Suffix Arrays

Recap from Last Time

Suffix Trees

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Suffix Trees

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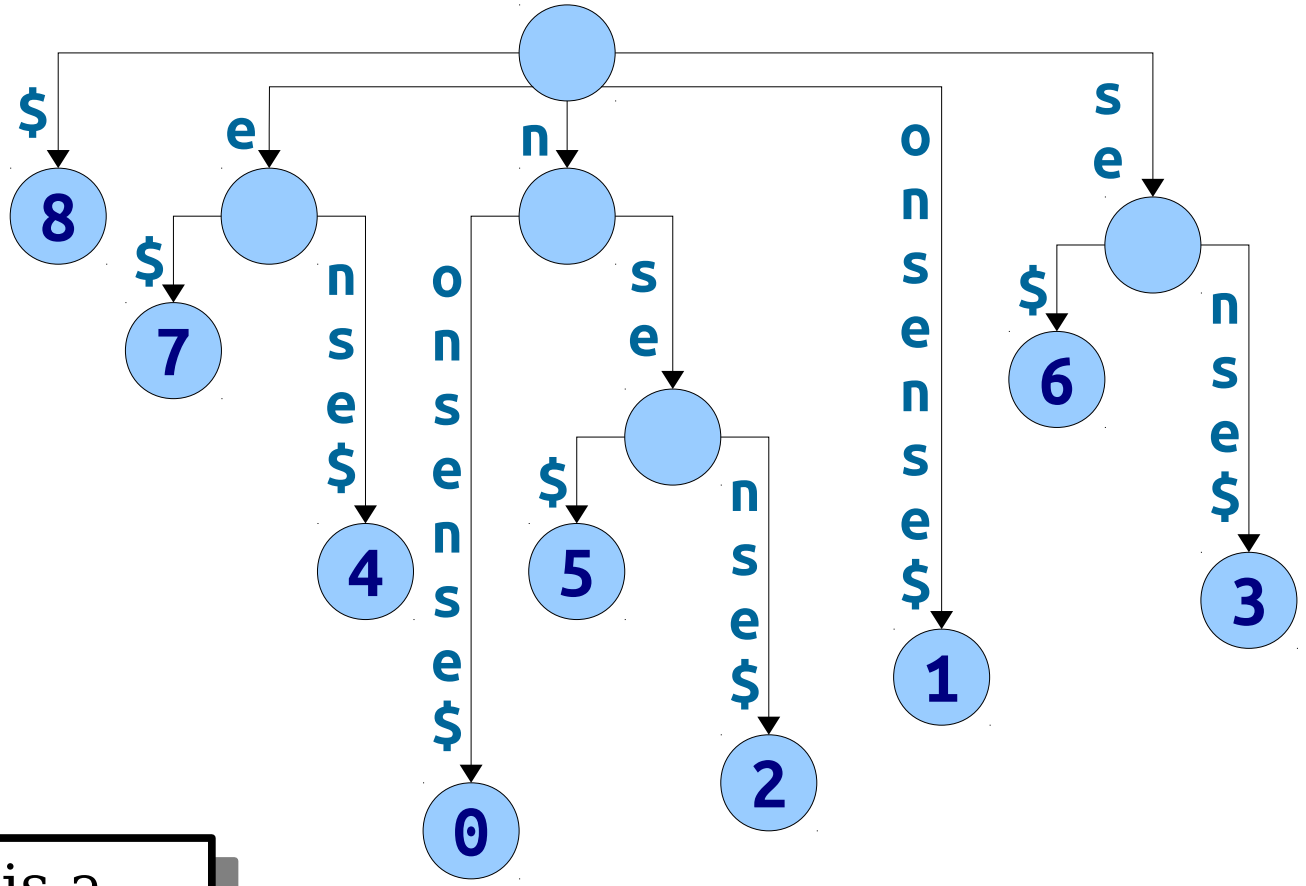

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Theorem: *w* is a substring of *x* if and only if *w* is a prefix of a suffix of *x*.

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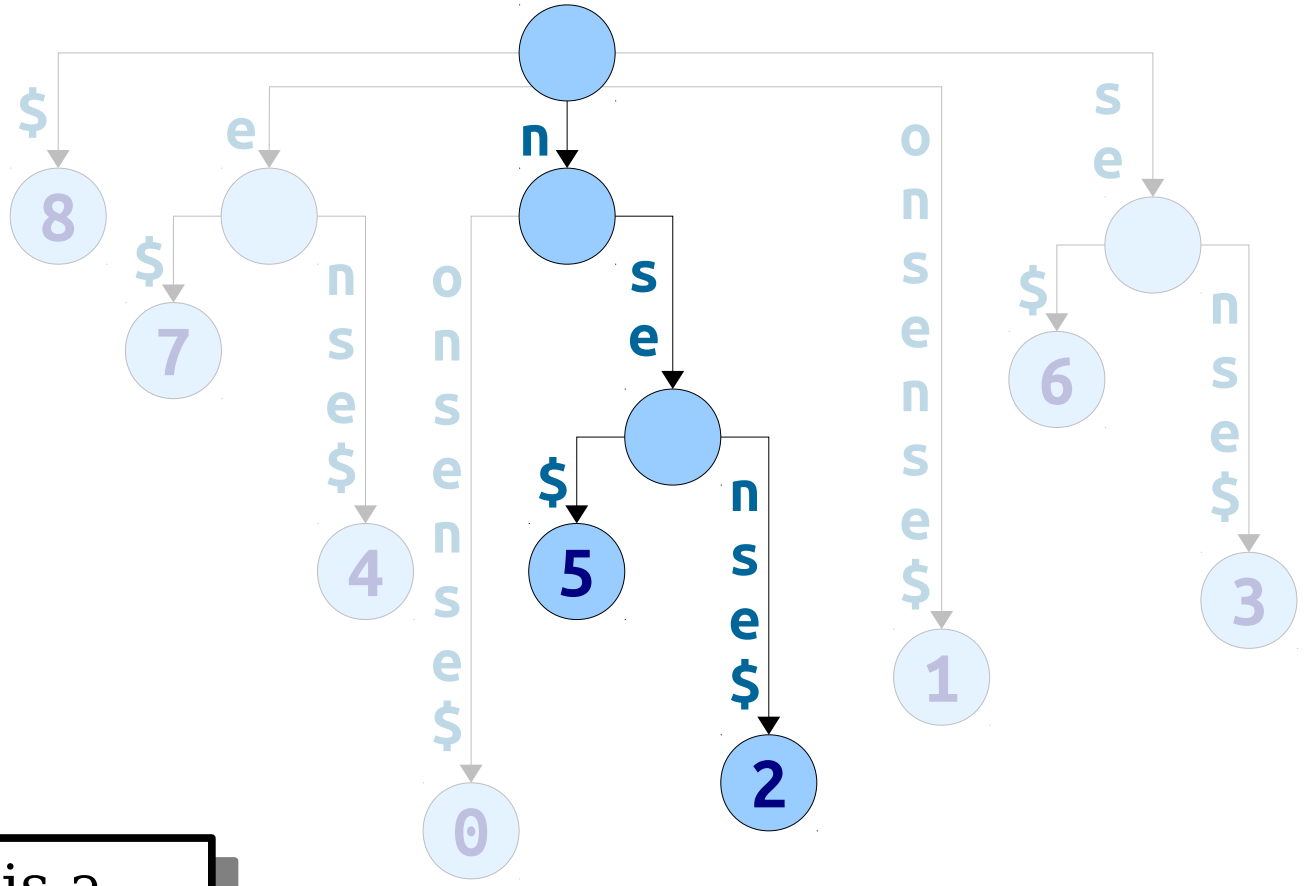

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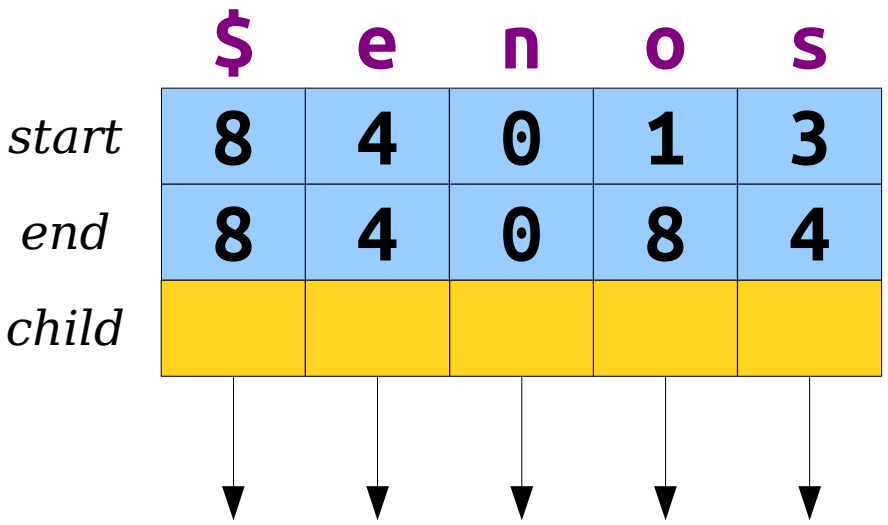
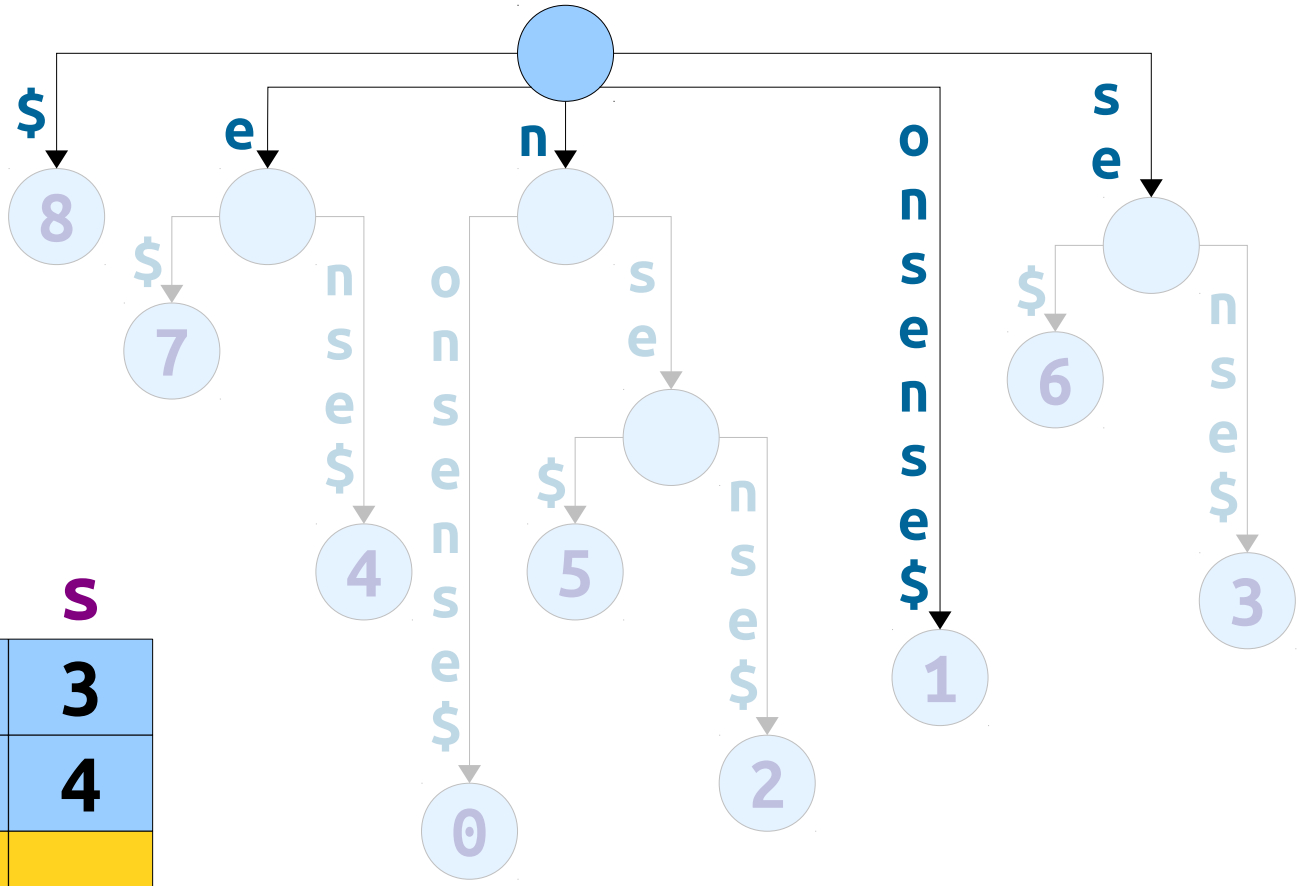
Representing a Suffix Tree





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New Stuff!

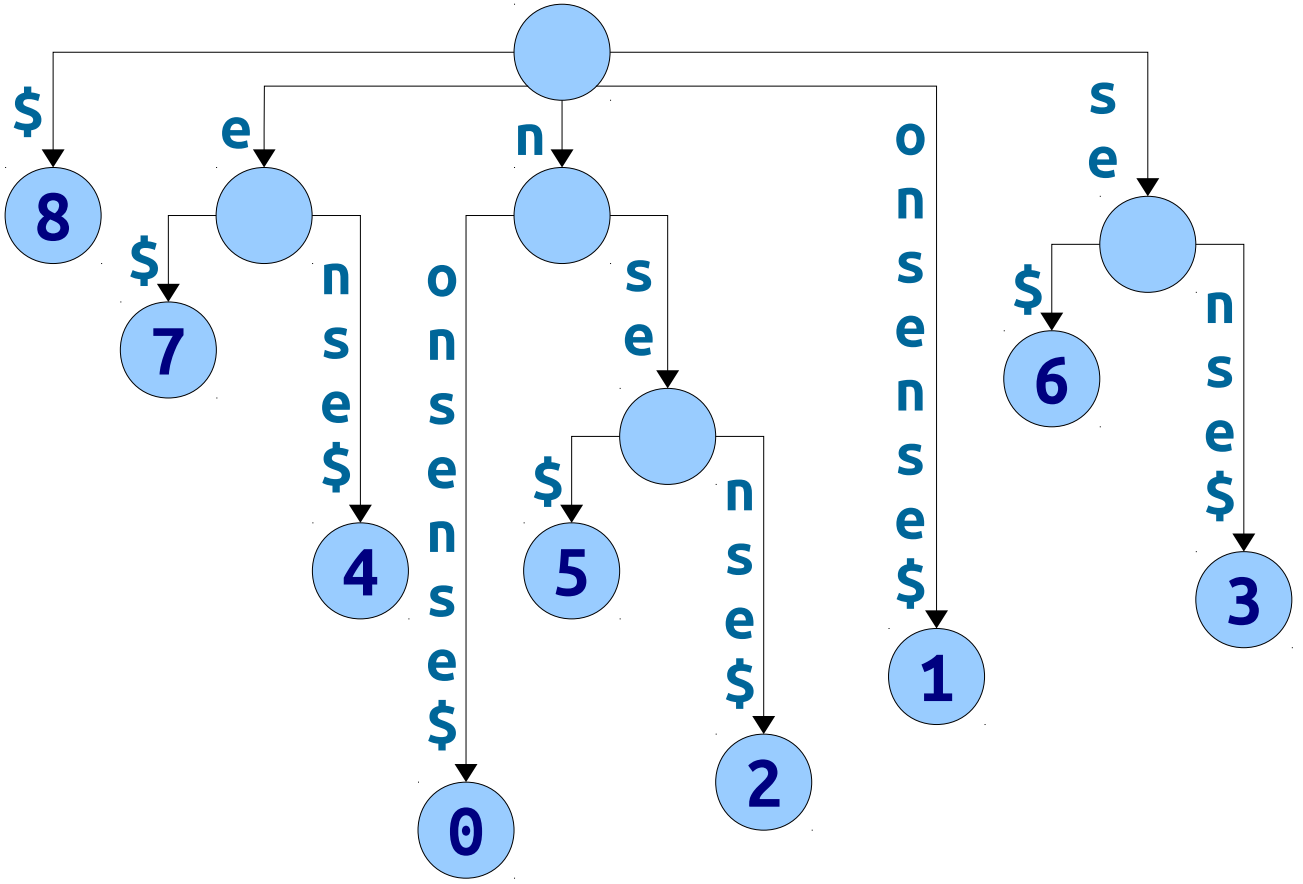
Suffix Trees: The Catch

Suffix Tree Space Usage

- Suffix tree edges take up a *lot* of space.
  - Two machine words per edge to denote the range of characters visited.
  - One machine word per edge for the pointer itself.
  - Number of edges ranges from n to $2n - 1$, so this is between $3n$ and $6n$ machine words *per character*.
- Example: a human genome is about three billion characters long.
  - With clever techniques, that can be packed into about 800MB.
  - On a 32-bit machine, the suffix tree needs about 48GB – too big to fit into memory!
  - On a 64-bit machine, the suffix tree needs about 96GB – way more than a typical machine can hold!

Key Question: Can we get the benefits of a suffix tree without the space penalty?

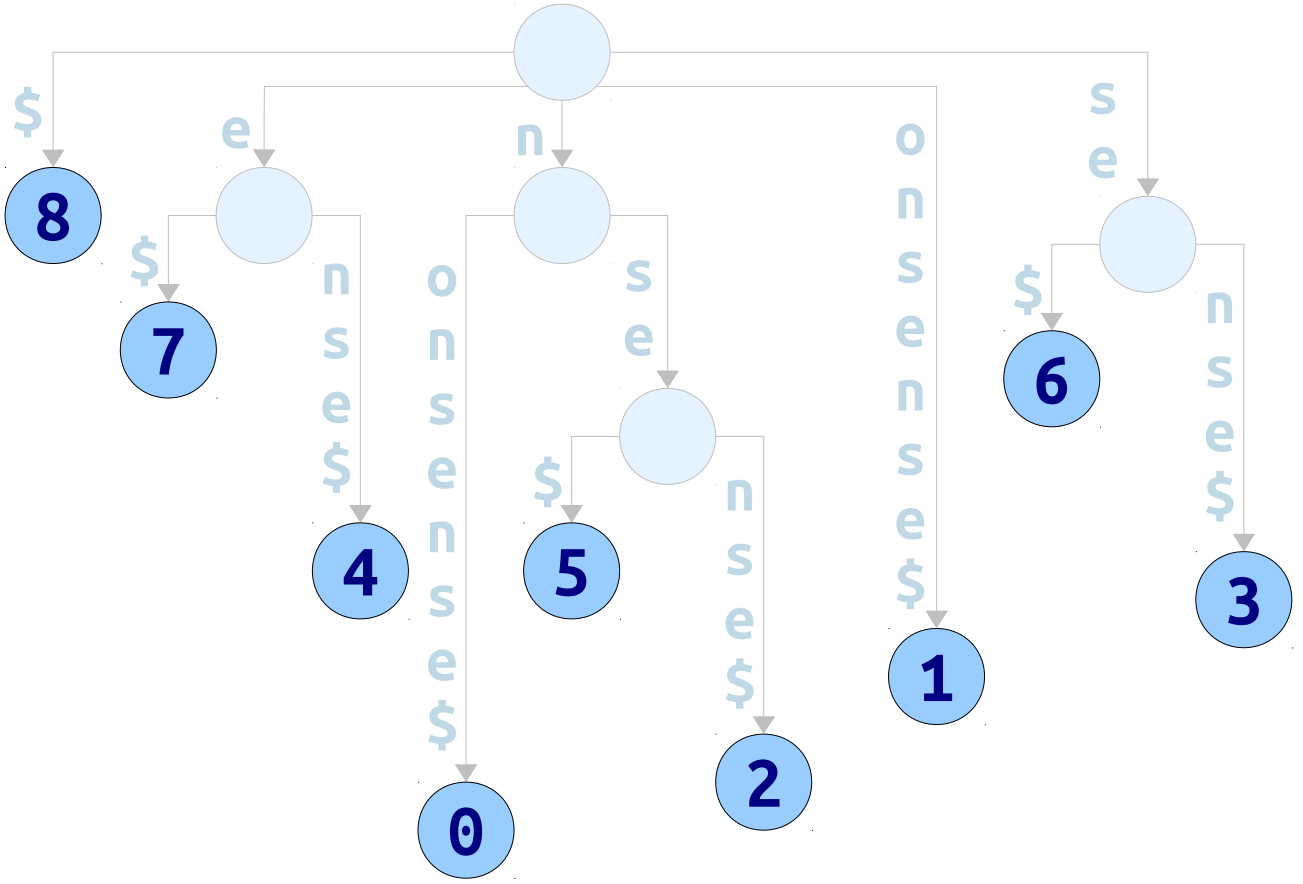
What is it about suffix trees that make them so useful algorithmically?



Theorem: There is a node labeled ω in a suffix tree for T

if and only if

ω is a suffix of T\$ or ω is a branching word in T.



