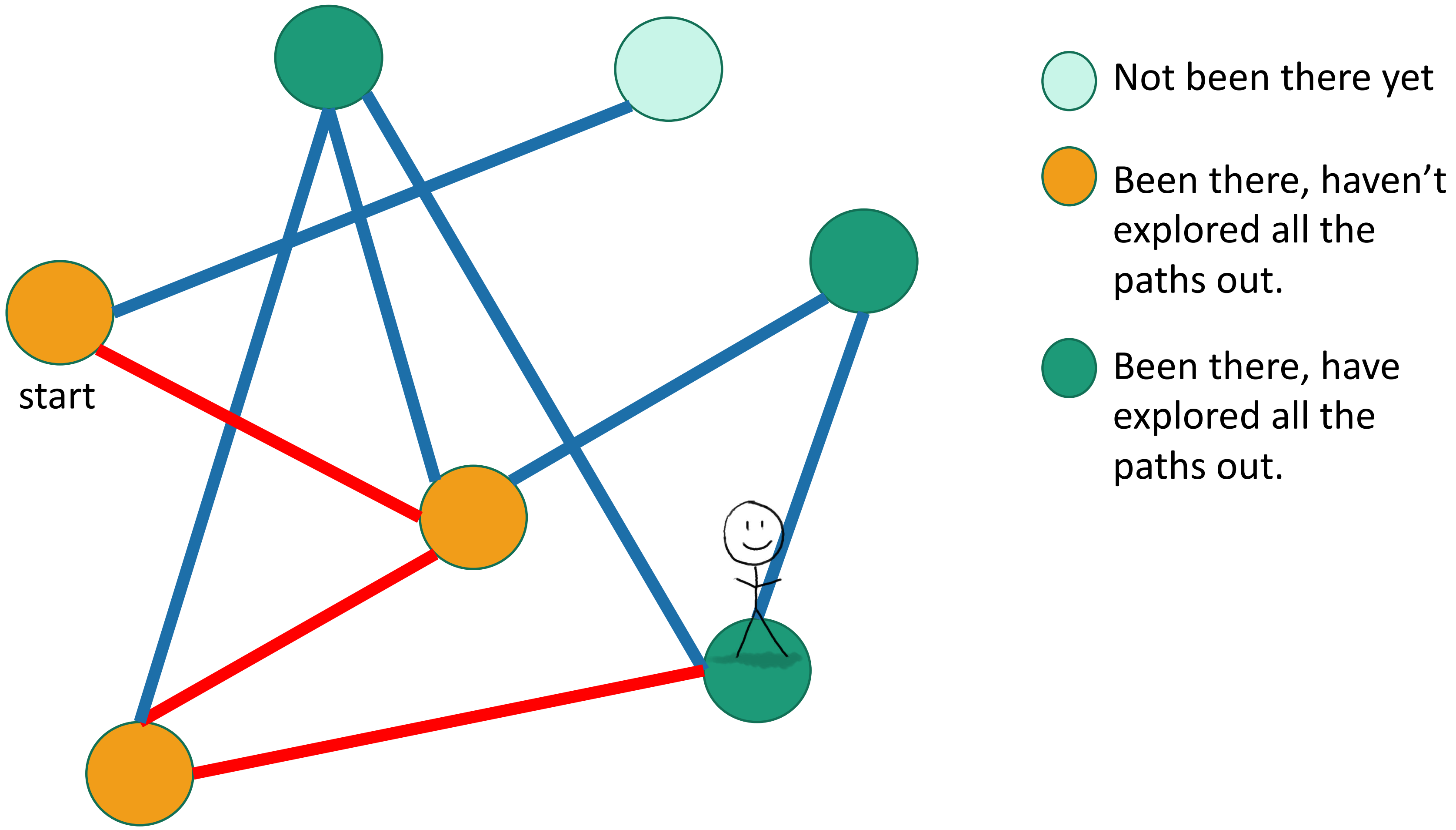

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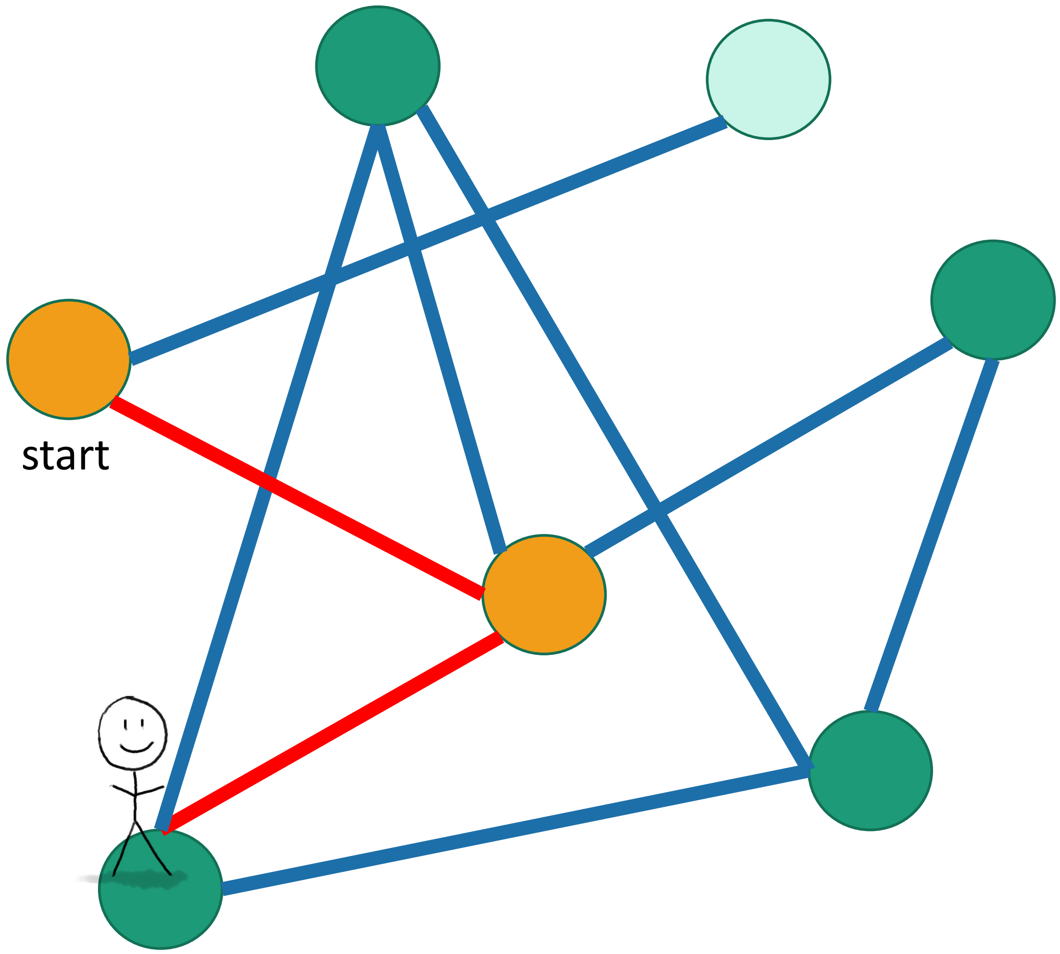


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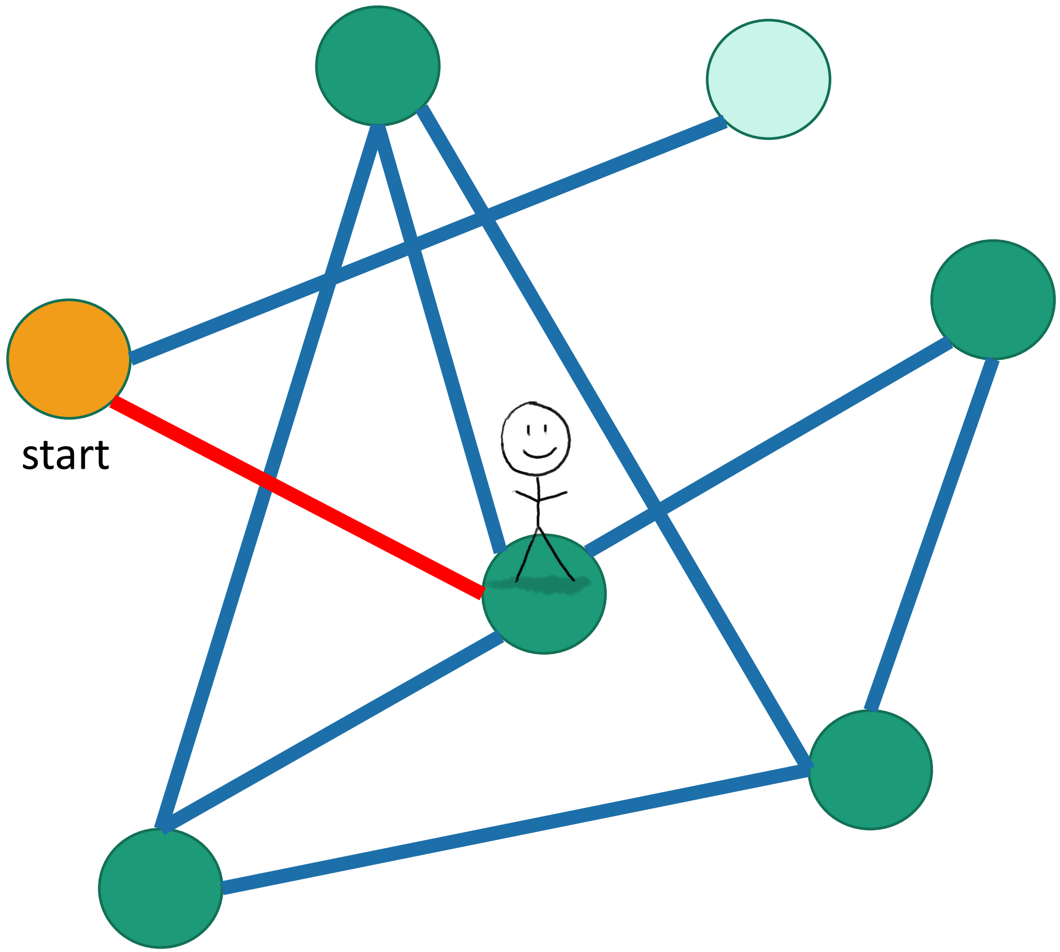


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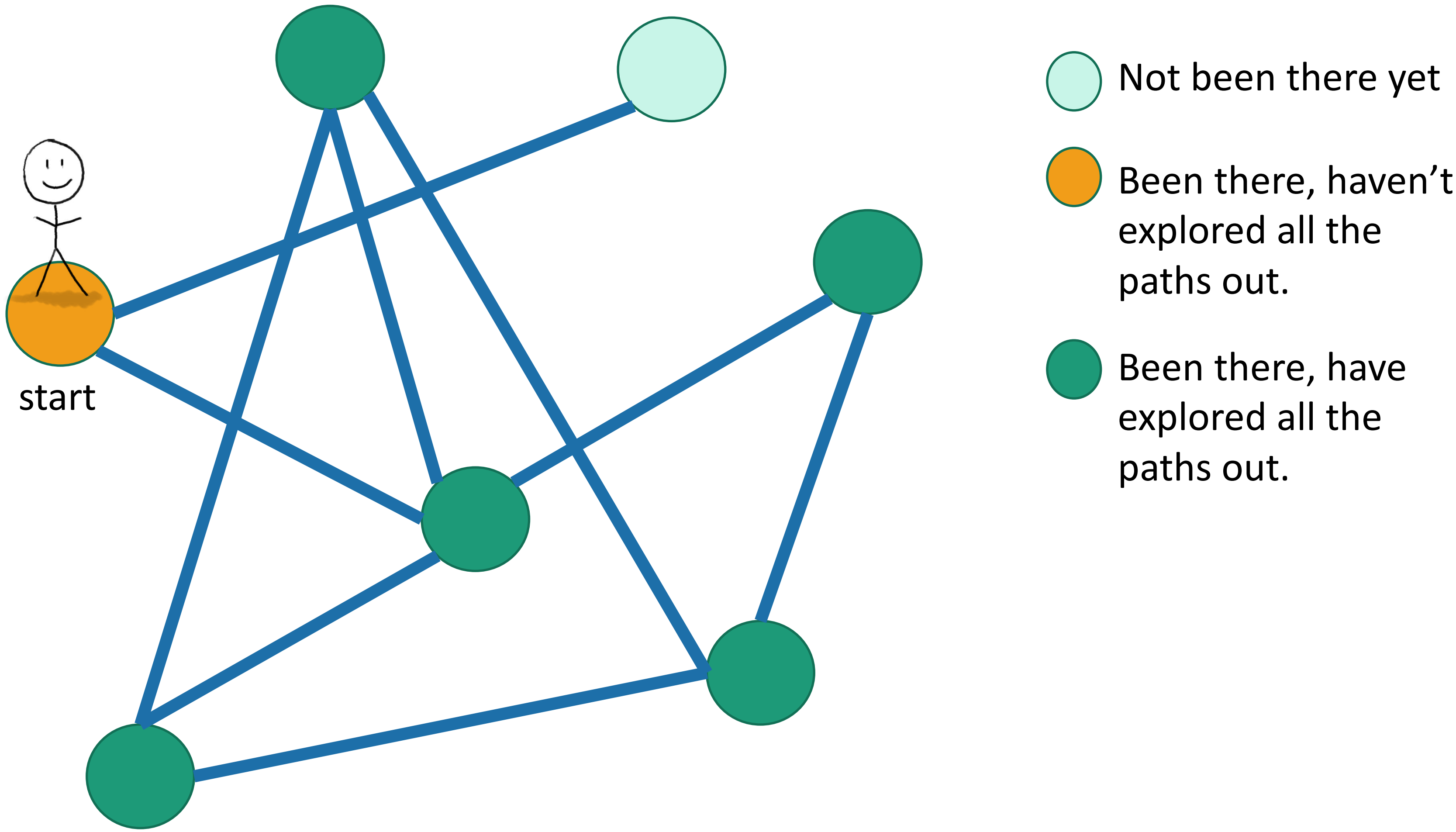

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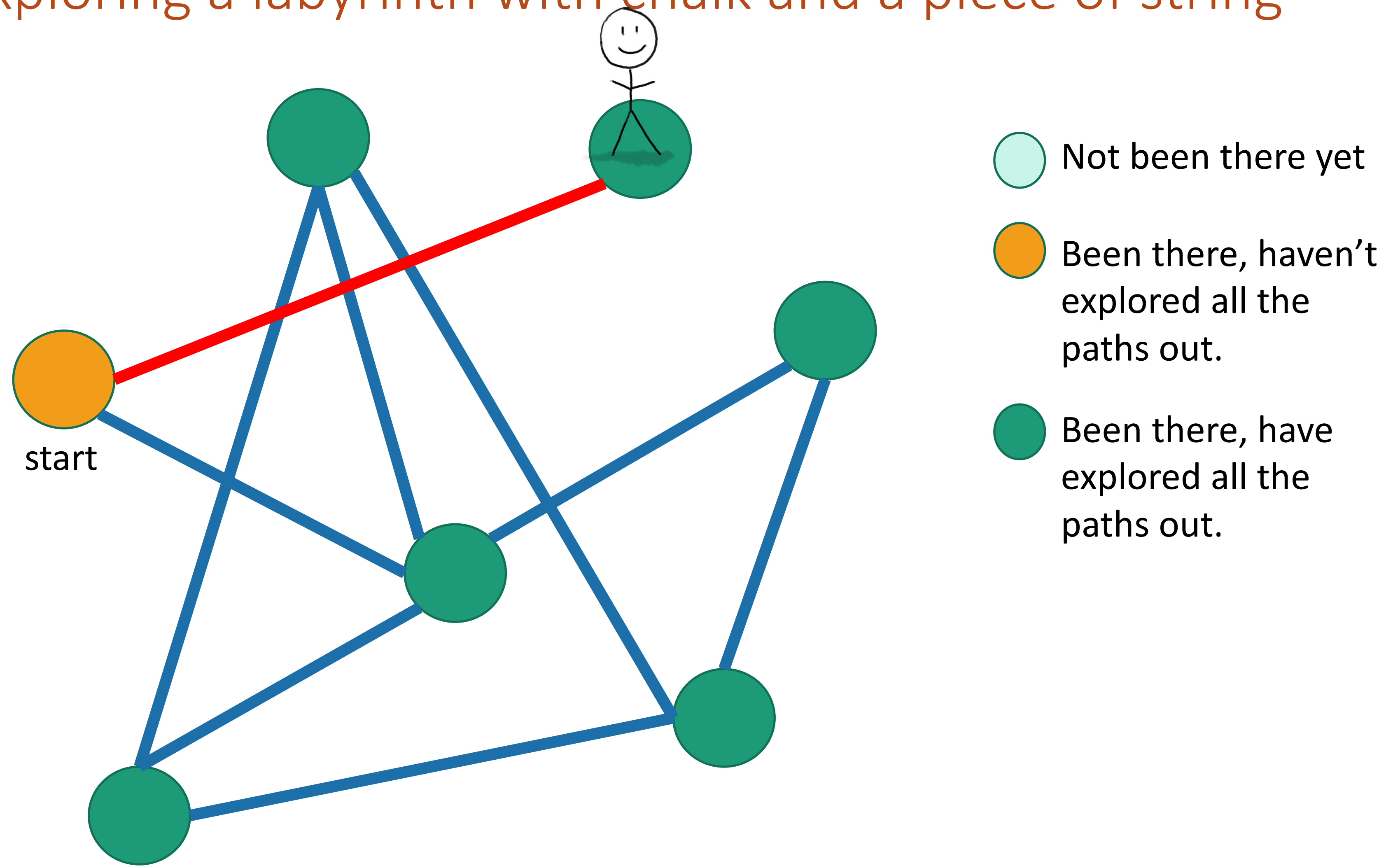

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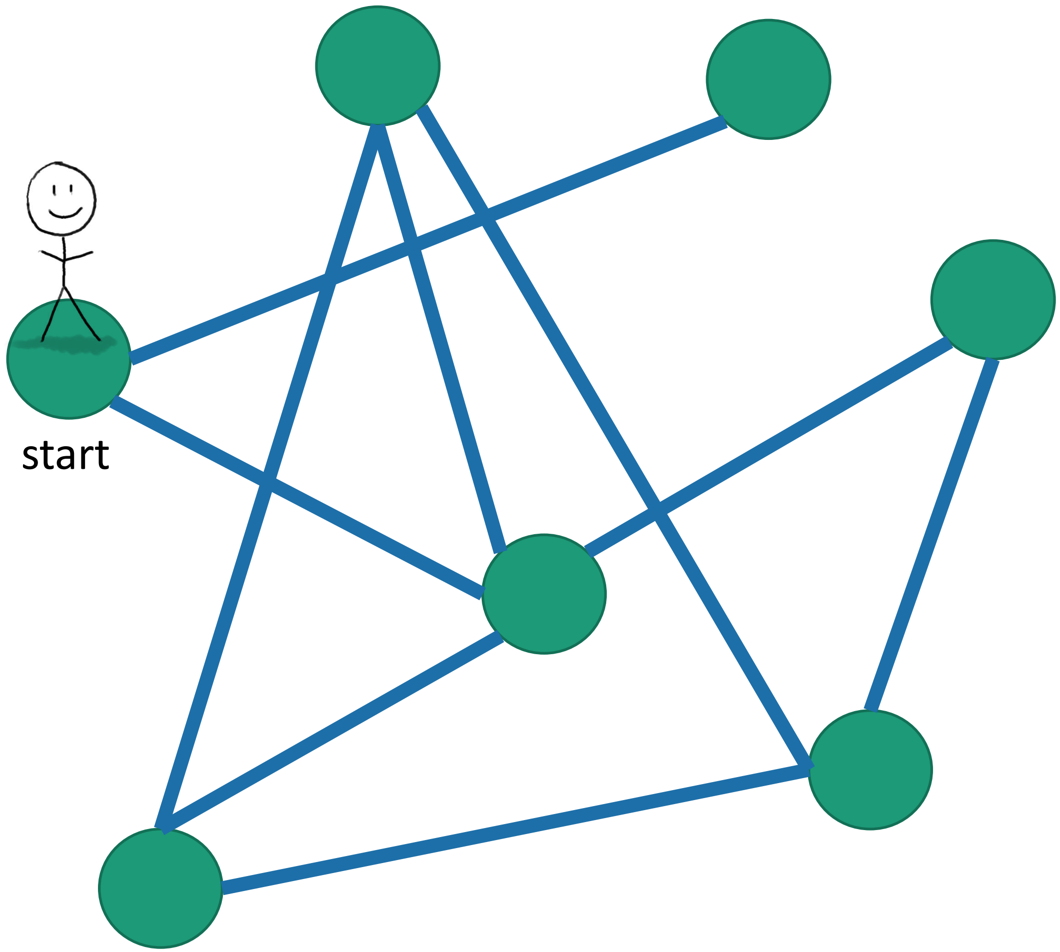


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Labyrinth:

**EXPLORED!**

# Depth First Search

## Exploring a labyrinth with pseudocode

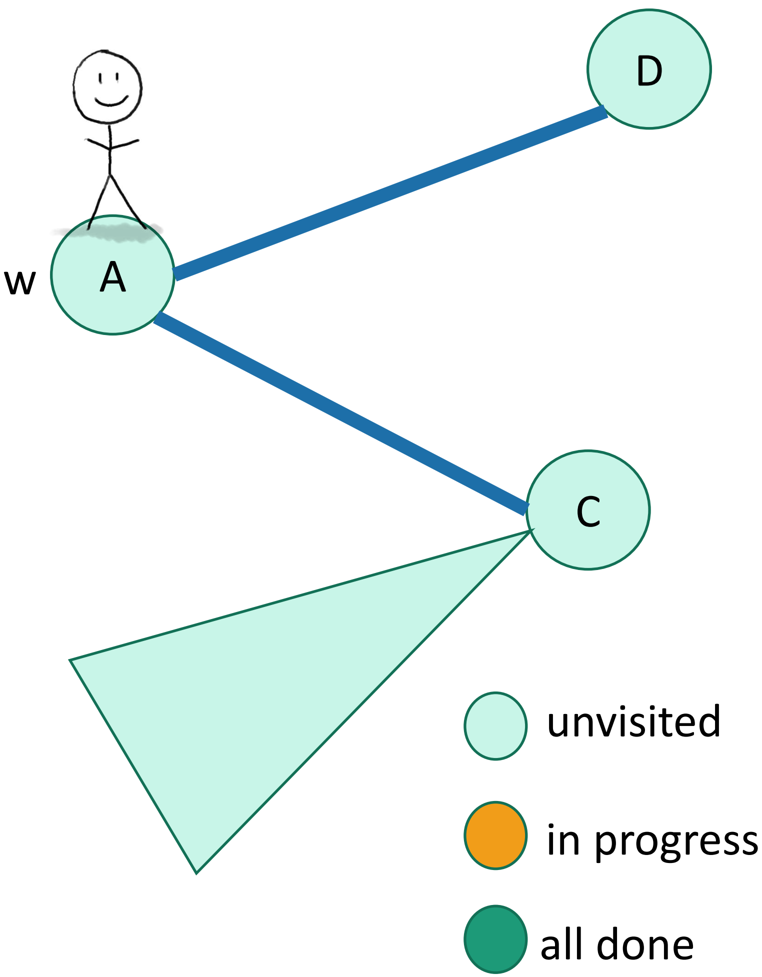
- Each vertex keeps track of whether it is:
  - Unvisited
  - In progress
  - All done

- Each vertex will also keep track of:
  - The time we **first enter it**.
  - The time we finish with it and mark it **all done**.

You might have seen other ways to implement DFS than what we are about to go through. This way has more bookkeeping, but more intuition – also, the bookkeeping will be useful later!

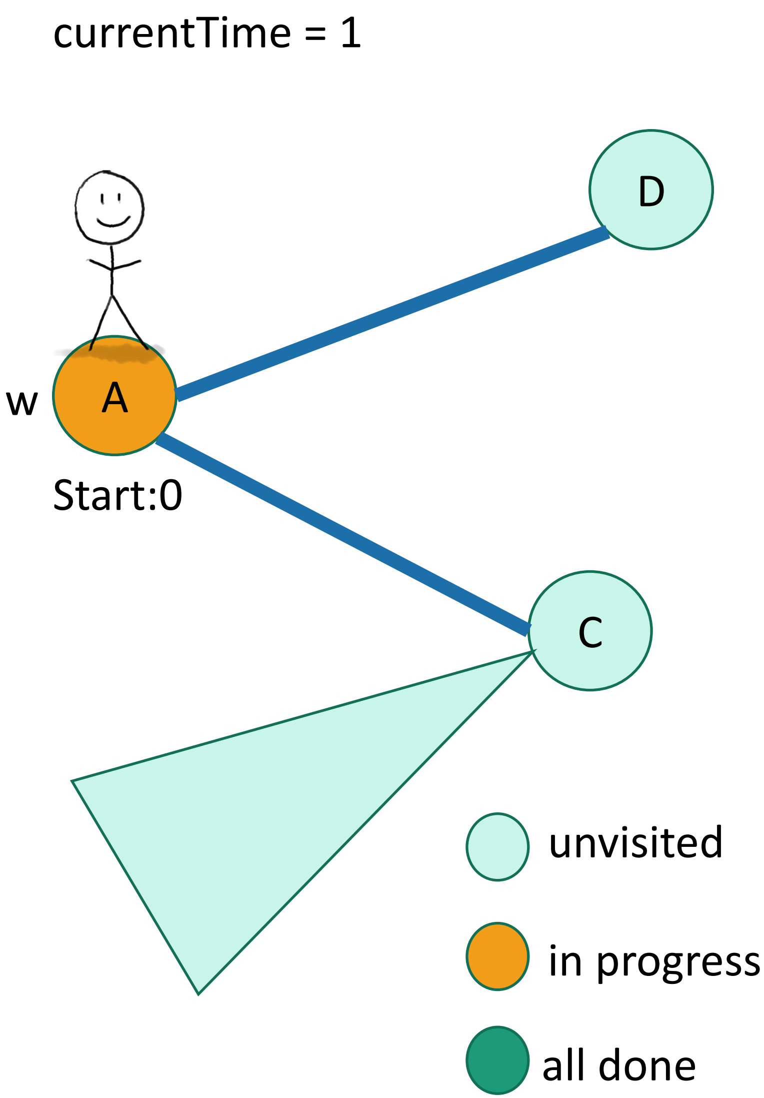
# Depth First Search

currentTime = 0




- **DFS**(w, currentTime):
  - w.startTime = currentTime
  - currentTime ++
  - Mark w as **in progress**.
  - **for** v in w.neighbors:
    - **if** v is **unvisited**:
      - currentTime
        = **DFS**(v, currentTime)
      - currentTime ++
  - w.finishTime = currentTime
  - Mark w as **all done**
  - **return** currentTime

# Depth First Search

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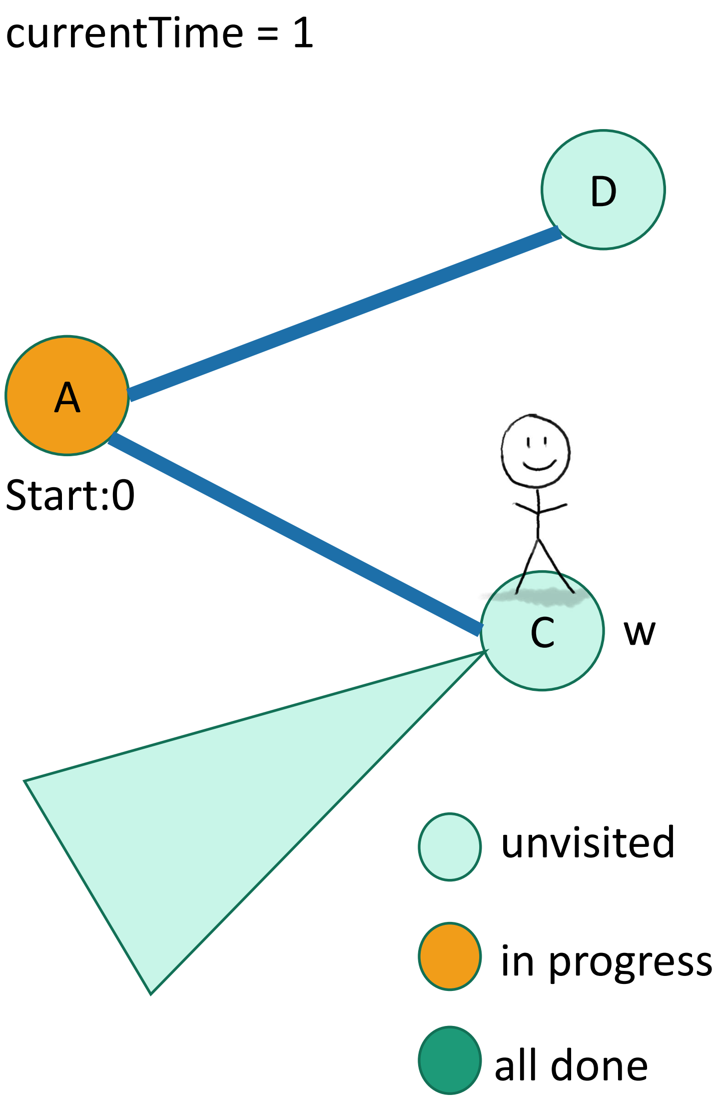


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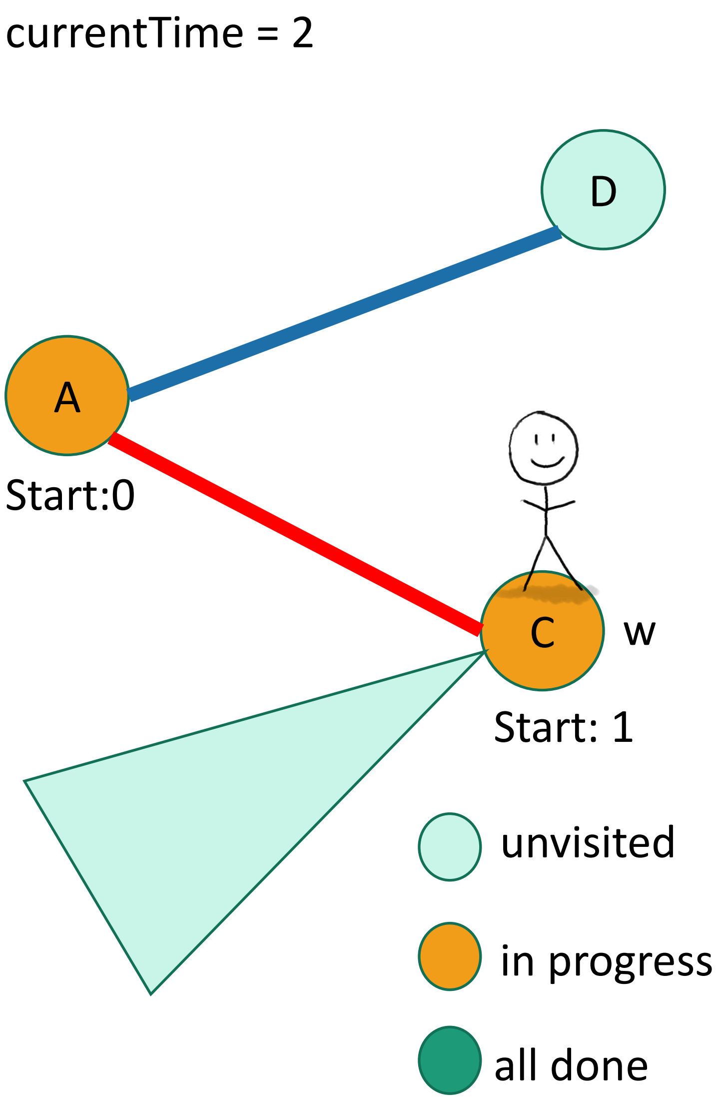


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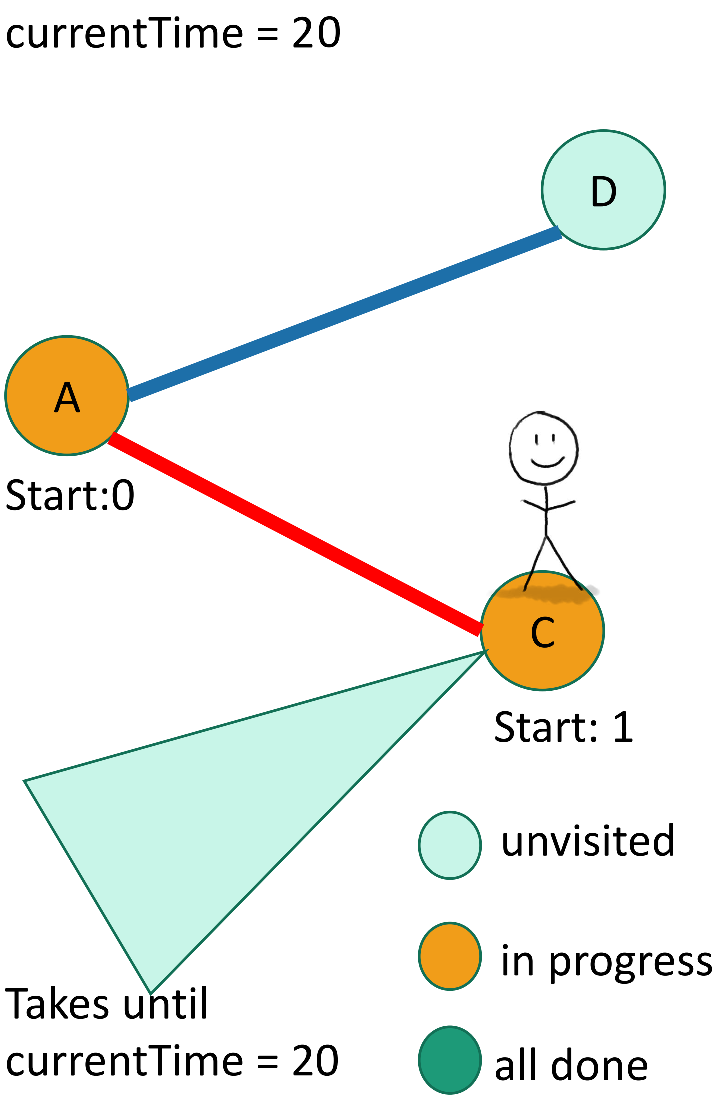


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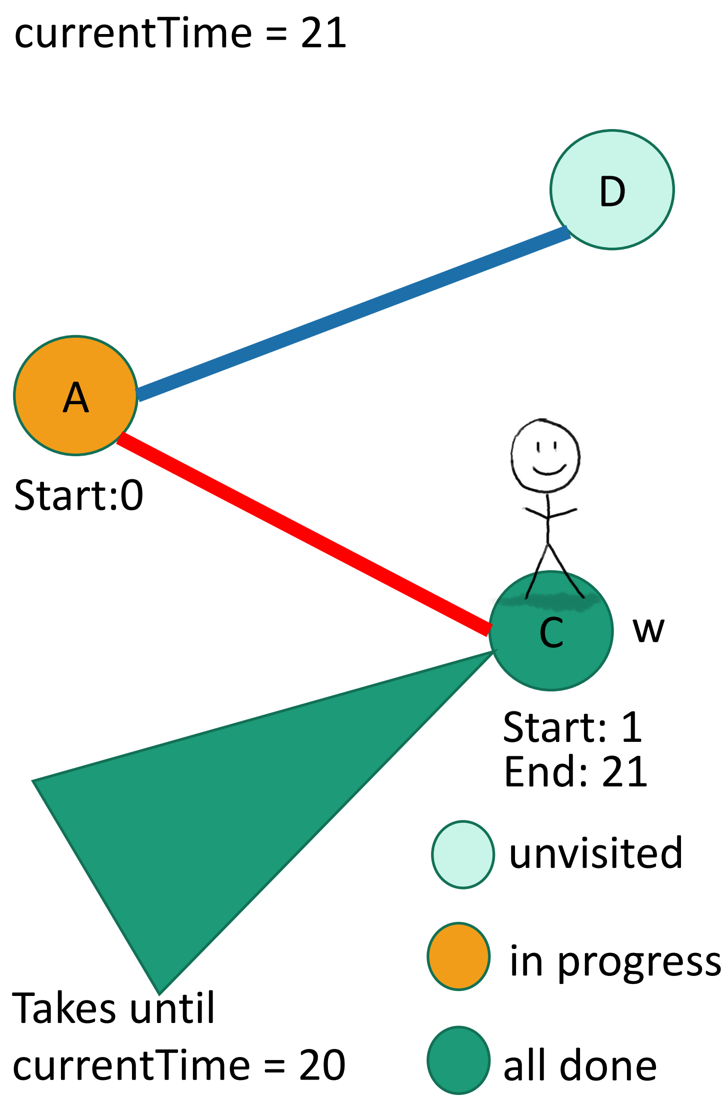


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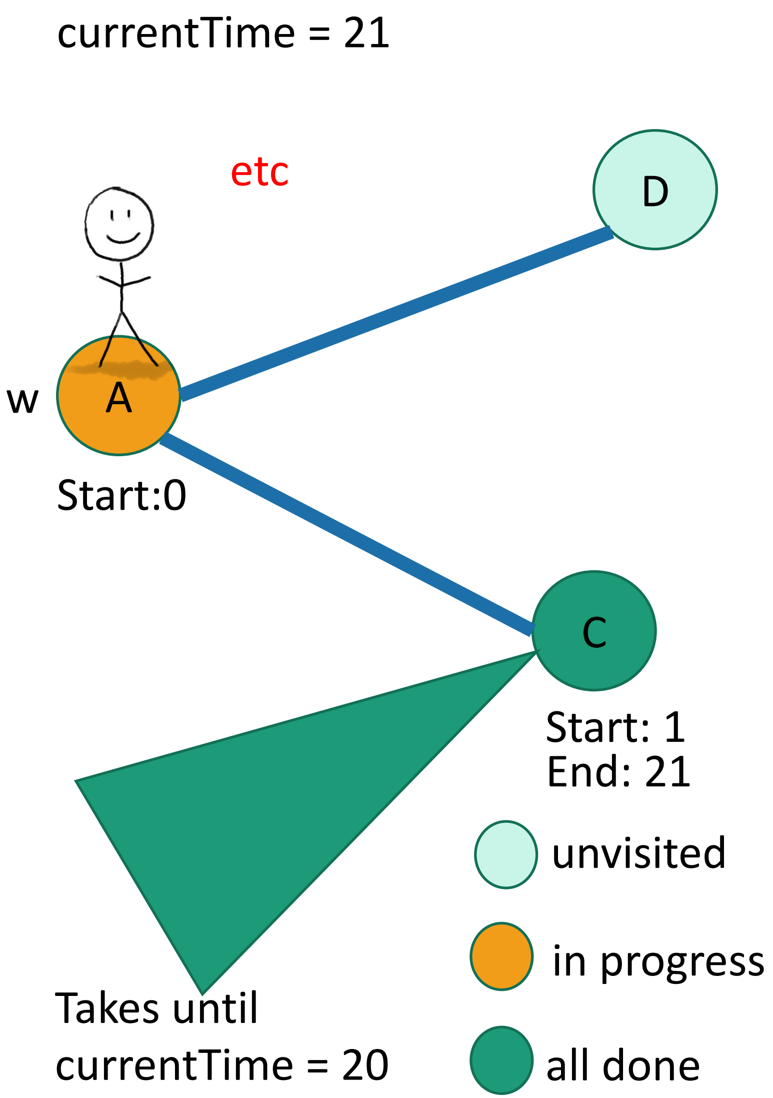
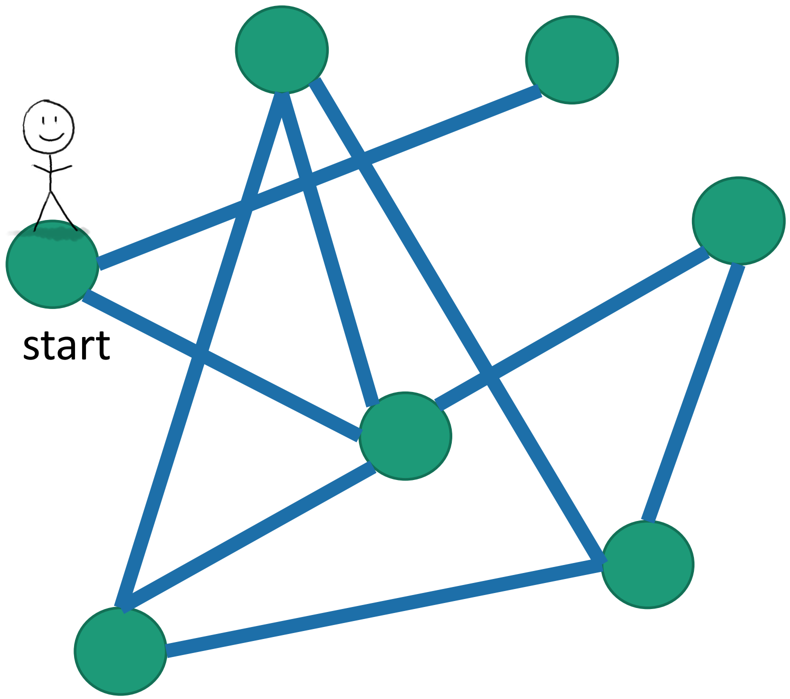
  - **return** currentTime

# Depth First Search

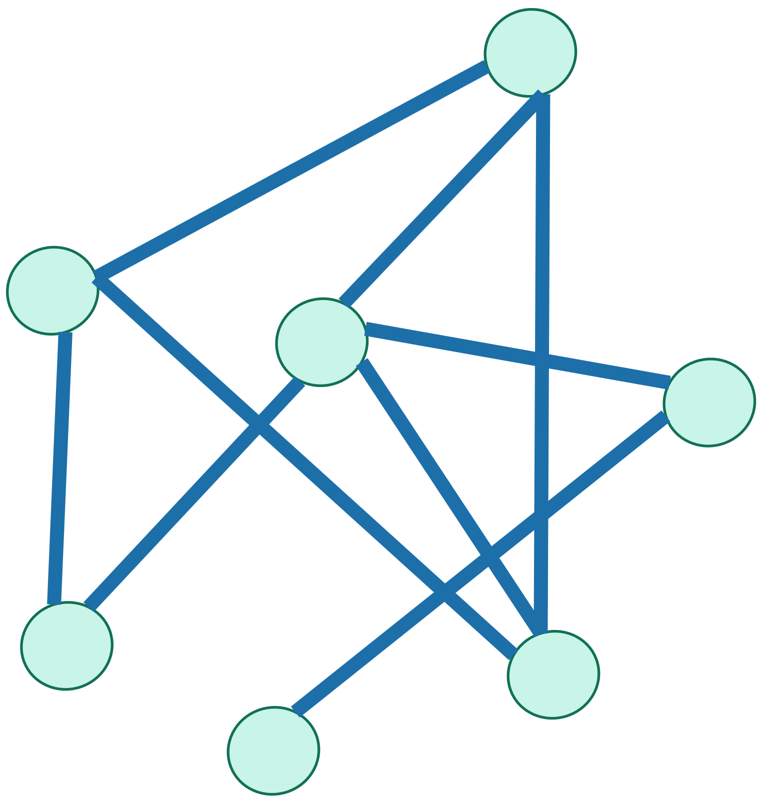



- **DFS**(w, currentTime):
  - w.startTime = currentTime
  - currentTime ++
  - Mark w as **in progress**.
  - **for** v in w.neighbors:
    - **if** v is **unvisited**:
      - currentTime
        = **DFS**(v, currentTime)
      - currentTime ++
  - w.finishTime = currentTime
  - Mark w as **all done**
  - **return** currentTime

# DFS finds all the nodes reachable from the starting point



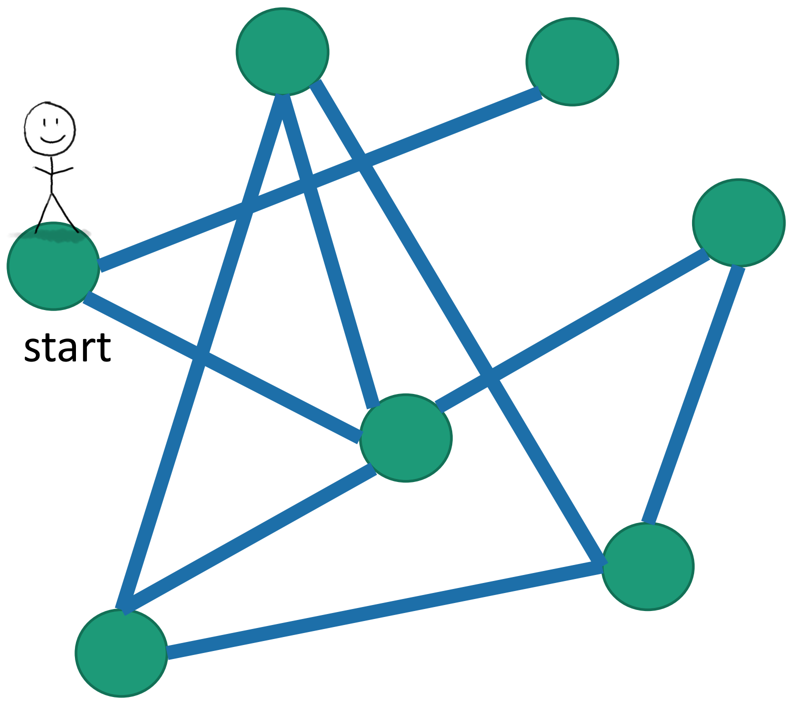

In an undirected graph, this is called a **connected component.**

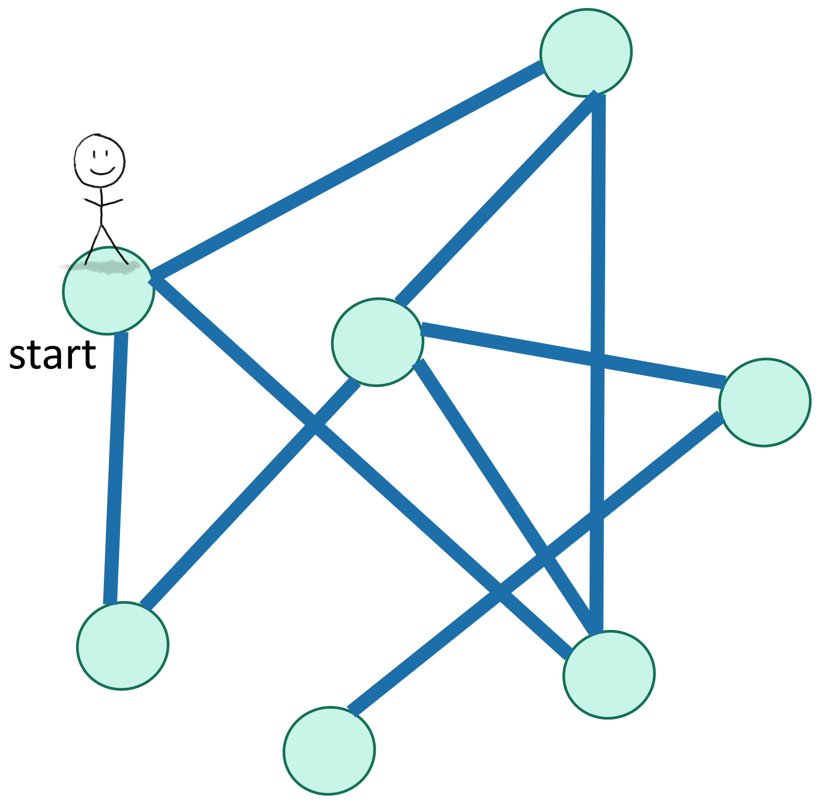


**One application**: finding connected components.

# To explore the whole graph

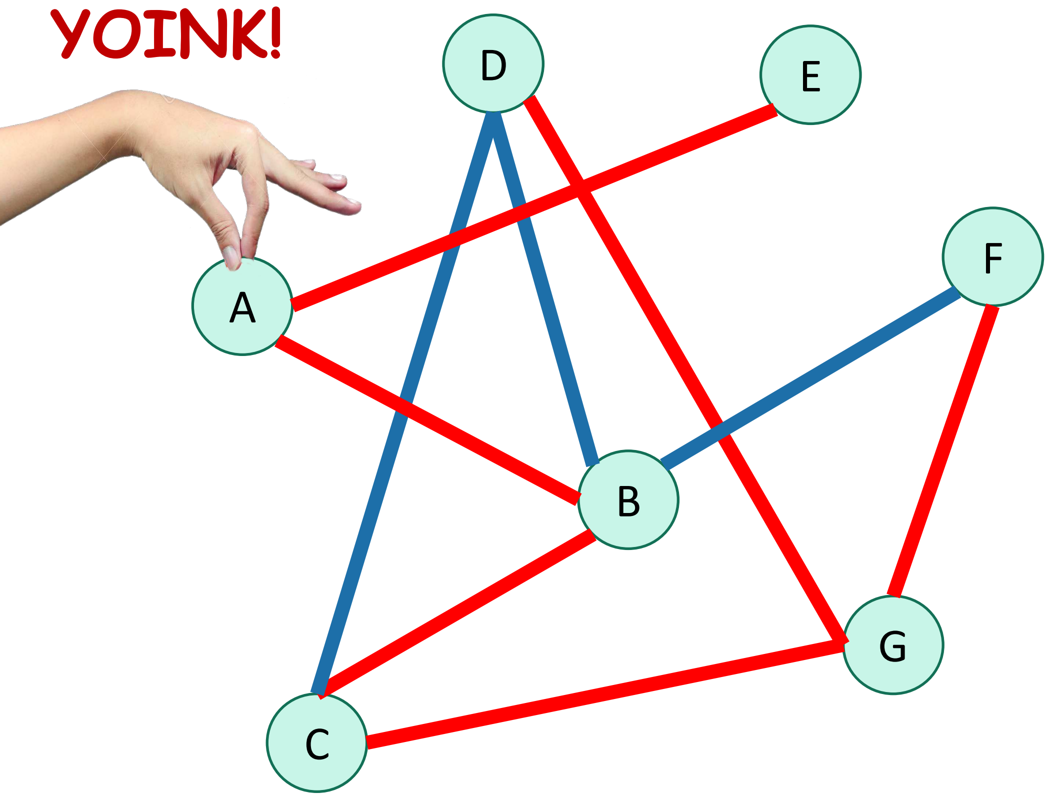
- Do it repeatedly!

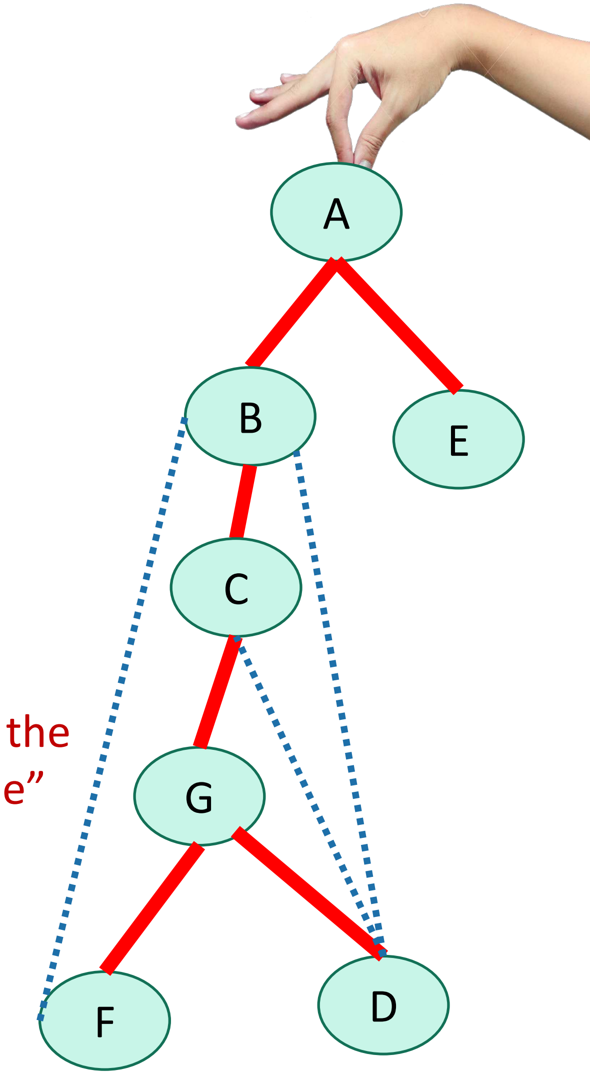


# Why is it called depth-first?

- We are implicitly building a tree:




Call this the "DFS tree"

- And first we go as deep as we can.

# Running time

To explore **just the connected component** we started in

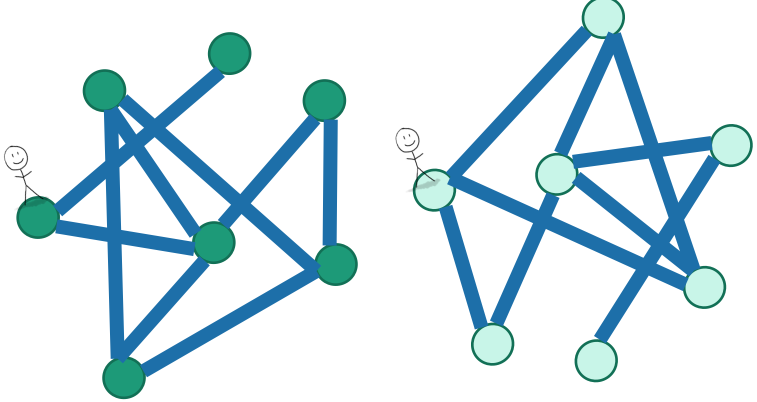
- We look at each edge only once.
- And basically don't do anything else.
- So…

$$O(m)$$

- (Assuming we are using the linked-list representation)
- (Details on board)

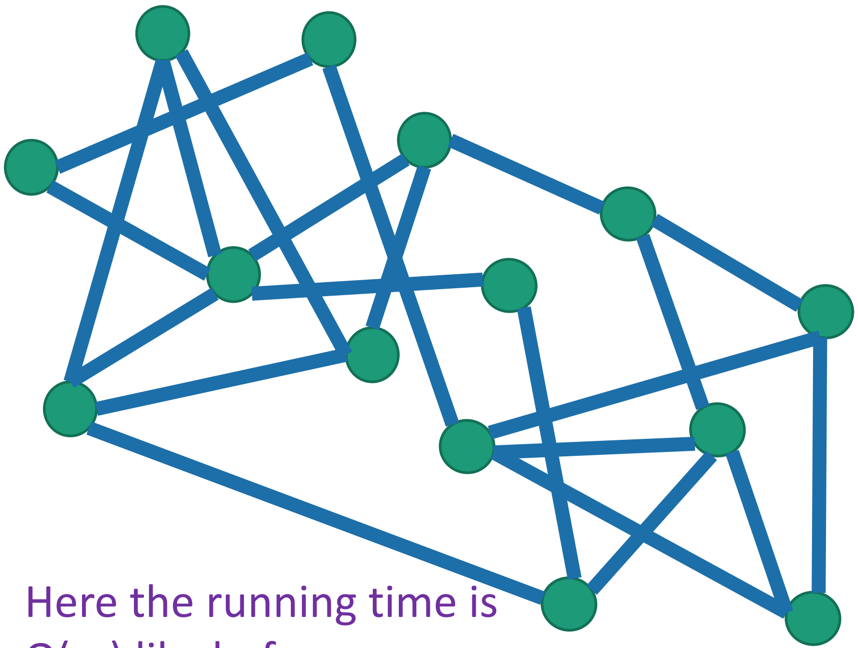
# Running time

To explore the whole thing



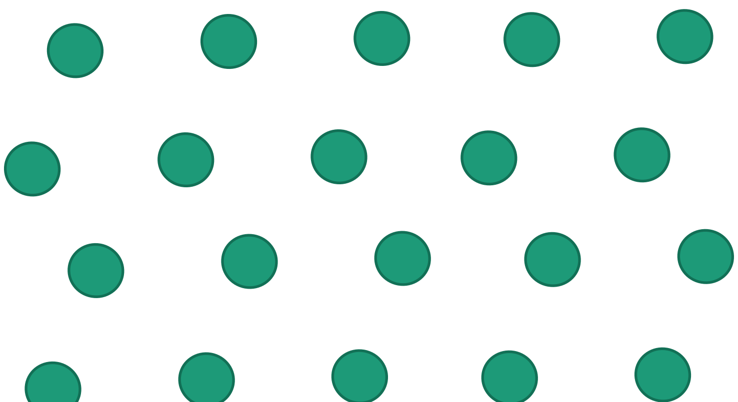
- Explore the connected components one-by-one.
- This takes time *[on board]*

$$O(n + m)$$



Here the running time is O(m) like before

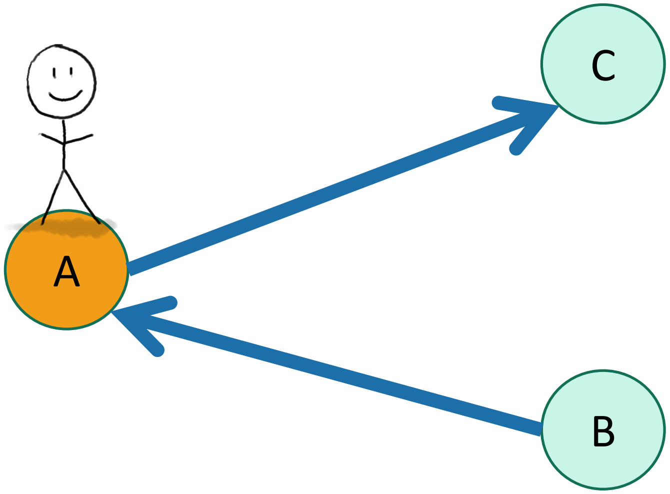
or



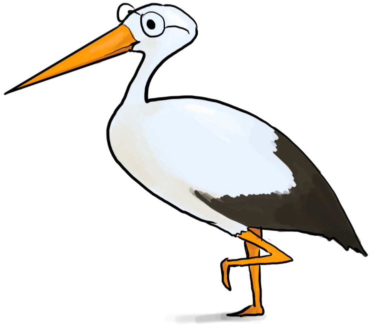
Here m=0 but it still takes time O(n) to explore the graph.

# You check:

DFS works fine on directed graphs too!



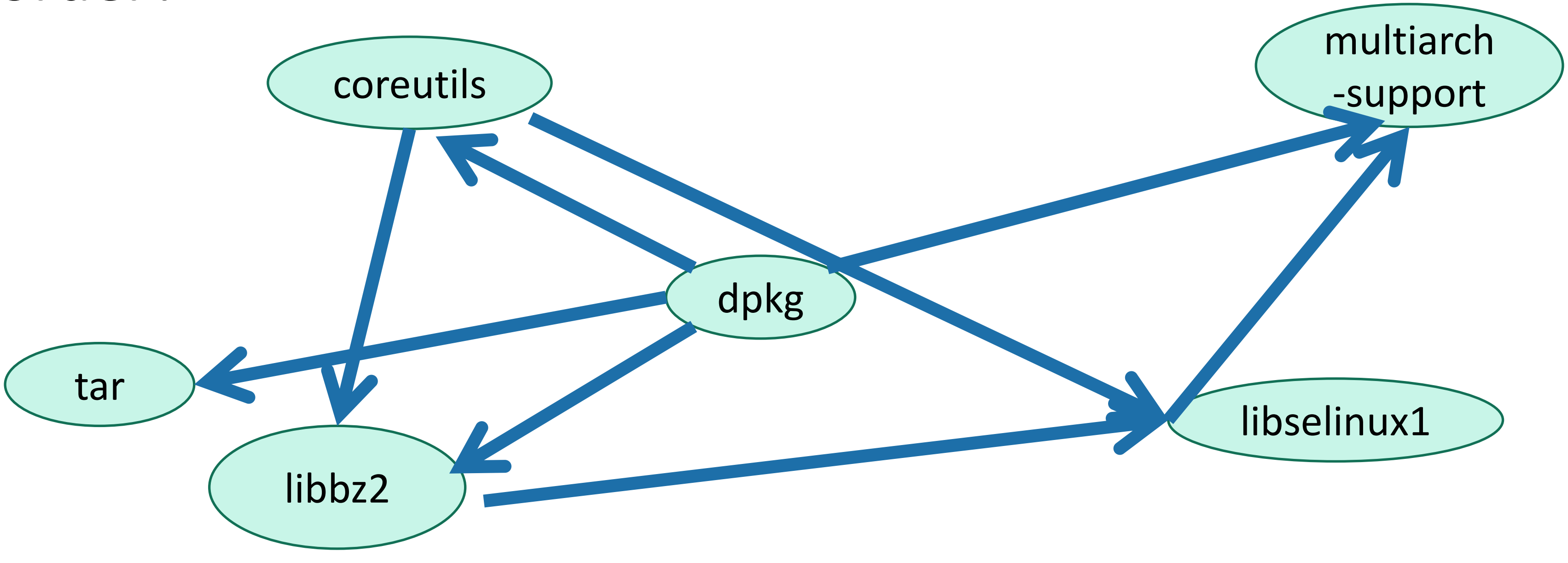

Only walk to C, not to B.