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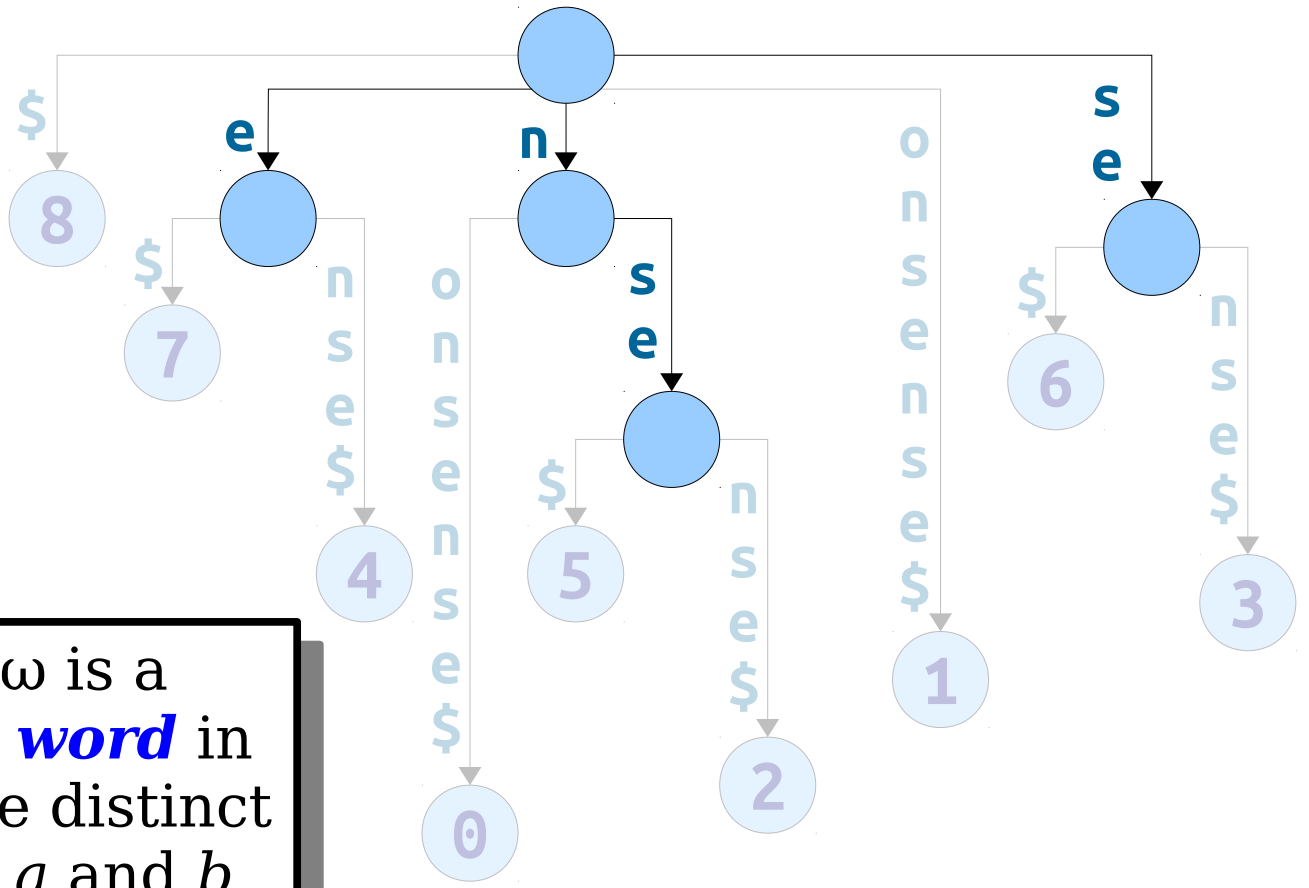
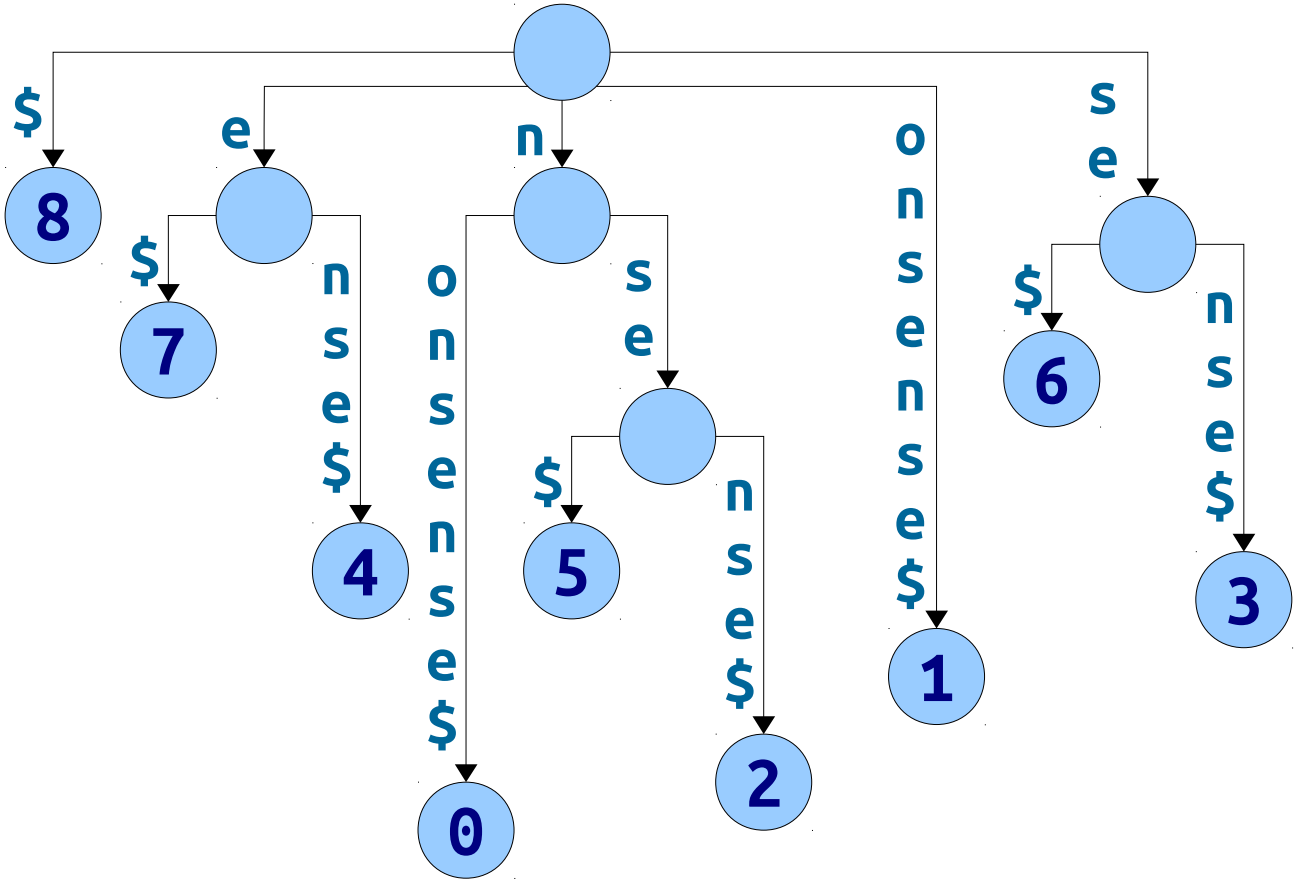


A string ω is a ***branching word*** in T if there are distinct characters a and b where ωa and ωb are substrings of T\$.

nonsense$

Theorem: There is a node labeled ω in a suffix tree for T *if and only if* ω is a suffix of T\$ or ω is a branching word in T.



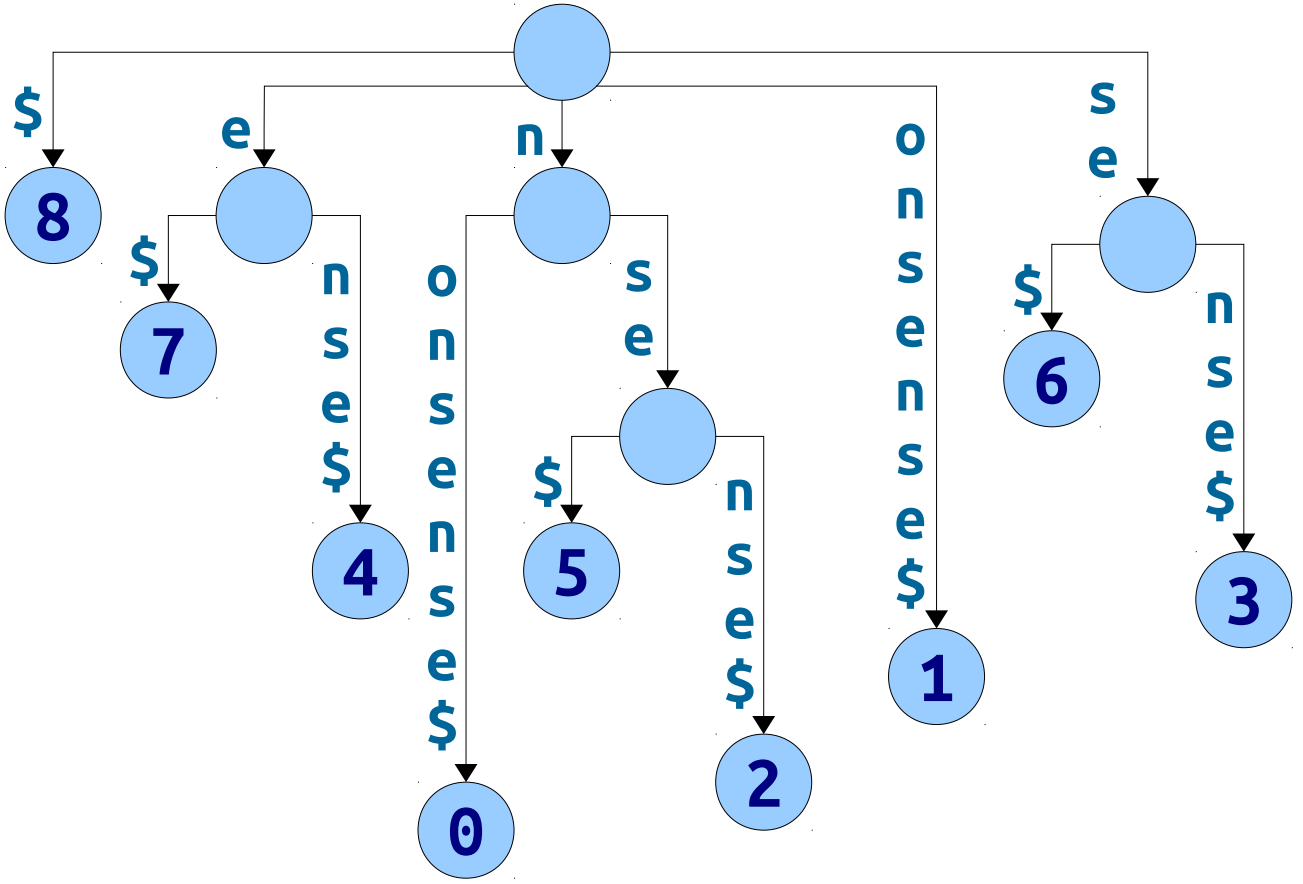
Key Intuition: The efficiency in a suffix tree is largely due to

1. keeping the suffixes in sorted order, and
2. exposing branching words.

Where We're Going

- Today, we'll see two data structures that encode much of the same information as suffix trees, but in much less space.
  - The ***suffix array*** stores information about the ordering of the suffixes of a string.
  - The ***LCP array*** stores information about the branching words of a string.
- Together, they'll provide algorithms that match or are comparable to the time bounds from last time.

Suffix Arrays



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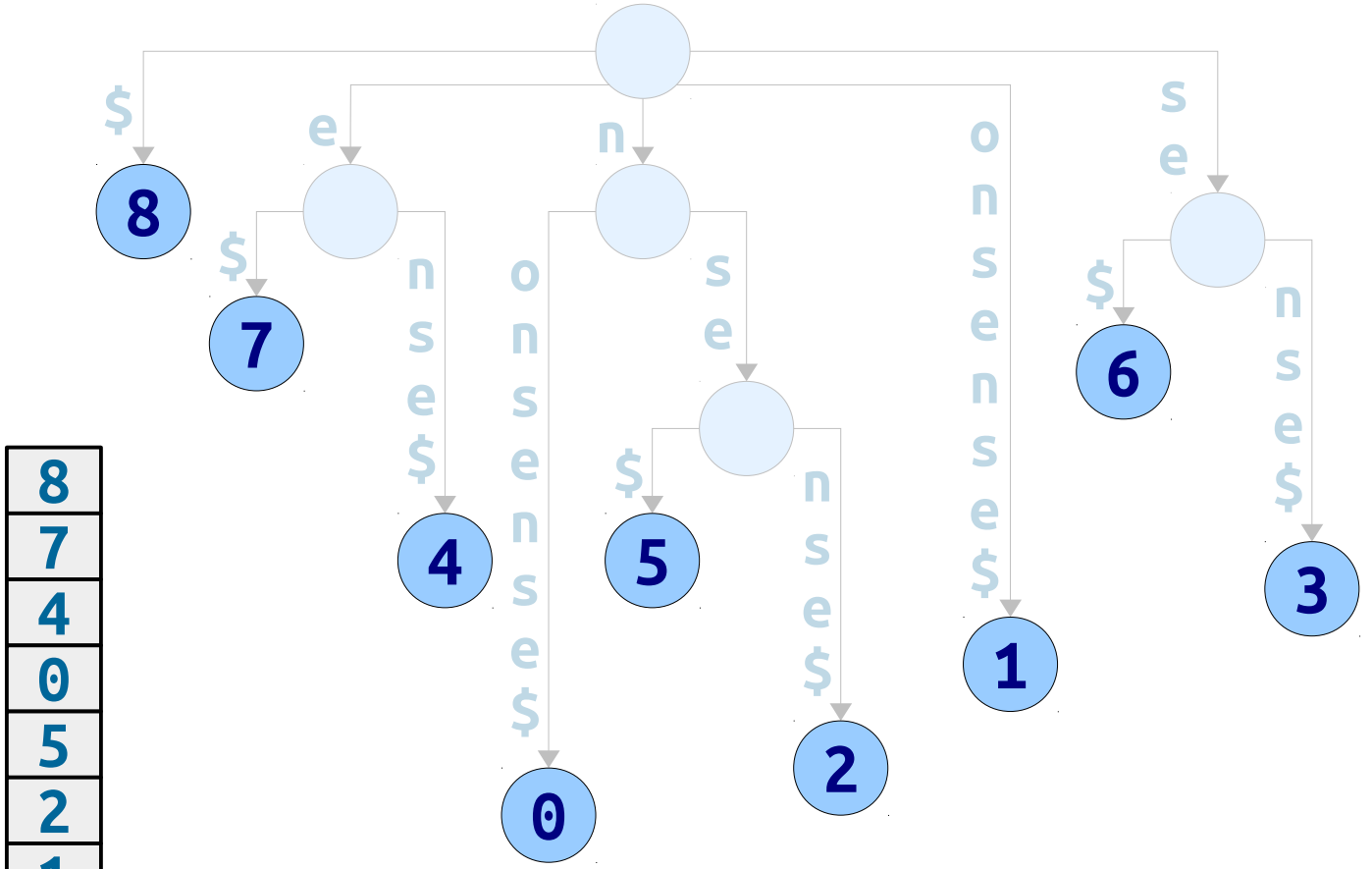

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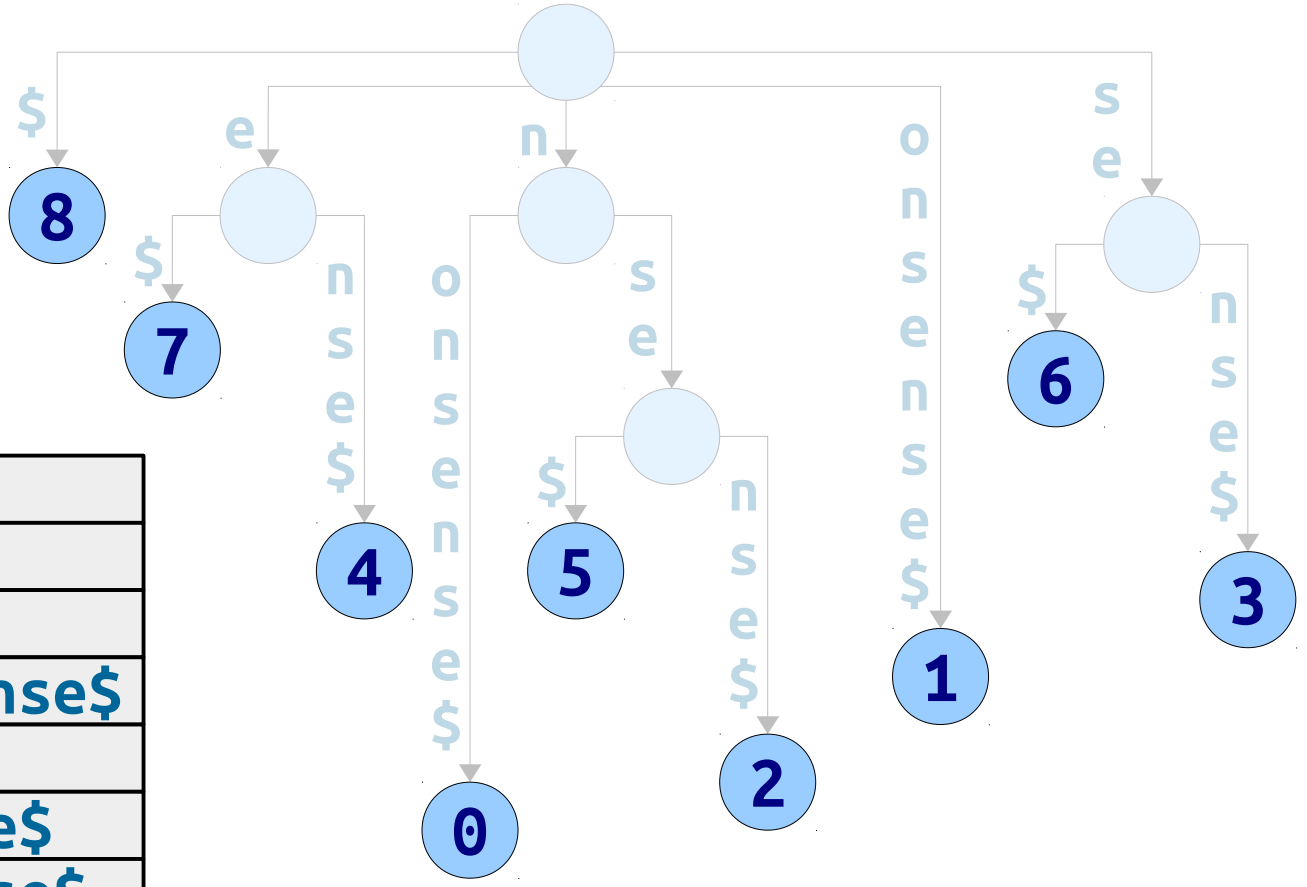

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Suffix Arrays

- A ***suffix array*** for a string T is a sorted array of the suffixes of the string T\$.
- Suffix arrays distill out just the first component of suffix trees: they store suffixes in sorted order.

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- A ***suffix array*** for a string T is a sorted array of the suffixes of the string T\$.
- Suffix arrays distill out just the first component of suffix trees: they store suffixes in sorted order.
- ***Non-obvious fact:*** Suffix arrays can be built in time O(***m***). Details next time!

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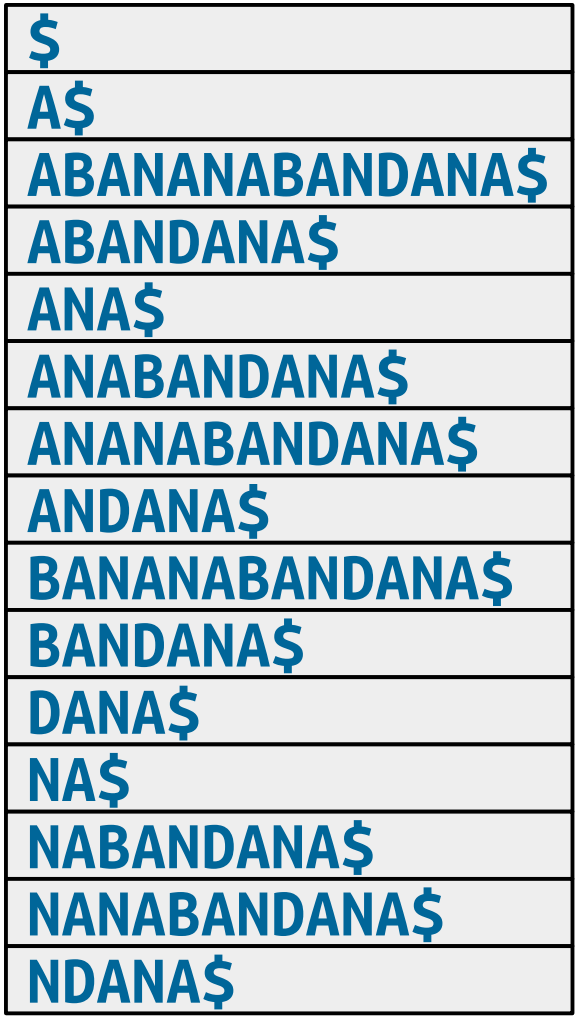
Suffix Arrays

- Last time, we saw how to find all instances of a pattern ***P*** in a text ***T*** using suffix *trees*.
- How could we do that with suffix *arrays*?
- ***Question:*** What's a good algorithm for finding an element of a sorted array?



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Suffix Arrays

- ***Reminder:*** Our text string ***T*** has length ***m***. Our pattern string ***P*** has length ***n***.
- ***Claim:*** With a suffix array, we can determine whether ***P*** matches in ***T*** in time O(***n*** log ***m***).

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  - Binary search has O(log ***m***) rounds.
  - Each probe takes time O(***n***).
- This bound can be made tight. *(How?)*
- Figure that ***m*** is often much bigger than ***n***, so this is a huge win over a raw scan.

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Suffix Arrays

- ***Claim:*** With a suffix array, we can find all matches of a pattern ***P*** in ***T*** in time O(***n*** log ***m*** + ***z***), where ***z*** is the number of matches.
- ***Idea:*** Binary search can be used to find a range of values equal to some key. Adapt that idea to find all suffixes beginning with the same prefix.

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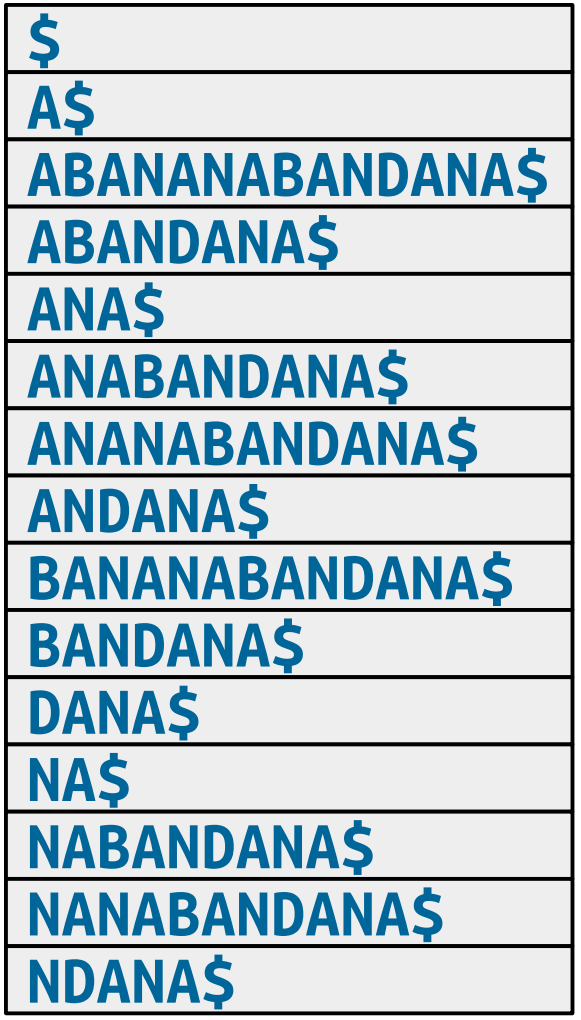
Storing Suffix Arrays

- The way we've drawn suffix arrays is terribly space-inefficient.
  - It always uses space $\Theta(m^2)$, since that's how many total characters occur in all suffixes.
- Can we do better?



ABANANABANDANA$

Storing Suffix Arrays

- We reduced the space usage of suffix trees by representing substrings, implicitly, as ranges within the original string.
- ***Idea:*** Don't store the suffixes themselves. Just store the starting positions of the suffixes.
- Space: $\Theta(m)$, and with only one machine word used per character of input.

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Storing Suffix Arrays

- Although the picture to the right is how we'd represent the suffix array in memory, for this lecture we'll draw things out the longer way.
- This is just to build intuition; we wouldn't actually do that in practice.

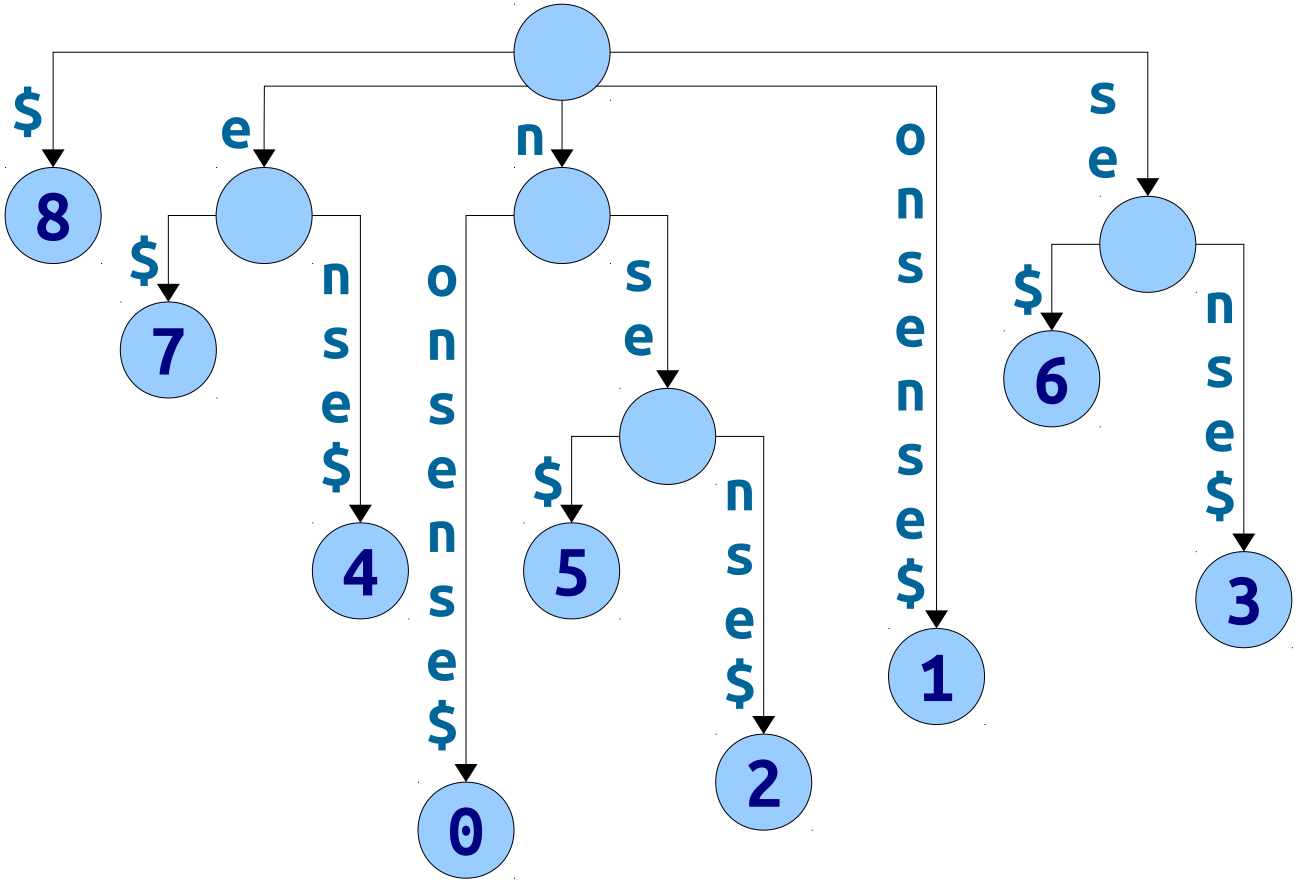
ABANANABANDANA$
01234567890123 4

The Story So Far

- Suffix arrays store all the suffixes of a string in sorted order.
- They provide an

$$\langle O(m), O(n \log m + z) \rangle$$

  solution to the substring search problem.
- ***Intuition:*** Suffix trees are valuable in large part because they just keep the suffixes sorted.
- What else are suffix trees doing?



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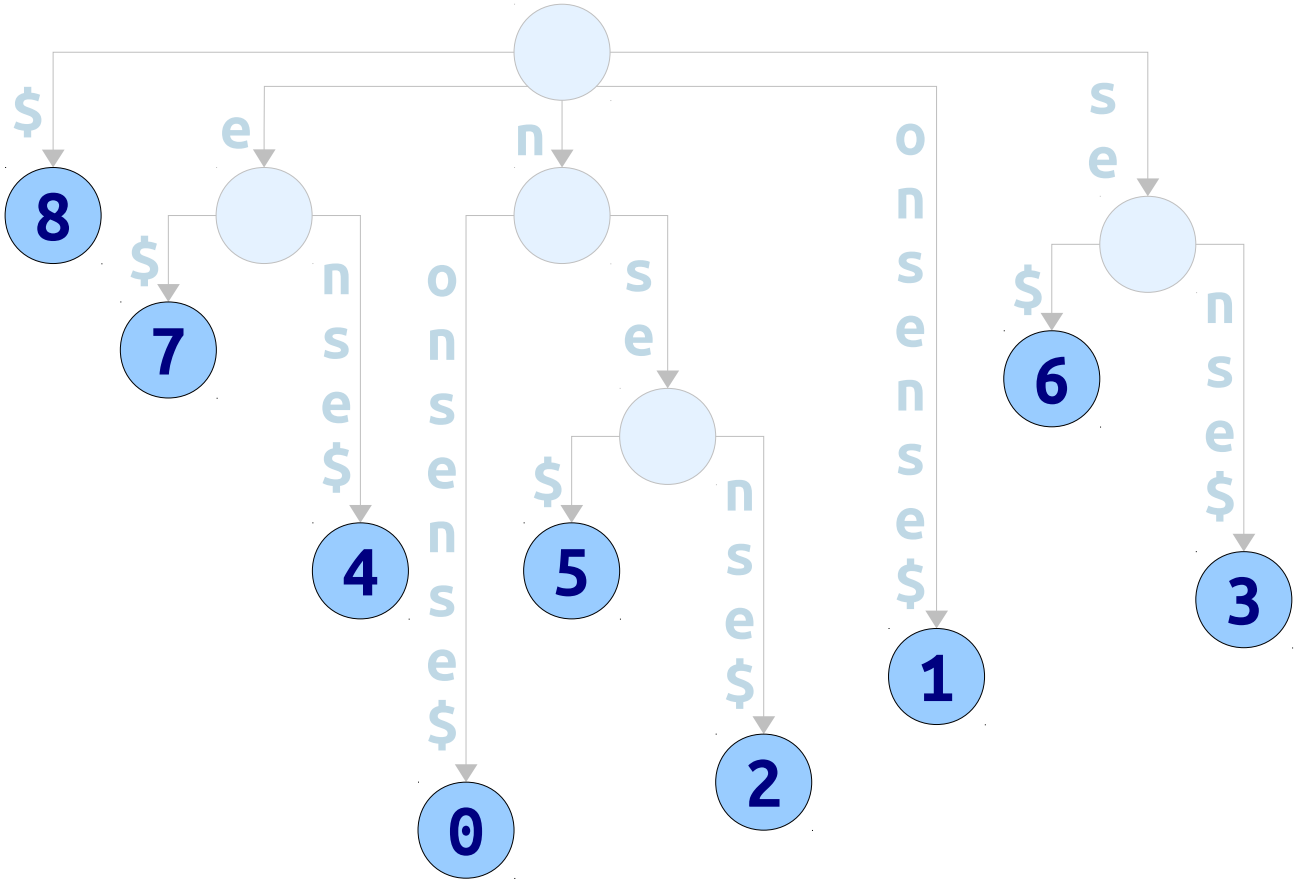

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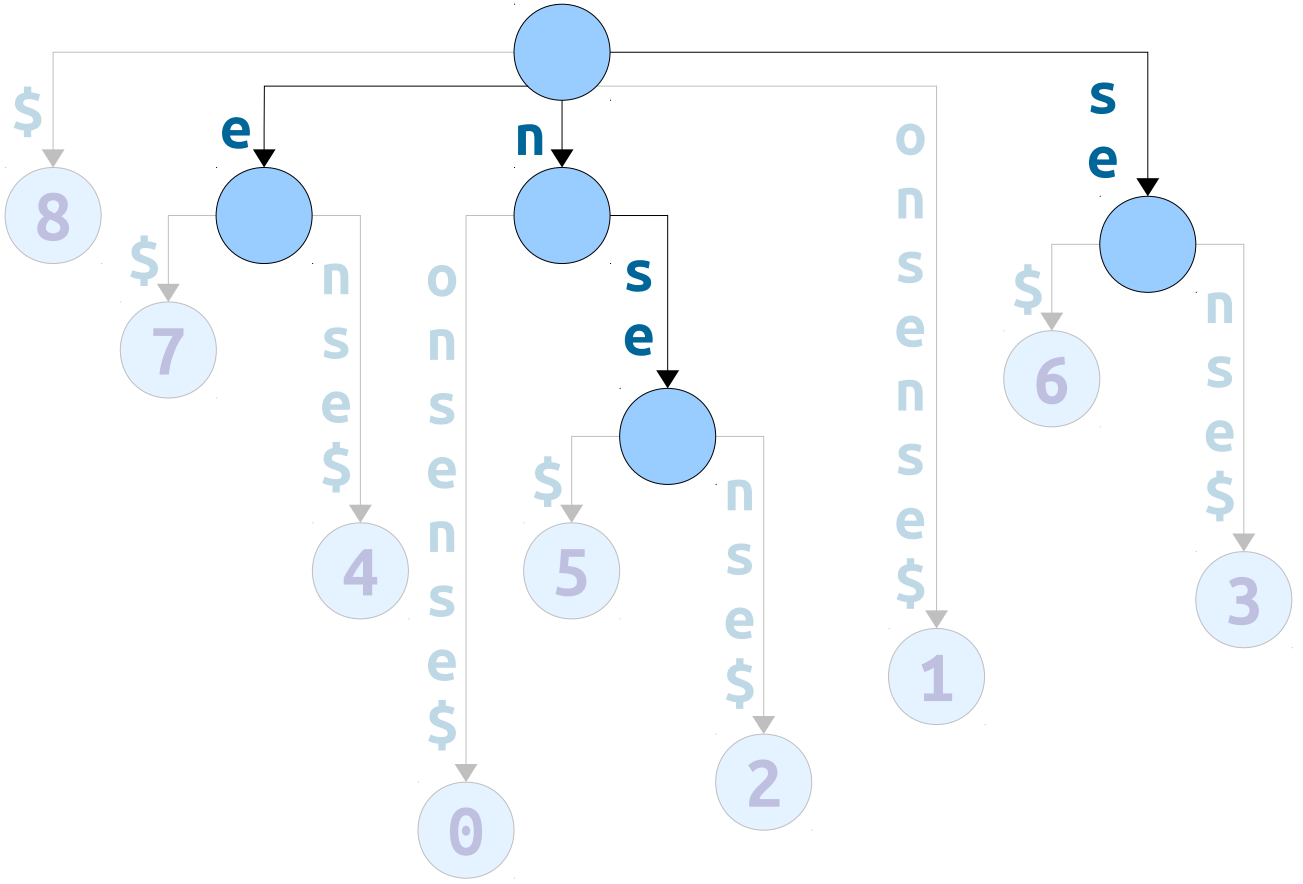

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Branching Words

- ***Recall:*** If T is a string, then ω is a ***branching word*** in T if there are characters $a \neq b$ such that ωa and ωb are substrings of T\$.

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Although ABA is a repeated substring, it is not a branching word because all appearances are followed by N.

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The substring ANANA only appears once, so it's not a branching word.

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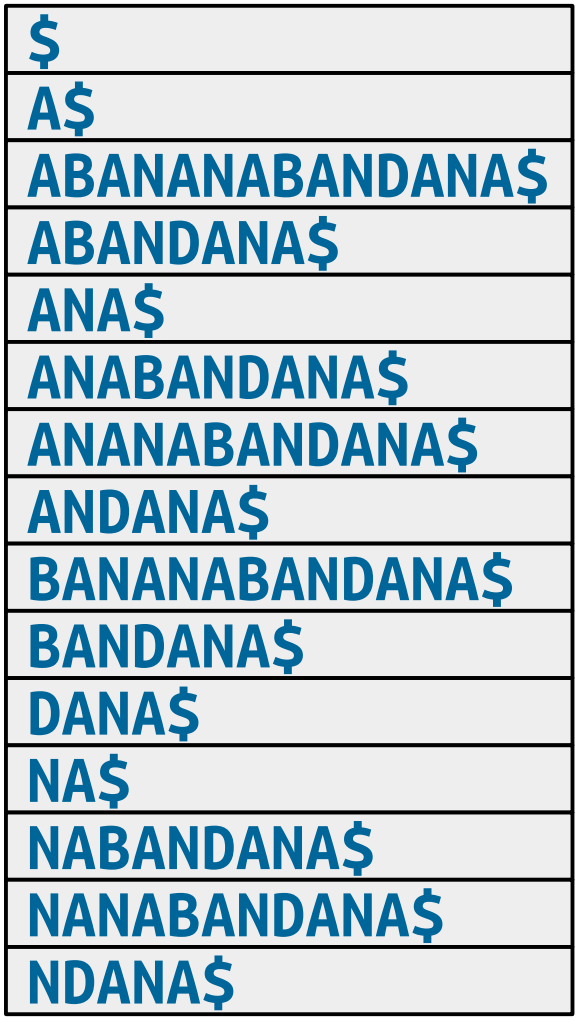
Branching Words

- Notice that, by sorting suffixes, we've made it easier to spot branching words.
- Specifically, all suffixes starting with a branching word will be adjacent in the suffix array.
- The branching word will be the ***longest common prefix*** (or ***LCP***) of those adjacent suffixes.



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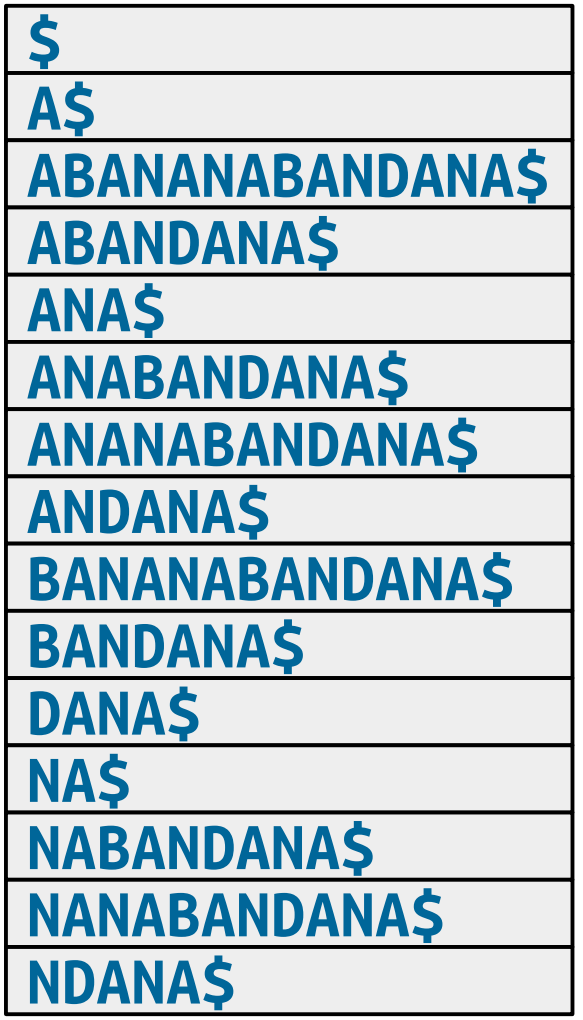

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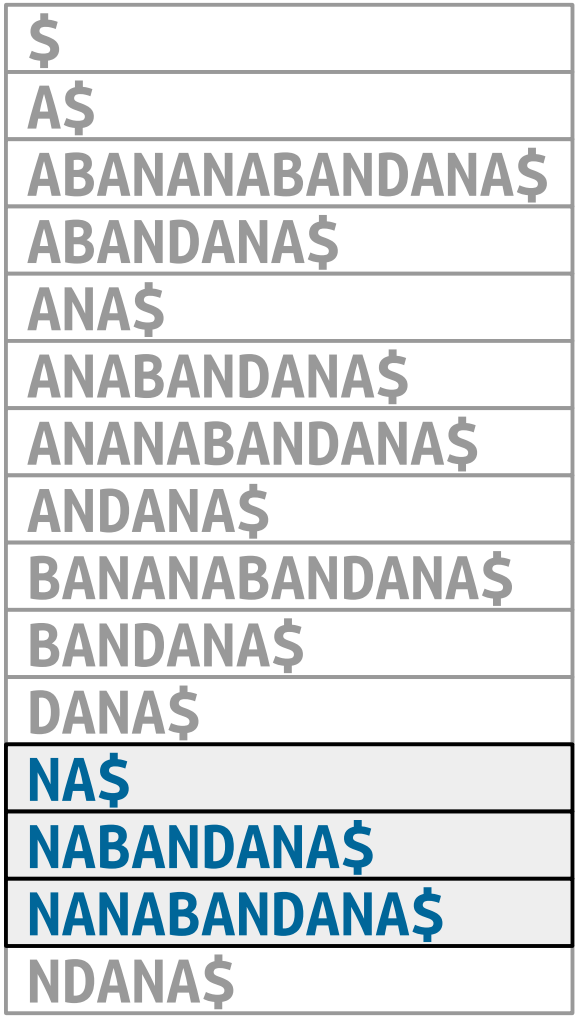

ABANANABANDANA$

Branching Words

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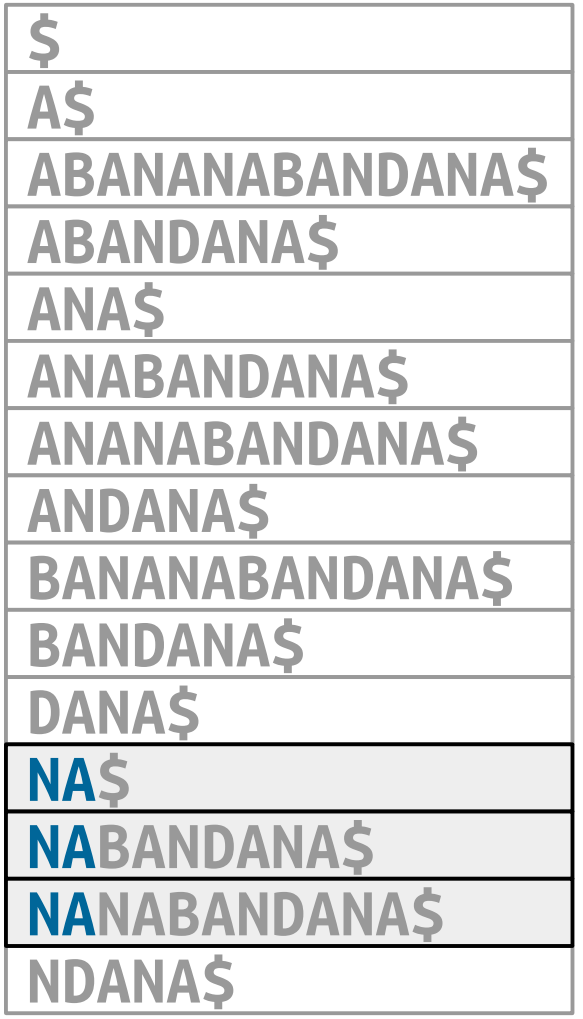