Siggi the studious stork

# Pre-lecture exercise

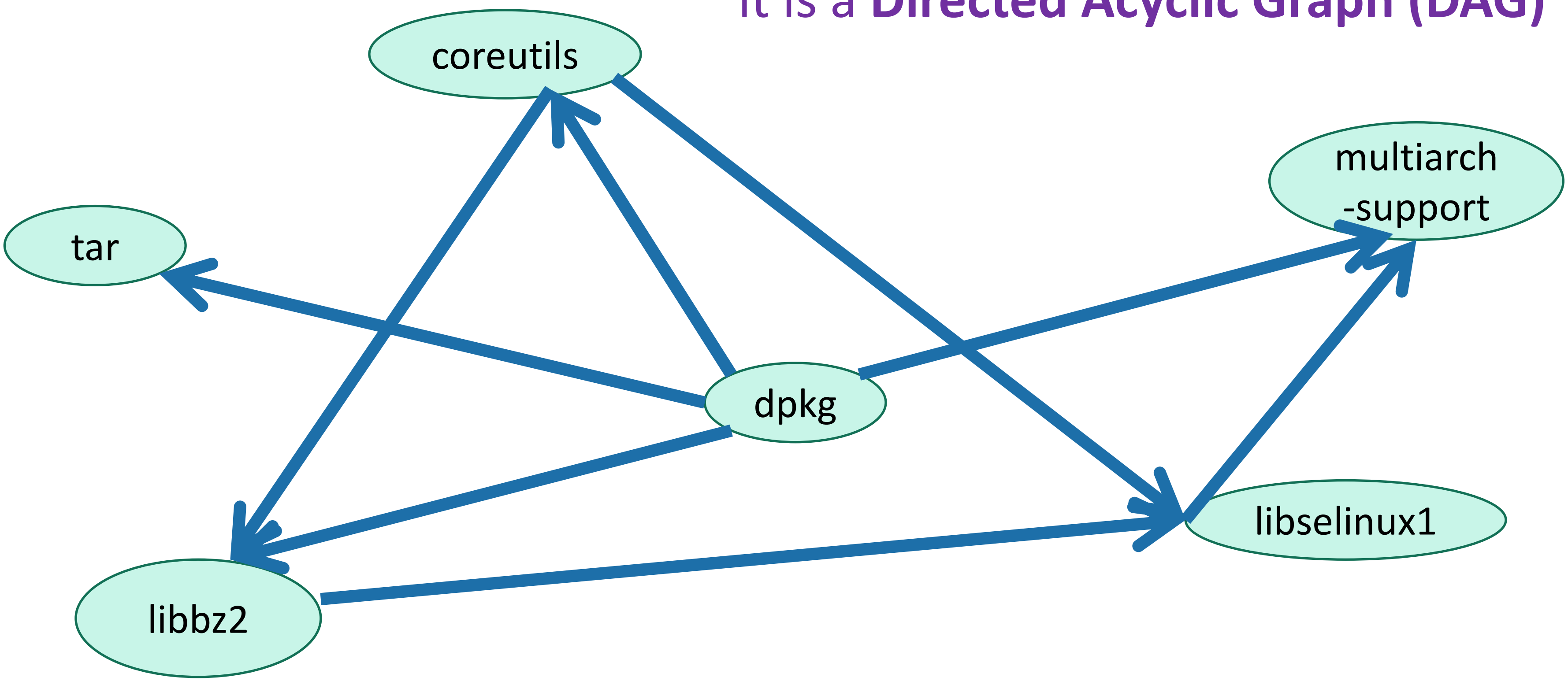
- How can you sign up for classes so that you never violate the pre-req requirements?
- More practically, given a package dependency graph, how do you install packages in the correct order?



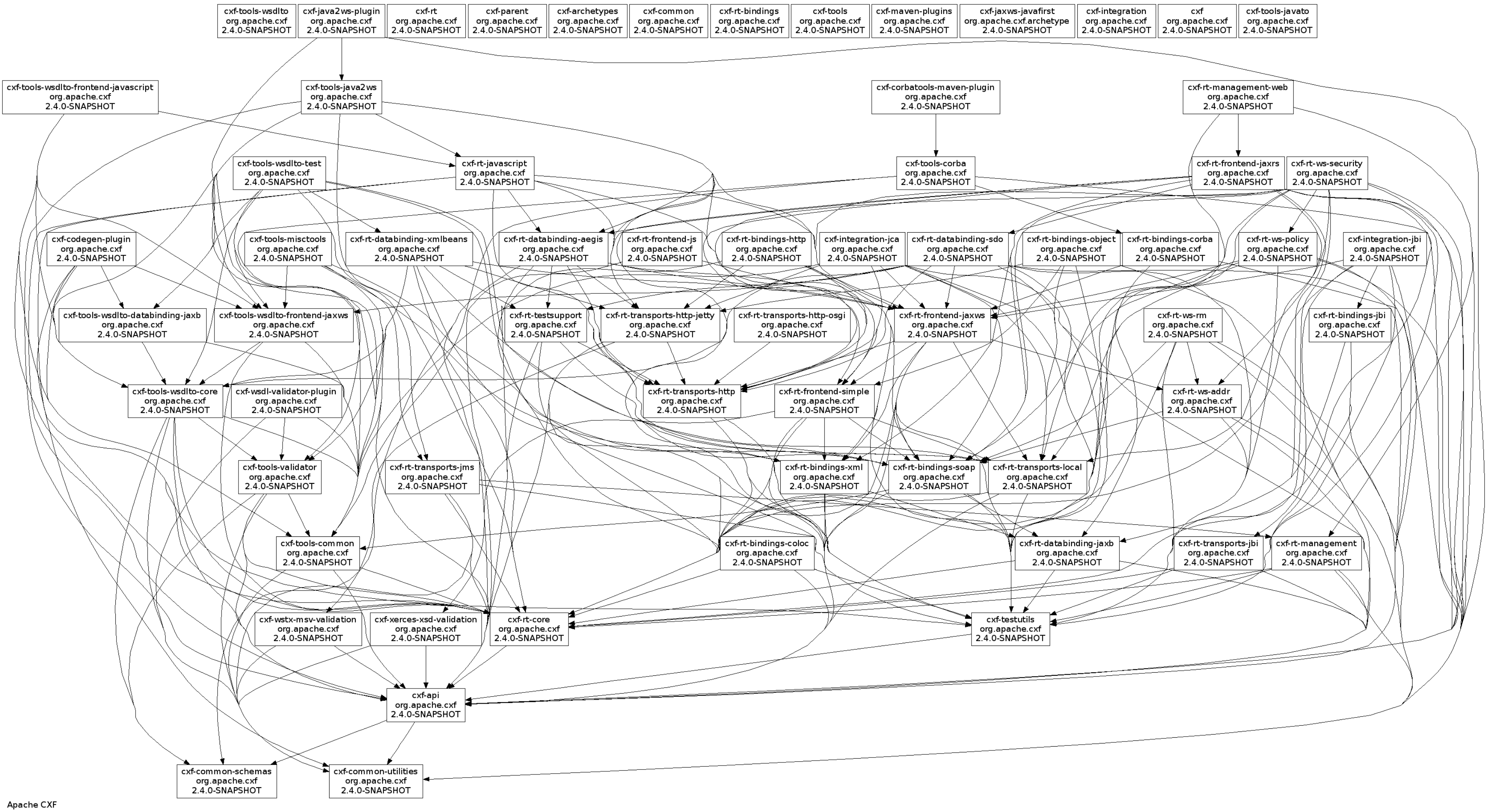
# Application: topological sorting

- Question: in what order should I install packages?

Suppose the dependency graph has no cycles:
it is a **Directed Acyclic Graph (DAG)**

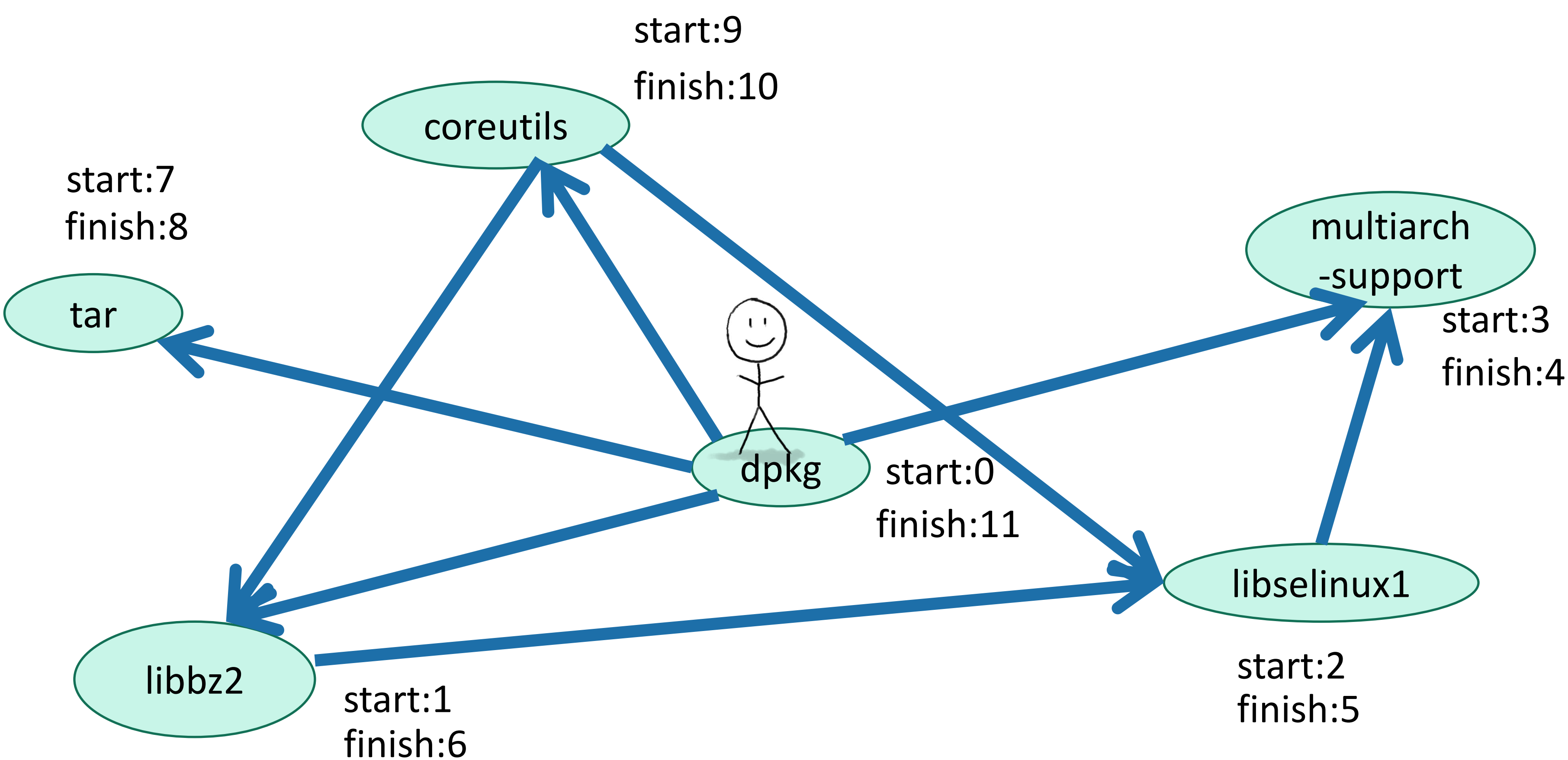


# Can’t always eyeball it.



# Let’s do DFS

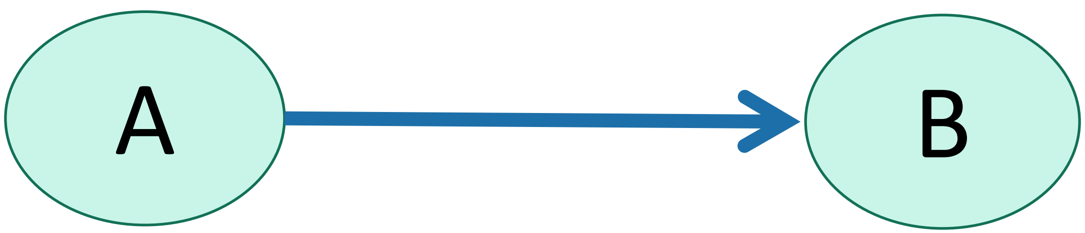
***Discussion and observations on board.***



Suppose the underlying graph has no cycles

# Finish times seem useful

**Claim:** In general, we'll always have:




finish: [larger]

finish: [smaller]

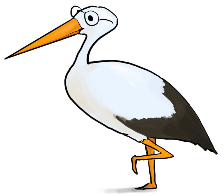
To understand why, let's go back to that DFS tree.

# A more general statement

(this holds even if there are cycles)

This is called the “parentheses theorem” in CLRS

(check this statement carefully!)

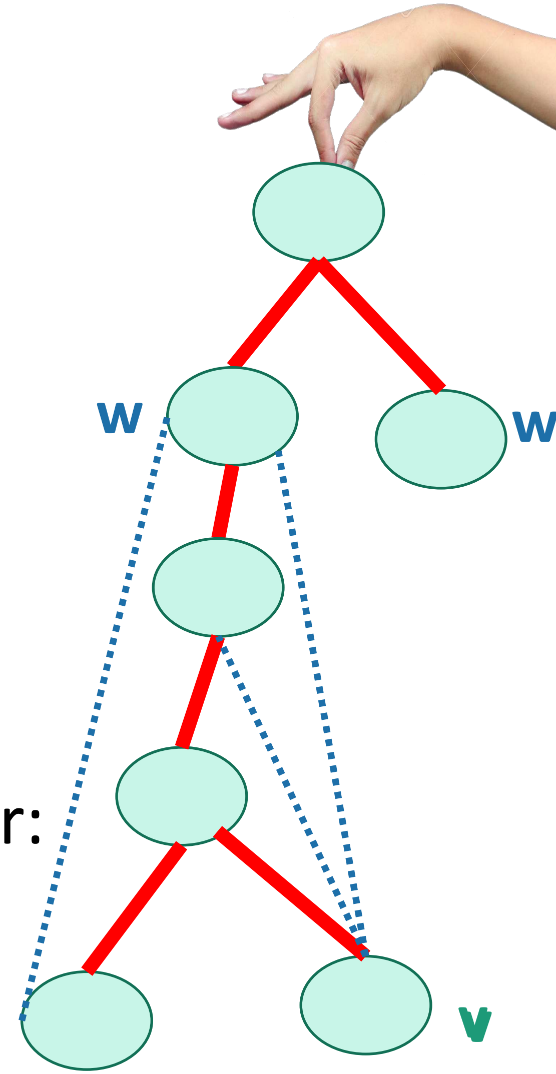


- If v is a descendant of w in this tree:

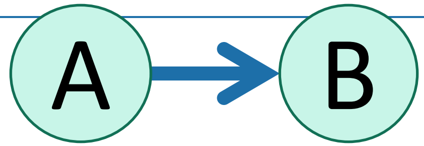

- If w is a descendant of v in this tree:


- If neither are descendants of each other:


(or the other way around)

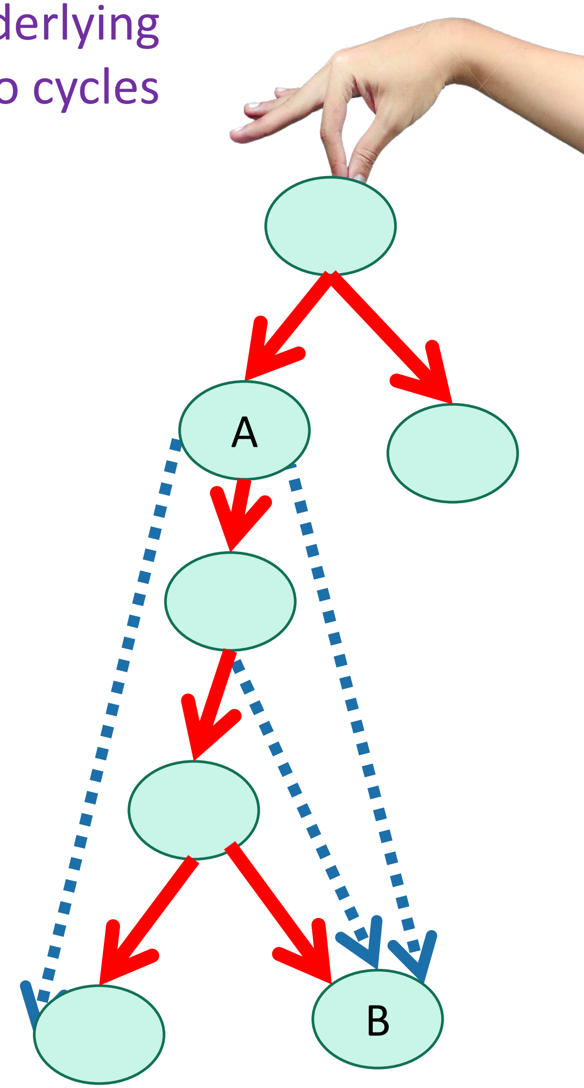


# So to prove this ->

If 


Then B.finishTime < A.finishTime

Suppose the underlying graph has no cycles

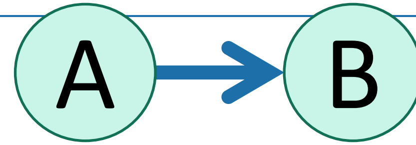



- **Case 1**: B is a descendant of A in the DFS tree.

- Then


- aka, B.finishTime < A.finishTime.

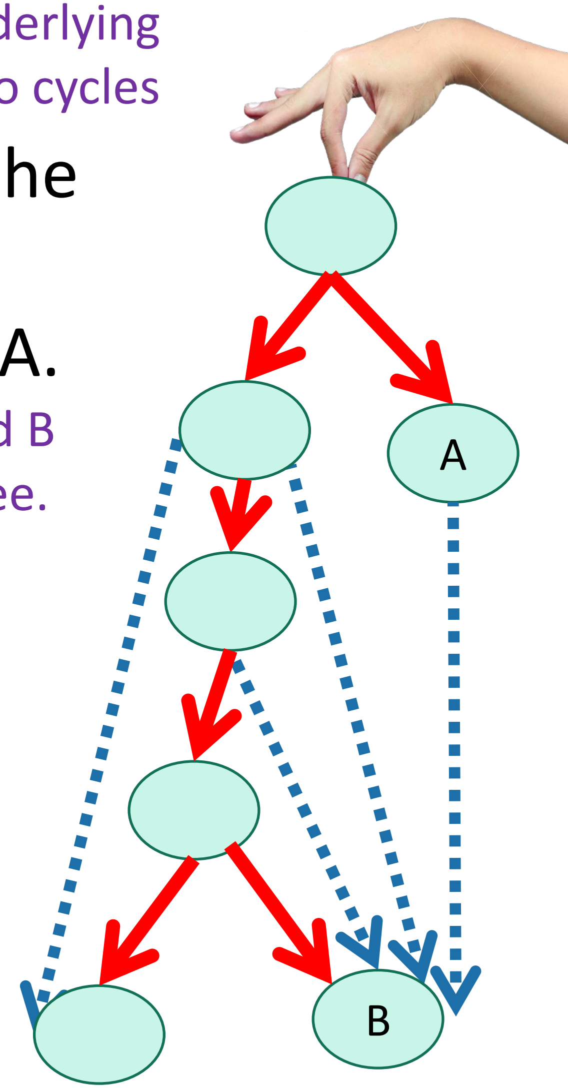
# So to prove this ->

If 


Then B.finishTime < A.finishTime

NOTE: In class this case was missing!!!
I messed up ☹
But it's here now.

Suppose the underlying graph has no cycles



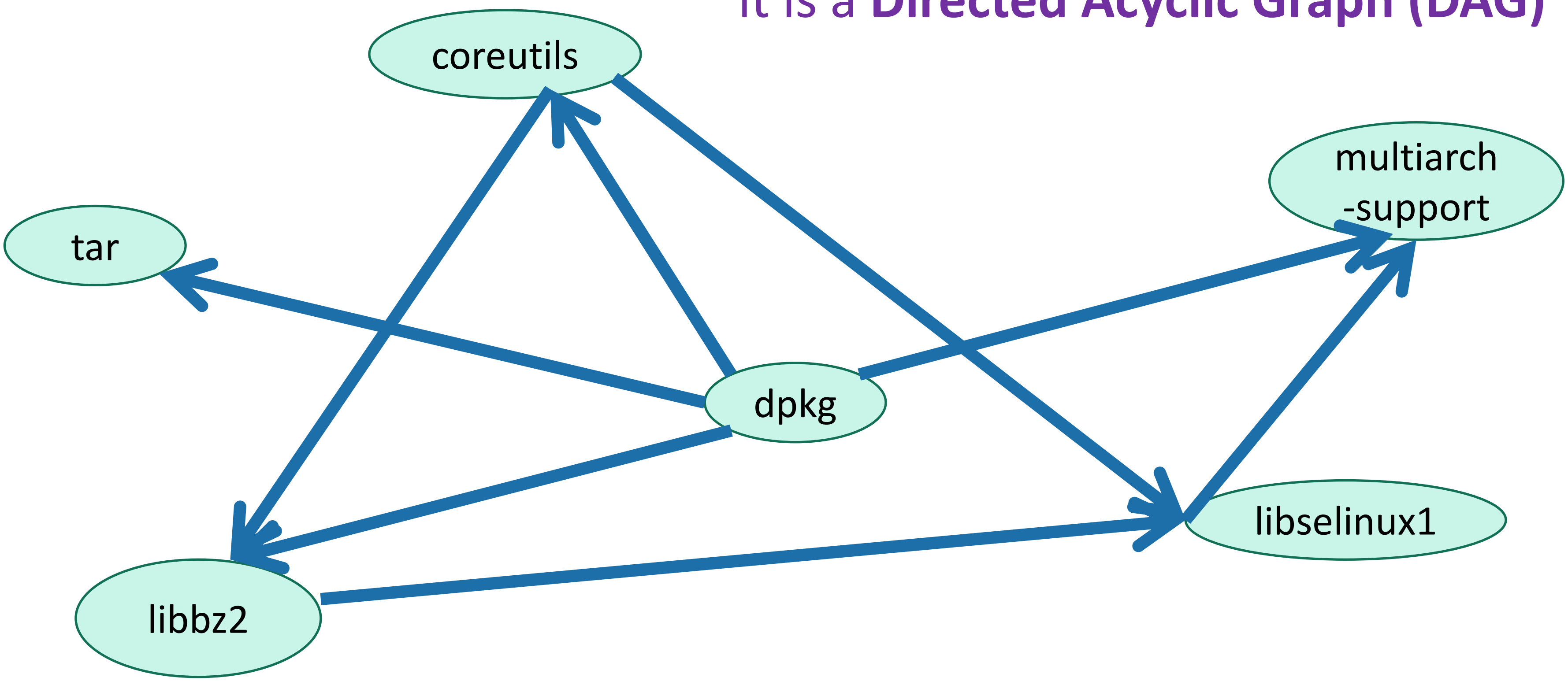

- **Case 2**: B is a NOT descendant of A in the DFS tree.
- Then we must have explored B before A.
  - Otherwise we would have gotten to B from A, and B would have been a descendant of A in the DFS tree.
- Then


- aka, B.finishTime < A.finishTime.

# Back to this problem

- Question: in what order should I install packages?

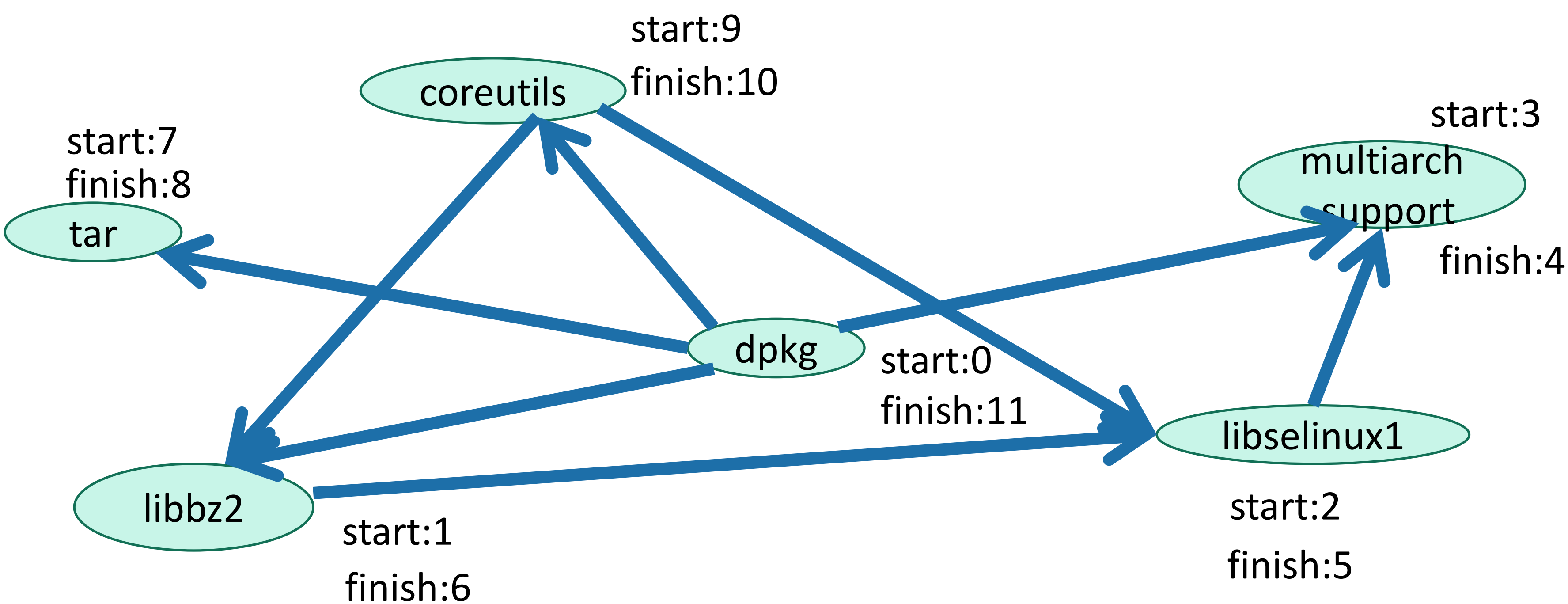
Suppose the dependency graph has no cycles: it is a **Directed Acyclic Graph (DAG)**



# In reverse order of finishing time

- Do DFS
- Maintain a list of packages, in the order you want to install them.
- When you mark a vertex as **all done**, put it at the **beginning** of the list.

```
• dpkg
• coreutils
• tar
• libbz2
• libselinux1
• multiarch_support
```



# For implementation,
## see IPython notebook

In [69]:
```
print(G)
```

```
CS161Graph with:
         Vertices:
        dkpg,coreutils,multiarch_support,libselinux1,libbz2,tar,
         Edges:
        (dkpg,multiarch_support) (dkpg,coreutils) (dkpg,tar) (dkpg,libbz2
) (coreutils,libbz2) (coreutils,libselinux1) (libselinux1,multiarch_suppo
rt) (libbz2,libselinux1)
```

In [71]:
```
V = topoSort(G)
for v in V:
    print(v)
```

```
dkpg
tar
coreutils
libbz2
libselinux1
multiarch_support
```

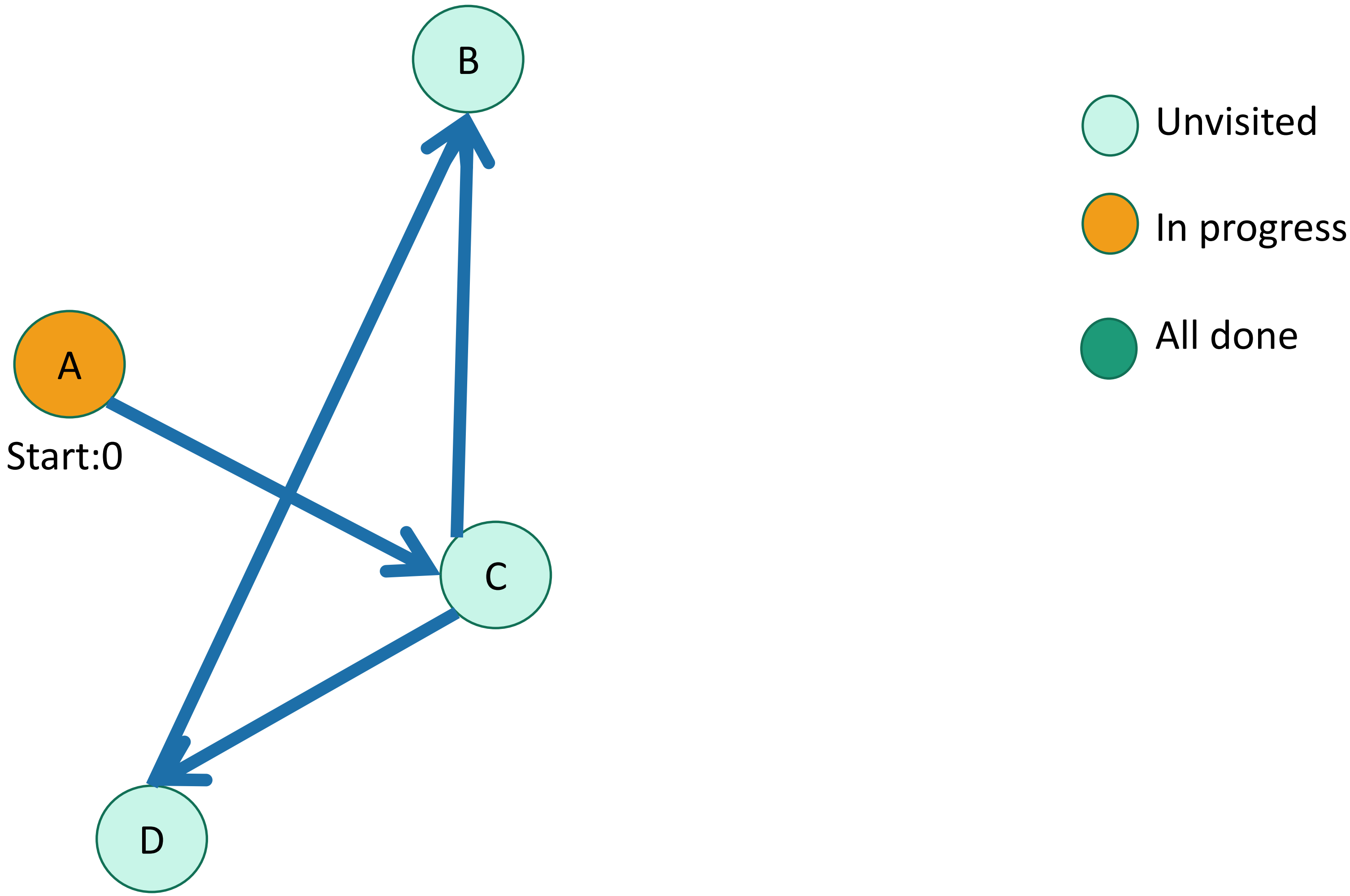
# What did we just learn?