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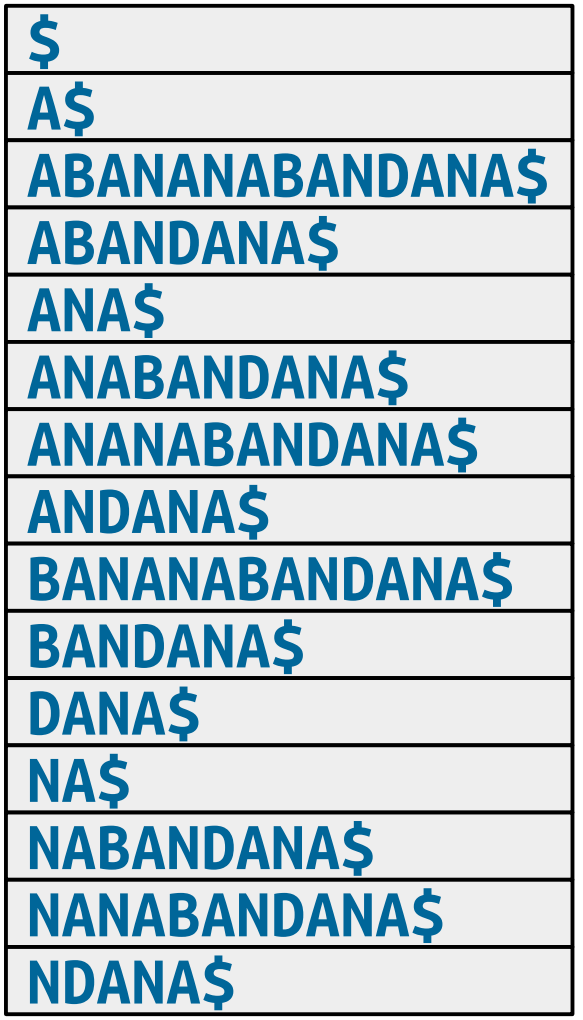

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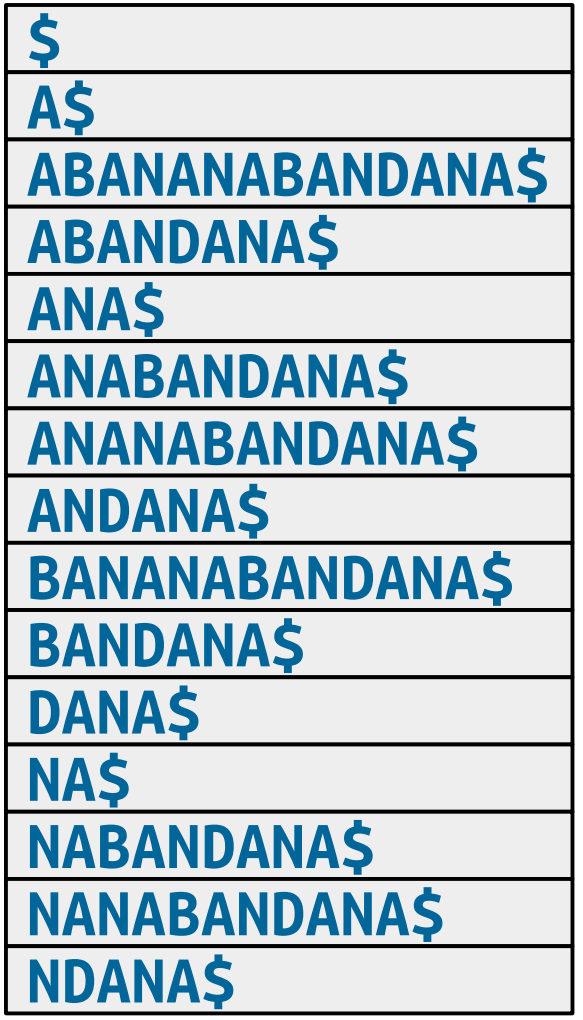
Branching Words

- ***Theorem:*** A string ω is a branching word in string T if and only if it's the longest common prefix of two adjacent suffixes in T's suffix array.
- ***Proof idea:*** If ω is the longest common prefix of two adjacent suffixes, let a and b be the characters immediately following ω in those two suffixes. Then ωa and ωb are substrings of T\$.

  If ω is branching, choose the lexicographically smallest a and b making the definition work. Then the last suffix starting with ωa and the first suffix starting with ωb are adjacent in the suffix array. ■



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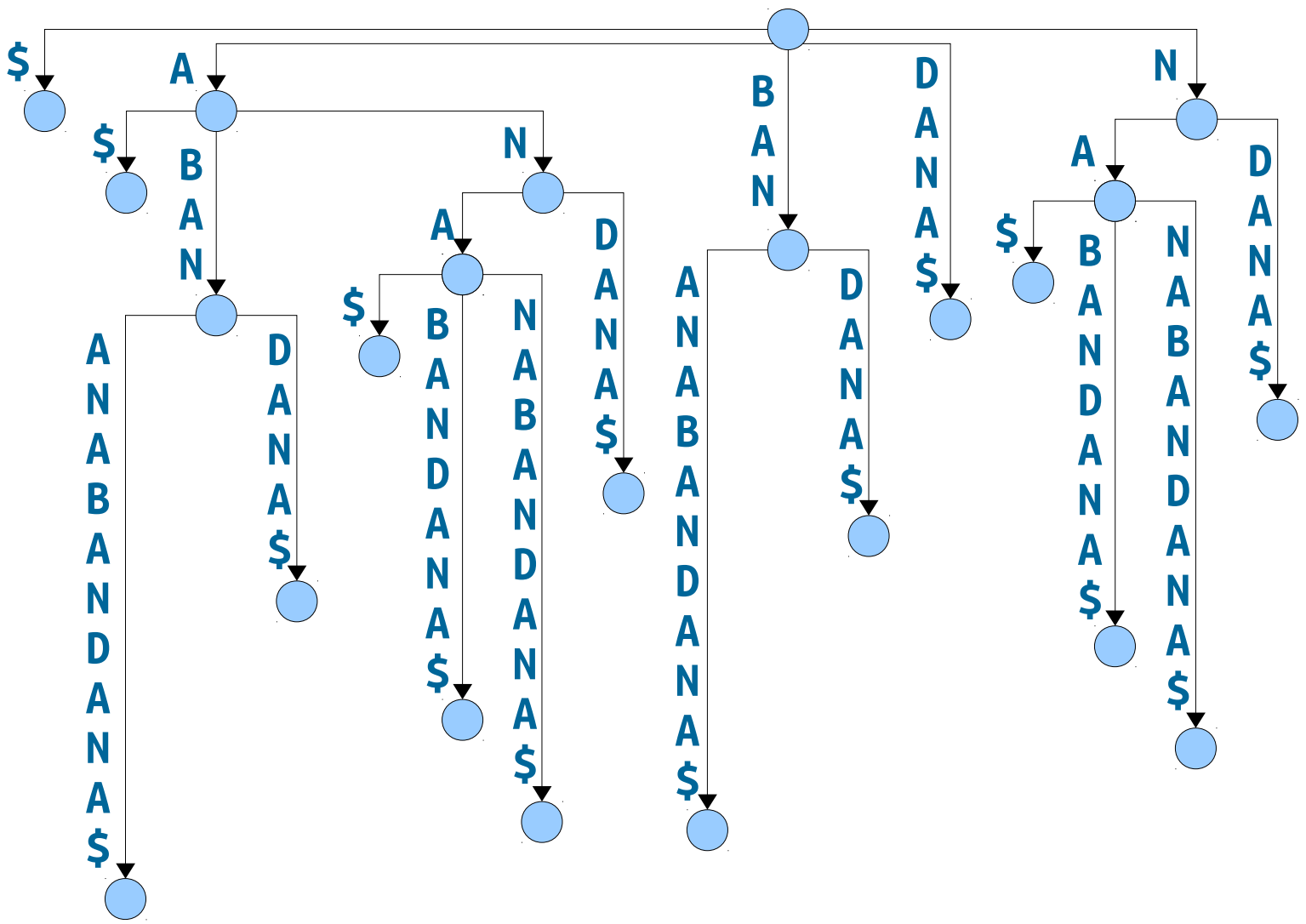


$
A$
ABANANABANDANA$
ABANDANA$
ANA$
ANABANDANA$
ANANABANDANA$
ANDANA$
BANANABANDANA$
BANDANA$
DANA$
NA$
NABANDANA$
NANABANDANA$
NDANA$

ABANANABANDANA$

ω is an internal node in the suffix tree for T

if and only if

ω is a branching word in T

if and only if

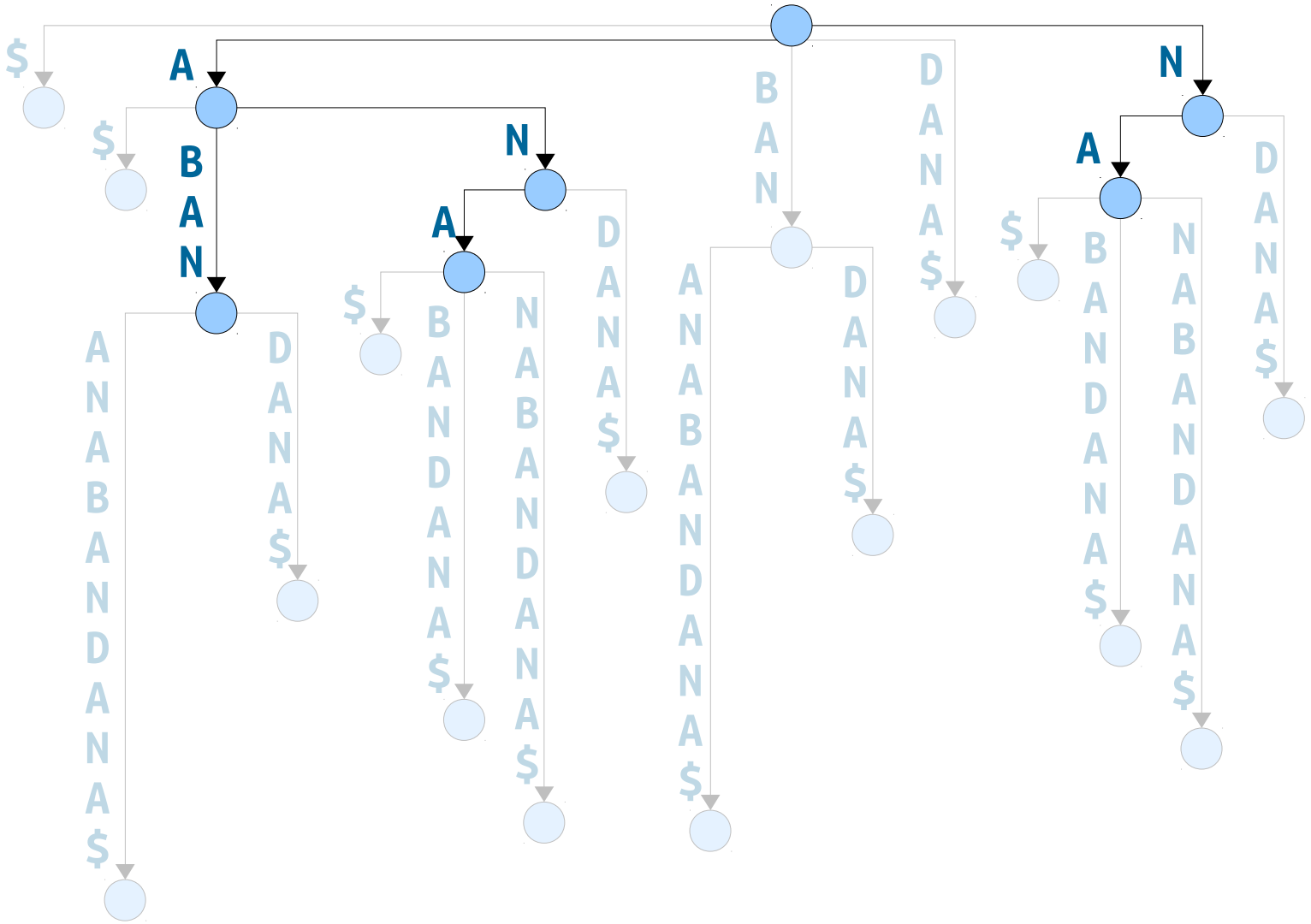
ω is the LCP of two adjacent suffixes in the suffix array for T

Key Intuition: Adjacent suffixes with long shared prefixes correspond to subtrees of the suffix tree.

Harnessing this Connection

Longest Repeated Substring

- Last time, we saw how to solve the longest repeated substring problem by using suffix trees.
- ***Algorithm:*** Find the internal node in the suffix tree with the longest label.
- ***Question:*** Can we do this with just a suffix array?



ABANANABANDANA$

Longest Repeated Substring

- We can list all branching words from a suffix array in time O(m^2).
  - O(m) pairs; each pair takes time O(m) to process.
- This worst-case bound can be realized. *(Do you see how?)*
- Contrast this with O(m) for a suffix tree.
- Can we do better?

$
A$
ABANANABANDANA$
ABANDANA$
ANA$
ANABANDANA$
ANANABANDANA$
ANDANA$
BANANABANDANA$
BANDANA$
DANA$
NA$
NABANDANA$
NANABANDANA$
NDANA$

ABANANABANDANA$

Longest Repeated Substring

- ***Observation:*** We don't actually need to know what all the branching words are to find the longest repeated substring.
- We just need to know how long they are.
- That way, we can figure out which is longest.
- Is there some nice way to do this?

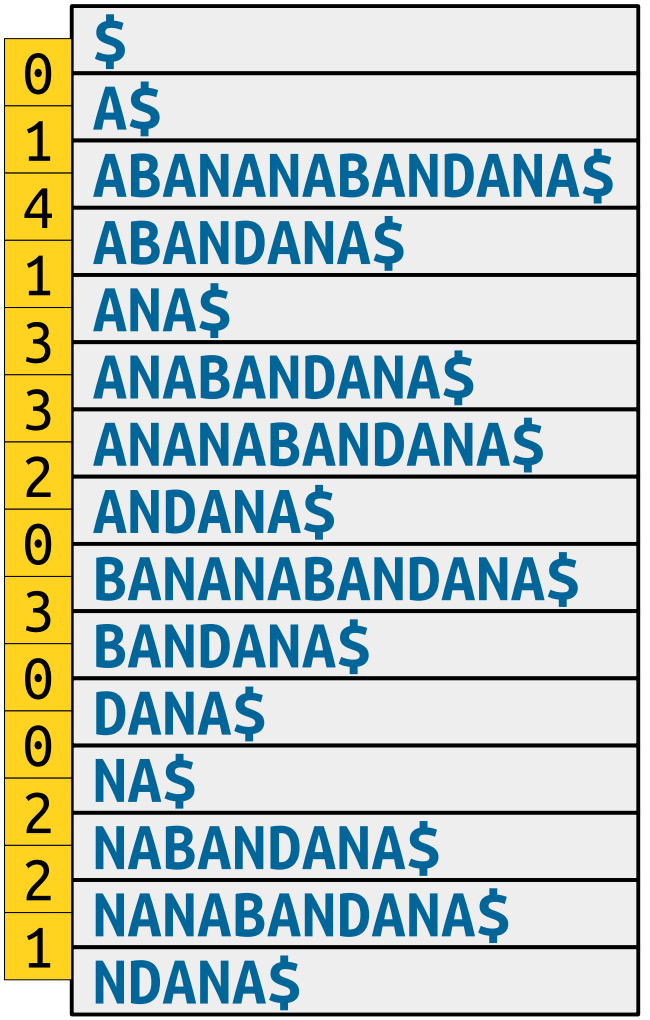
$
A$
ABANANABANDANA$
ABANDANA$
ANA$
ANABANDANA$
ANANABANDANA$
ANDANA$
BANANABANDANA$
BANDANA$
DANA$
NA$
NABANDANA$
NANABANDANA$
NDANA$

ABANANABANDANA$

LCP Arrays

LCP Arrays

- The ***LCP array***, often denoted ***H***, is an array where $H[i]$ is the length of the LCP of the ith and $(i+1)$st suffixes in the suffix array.
- (The letter H comes from “height.”)



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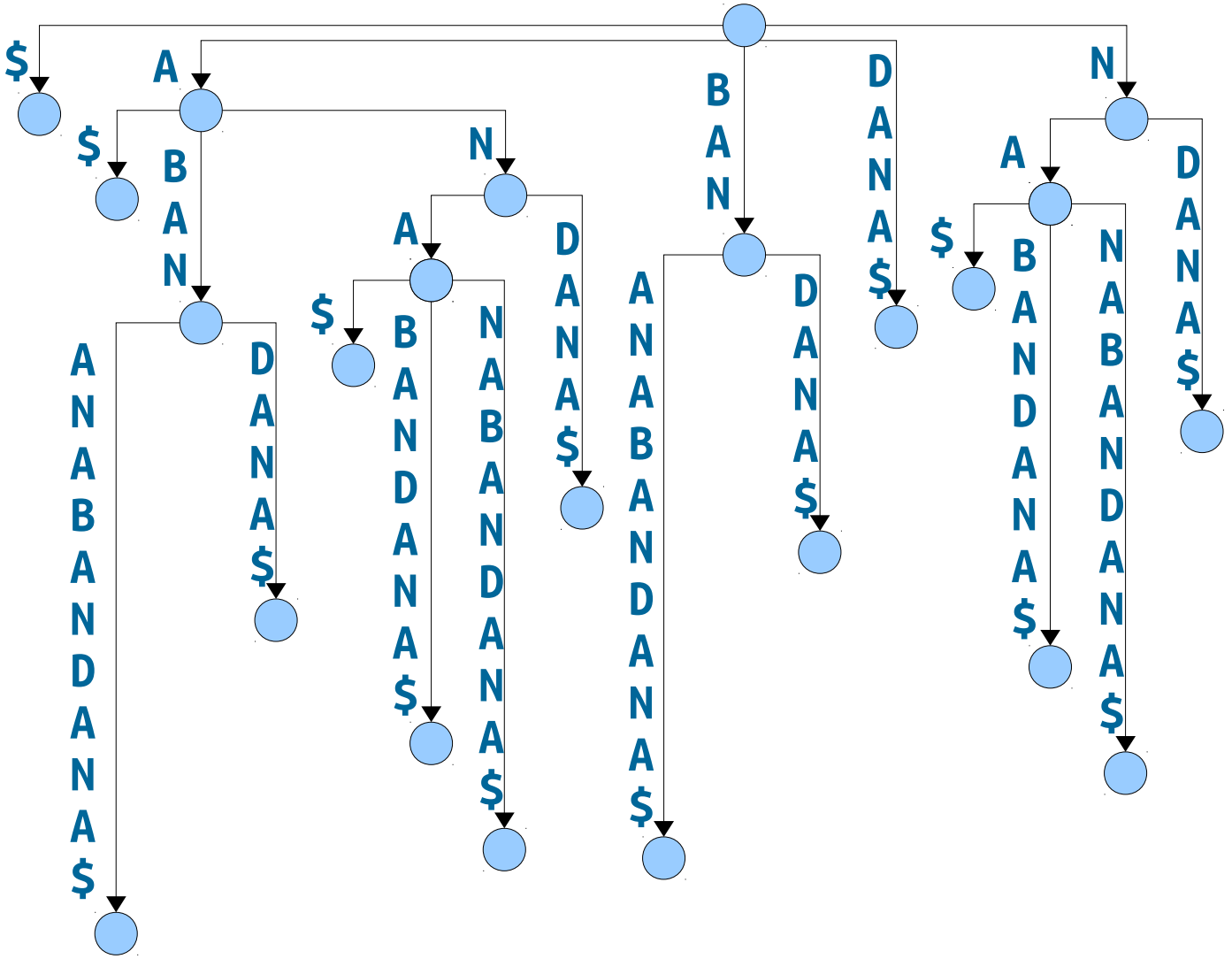


0	$
1	A$
4	ABANANABANDANA$
1	ABANDANA$
3	ANA$
3	ANABANDANA$
2	ANANABANDANA$
0	ANDANA$
3	BANANABANDANA$
0	BANDANA$
0	DANA$
2	NA$
2	NABANDANA$
1	NANABANDANA$
	NDANA$

ABANANABANDANA$

Key intuition: The suffix array gives the leaves of the suffix tree. The LCP array gives the internal nodes of the suffix tree.

Using LCP Arrays

- If you already have a suffix array and LCP array, you can solve longest repeated substring in time O(***m***):
  - Find the largest element in the LCP array.
  - Return the string it corresponds to.
- ***Question:*** How fast can we construct an LCP array?

0	$
1	A$
4	ABANANABANDANA$
1	ABANDANA$
3	ANA$
3	ANABANDANA$
2	ANANABANDANA$
0	ANDANA$
3	BANANABANDANA$
0	BANDANA$
0	DANA$
2	NA$
2	NABANDANA$
1	NANABANDANA$
	NDANA$

ABANANABANDANA$

Time-Out for Announcements!

HackOverflow

- WiCS is running a hackathon, ***HackOverflow***, on Saturday, April 20th.
- Interested in attending? Use this link to sign up.
- Interested in serving as a mentor? Use this link to volunteer.
- Everyone is welcome to attend!