- DFS can help you solve the **TOPOLOGICAL SORTING PROBLEM**
  - That's the fancy name for the problem of finding an ordering that respects all the dependencies

- Thinking about the DFS tree is helpful.

# Example:

This example skipped in class – here for reference.



# Example

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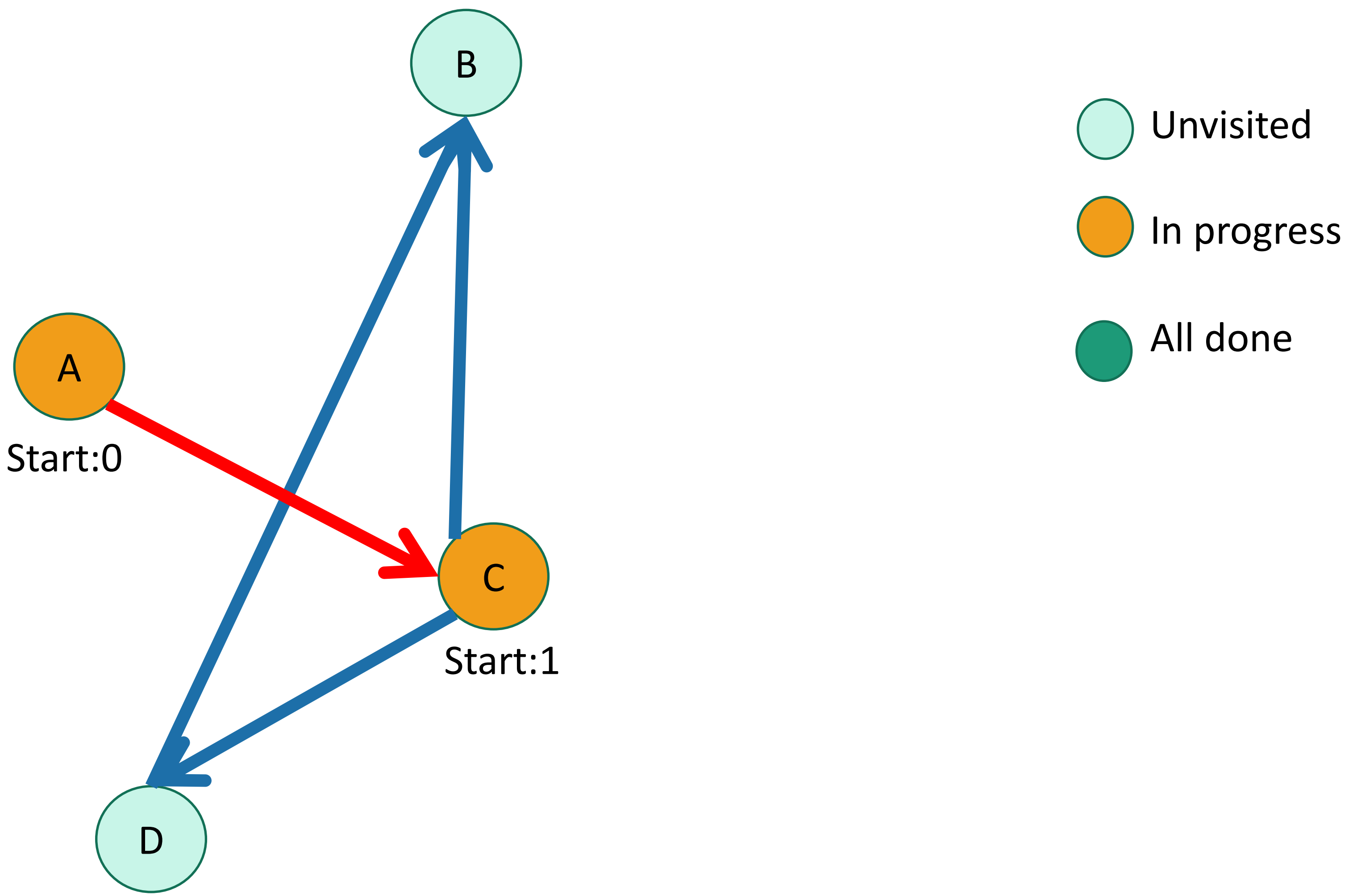

# Example

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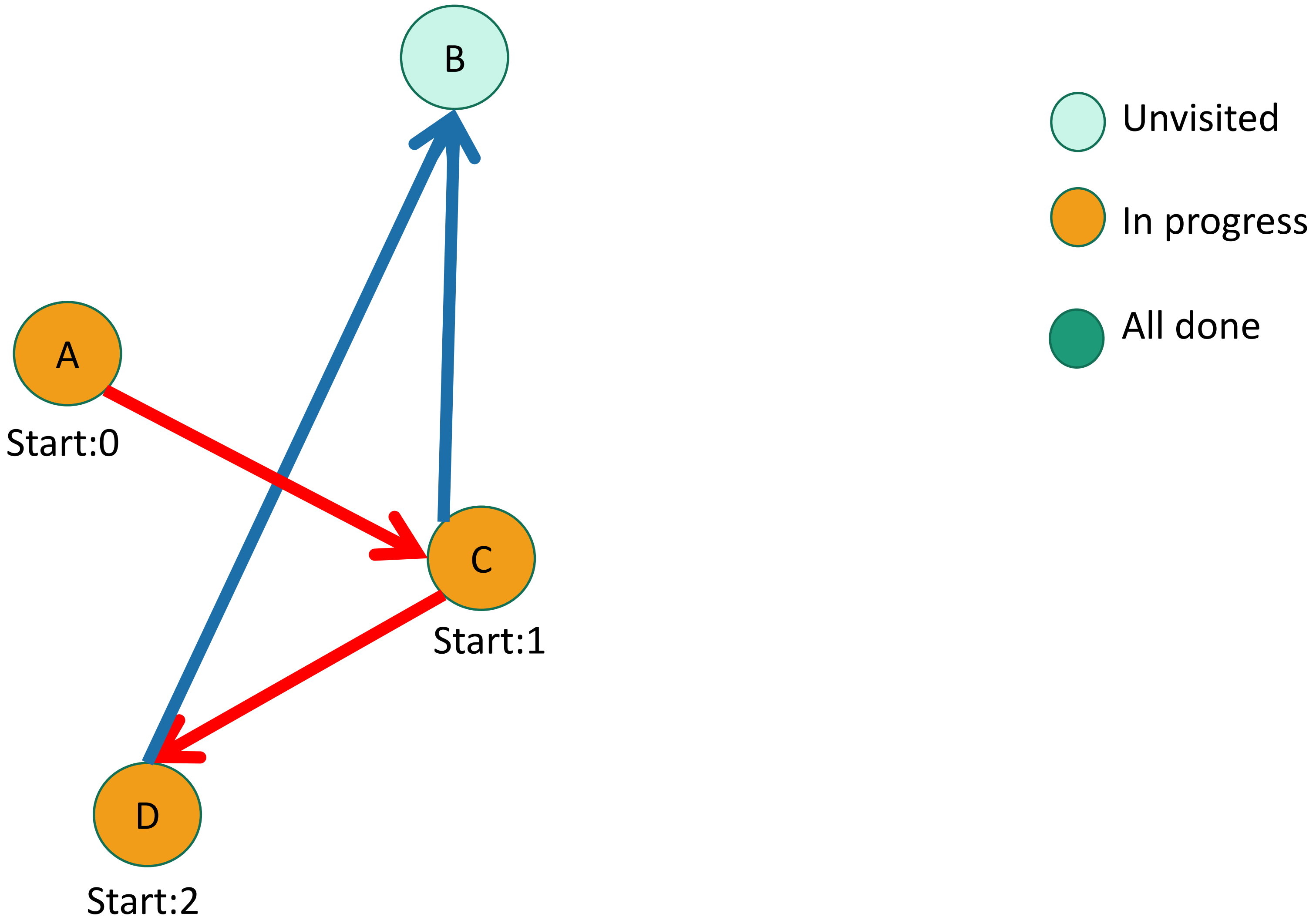
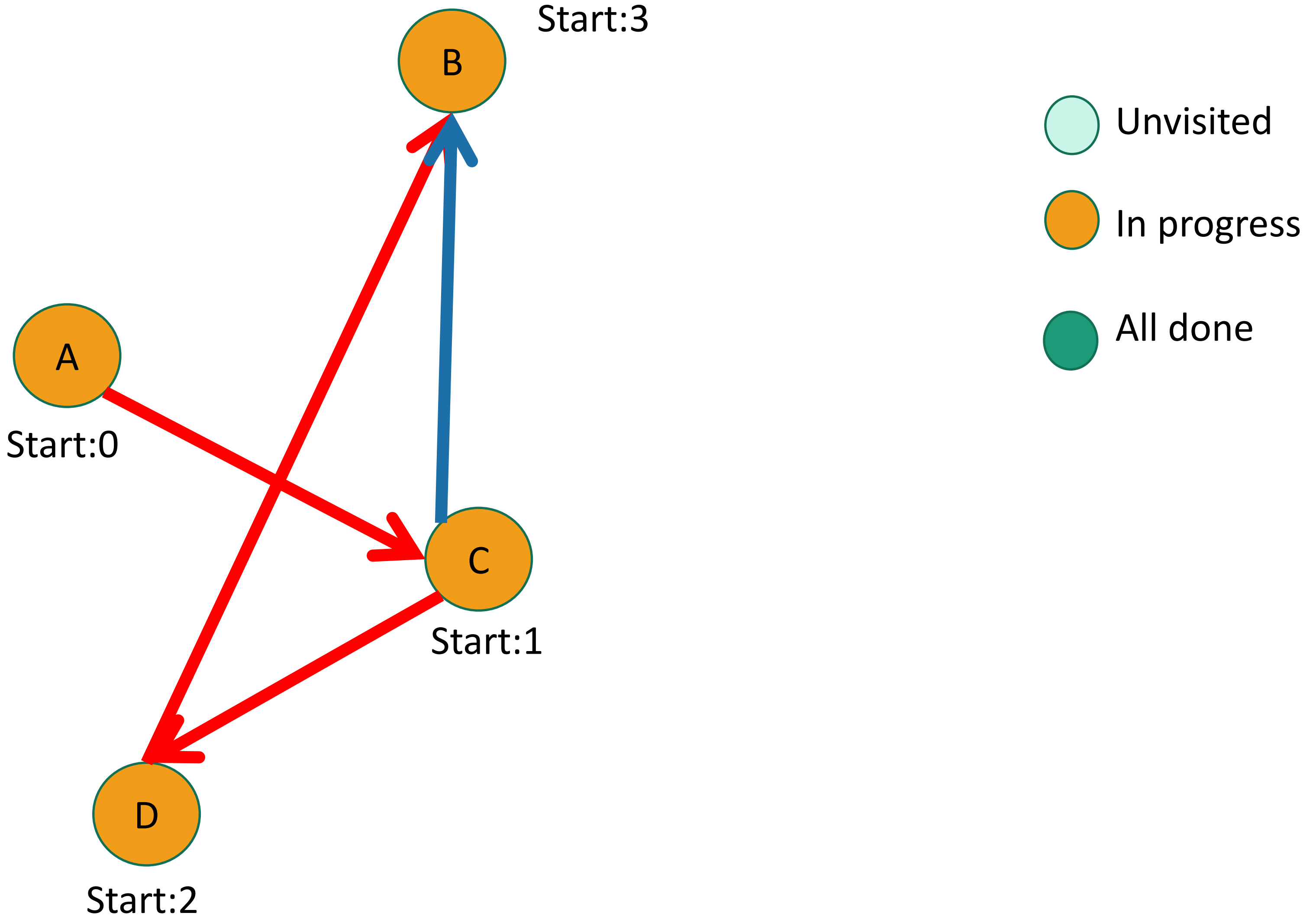


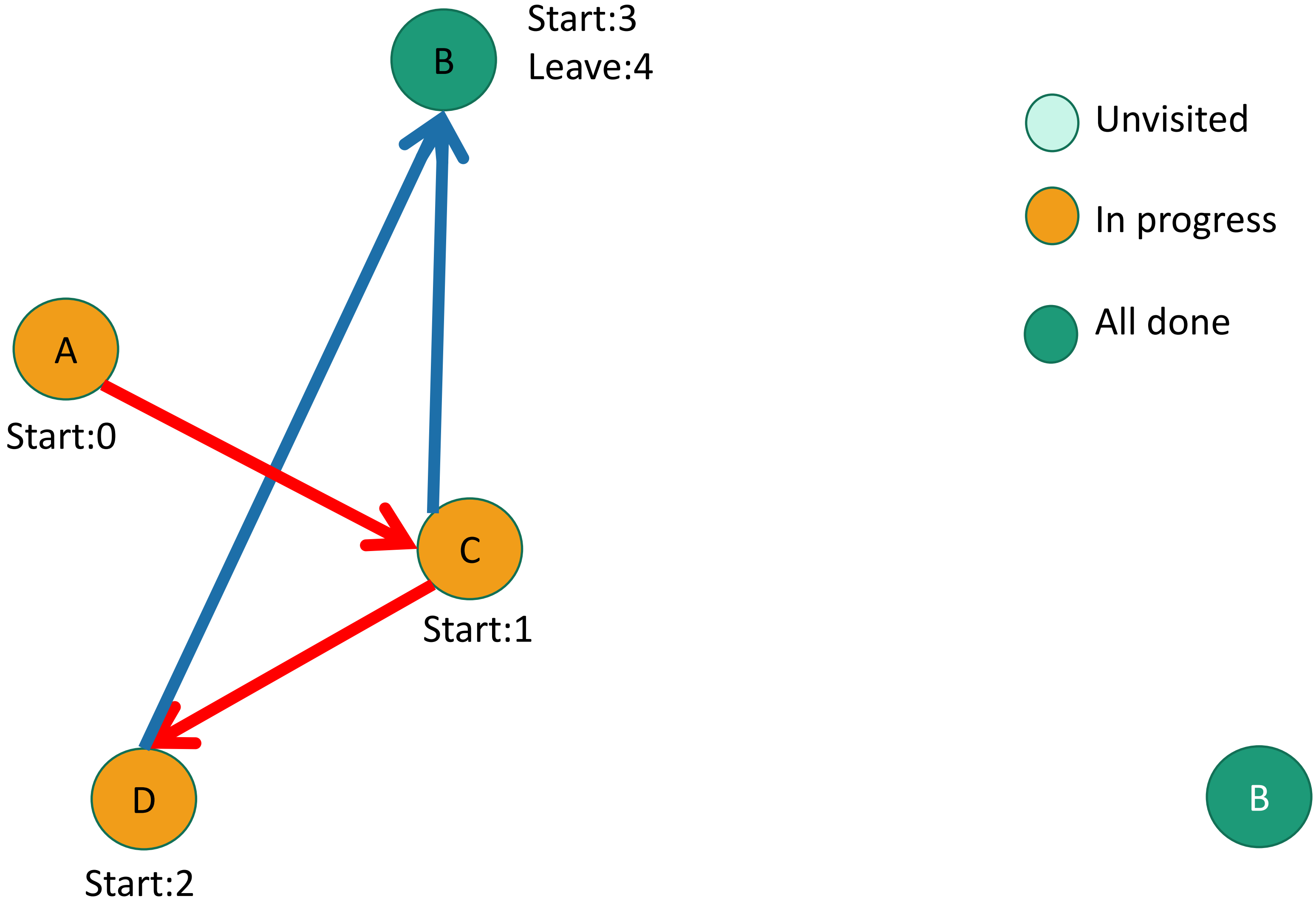
# Example

This example skipped in class – here for reference.



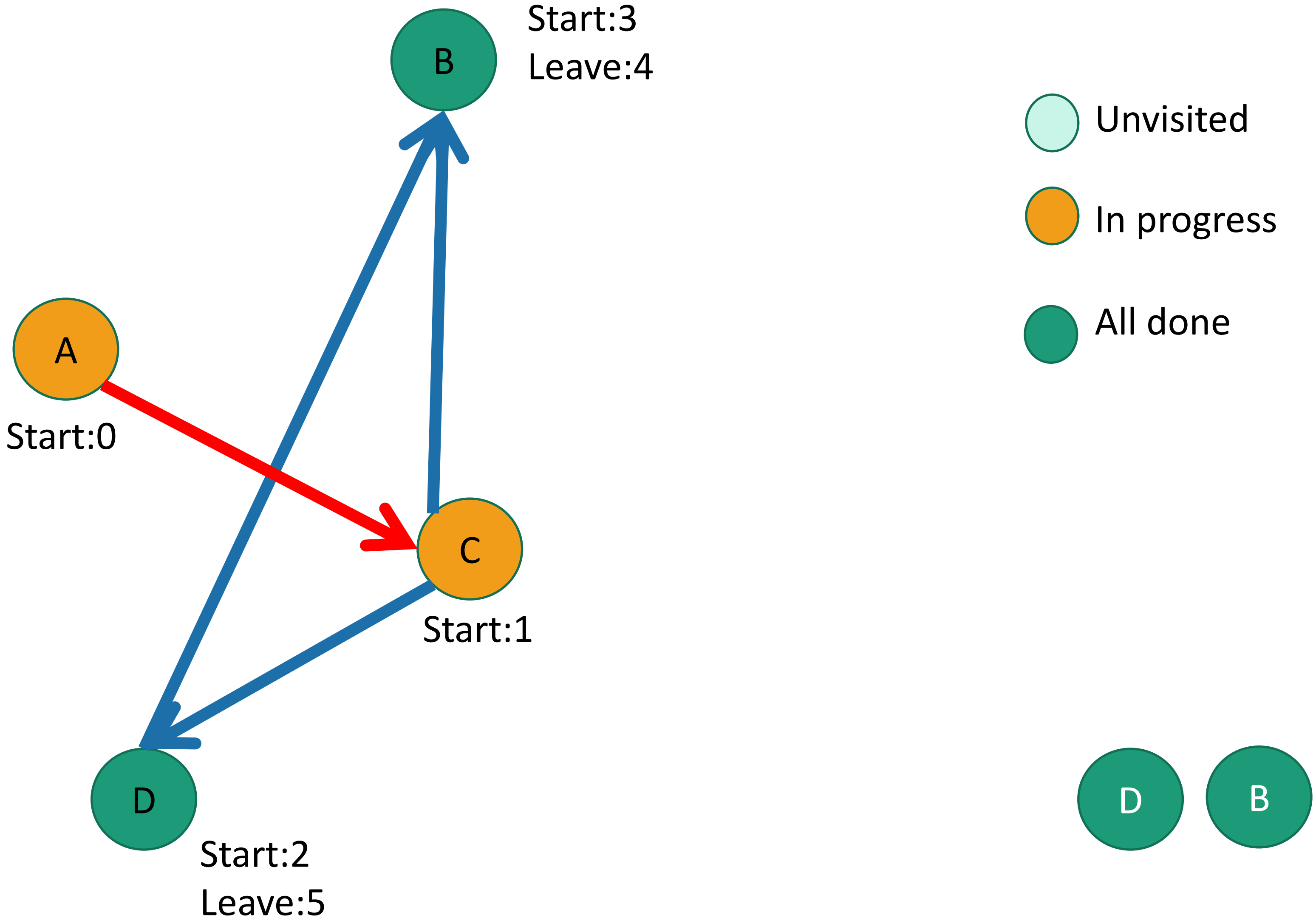
# Example

This example skipped in class – here for reference.



# Example

This example skipped in class – here for reference.



# Example

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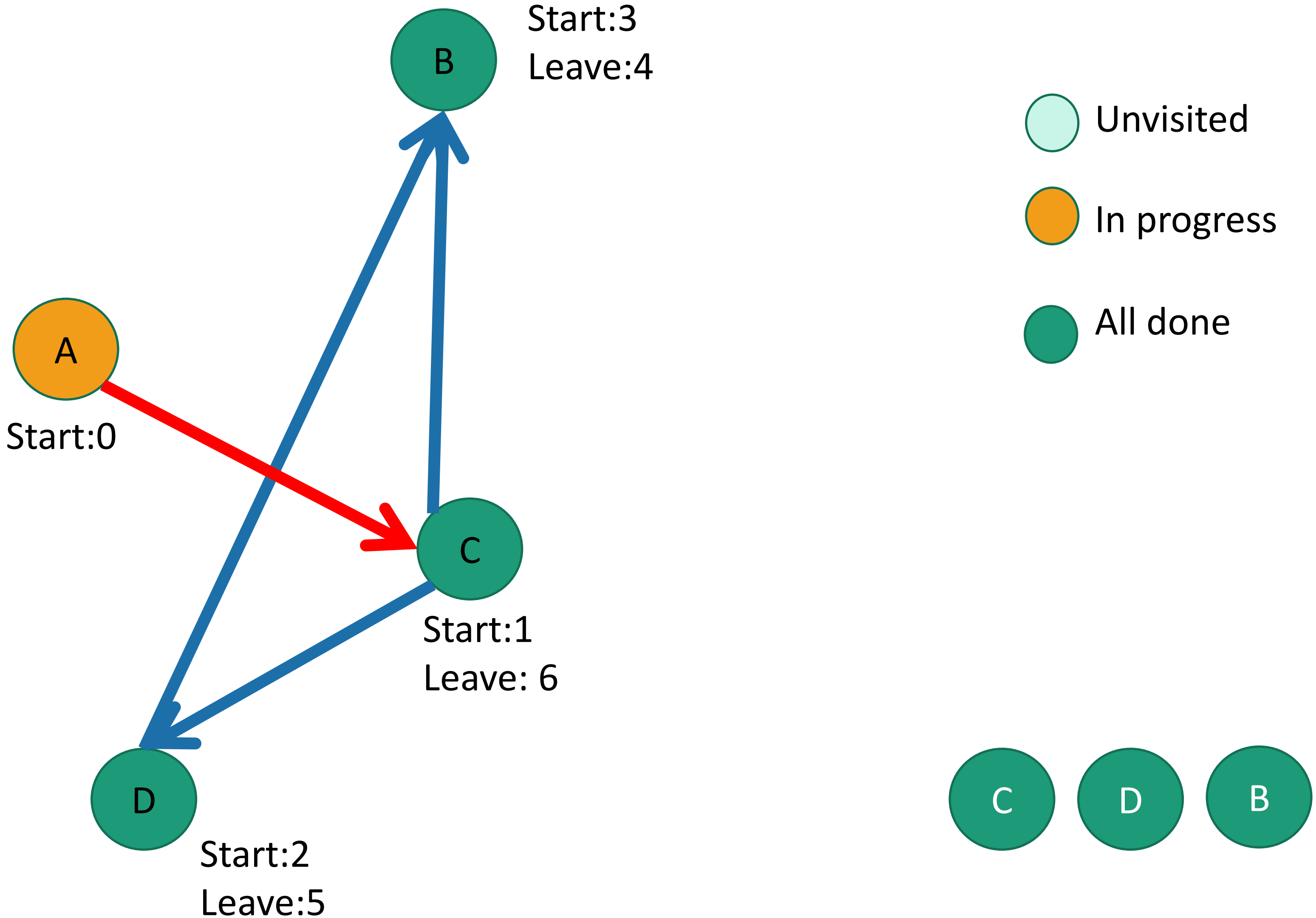
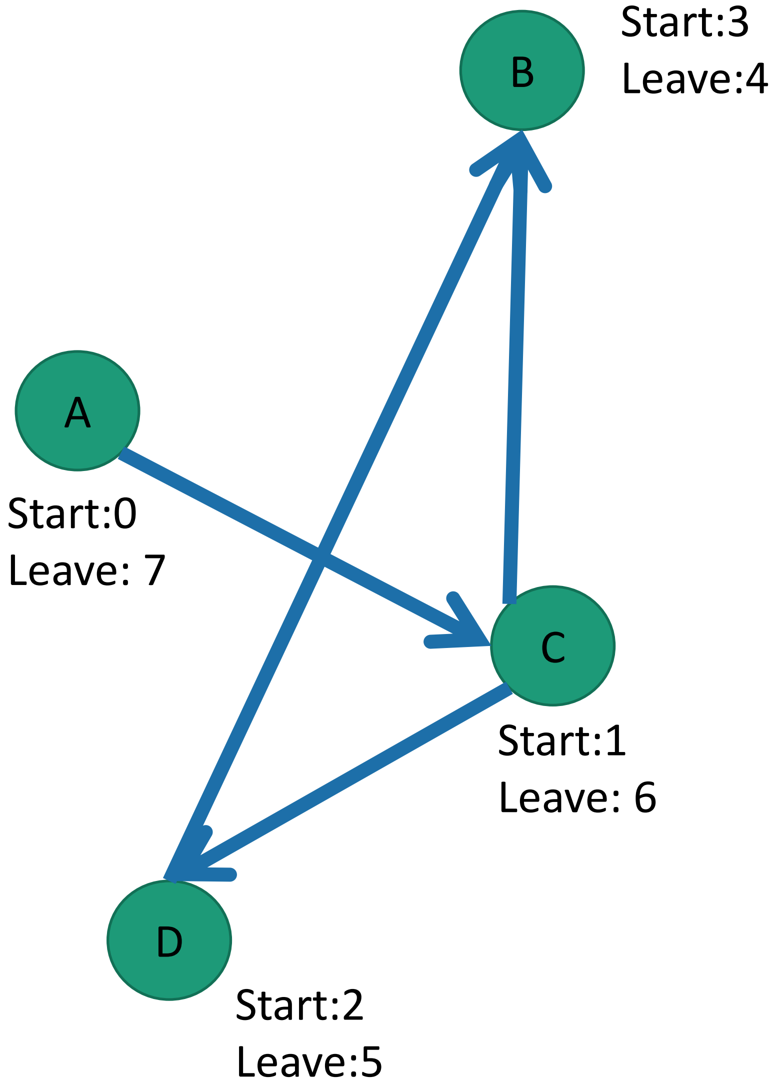


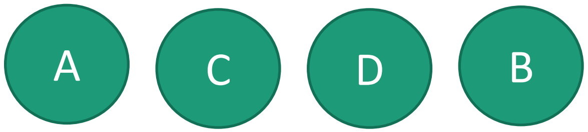
# Example

This example skipped in class – here for reference.