Problem Set One

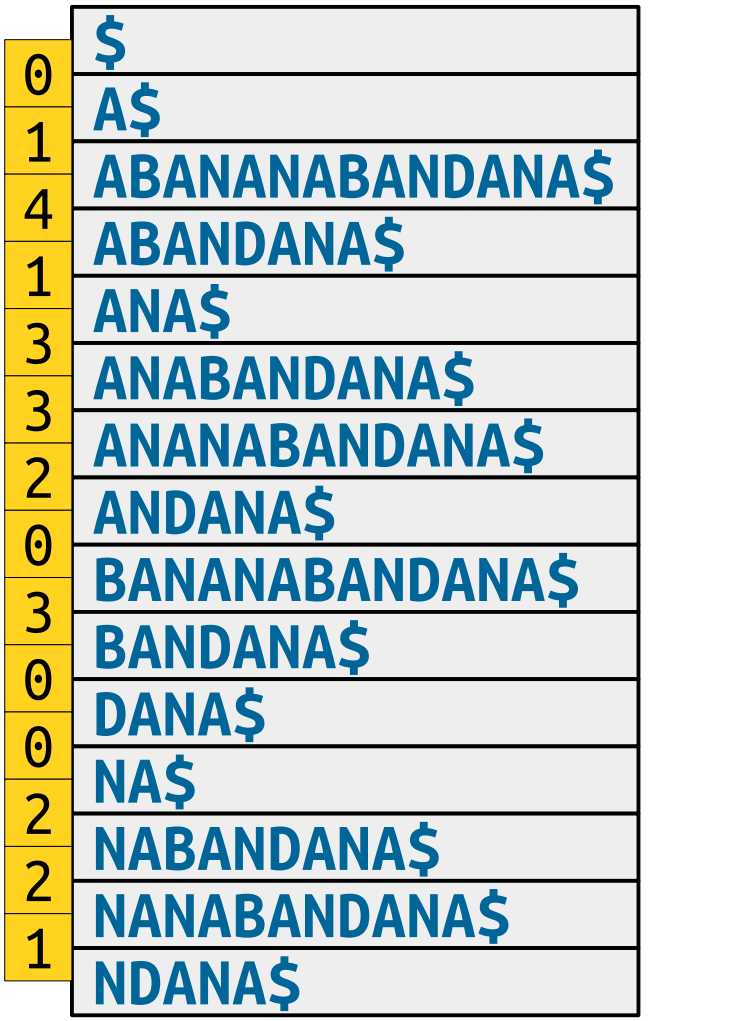
- Problem Set One is due on Tuesday at 2:30PM.
  - We encourage you to work with partners on the problem sets. It's really helpful to bounce ideas off of one another!
  - Need to find a partner? Visit Piazza and use the "Search for Teammates" feature!
- Solutions to Problem Set Zero are now up on the course website.

Back to CS166!

Building LCP Arrays

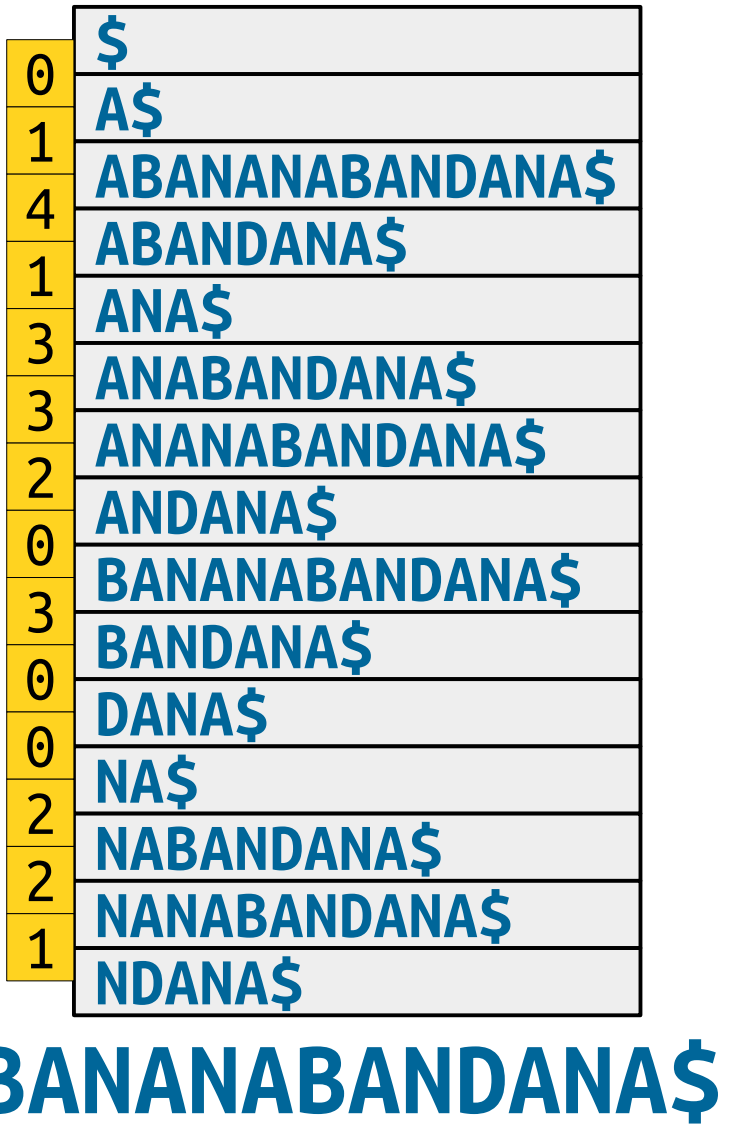
Building LCP Arrays

- It never hurts to start with the naive algorithm and see what happens!
- ***Algorithm:*** For each consecutive pair of strings in the suffix array, compute the length of their longest common prefix.
- We can upper-bound the runtime at $O(m^2)$.
- ***Question:*** Can we realize this upper bound?



Building LCP Arrays

- Why is our naive algorithm slow?
- ***Intuition:*** We aren't able to carry work from one suffix over to the next.



Building LCP Arrays

- ***Key intuition:*** Suffixes overlap one another! It should be possible to share LCP information across suffixes.

$
A$
ABANANABANDANA$
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ANA$
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Building LCP Arrays

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- For example, suppose we compute the LCP entry shown here.

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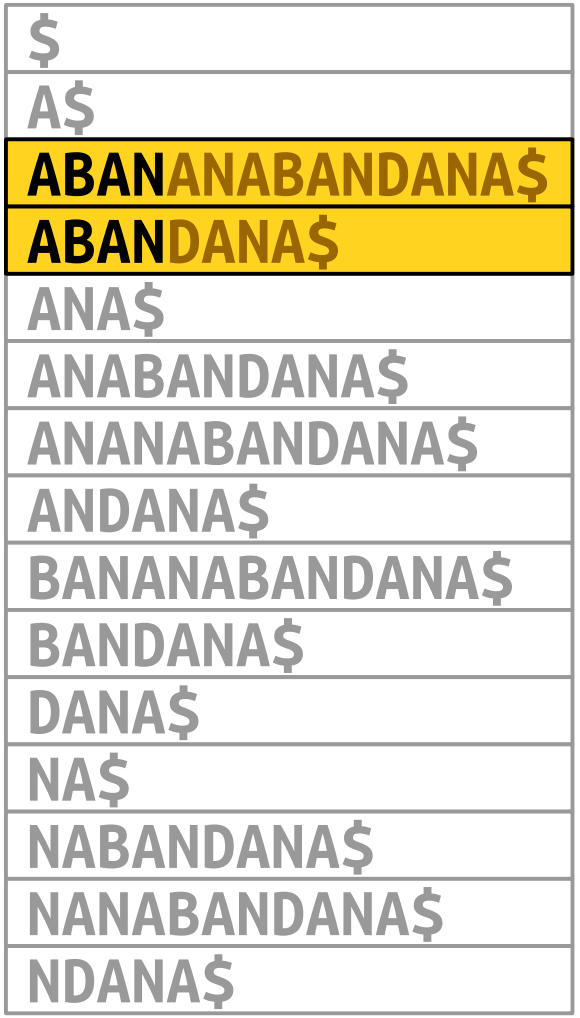

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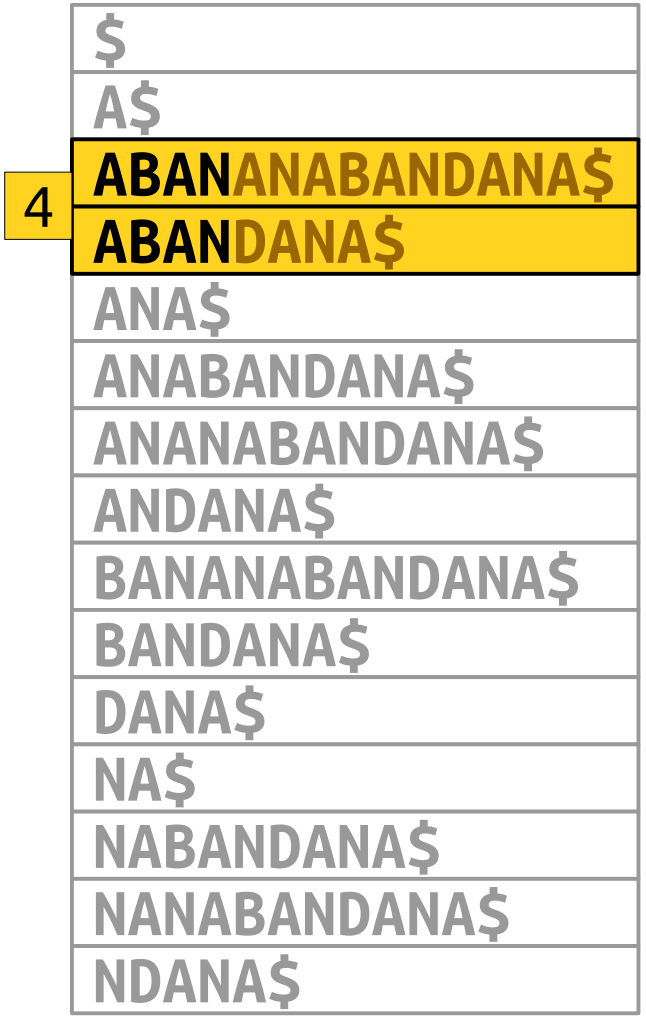

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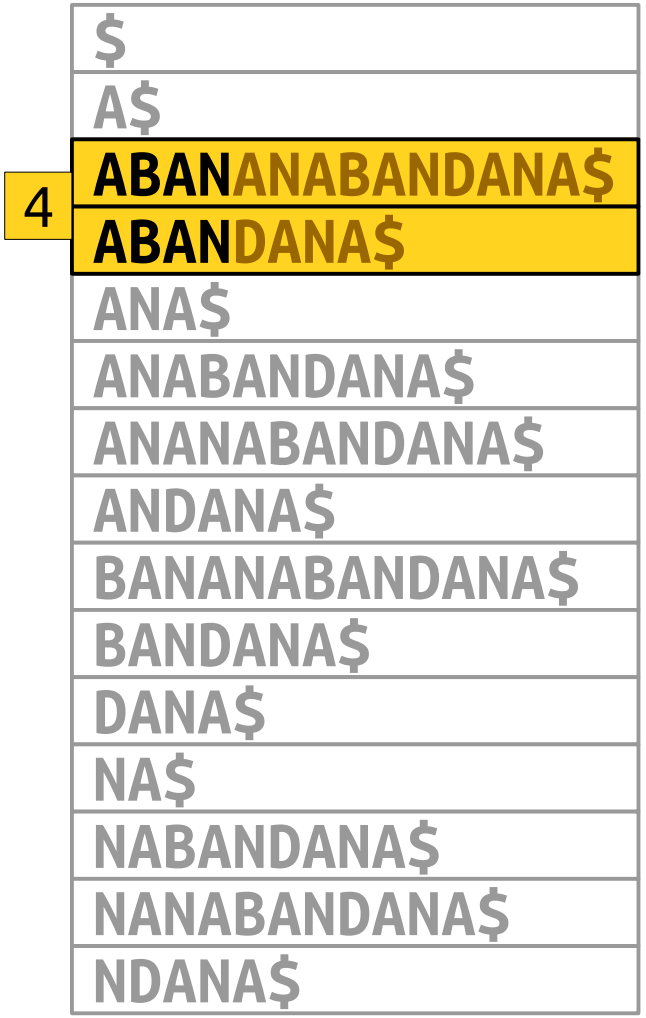

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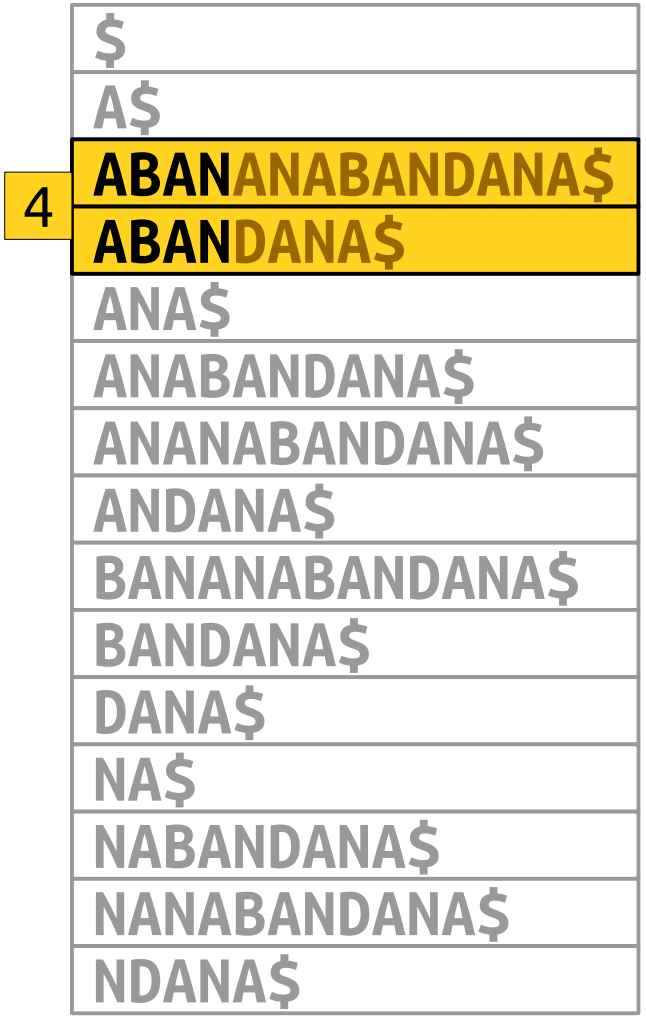

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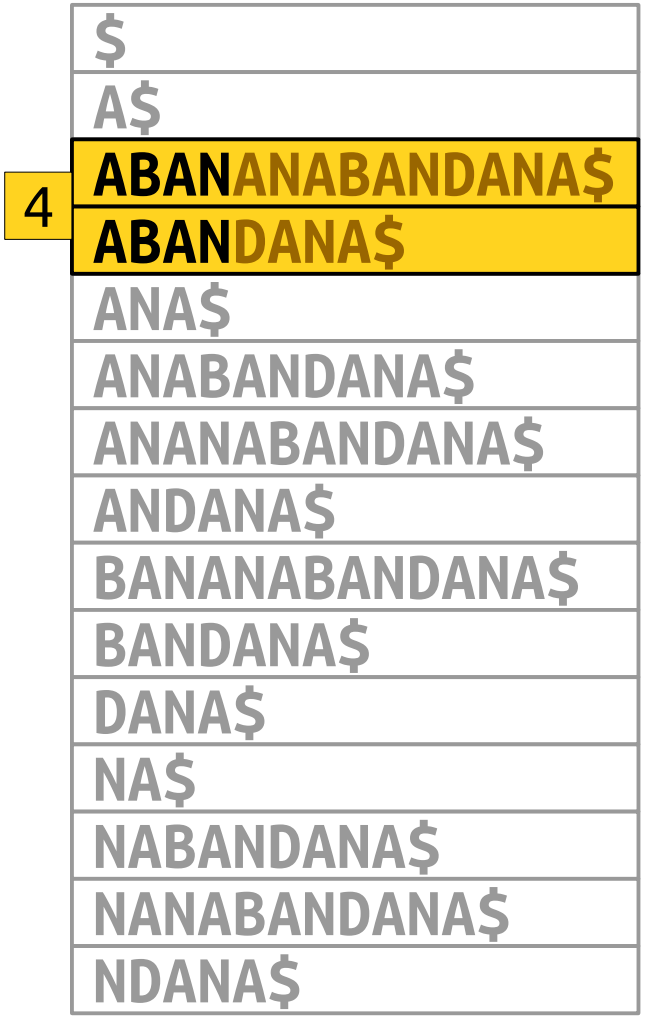

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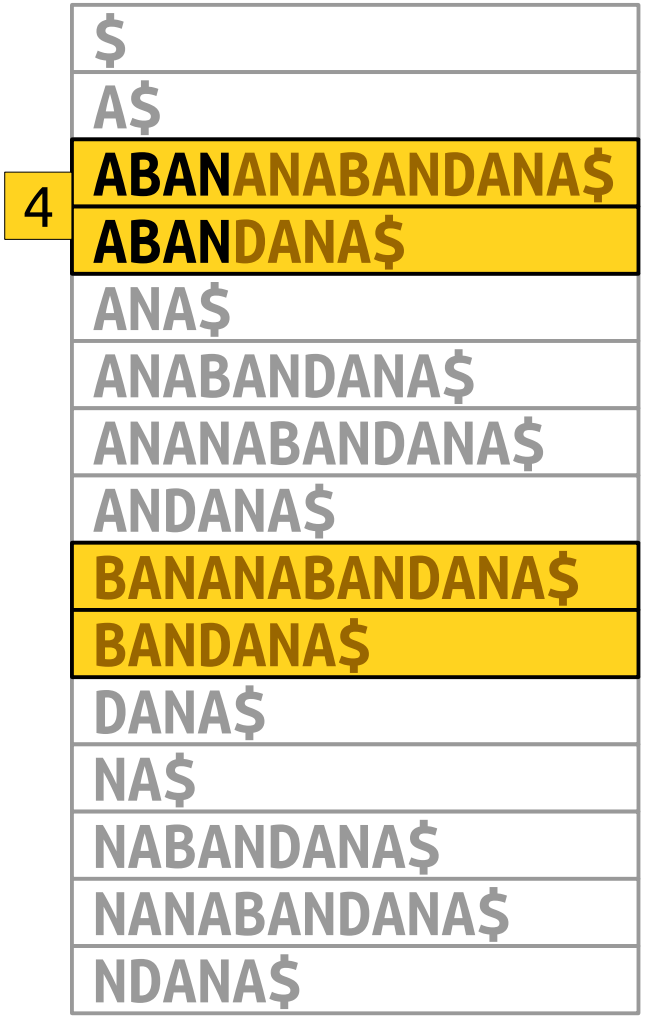

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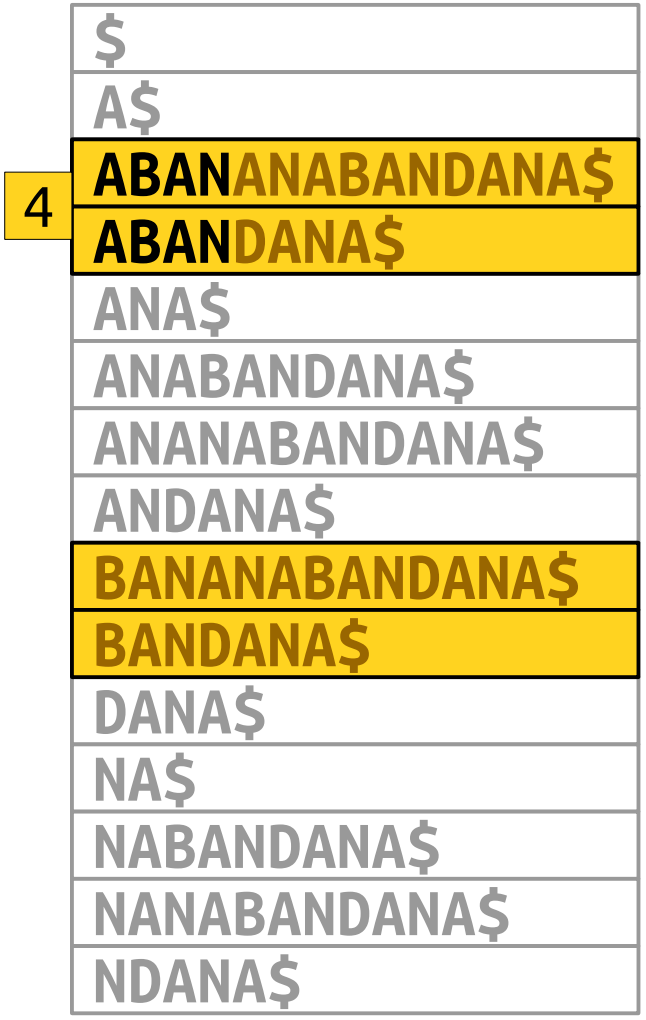

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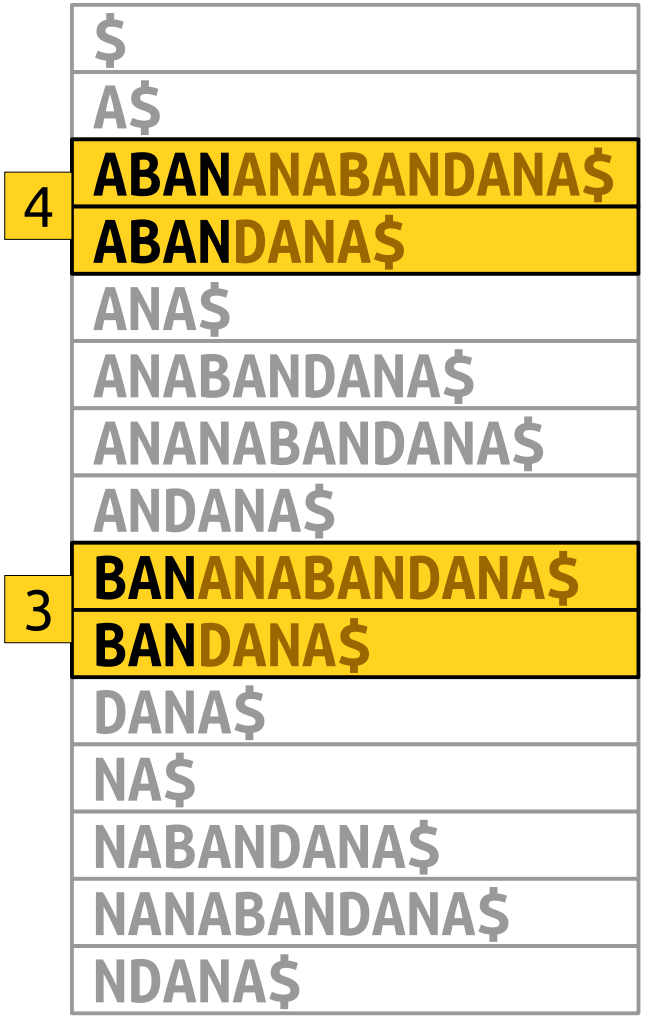