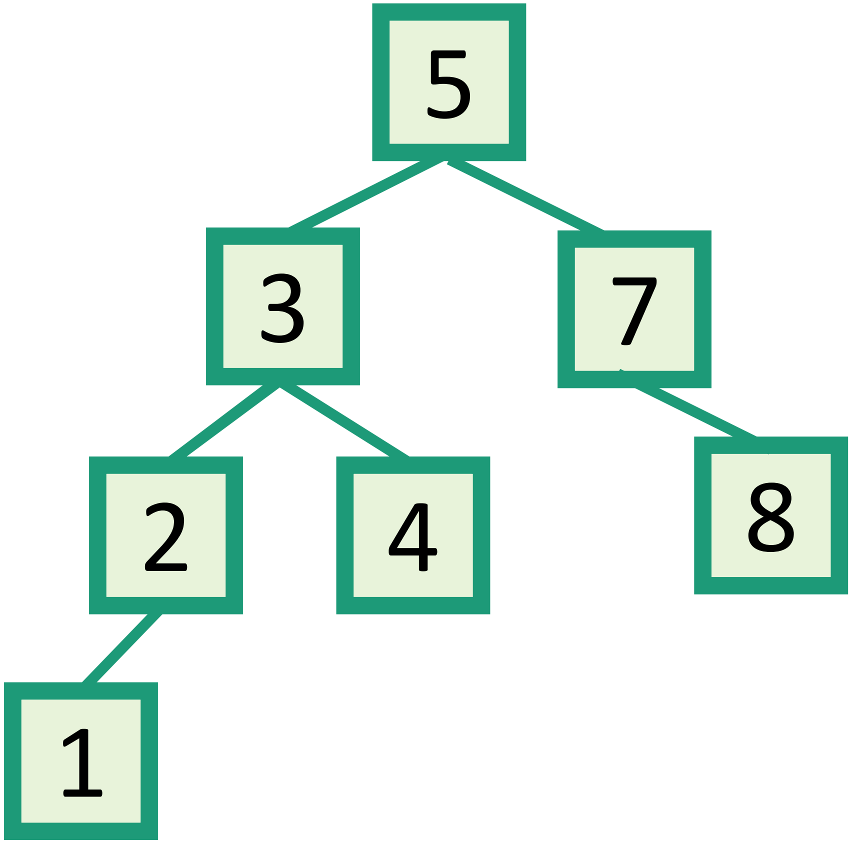


Do them in this order:



# Another use of DFS

- In-order enumeration of binary search trees



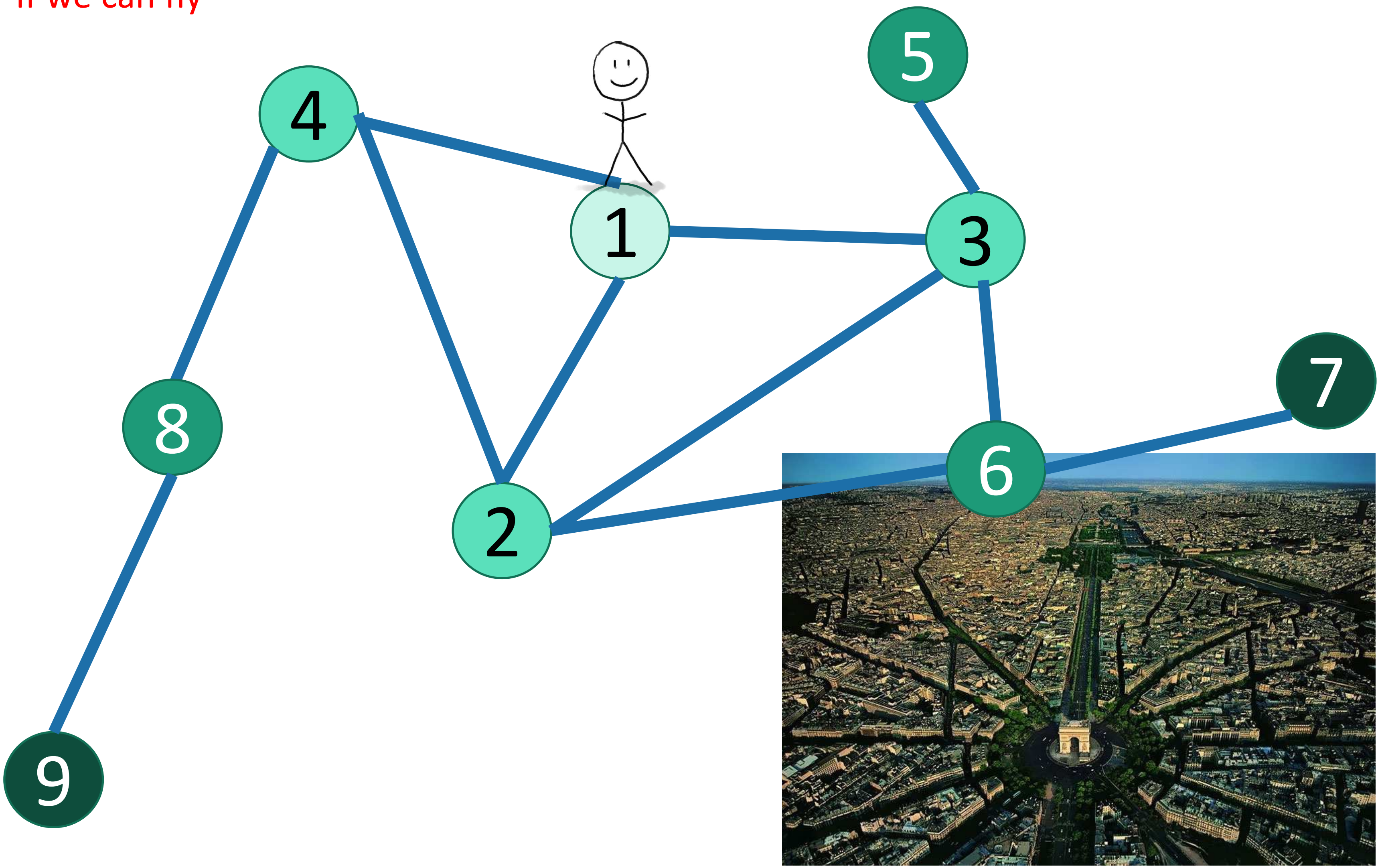

Given a binary search tree, output all the nodes **in order.**

Instead of outputting a node when you are done with it, output it when you are done with the left child and before you begin the right child.

# Part 2: breadth-first search

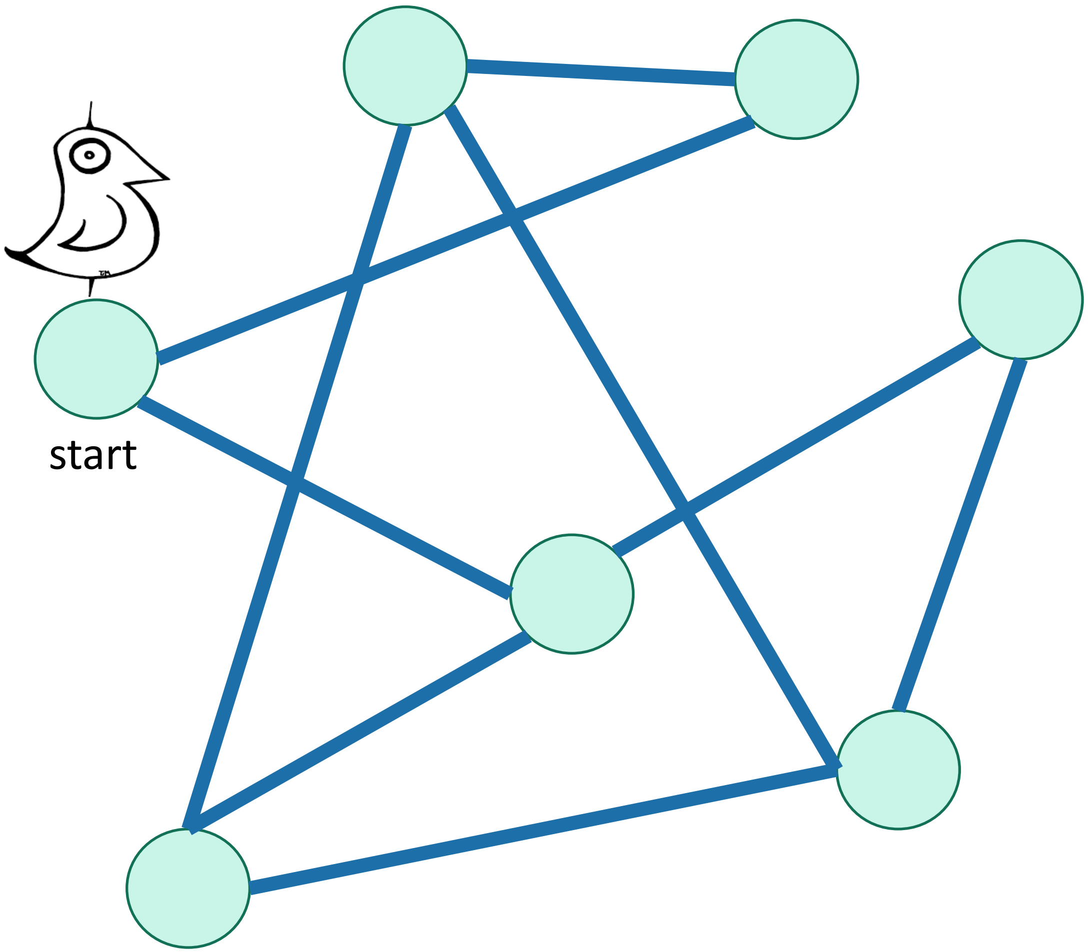
# How do we explore a graph?

If we can fly



# Breadth-First Search

## Exploring the world with a bird's-eye view




Not been there yet

Can reach there in zero steps

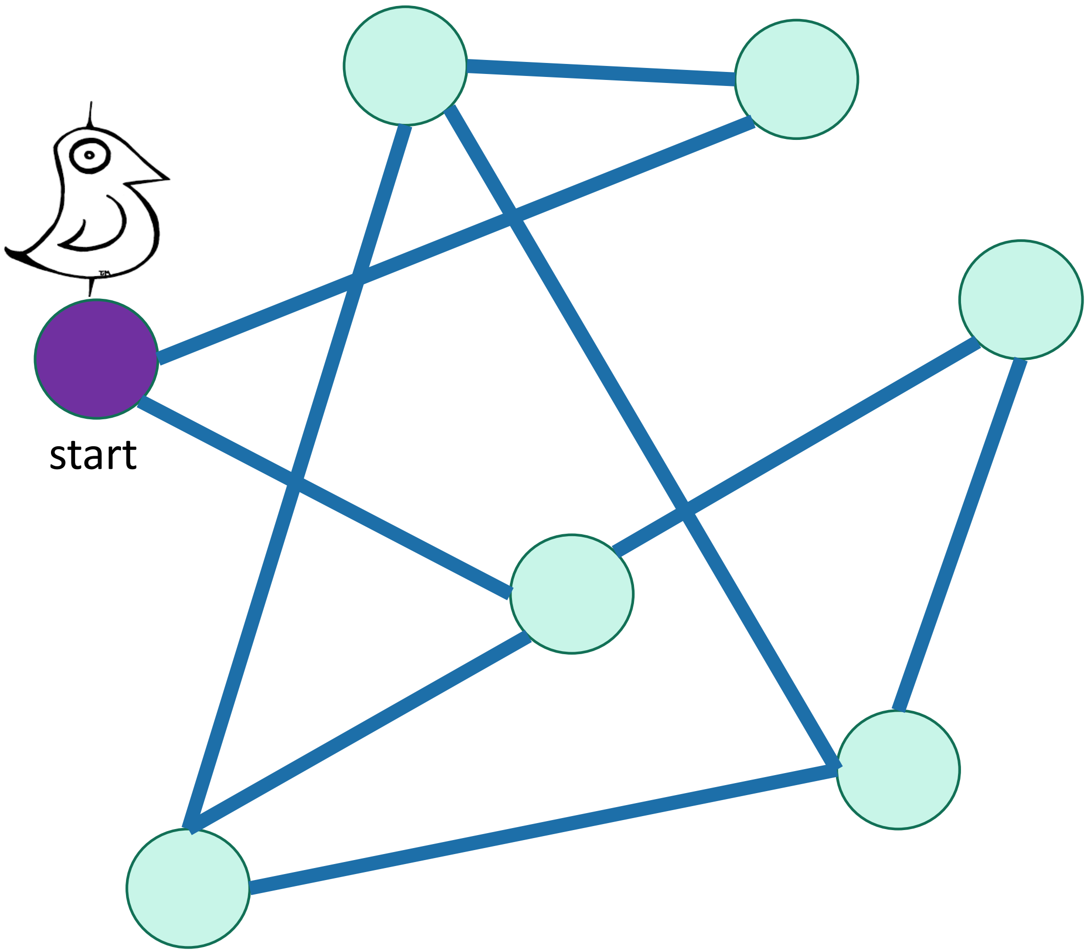
Can reach there in one step

Can reach there in two steps

Can reach there in three steps

# Breadth-First Search

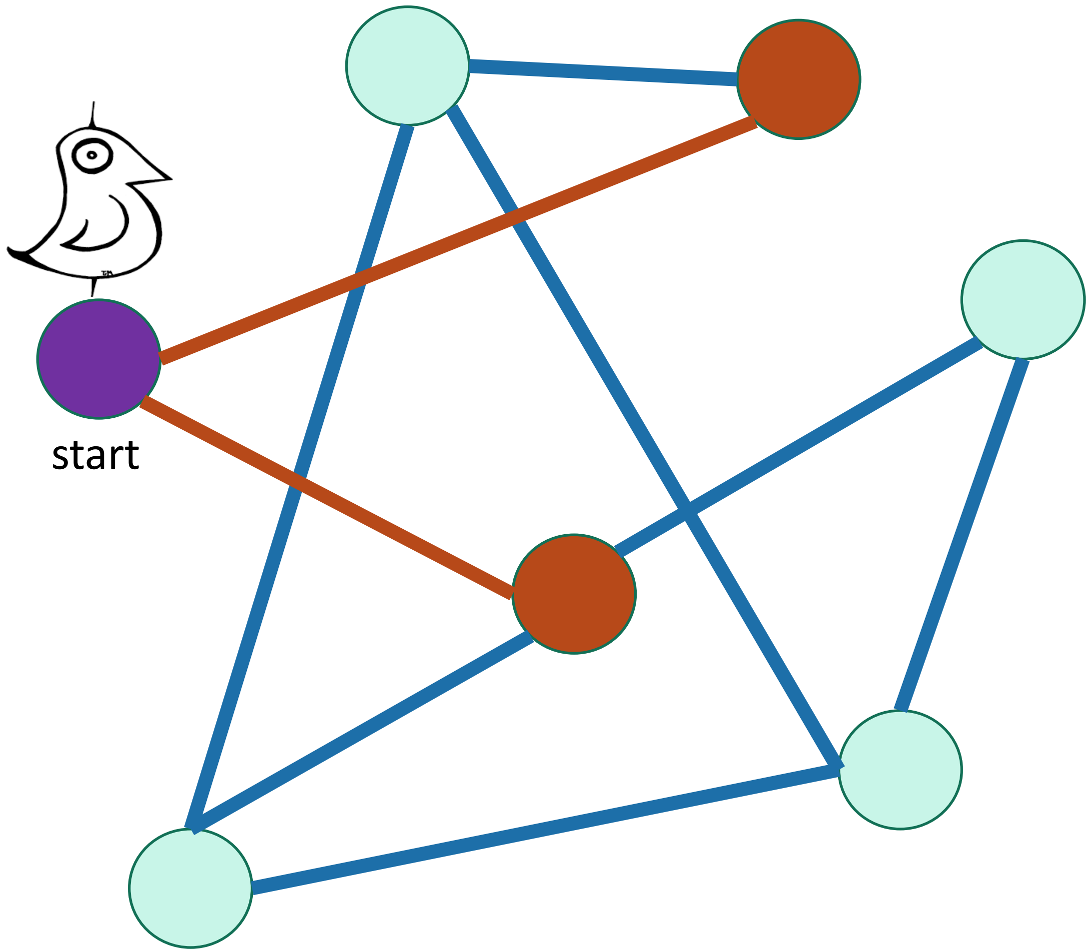
## Exploring the world with a bird's-eye view




- Not been there yet
- Can reach there in zero steps
- Can reach there in one step
- Can reach there in two steps
- Can reach there in three steps

# Breadth-First Search

## Exploring the world with a bird's-eye view




- Not been there yet
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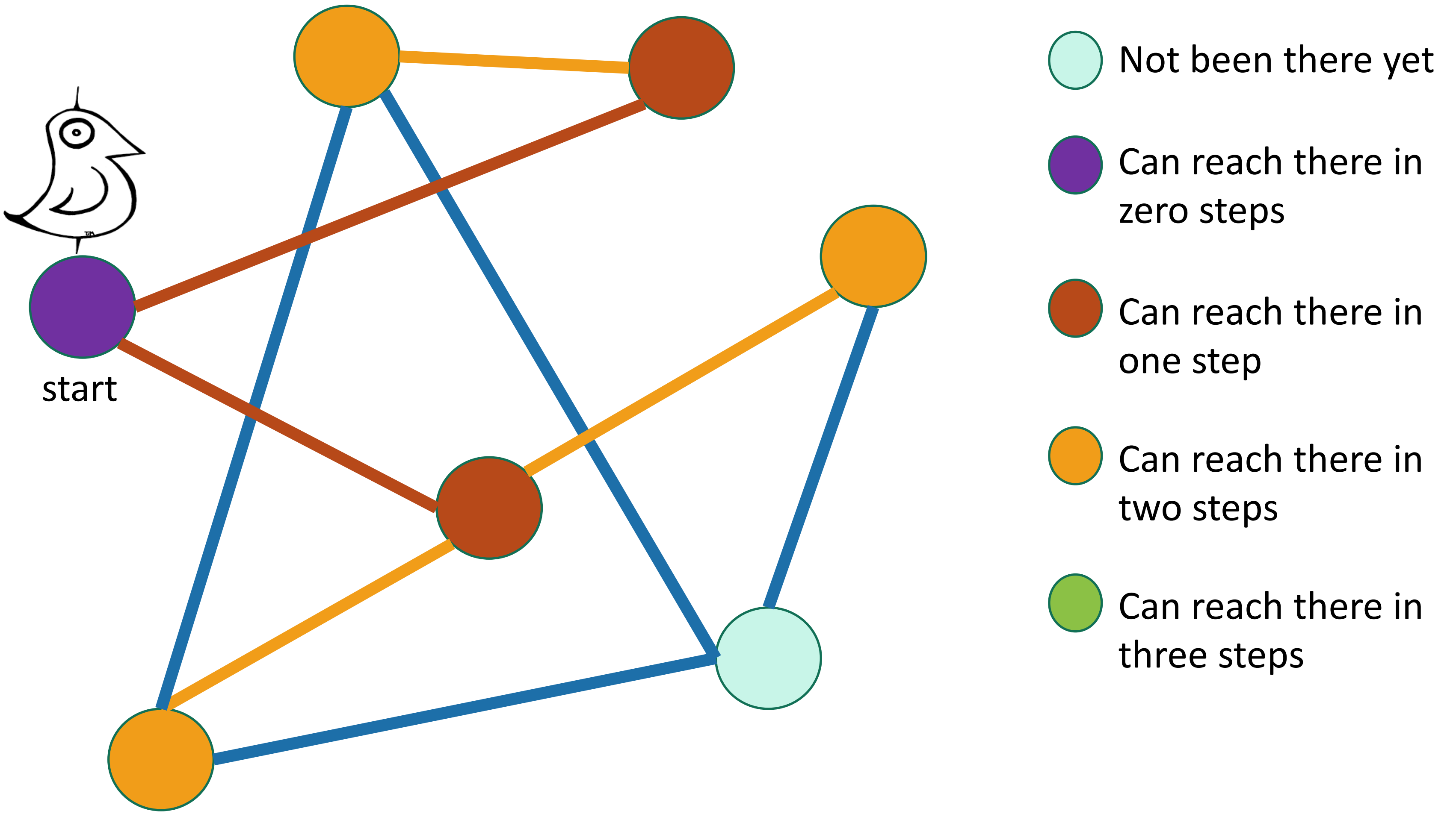
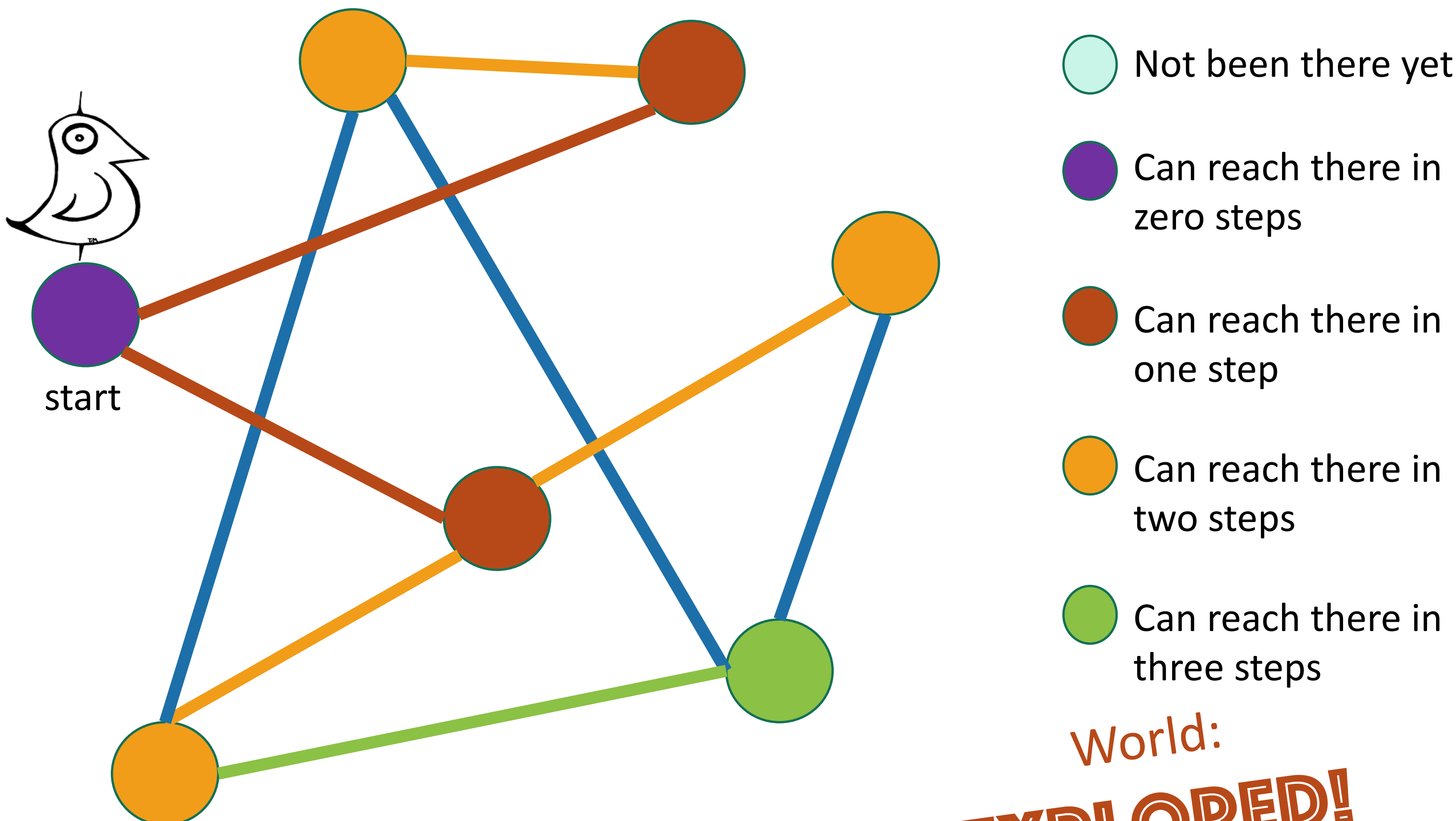
# Breadth-First Search

## Exploring the world with a bird's-eye view



# Breadth-First Search

## Exploring the world with a bird's-eye view




World:

**EXPLORED!**

Same disclaimer as for DFS: you may have seen other ways to implement this, this will be convenient for us.

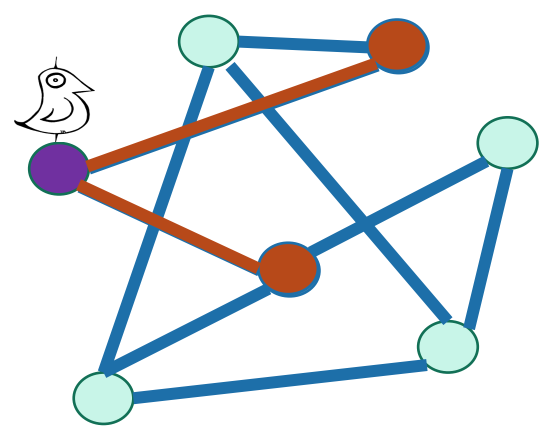
# Breadth-First Search

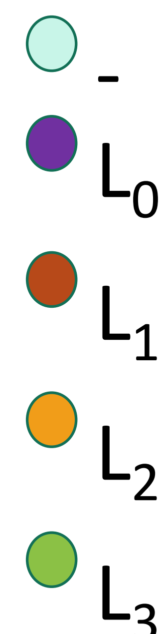
## Exploring the world with pseudocode

- Set $L_i$ = [] for i=1,…,n
- $L_0$ = {w}, where w is the start node
- **For** i = 0, …, n-1:
    - **For** u in $L_i$:
        - **For** each v which is a neighbor of u:
            - **If** v isn't yet visited:
                - mark v as visited, and put it in $L_{i+1}$

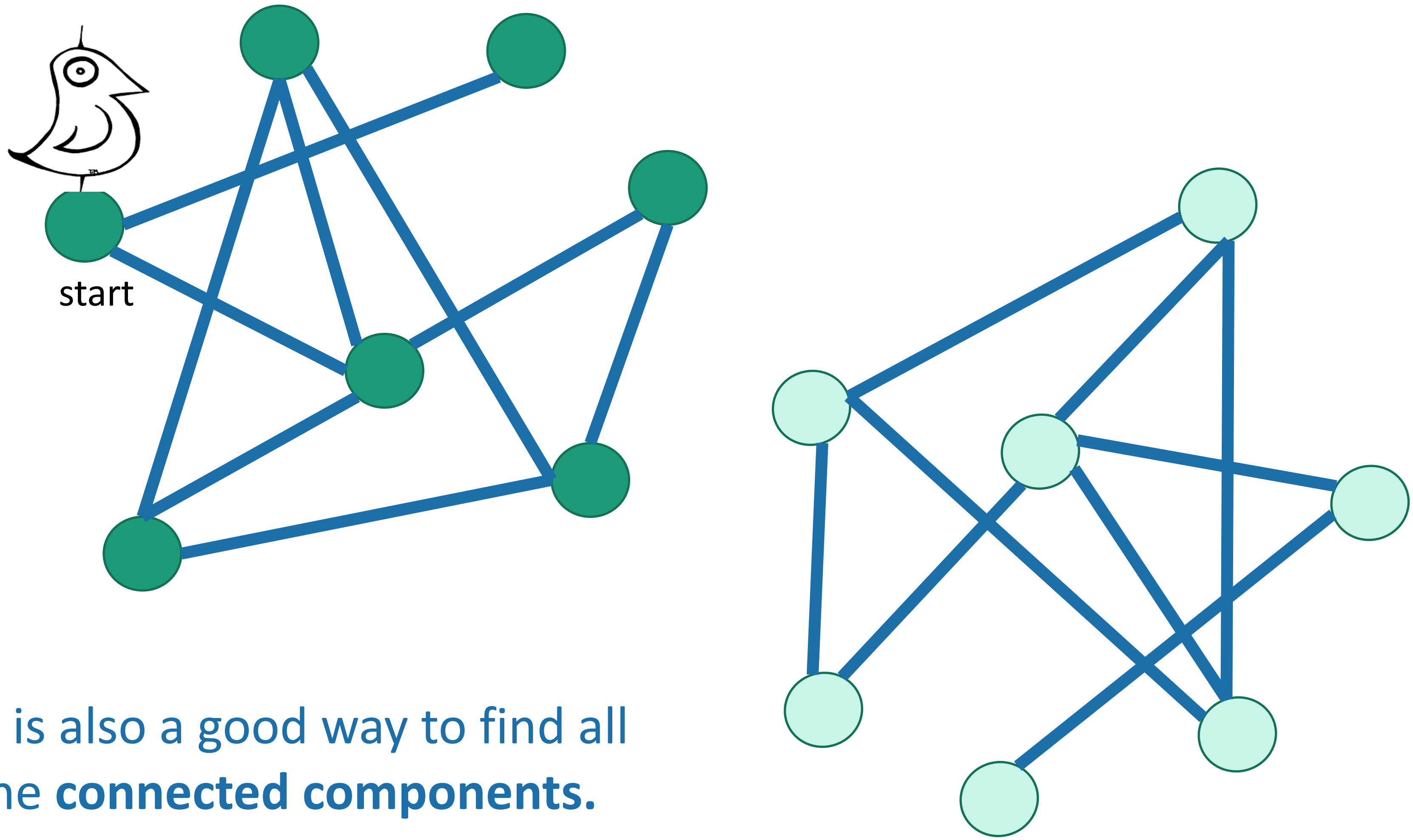
$L_i$ is the set of nodes we can reach in i steps from w

Go through all the nodes in $L_i$ and add their unvisited neighbors to $L_{i+1}$





# BFS also finds all the nodes reachable from the starting point




It is also a good way to find all the **connected components.**

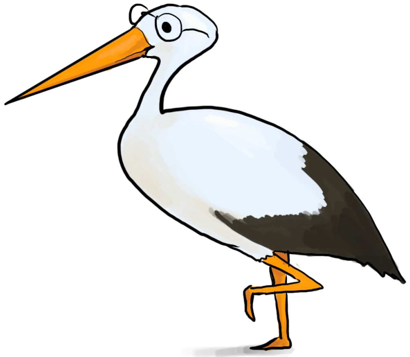
# Running time

## To explore the whole thing

- Explore the connected components one-by-one.
- Same argument as DFS: running time is

$$O(n + m)$$

Verify these!

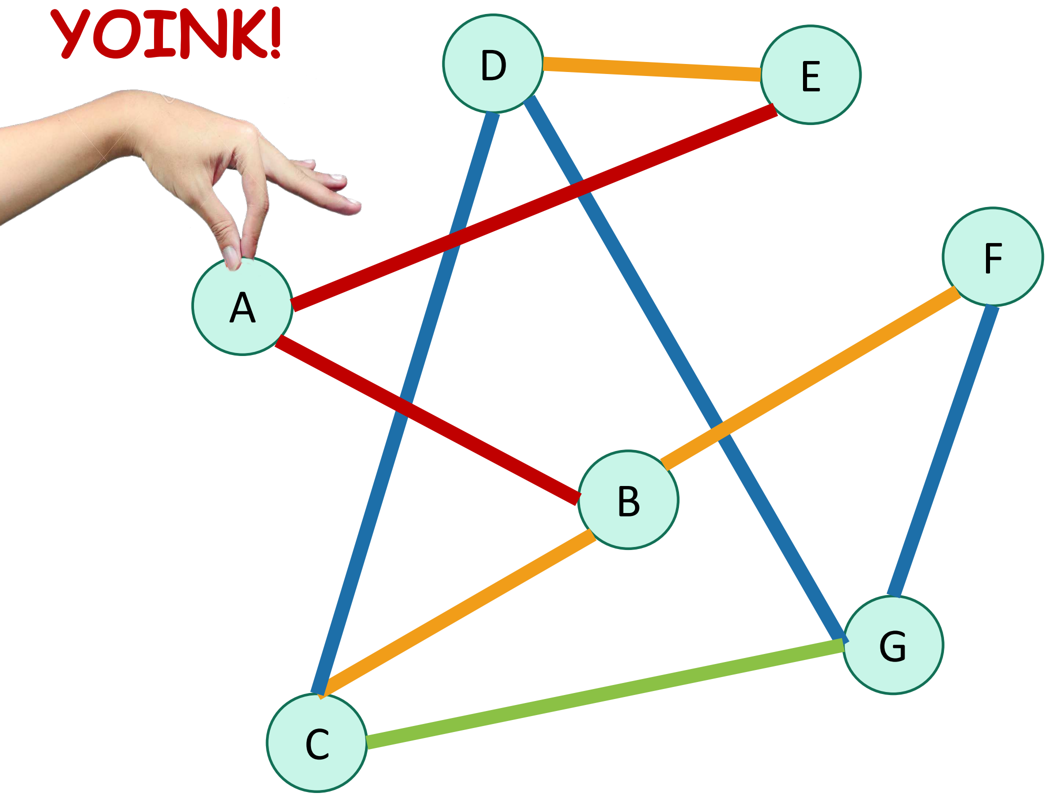


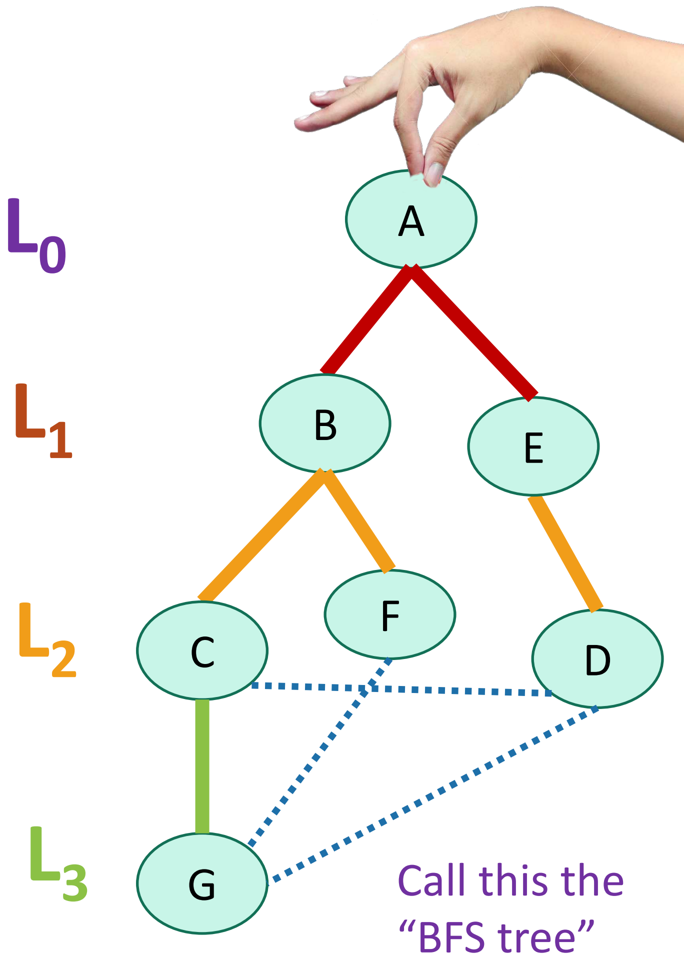
Siggi the Studious Stork

- Like DFS, BFS also works fine on directed graphs.

# Why is it called breadth-first?

- We are implicitly building a tree:

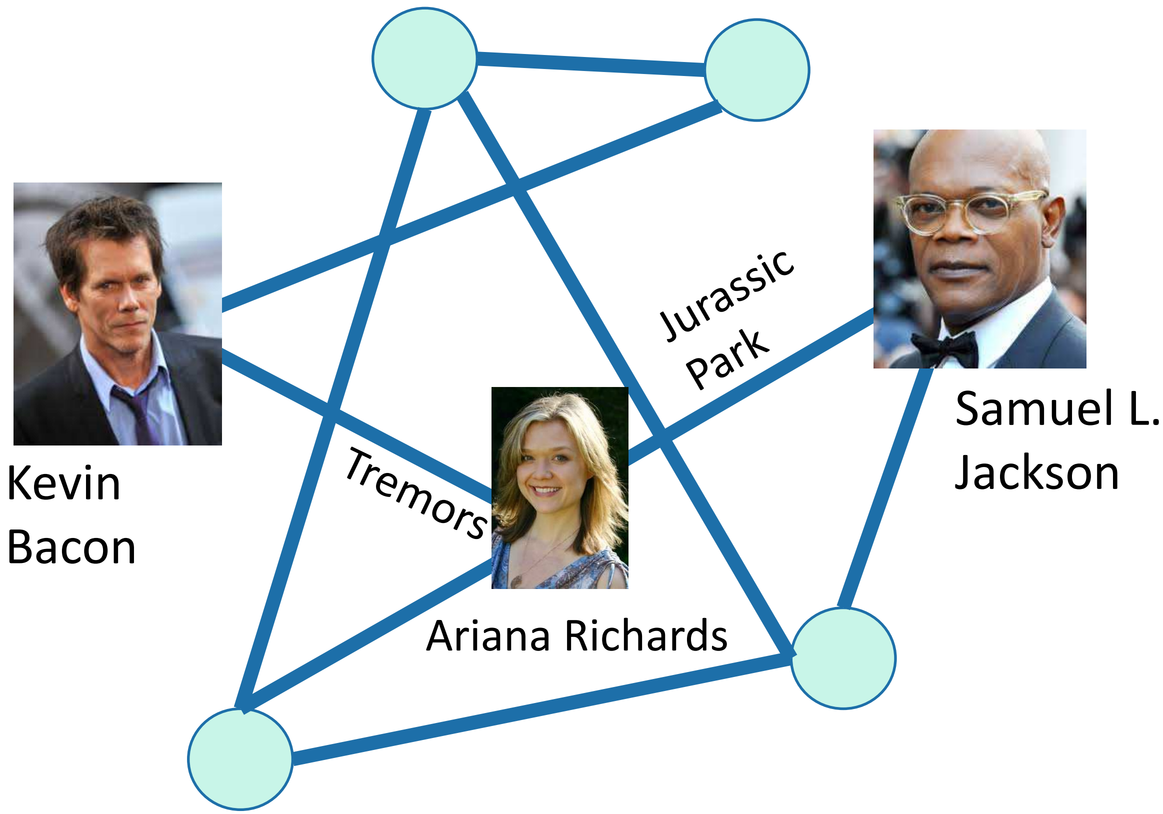



- And first we go as broadly as we can.

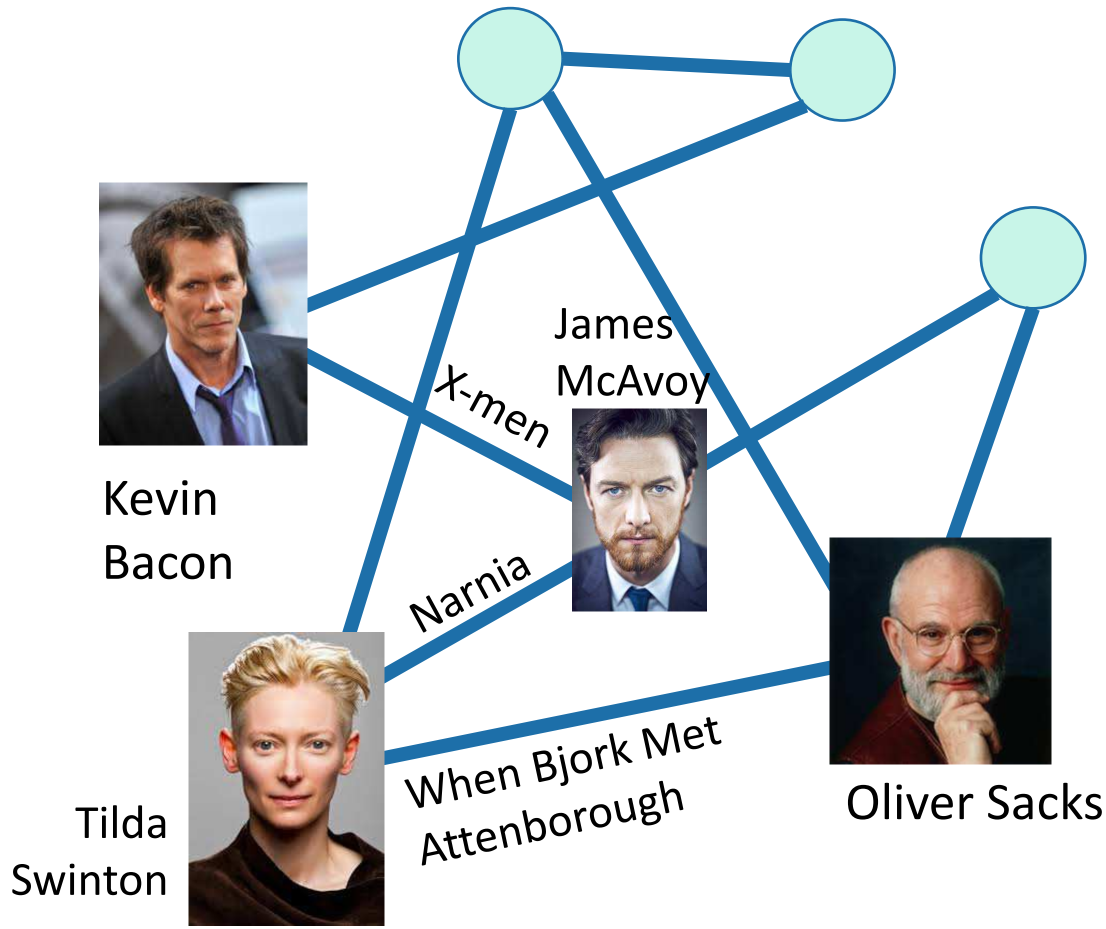
# Pre-lecture exercise

- What Samuel L. Jackson's Bacon number?




(Answer: 2)

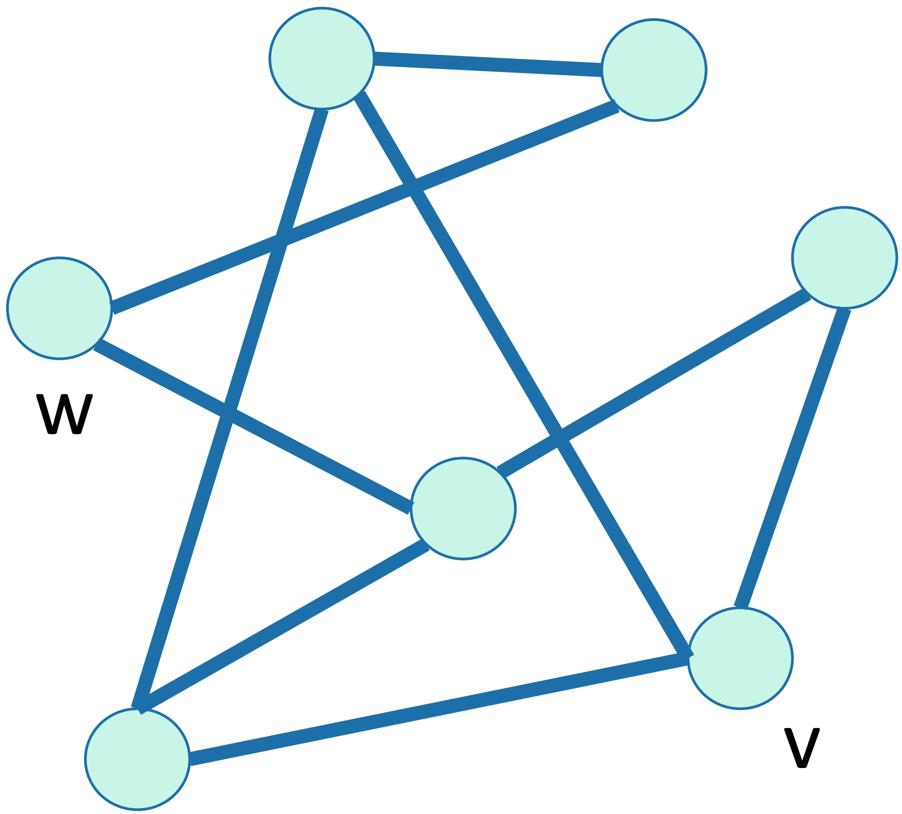
# I wrote the pre-lecture exercise before I realized that I really wanted an example with distance 3




It is really hard to find people with Bacon number 3!

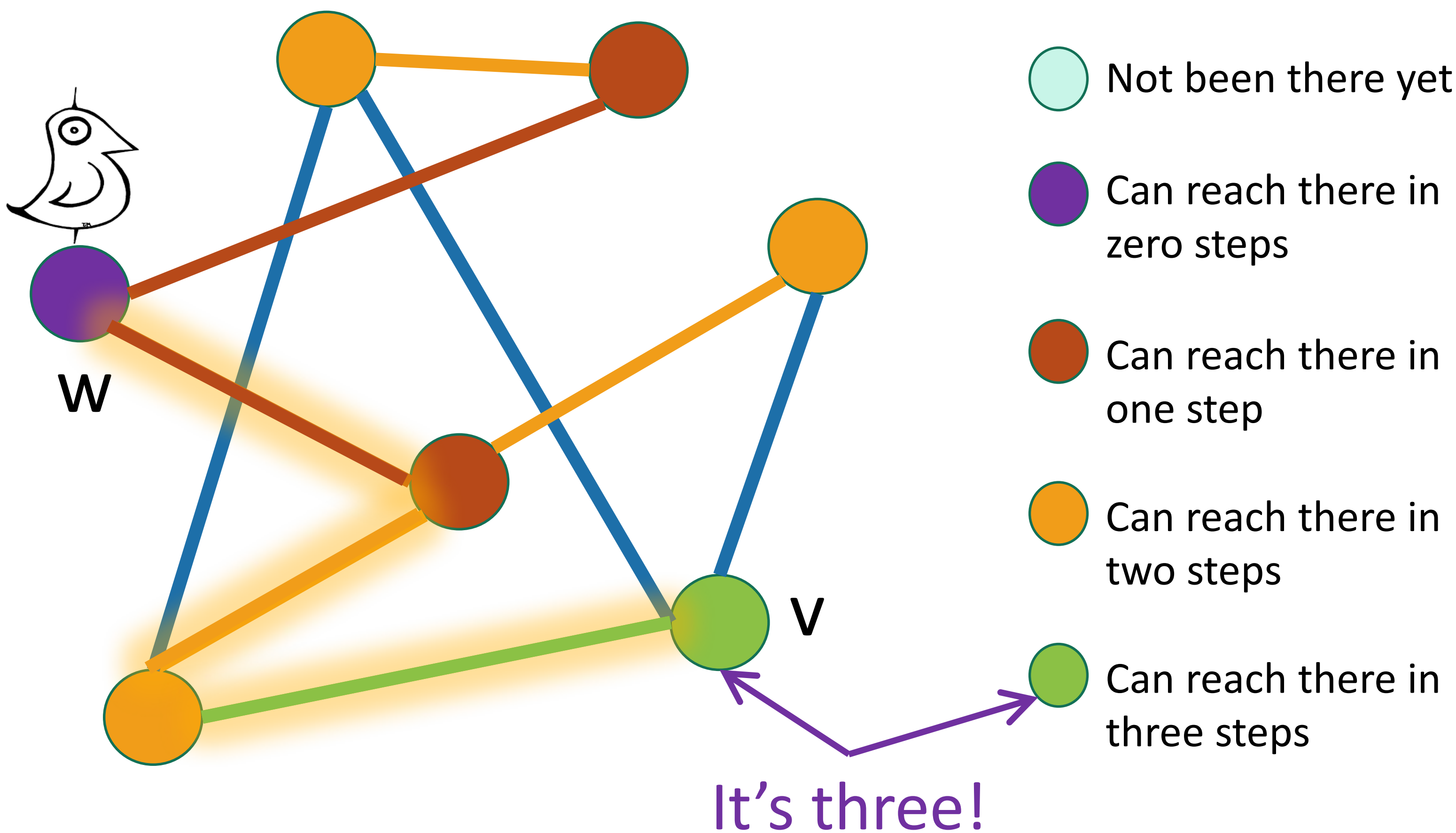
# Application: shortest path

- How long is the shortest path between w and v?



# Application: shortest path

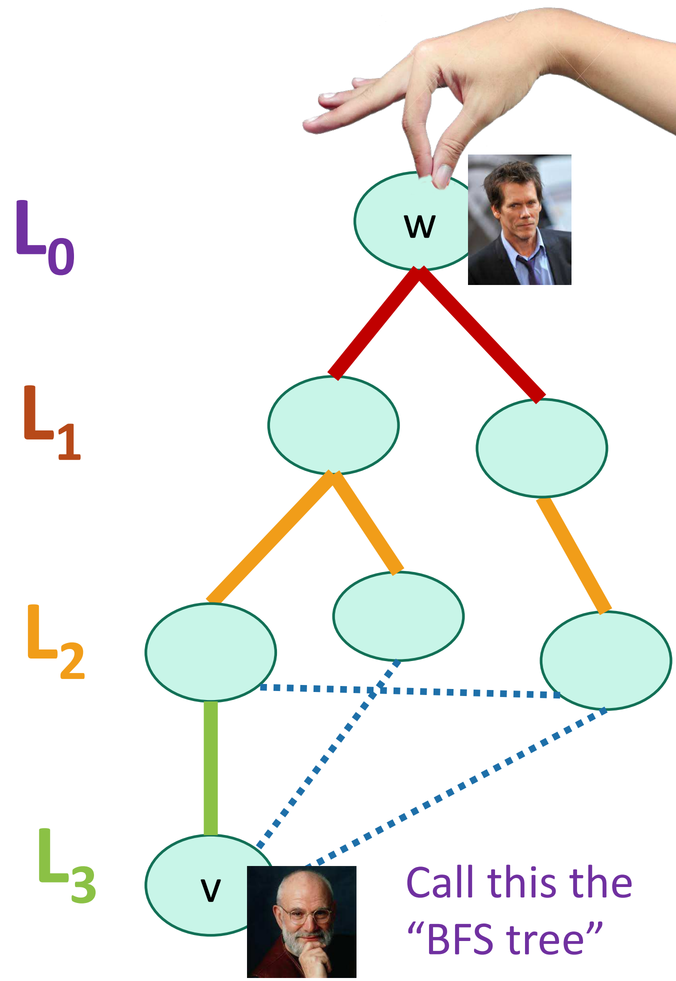
- How long is the shortest path between w and v?



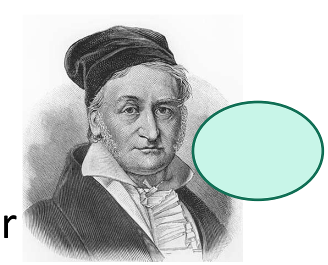
# To find the distance between w and all other vertices v

The **distance** between two vertices is the length of the shortest path between them.

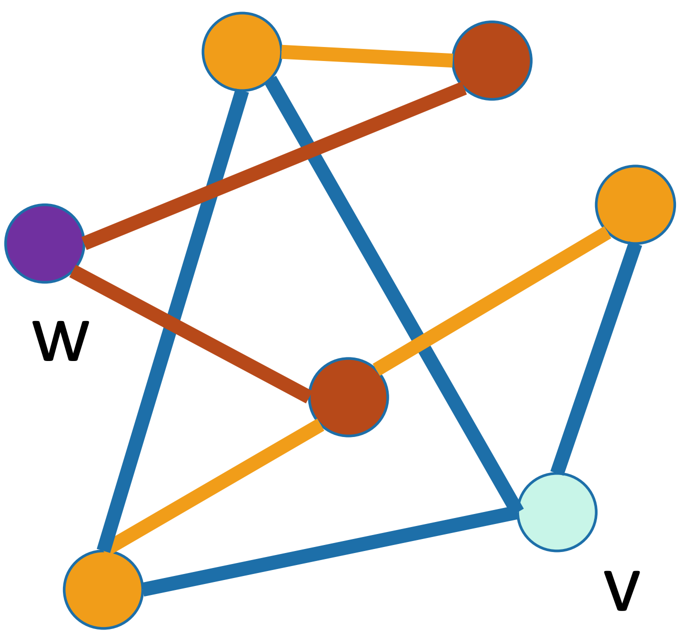
- Do a BFS starting at w
- For all v in $L_i$
  - The shortest path between w and v has length i
  - A shortest path between w and v is given by the path in the BFS tree.
- If we never found v, the distance is infinite.




Gauss has no Bacon number



# Proof idea (on board)




Not been there

Can reach there in zero steps

Can reach there in one step

Can reach there in two steps

Can reach there in three steps

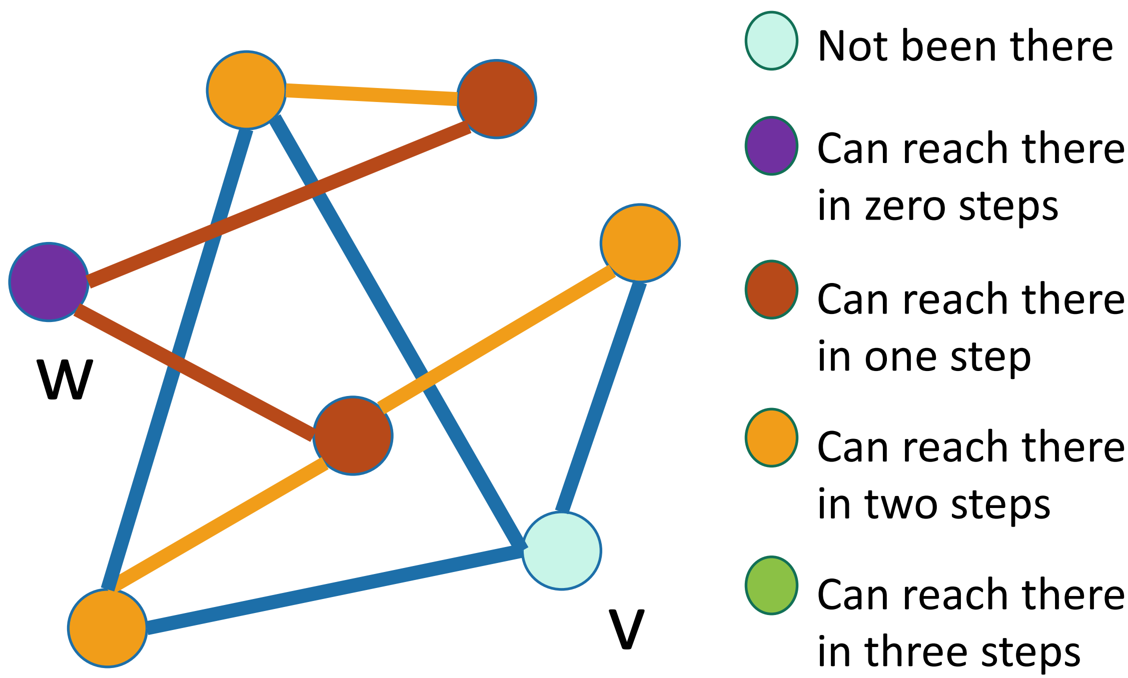
# Proof idea

***THIS SLIDE SKIPPED IN CLASS***

Just the idea…see CLRS for details!

- Suppose by **induction** it's true for vertices in $L_0$, $L_1$, $L_2$
  - For all i < 3, the vertices in $L_i$ have distance i from v.
- **Want to show**: it's true for vertices of distance 3 also.
  - aka, the shortest path between w and v has length 3.

- **Well, it has distance at most 3**
  - Since we just found a path of length 3
- **And it has distance at least 3**
  - Since if it had distance i < 3, it would have been in Li.



# What did we just learn?

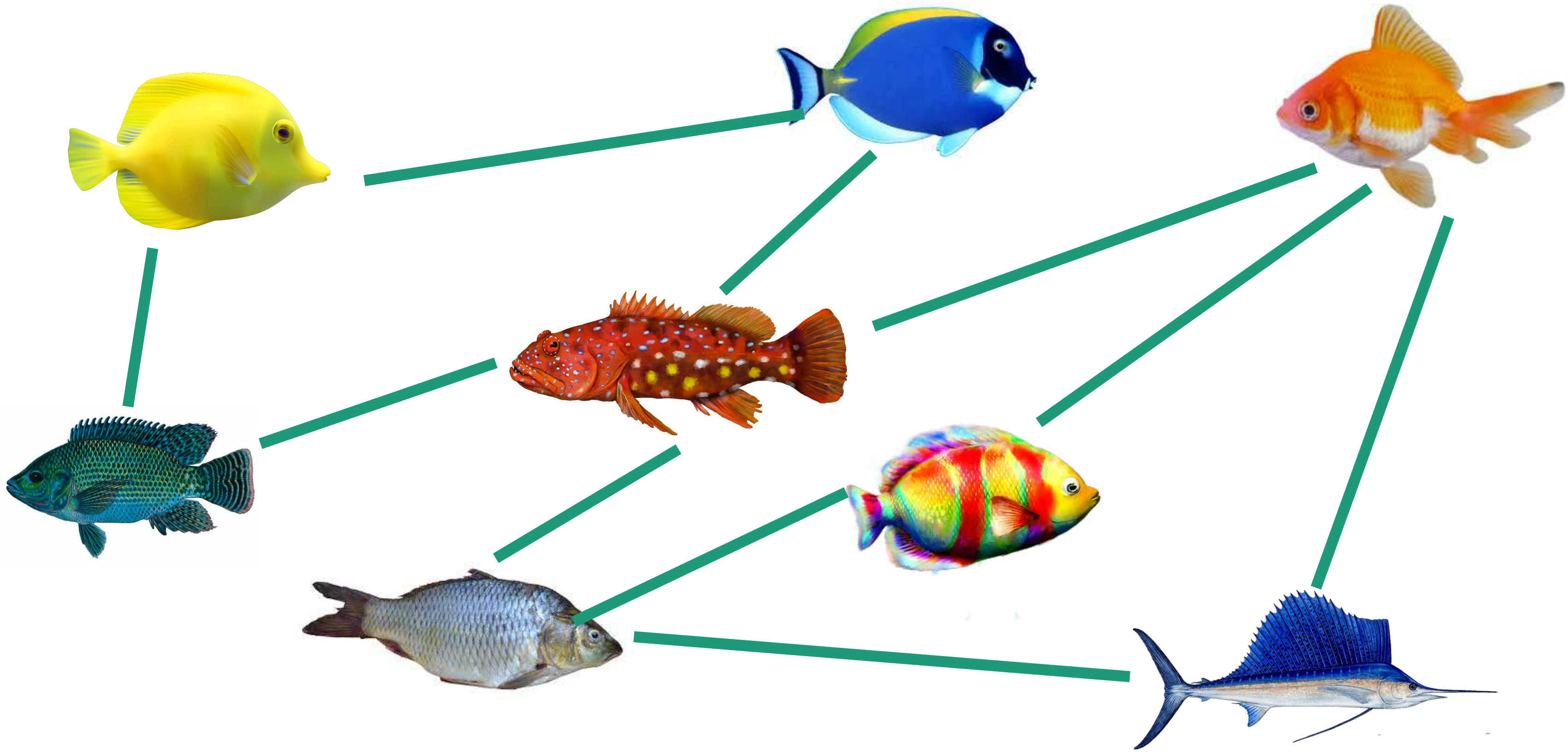
- The BFS tree is useful for computing distances between pairs of vertices.
- We can find the shortest path between u and v in time O(m).

# The BSF tree is also helpful for:

- Testing if a graph is bipartite or not.

# Pre-lecture exercise: fish

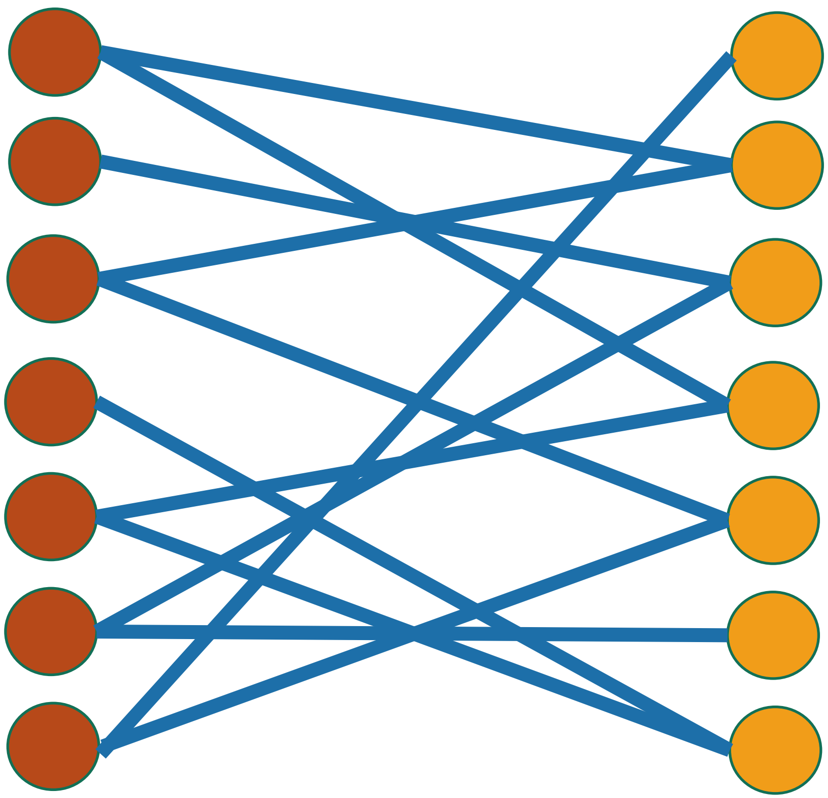
- Some pairs of species will fight if put in the same tank.
- You only have two tanks.
- Connected fish will fight.