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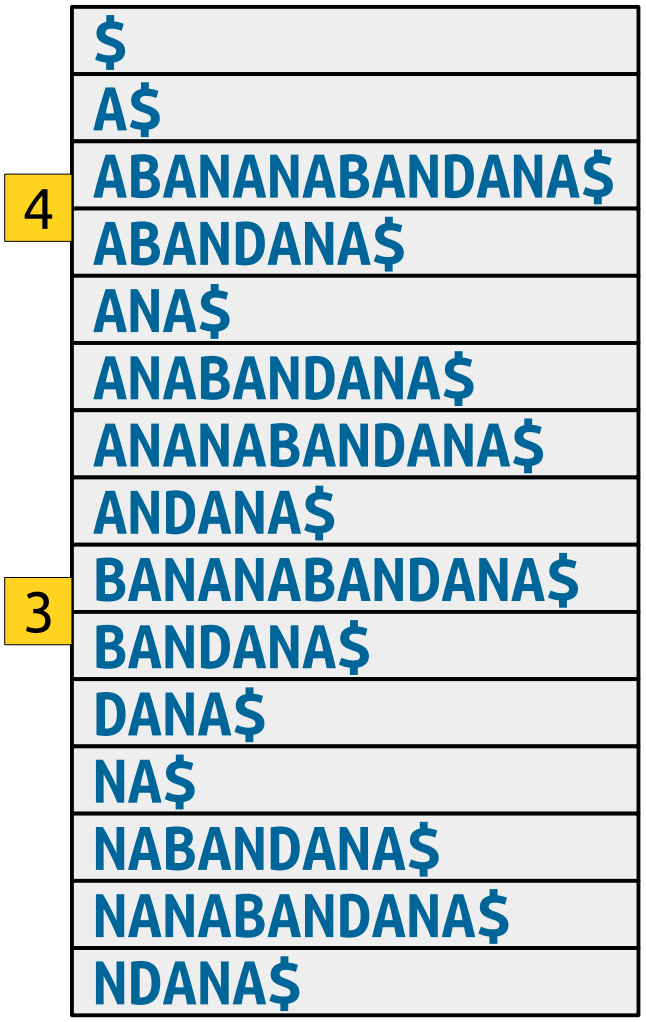

ABANANABANDANA$

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Building LCP Arrays

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$
A$
ABANANABANDANA$
ABANDANA$
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ANANABANDANA$
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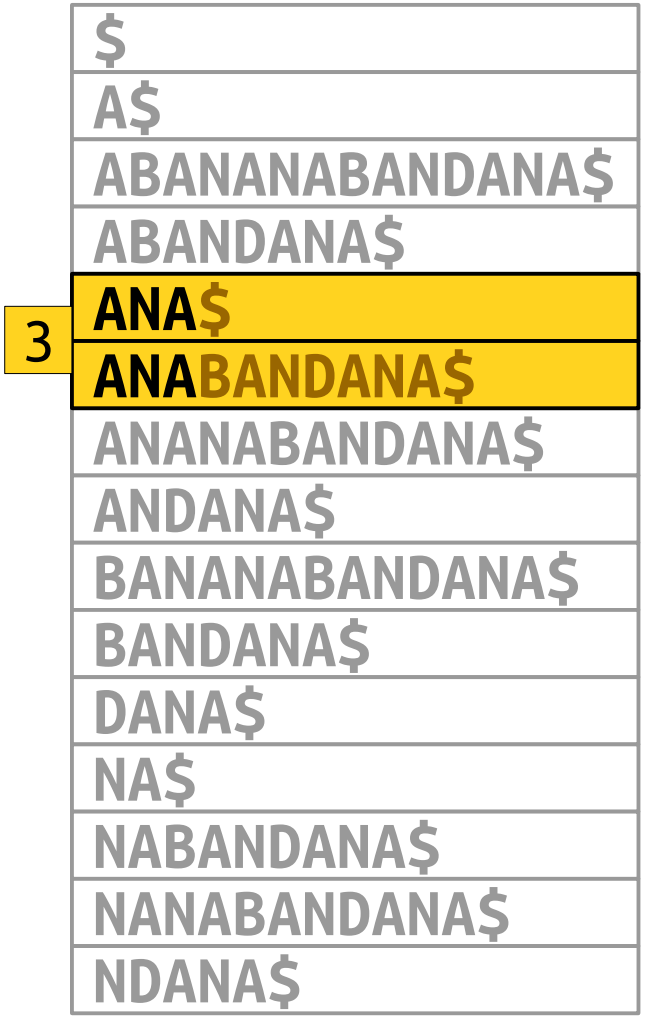

Building LCP Arrays

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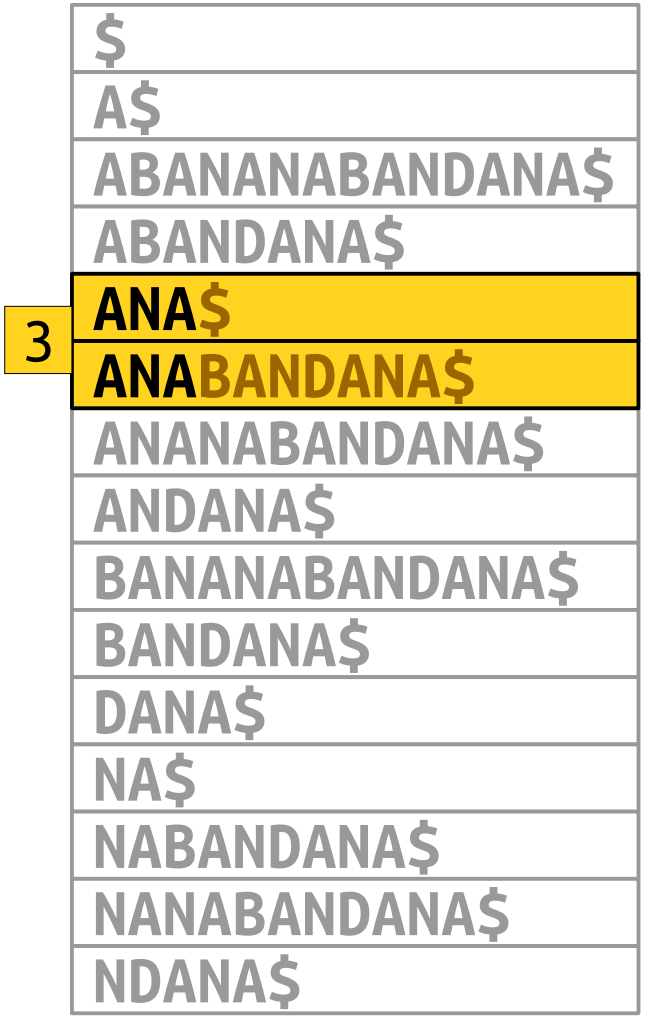
- As before, drop the first letter from each suffix.



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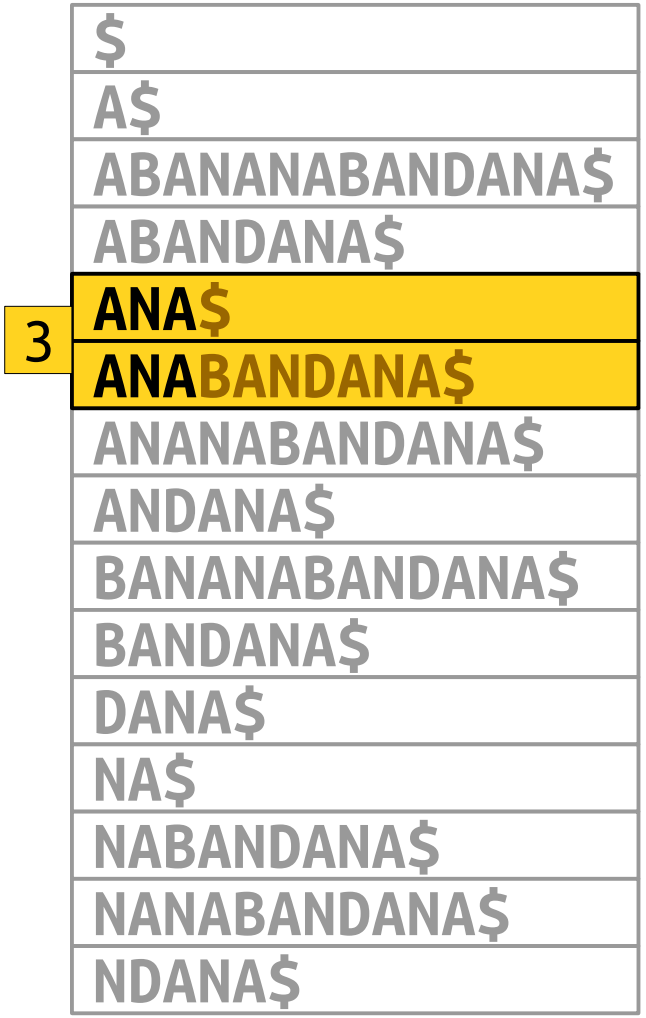

ANA$

ANABANDANA$

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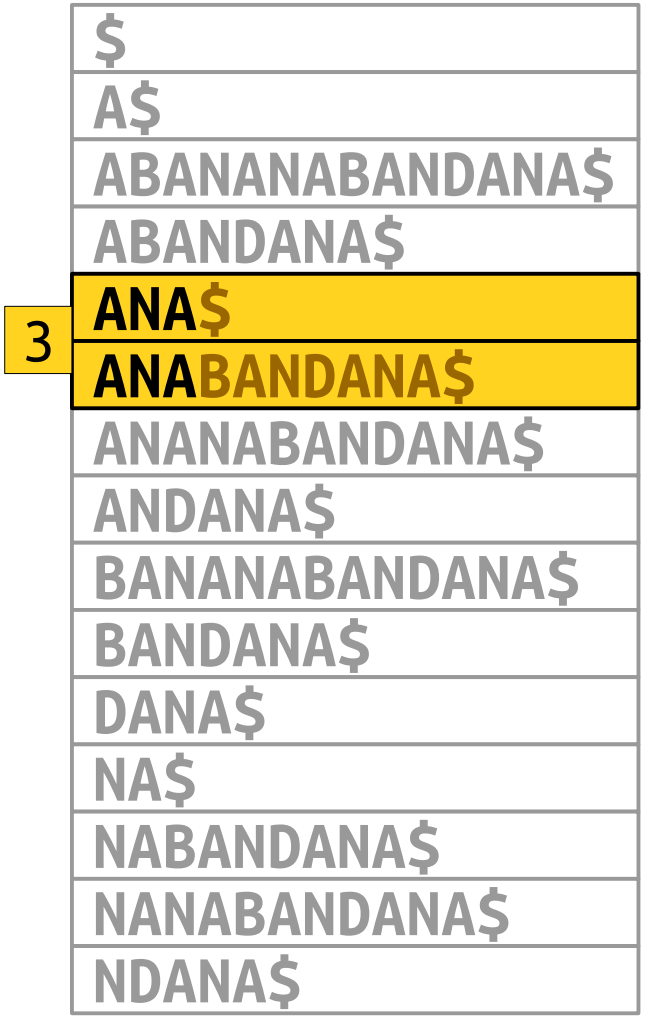

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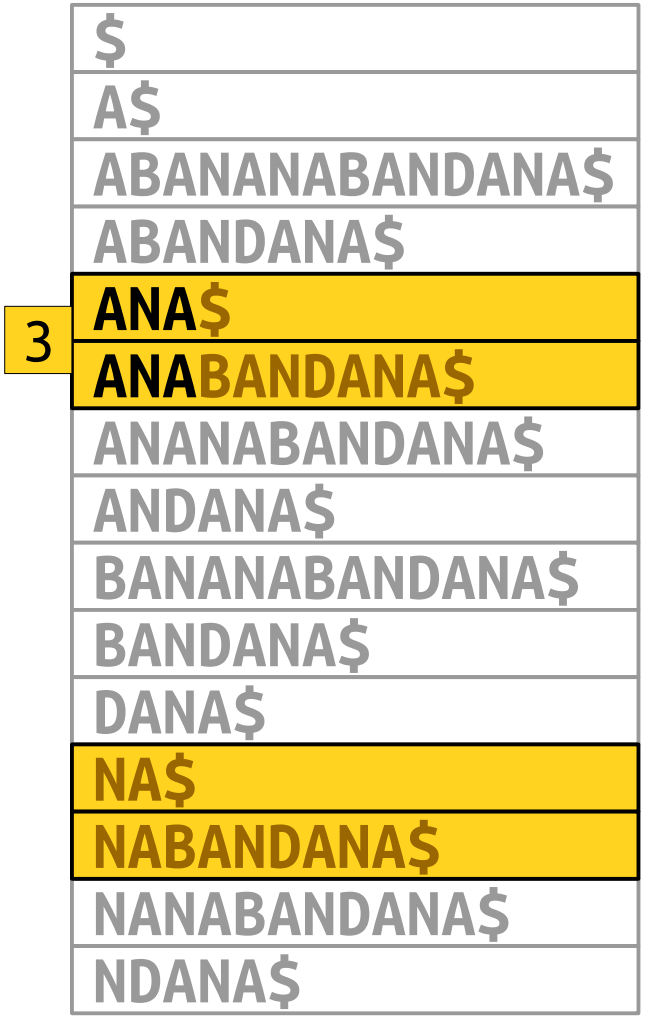

ANA$

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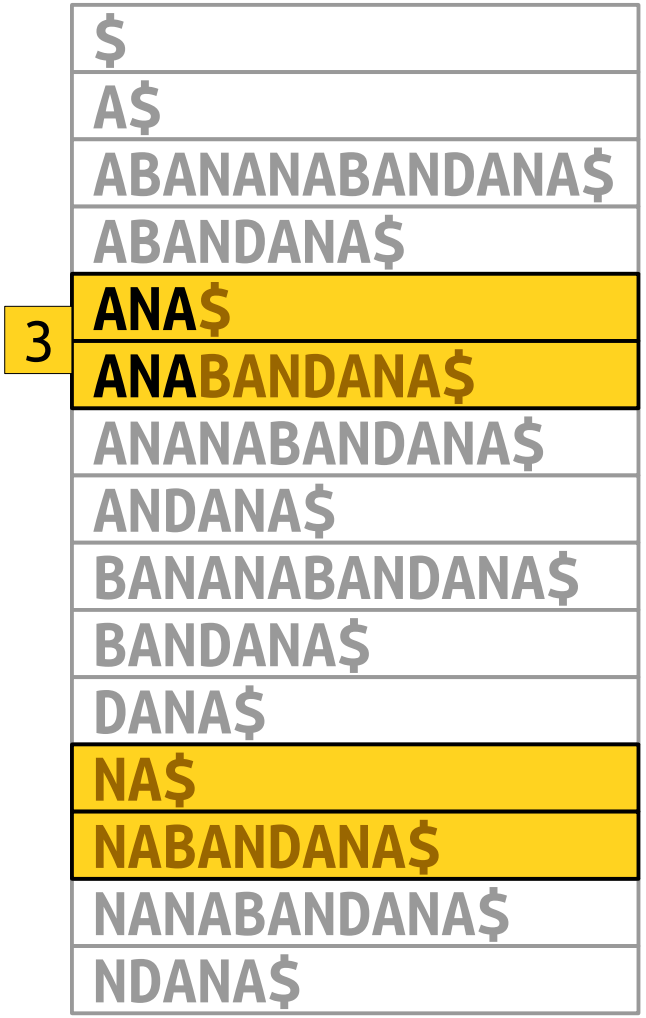
Building LCP Arrays

- Let’s do another example. Suppose we know the LCP of these suffixes.
- As before, drop the first letter from each suffix.
- What can we say about the LCP of the resulting suffixes?



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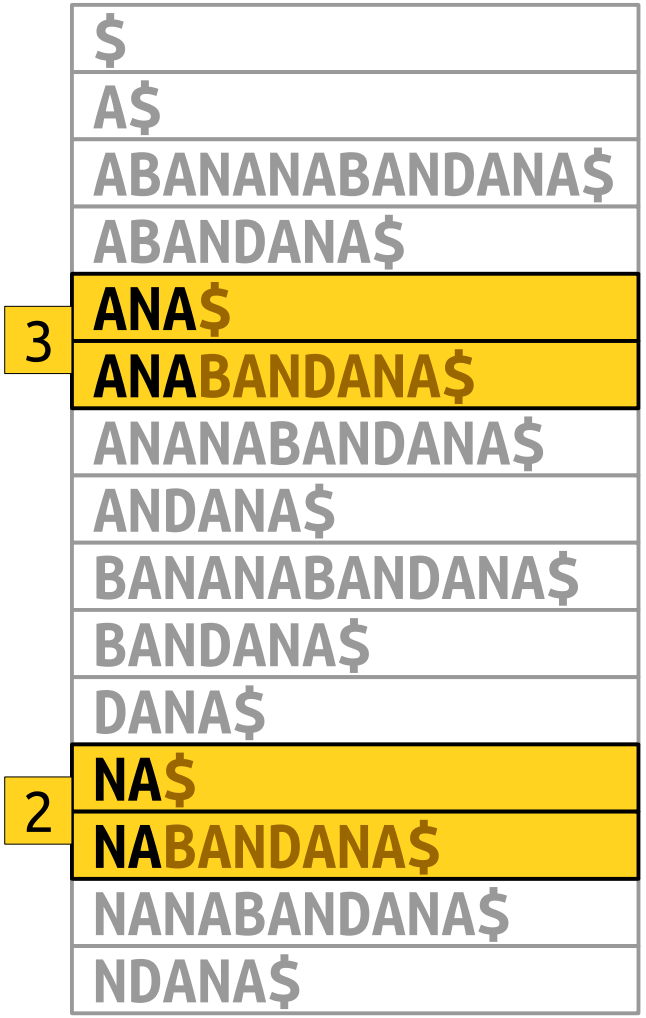

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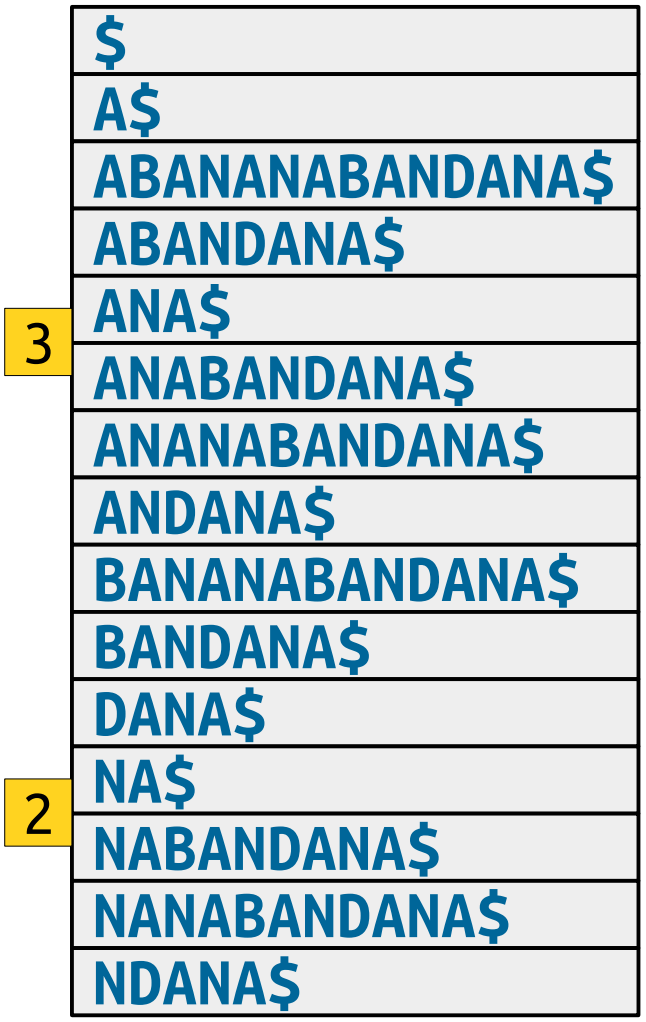

ANA$

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Building LCP Arrays

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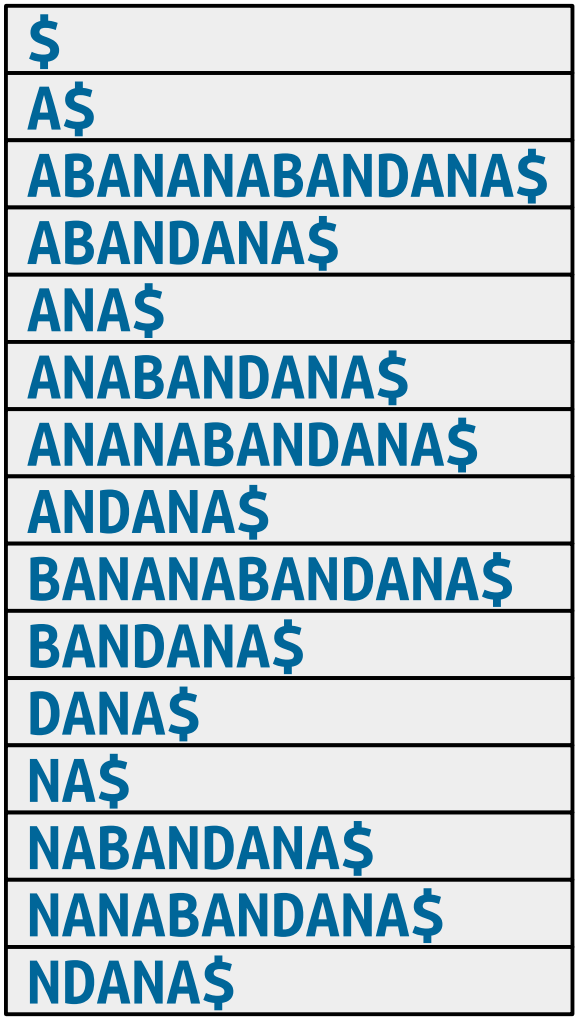

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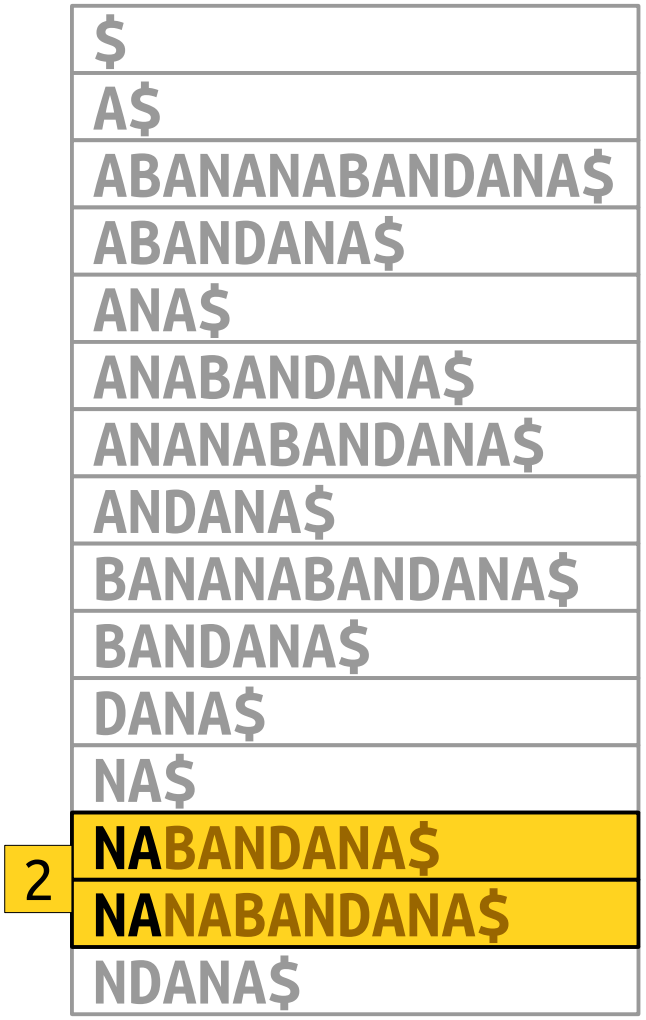

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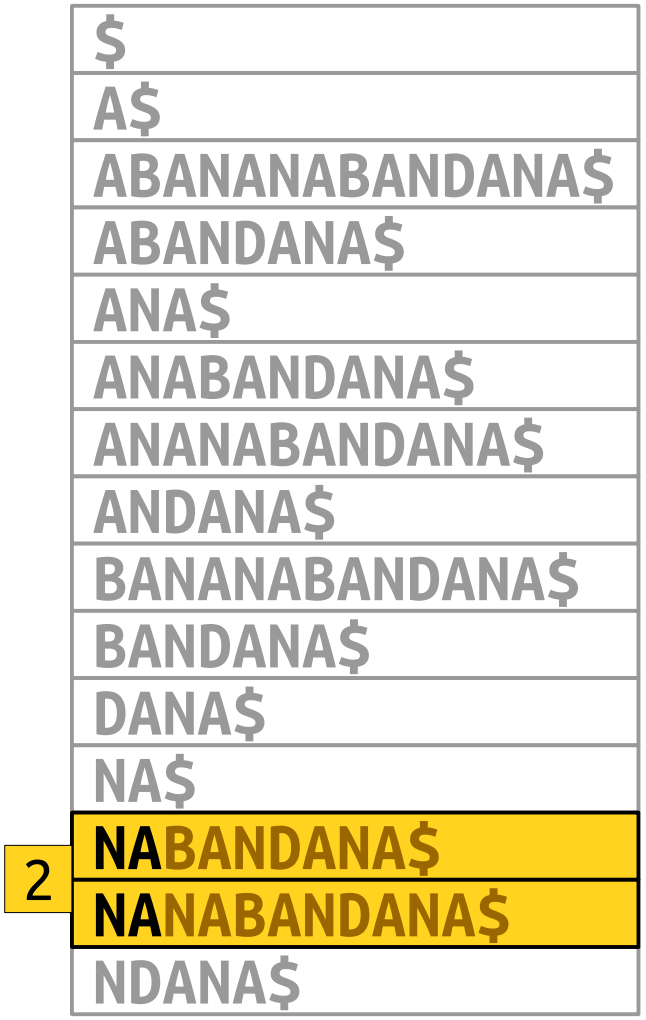

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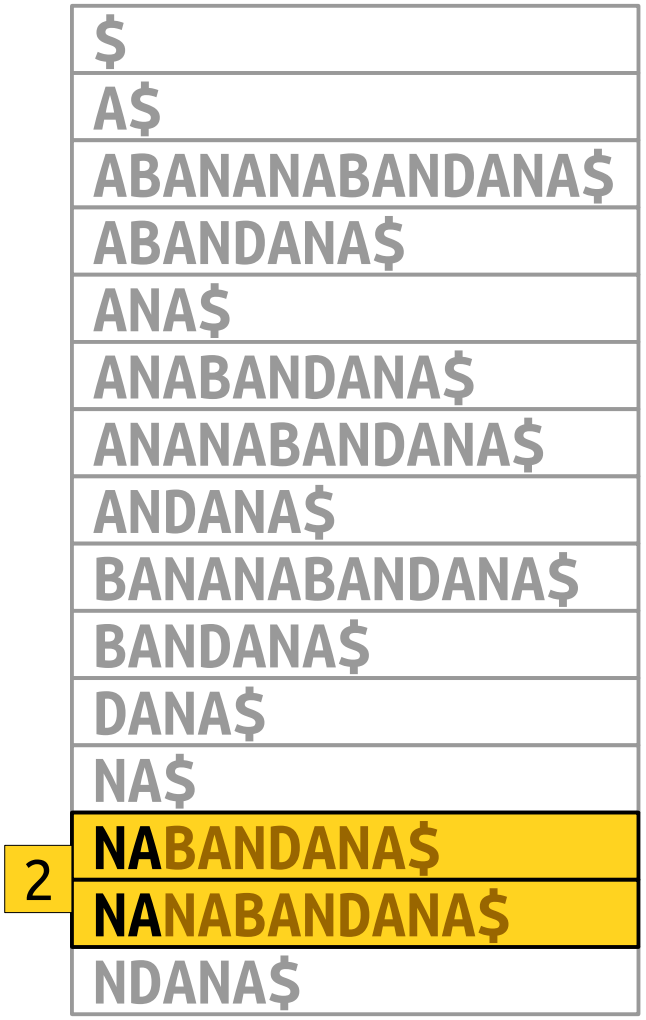

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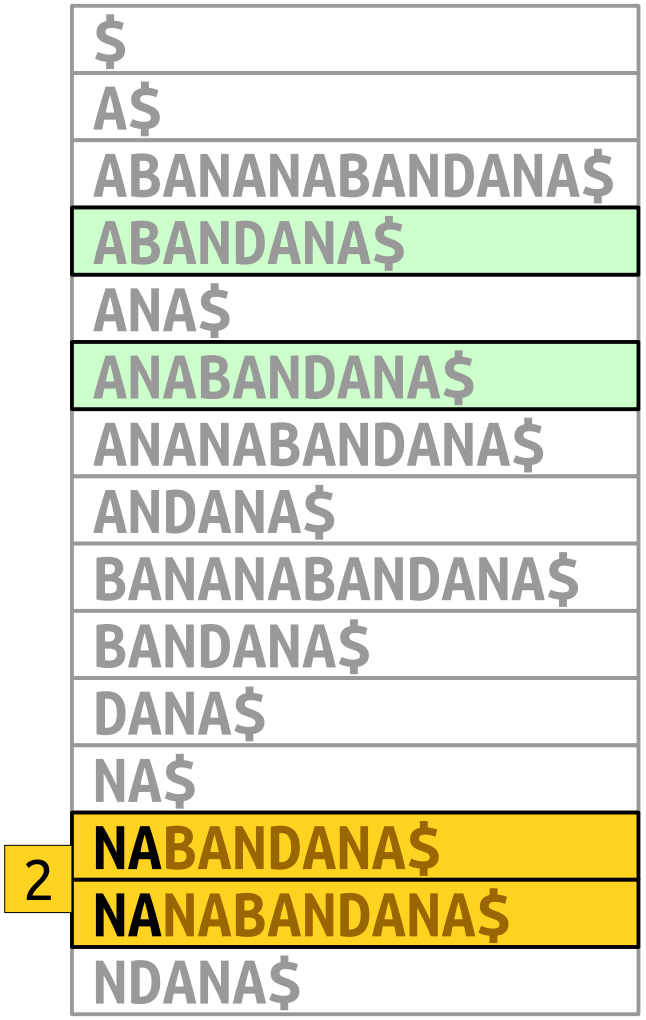

Building LCP Arrays

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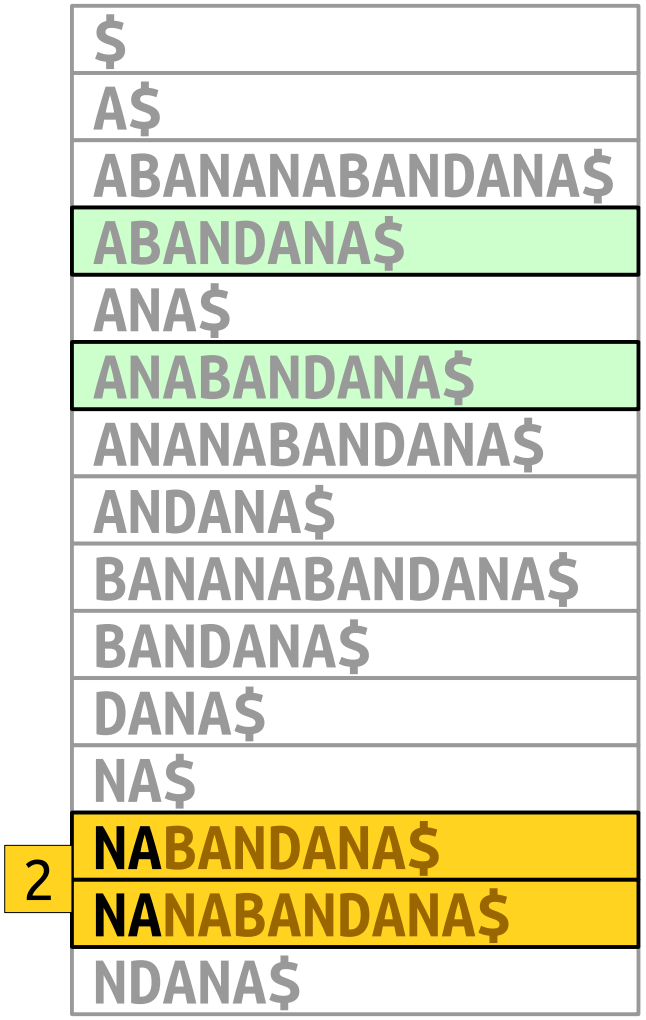
- ***Claim:*** Look at the second suffix in the pair. Its LCP with the suffix before it is at least the previous LCP minus one.



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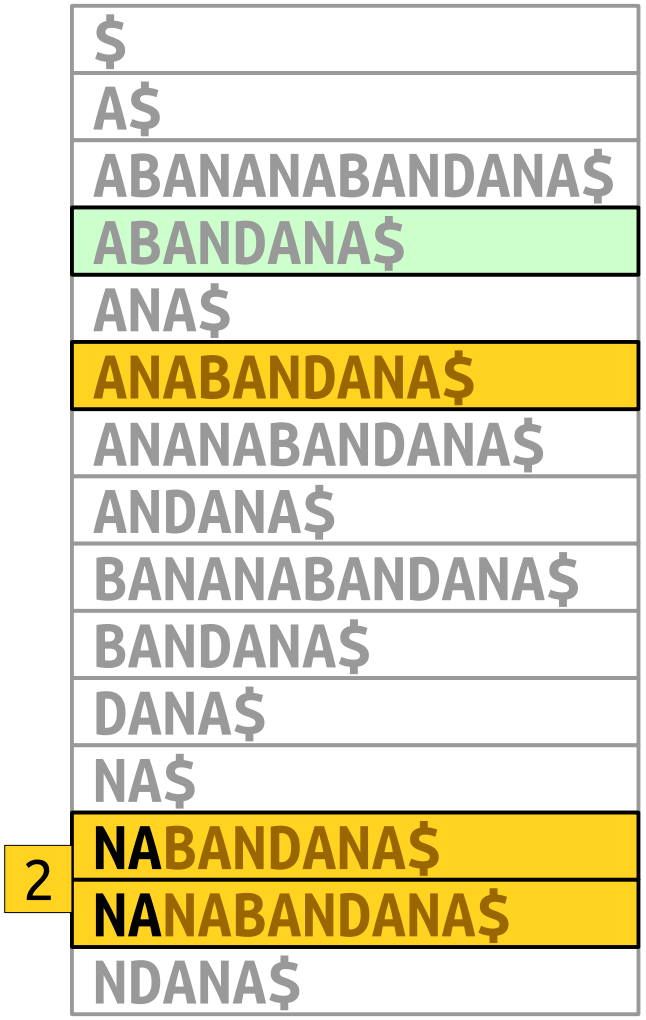

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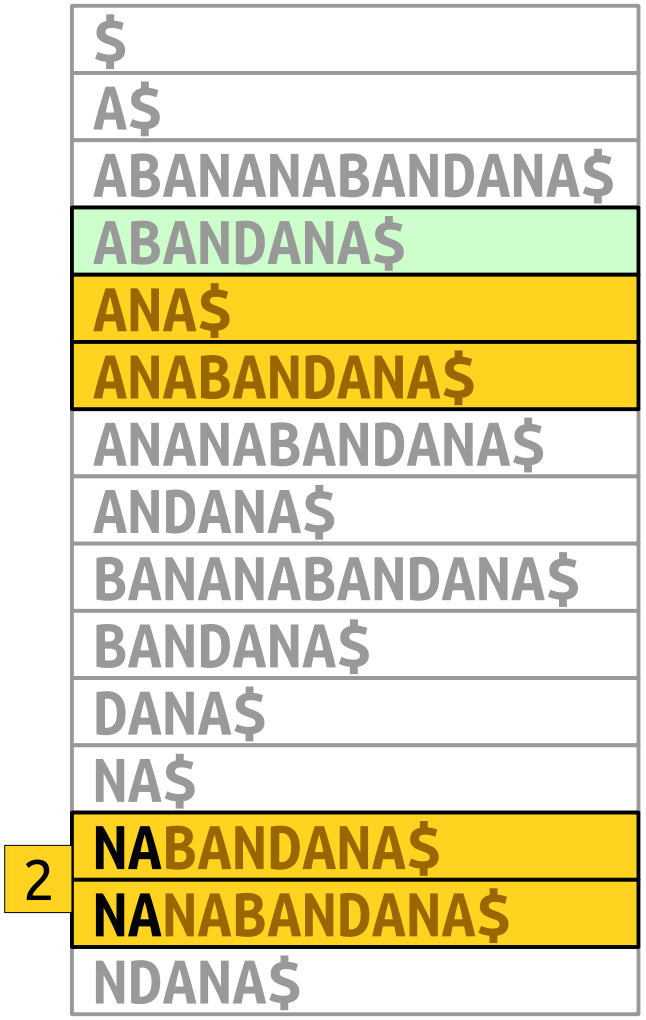

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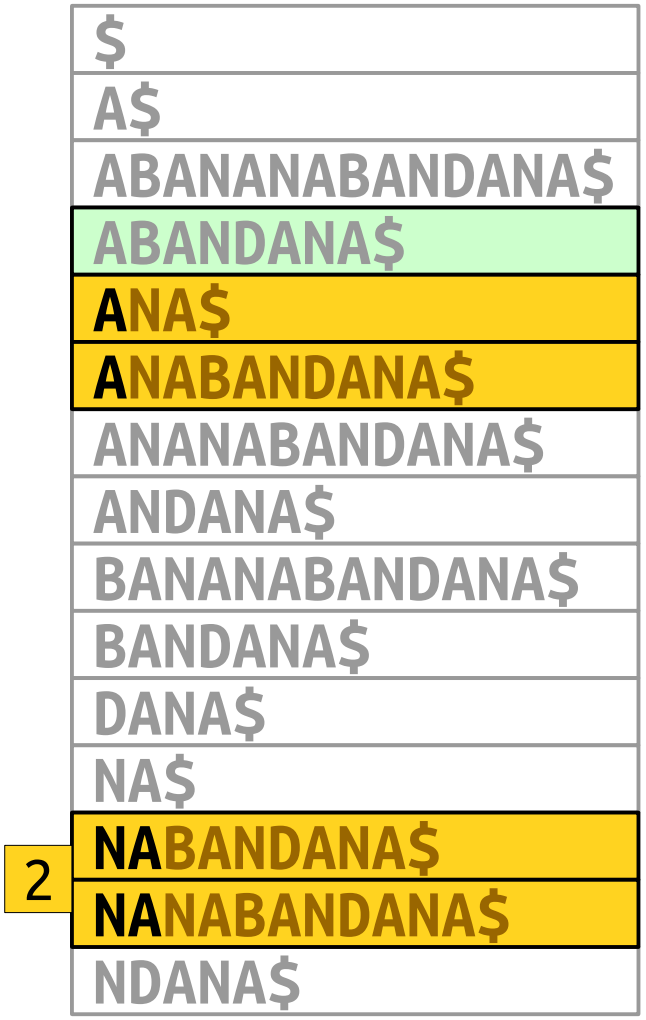

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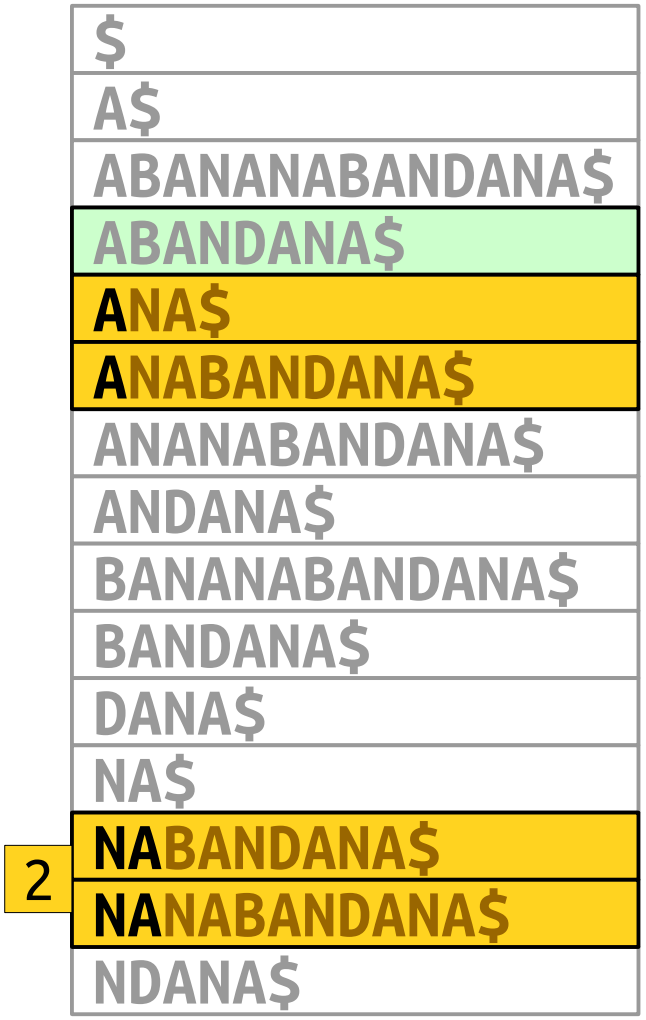

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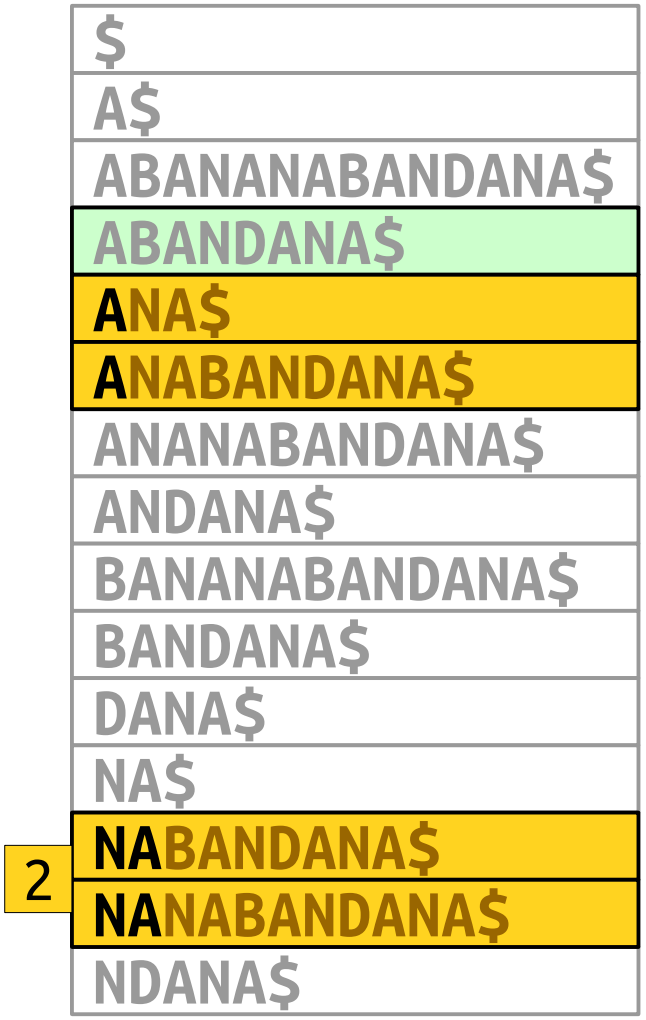