# Application: testing if a graph is bipartite

- Bipartite means it looks like this:

Can color the vertices red and orange so that there are no edges between any same-colored vertices



**Example:**

are in tank A

are in tank B

if the fish fight

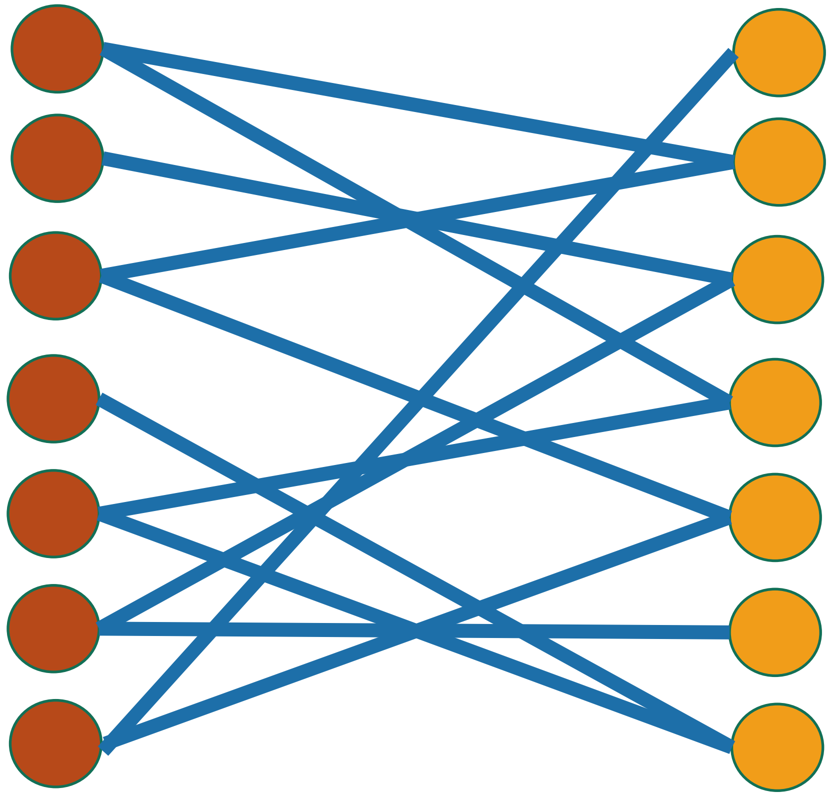
**Example:**

are students

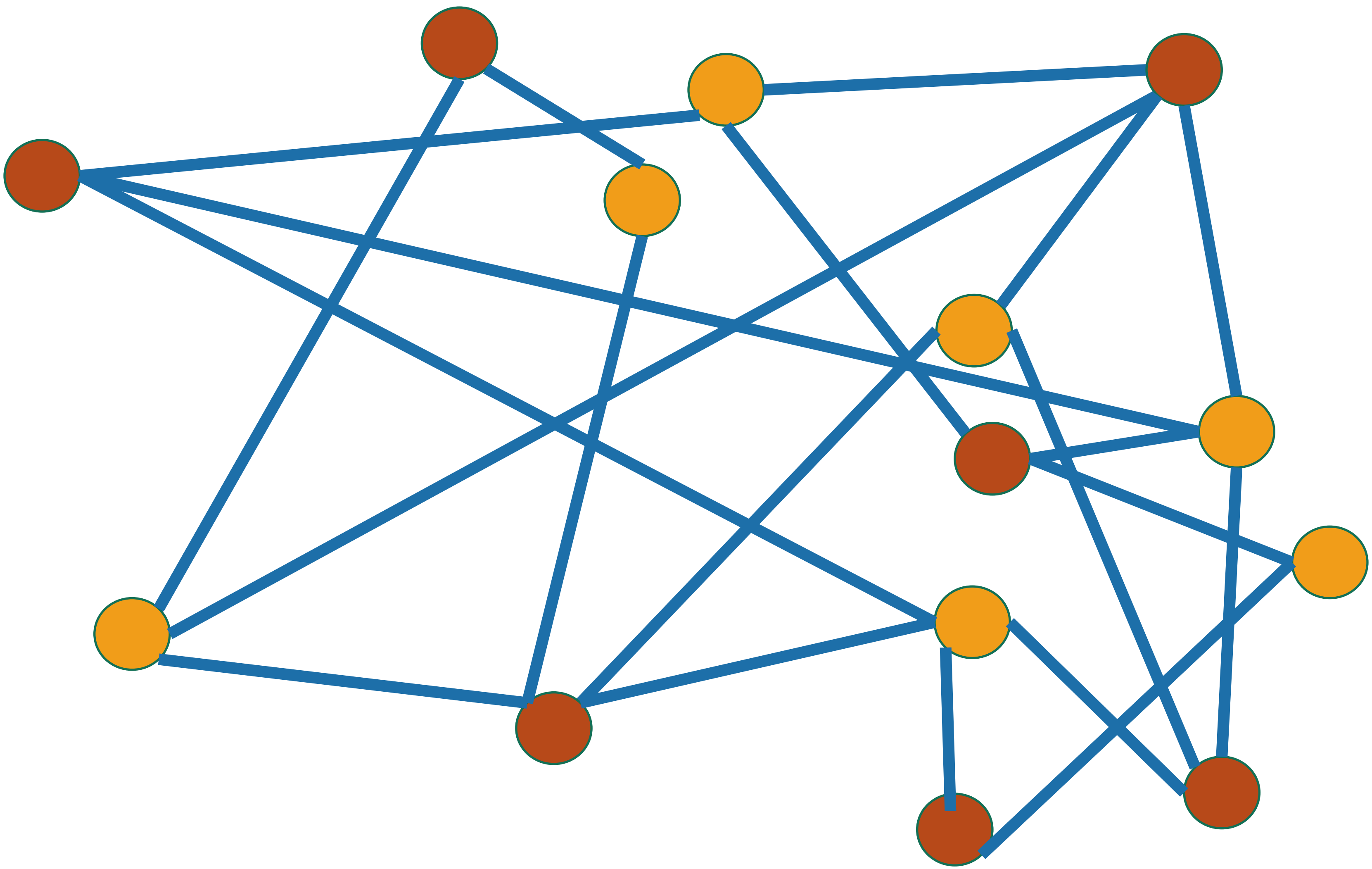
are classes

if the student is enrolled in the class

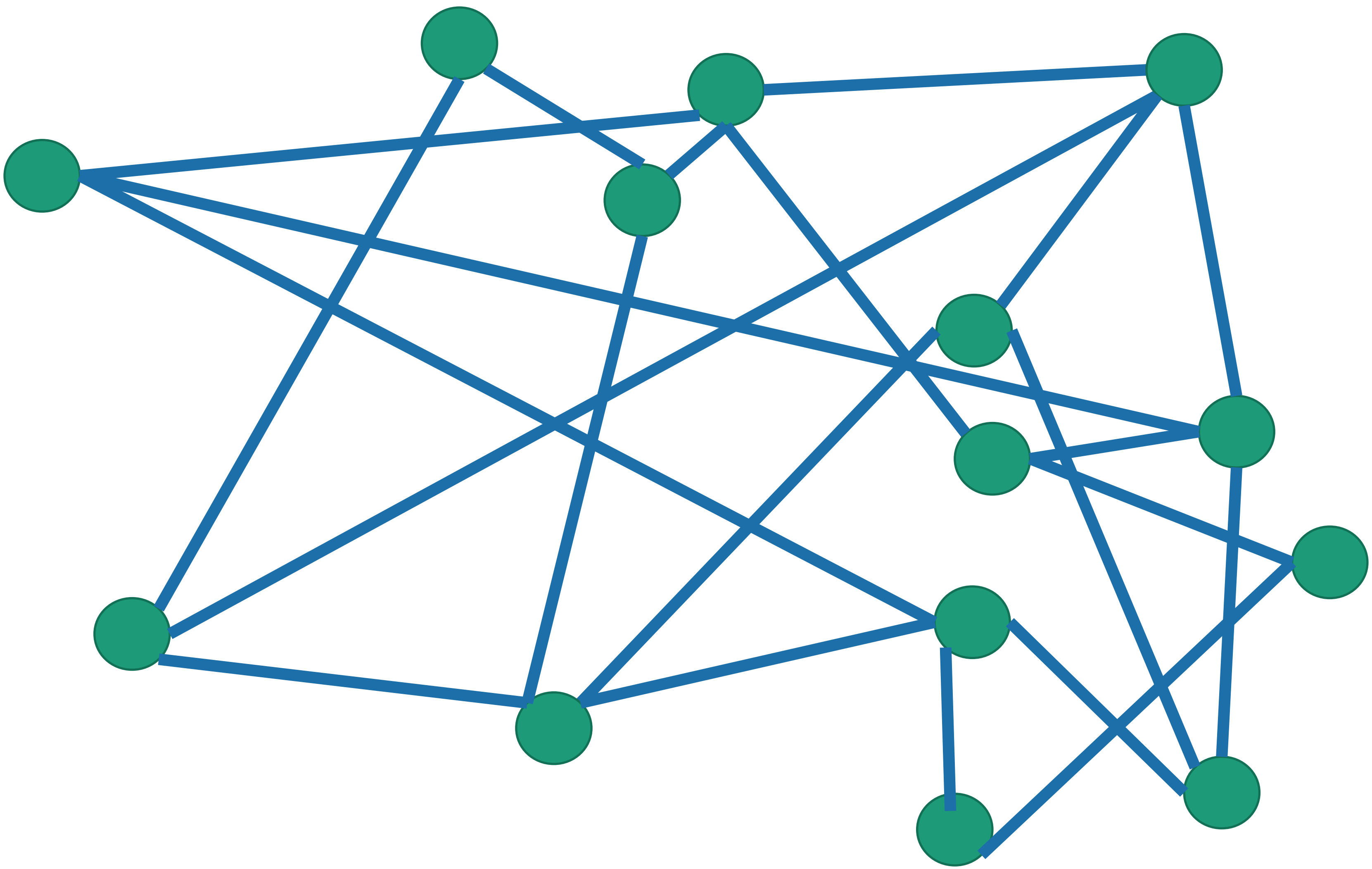
# Is this graph bipartite?



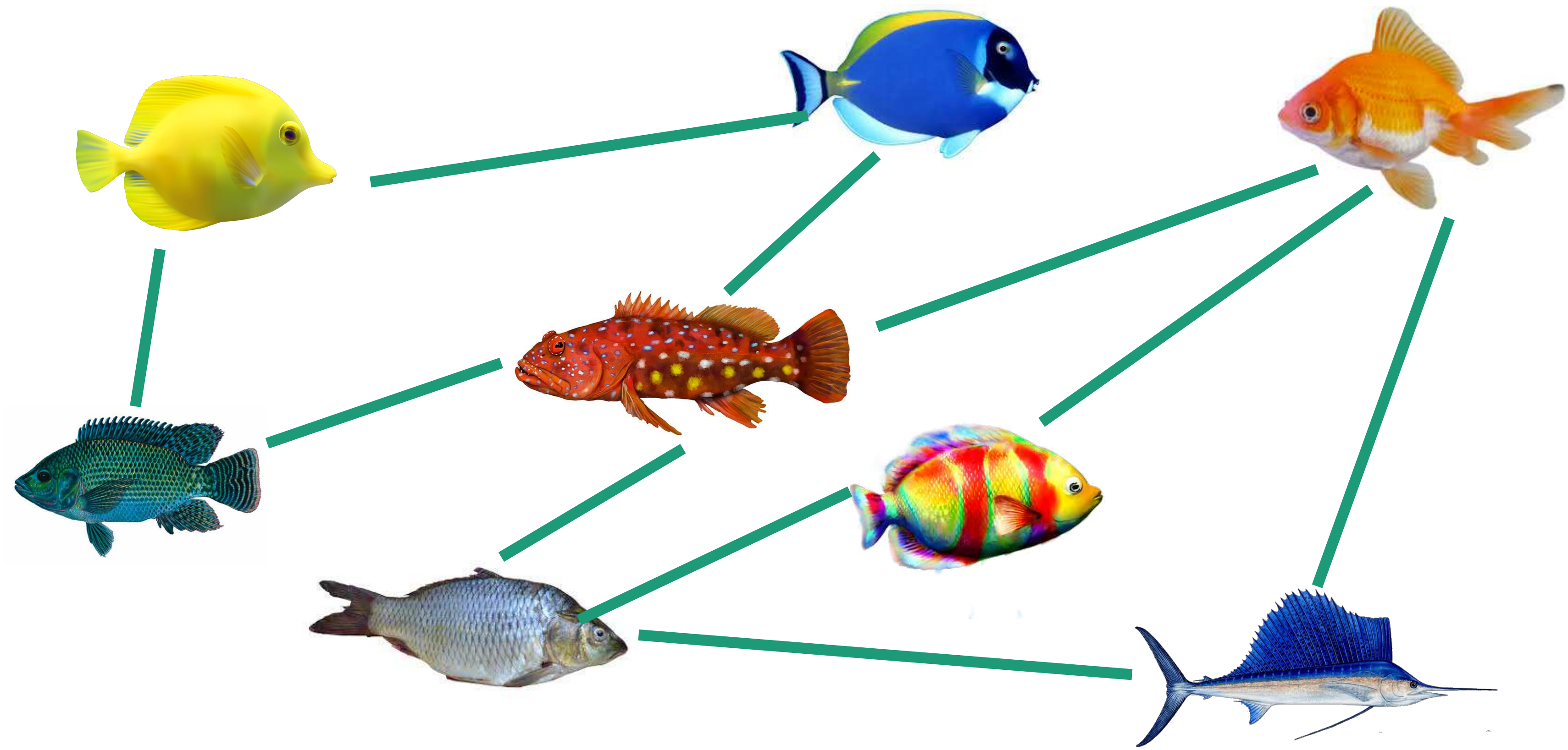
# How about this one?



# How about this one?

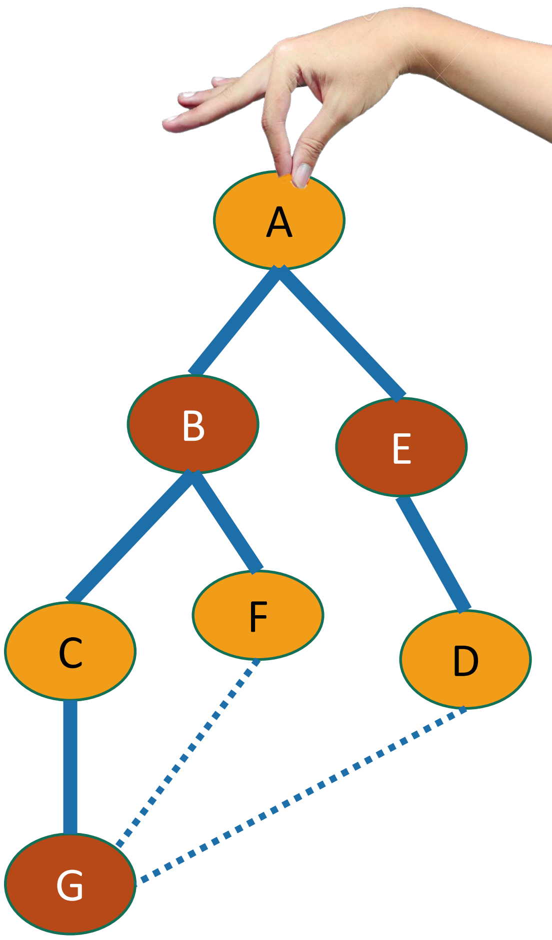


# This one?



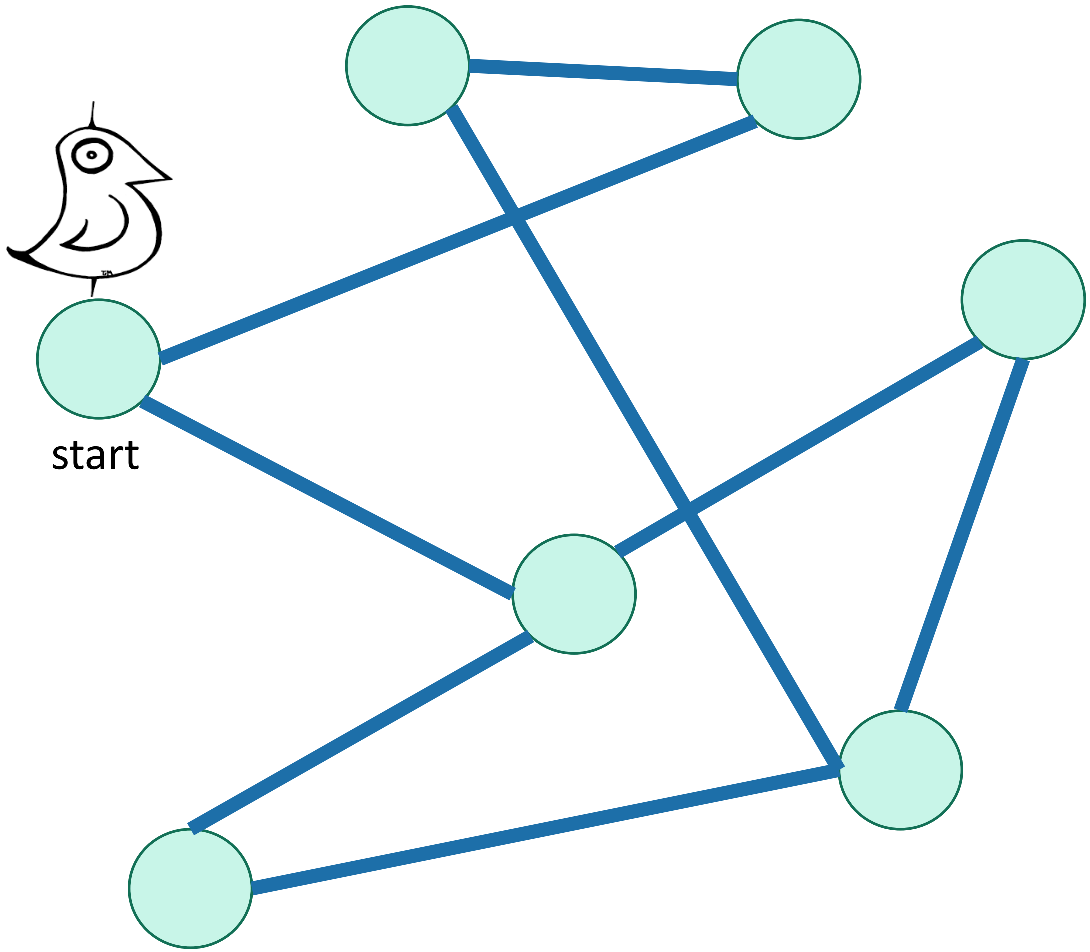
# Solution using BFS

- Color the levels of the BFS tree in alternating colors.
- If you never color two connected nodes the same color, then it is bipartite.
- Otherwise, it's not.



# Breadth-First Search

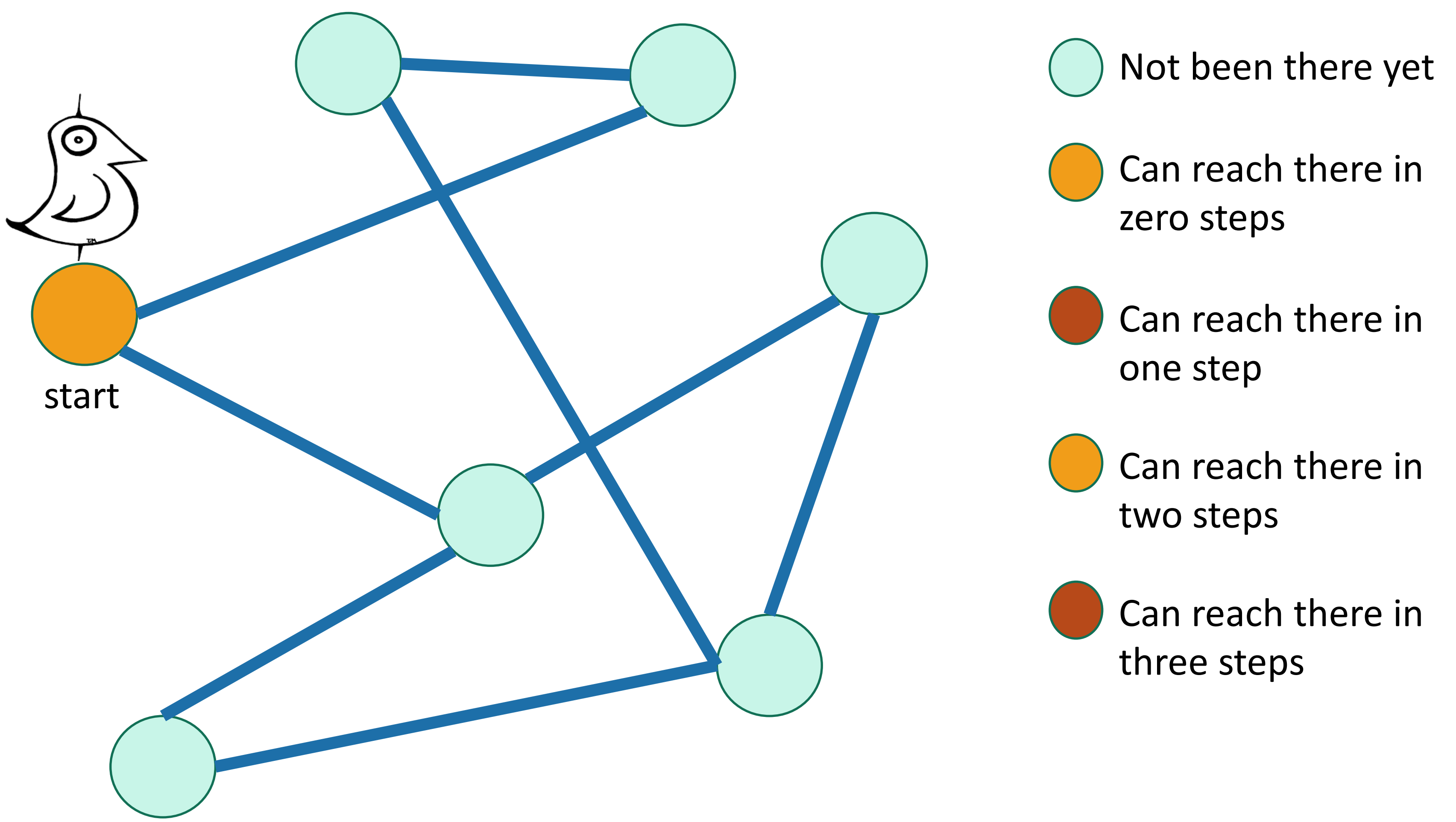
## For testing bipartite-ness




- Not been there yet
- Can reach there in zero steps
- Can reach there in one step
- Can reach there in two steps
- Can reach there in three steps

# Breadth-First Search

## For testing bipartite-ness



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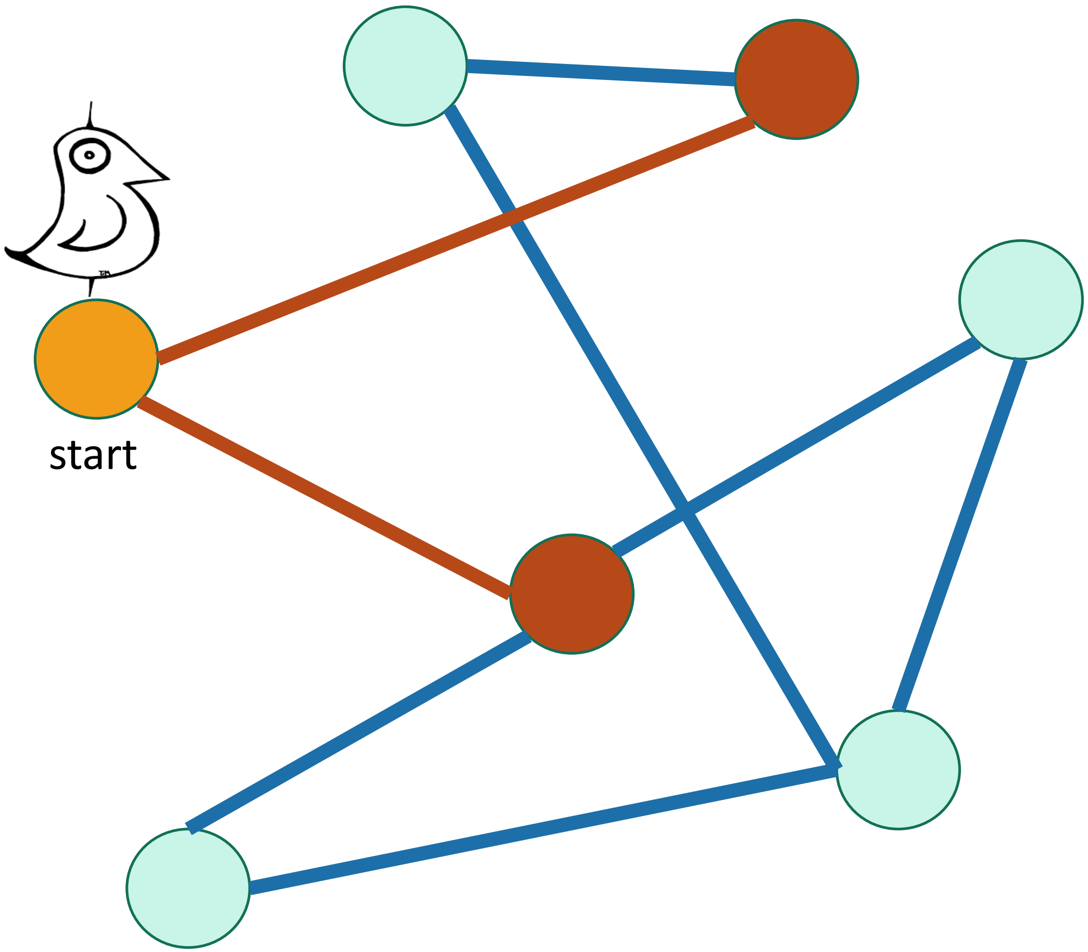


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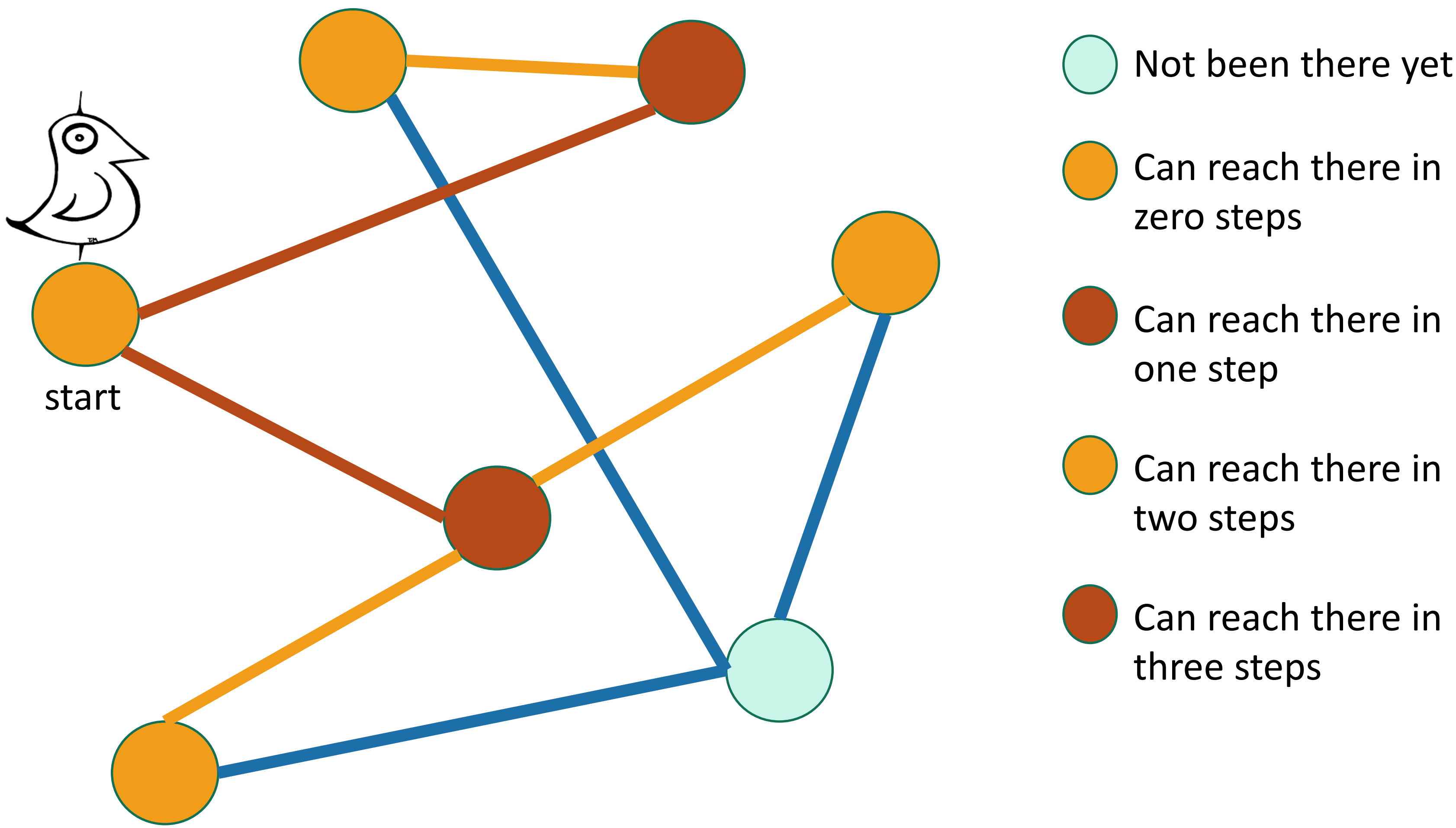

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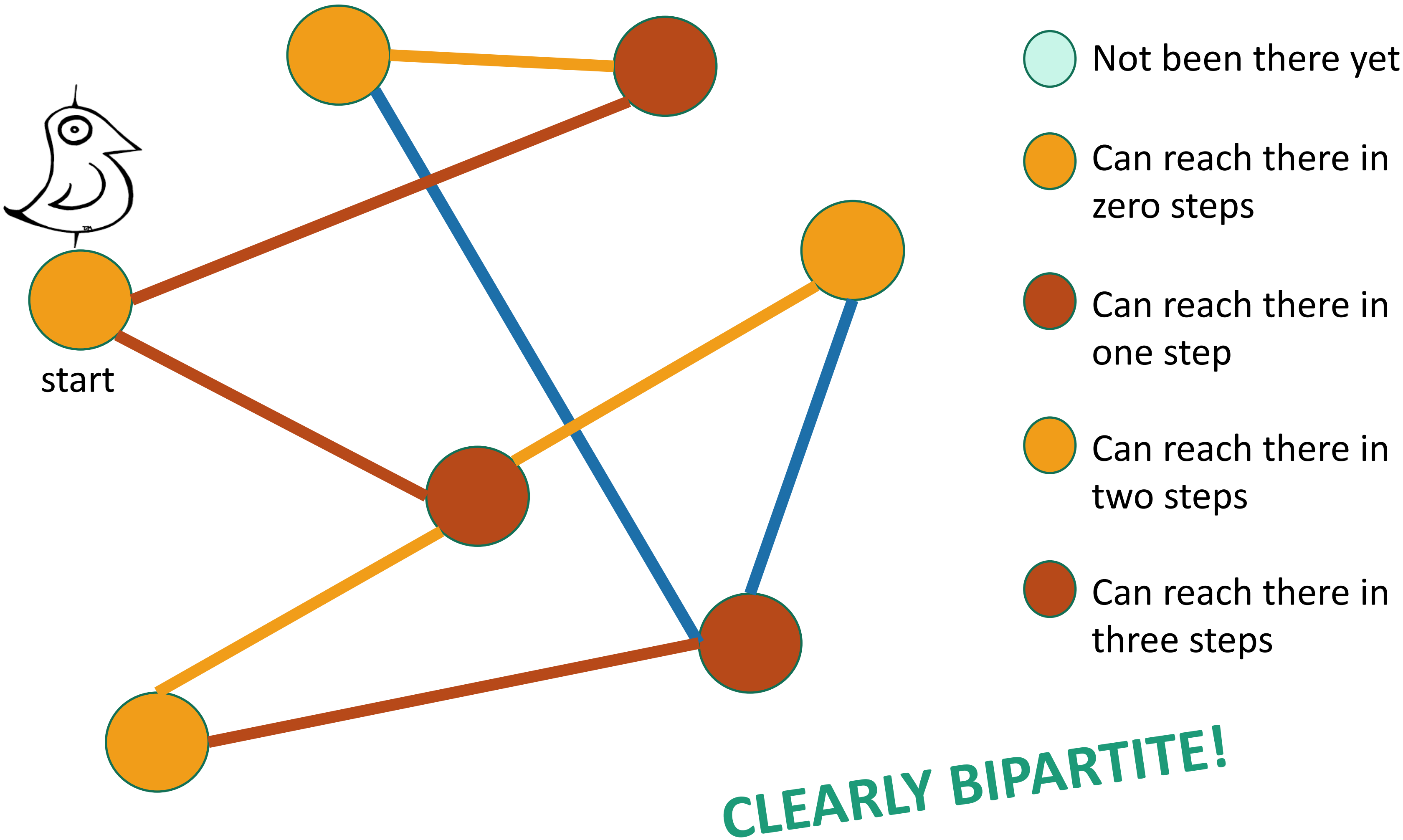

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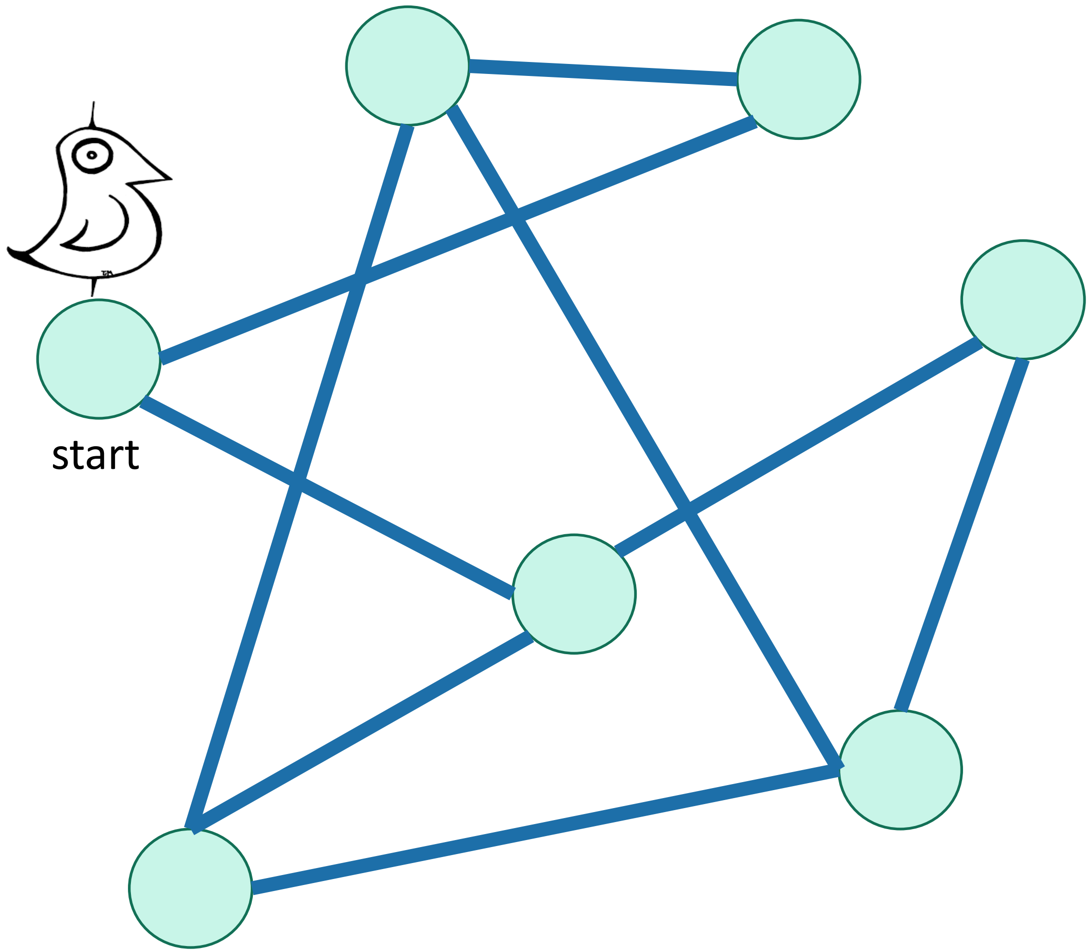
## For testing bipartite-ness




Not been there yet

Can reach there in zero steps

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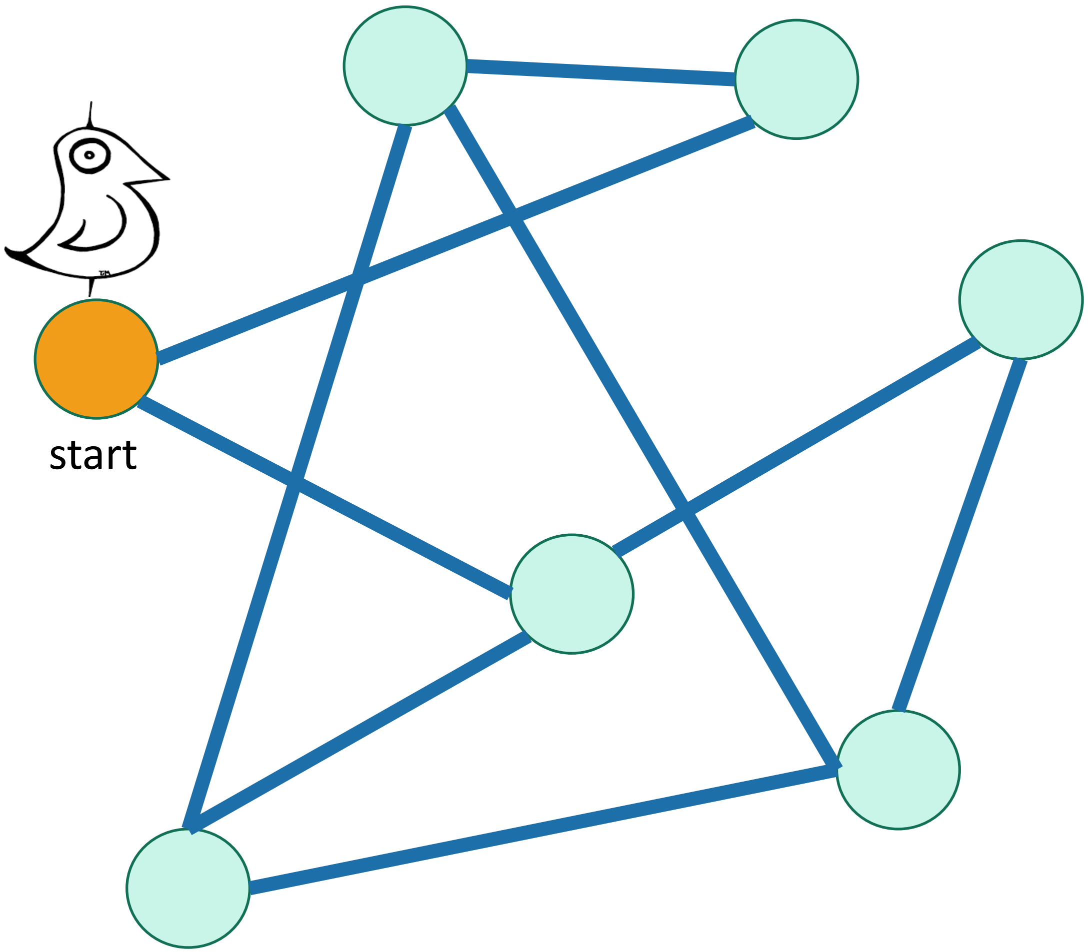
# Breadth-First Search

## For testing bipartite-ness

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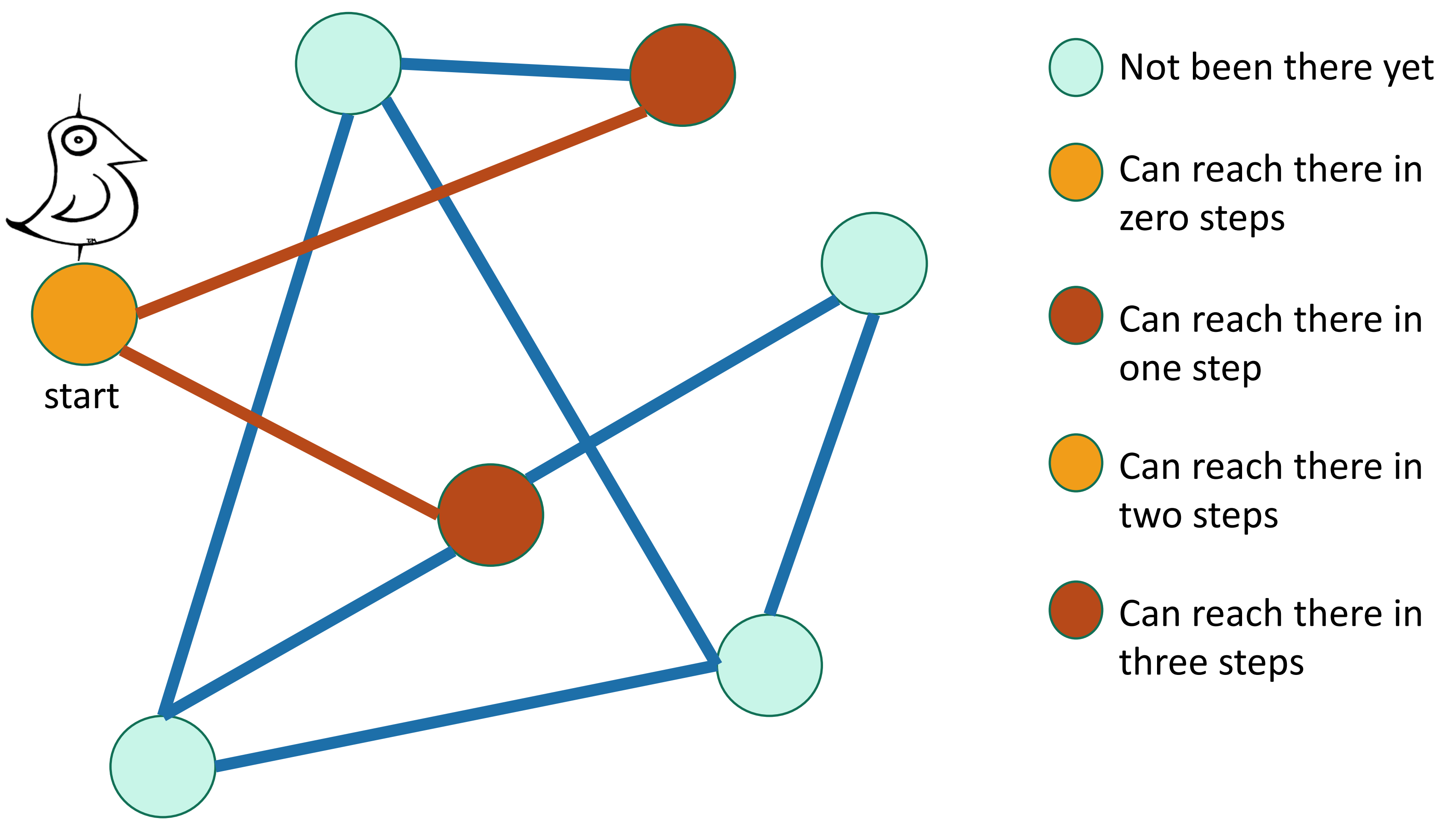
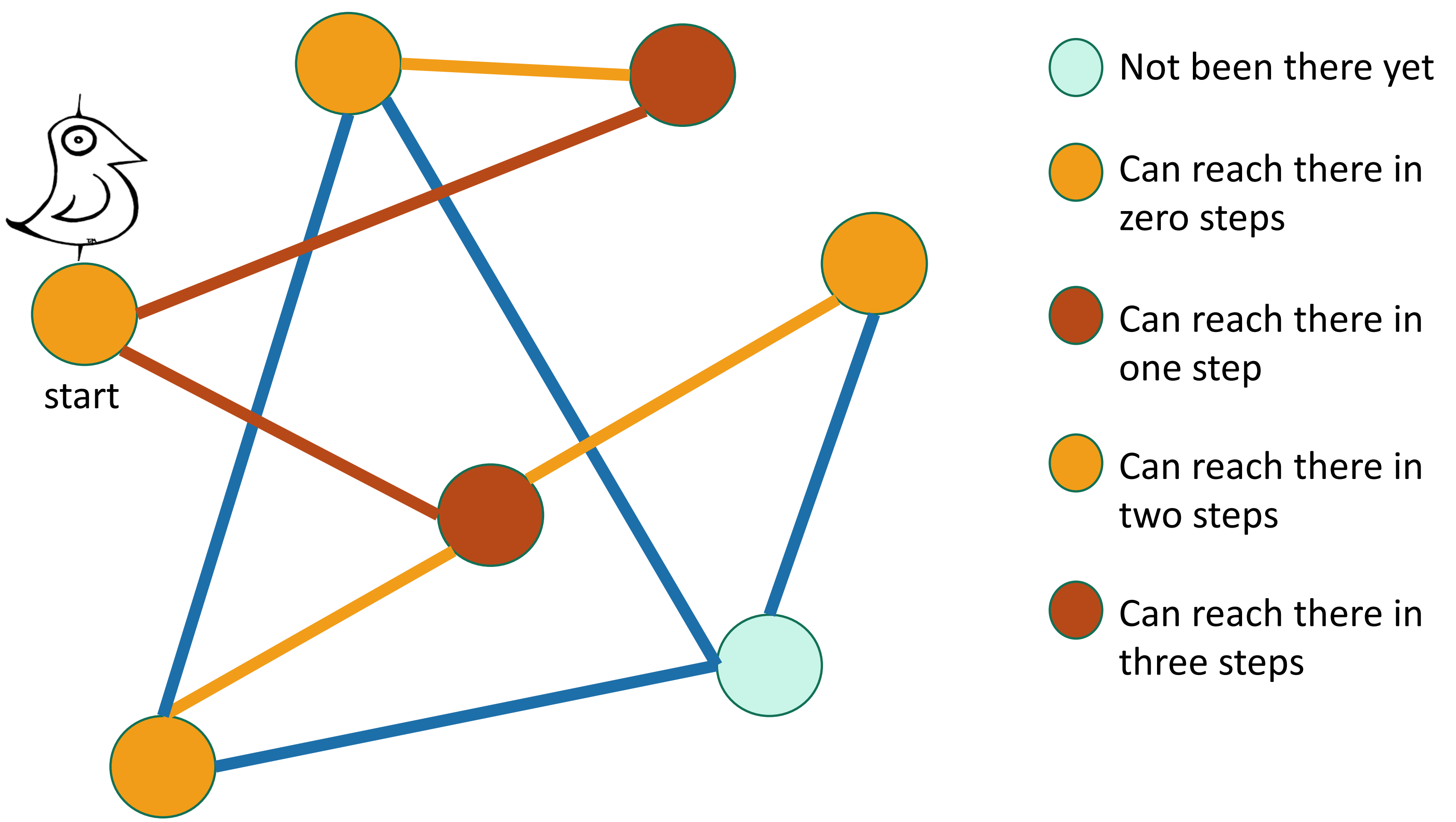
## For testing bipartite-ness



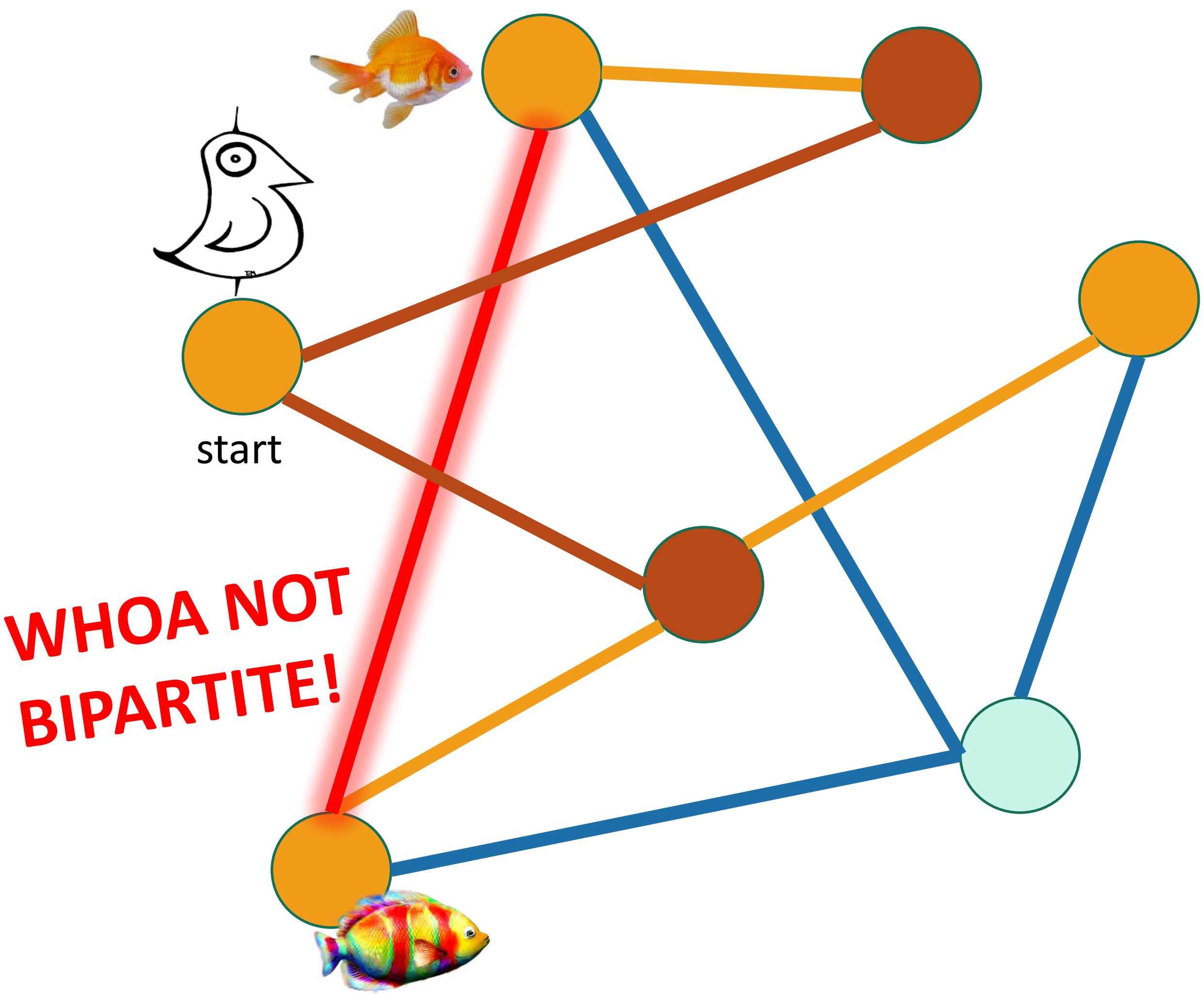
# Breadth-First Search

## For testing bipartite-ness



# Breadth-First Search

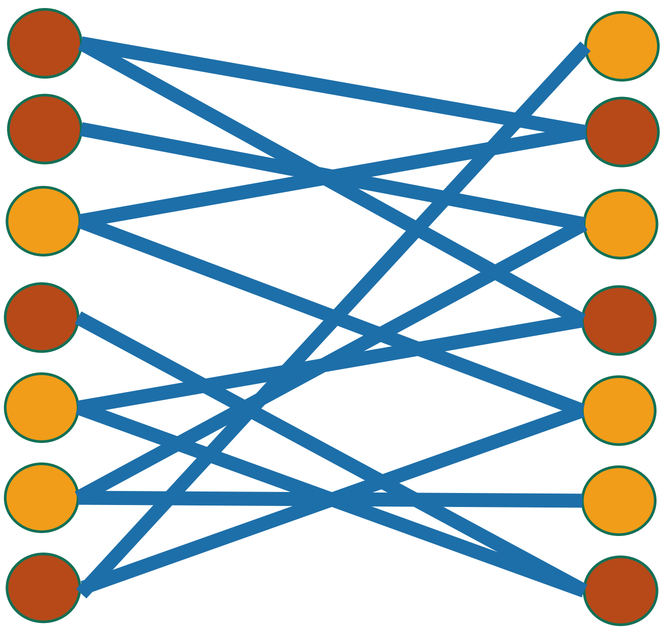
## For testing bipartite-ness




- Not been there yet
- Can reach there in zero steps
- Can reach there in one step
- Can reach there in two steps
- Can reach there in three steps

# Hang on now.

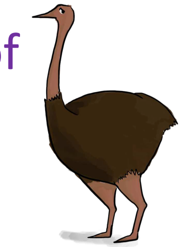
- Just because **this** coloring doesn't work, why does that mean that there is **no** coloring that works?



I can come up with plenty of bad colorings on this legitimately bipartite graph…

Plucky the pedantic penguin

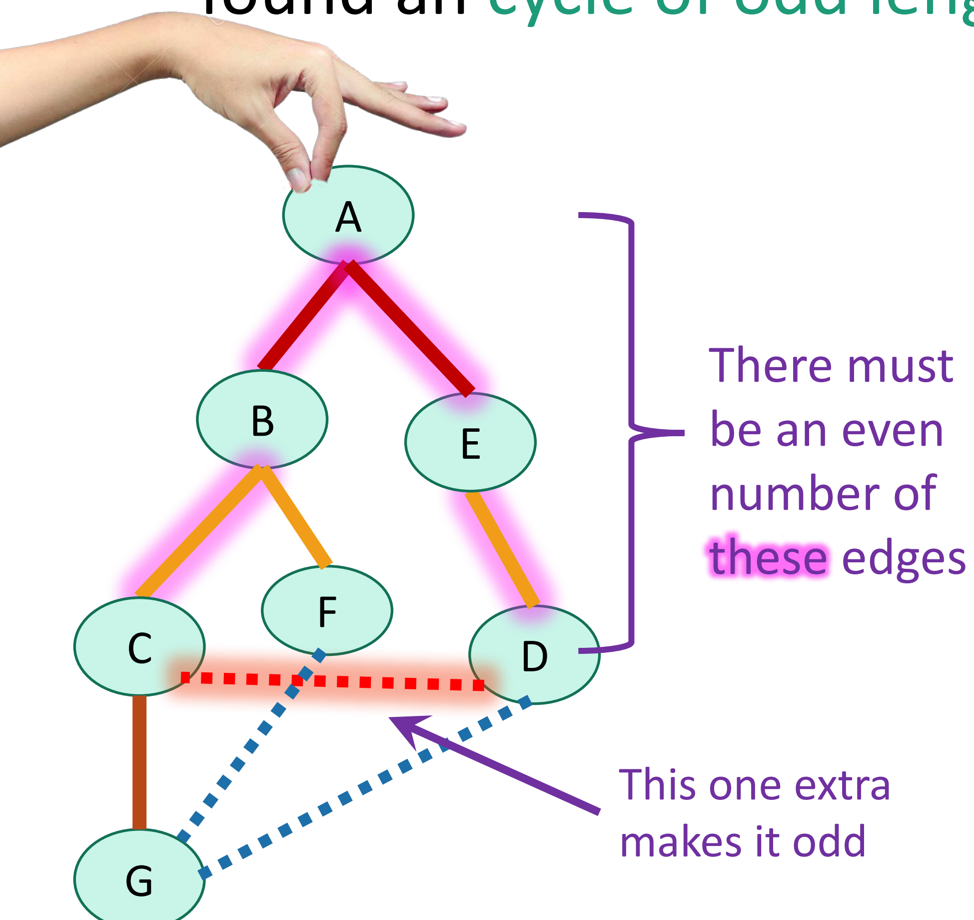
Make this proof sketch formal!

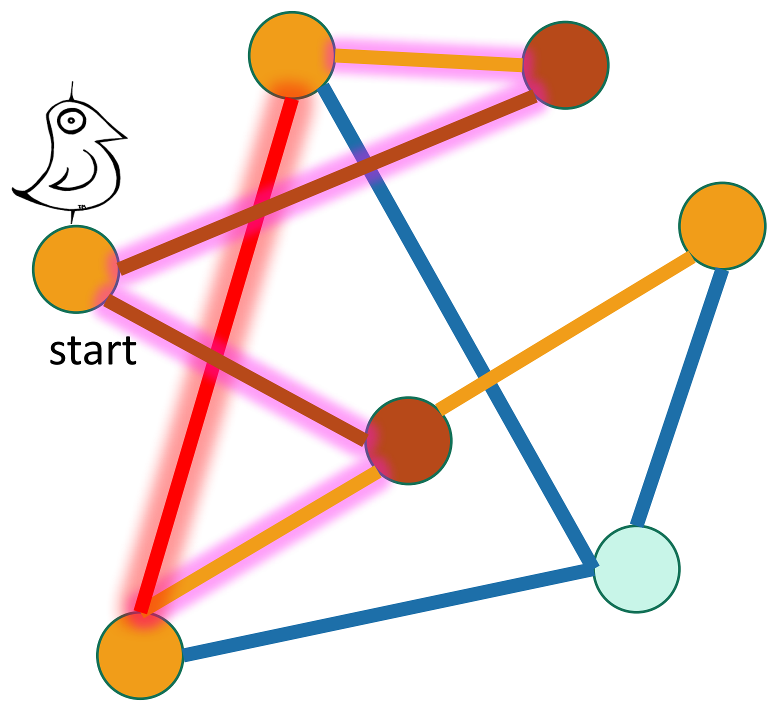


Ollie the over-achieving ostrich

# Some proof required

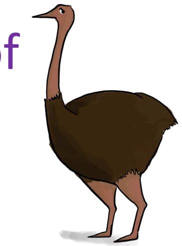
- If BFS colors two neighbors the same color, then it's found an cycle of odd length in the graph.



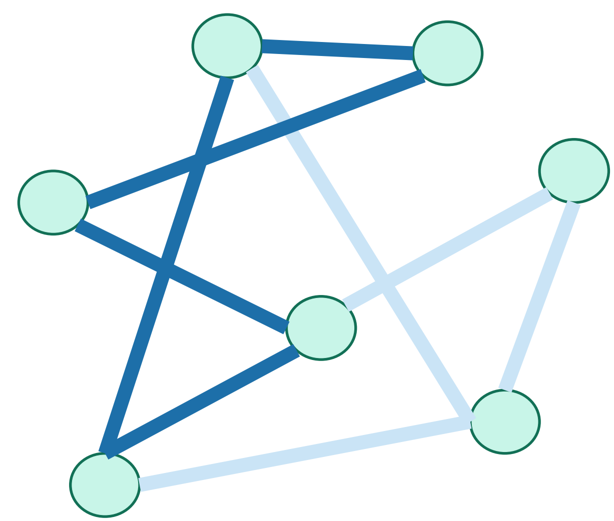
# Some proof required

Make this proof sketch formal!



Ollie the over-achieving ostrich

- If BFS colors two neighbors the same color, then it's found an cycle of odd length in the graph.
- So the graph has an odd cycle as a subgraph.
- But you can **never** color an odd cycle with two colors so that no two neighbors have the same color.
  - [Fun exercise!]
- So you can't legitimately color the whole graph either.
- **Thus it's not bipartite.**



# What did we just learn?

BFS can be used to detect bipartite-ness in time $O(n + m)$.

# Outline

- Part 0: Graphs and terminology
- Part 1: Depth-first search
  - Application: topological sorting
  - Application: in-order traversal of BSTs
- Part 2: Breadth-first search
  - Application: shortest paths
  - Application (if time): is a graph bipartite?

Recap

# Recap

- Depth-first search
  - Useful for topological sorting
  - Also in-order traversals of BSTs
- Breadth-first search
  - Useful for finding shortest paths
  - Also for testing bipartiteness
- Both DFS, BFS:
  - Useful for exploring graphs, finding connected components, etc

# Still open (next few classes)

- We can now find components in undirected graphs…
  - What if we want to find strongly connected components in directed graphs?
- How can we find shortest paths in weighted graphs?
- What is Samuel L. Jackson's Erdos number?
  - (Or, what if I want everyone's everyone-else number?)

# Next Time

- Strongly Connected Components

# Before Next Time

- Pre-lecture exercise: Strongly Connected What-Now?