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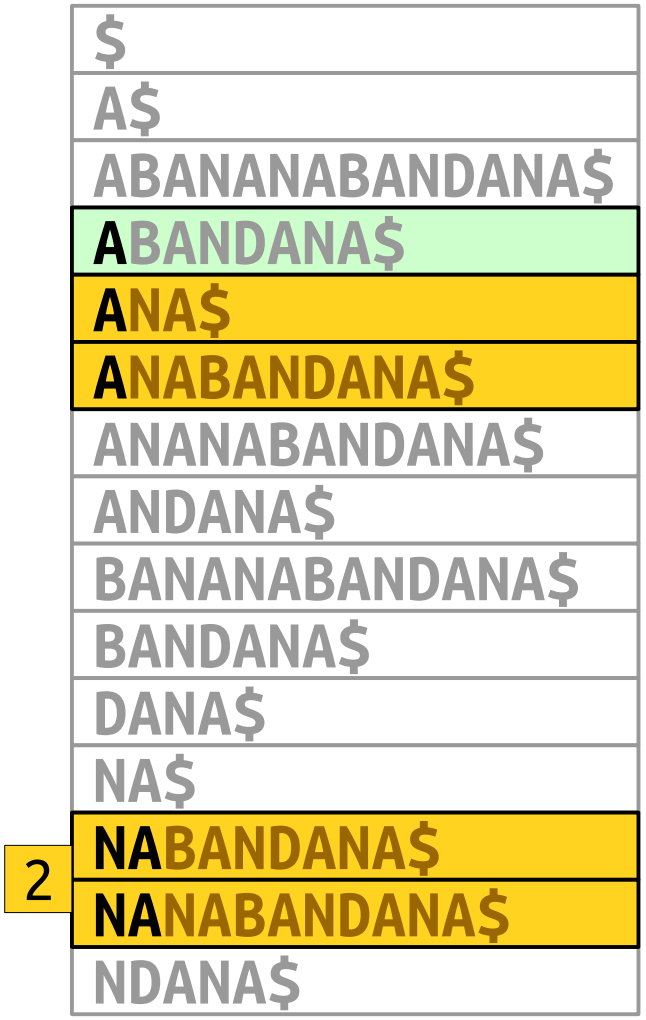

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- We know that these two new suffixes must have an LCP of at least 1, because the two old suffixes have an LCP of 2.
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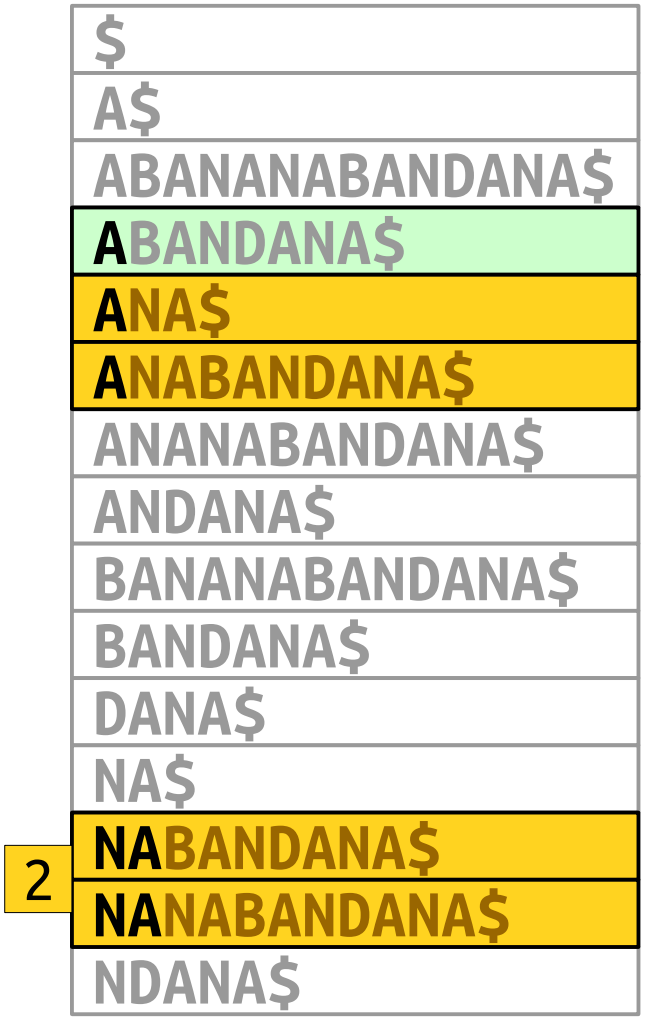

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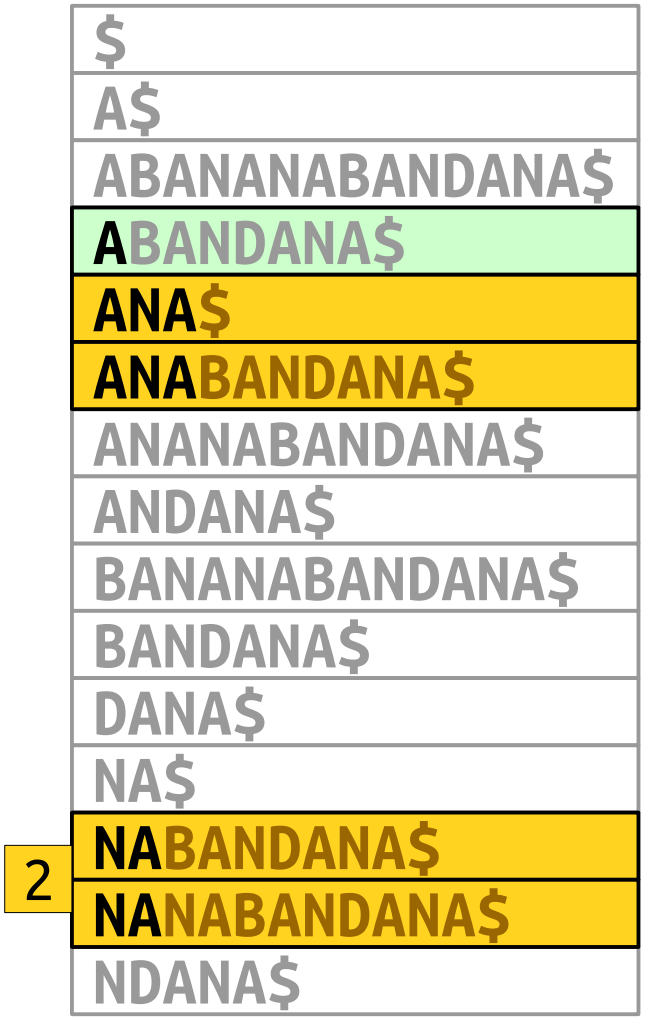

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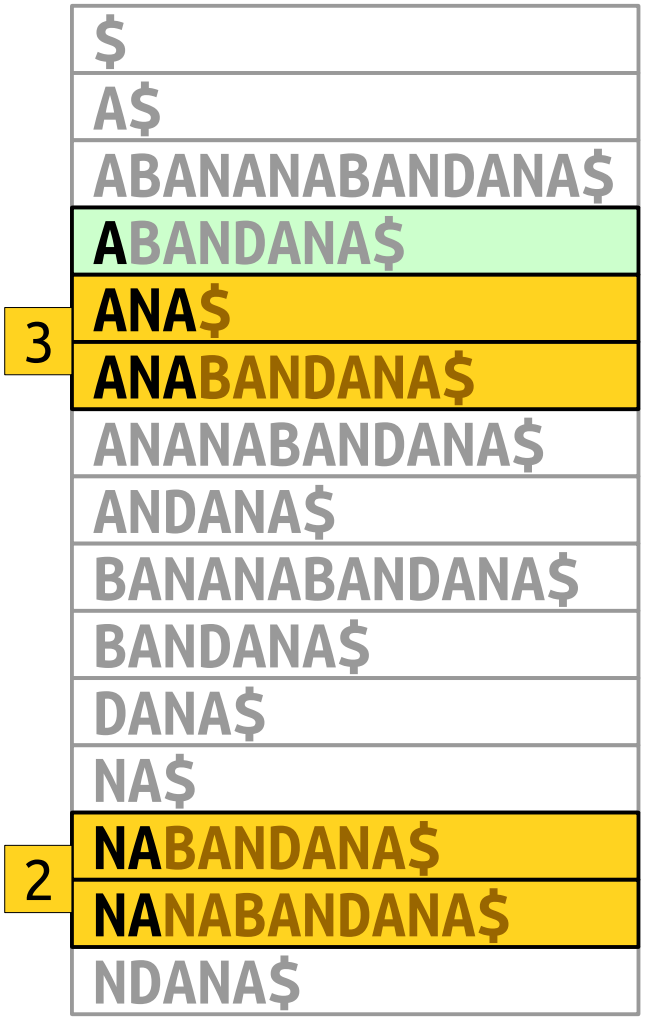

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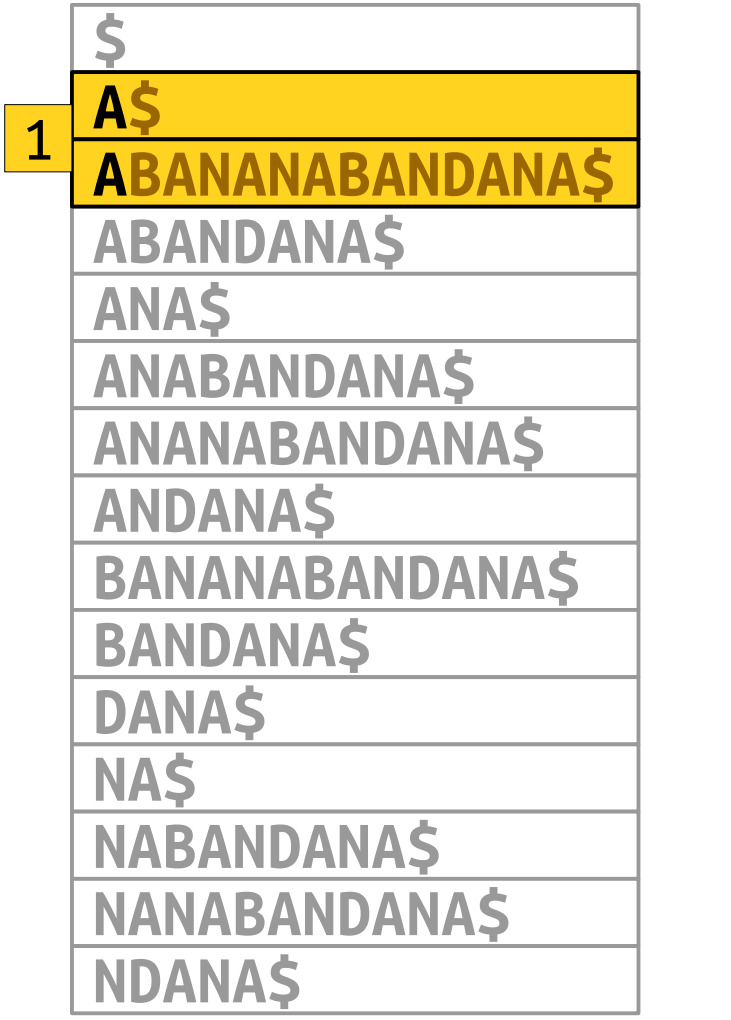

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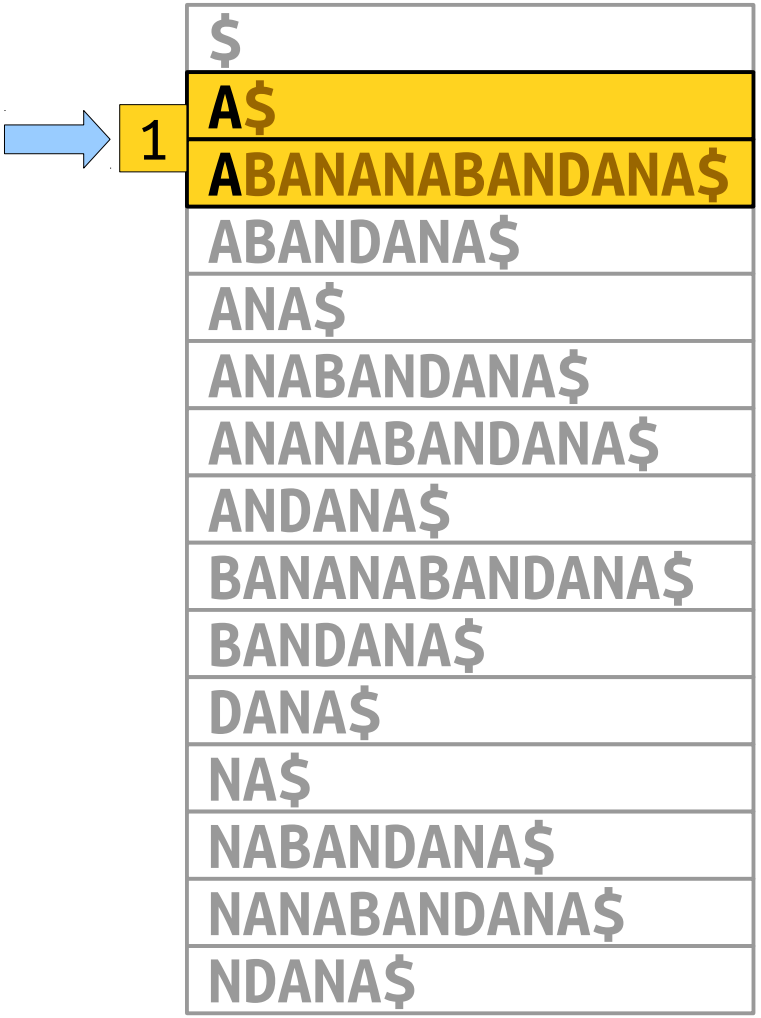

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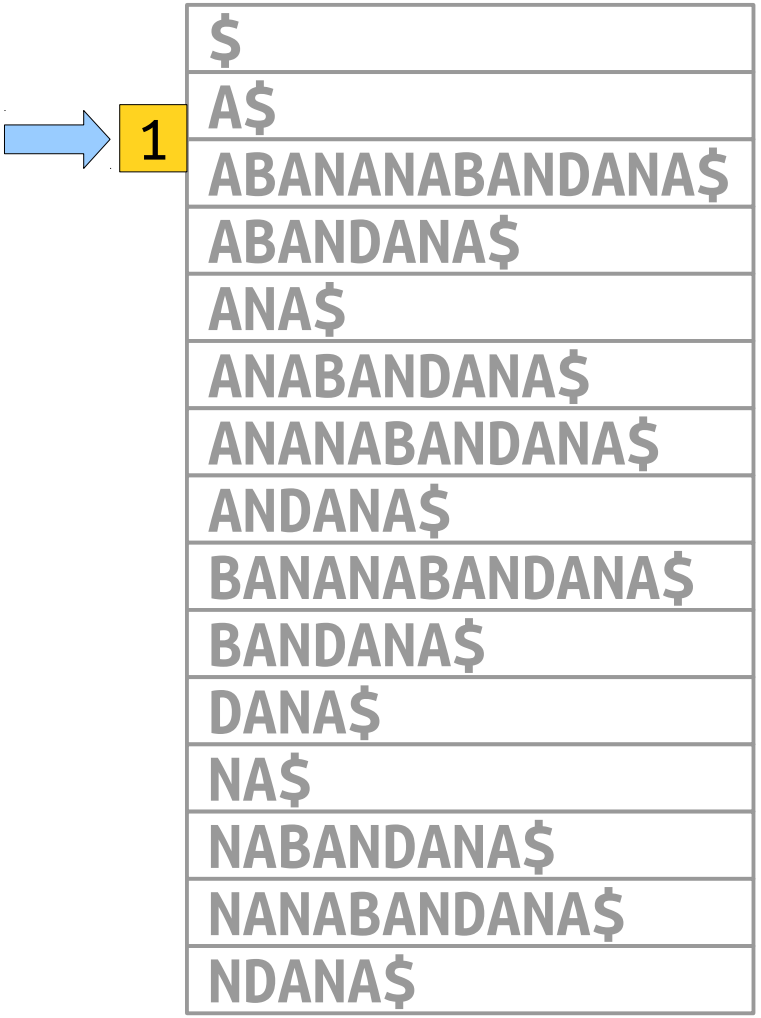

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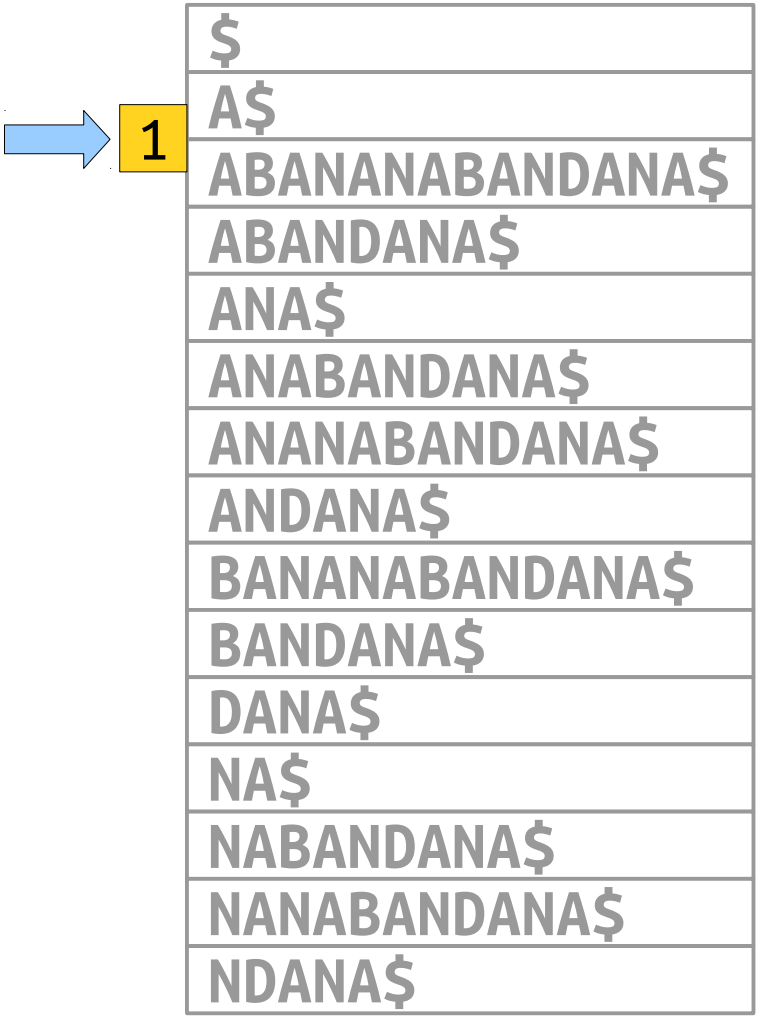

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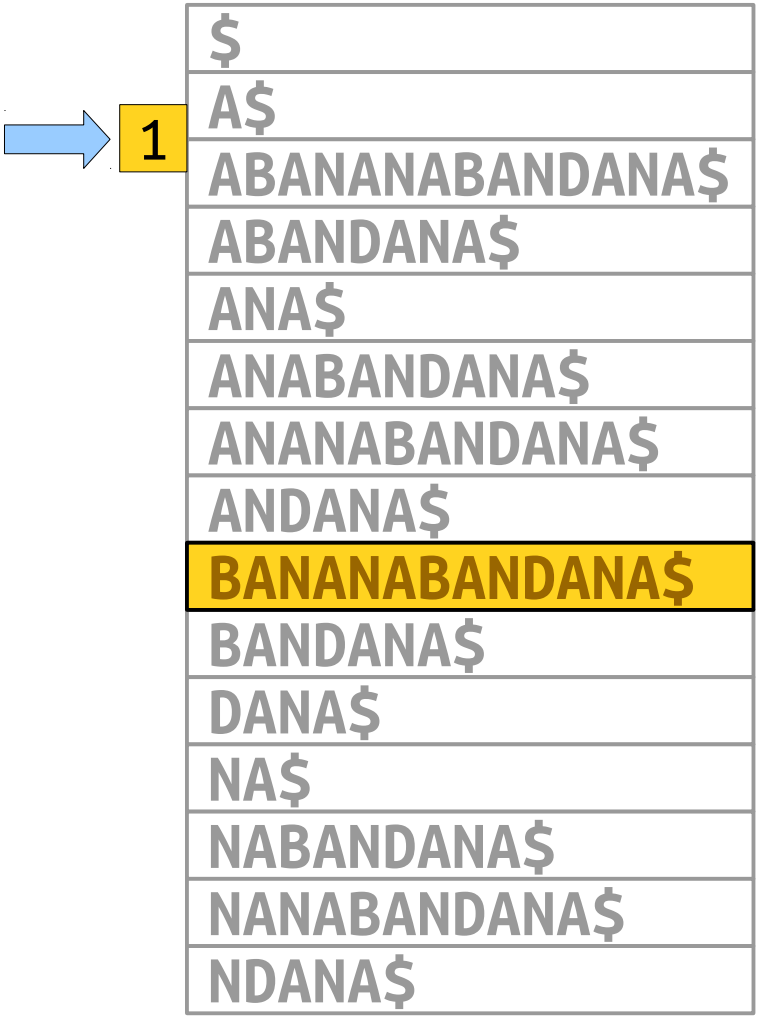

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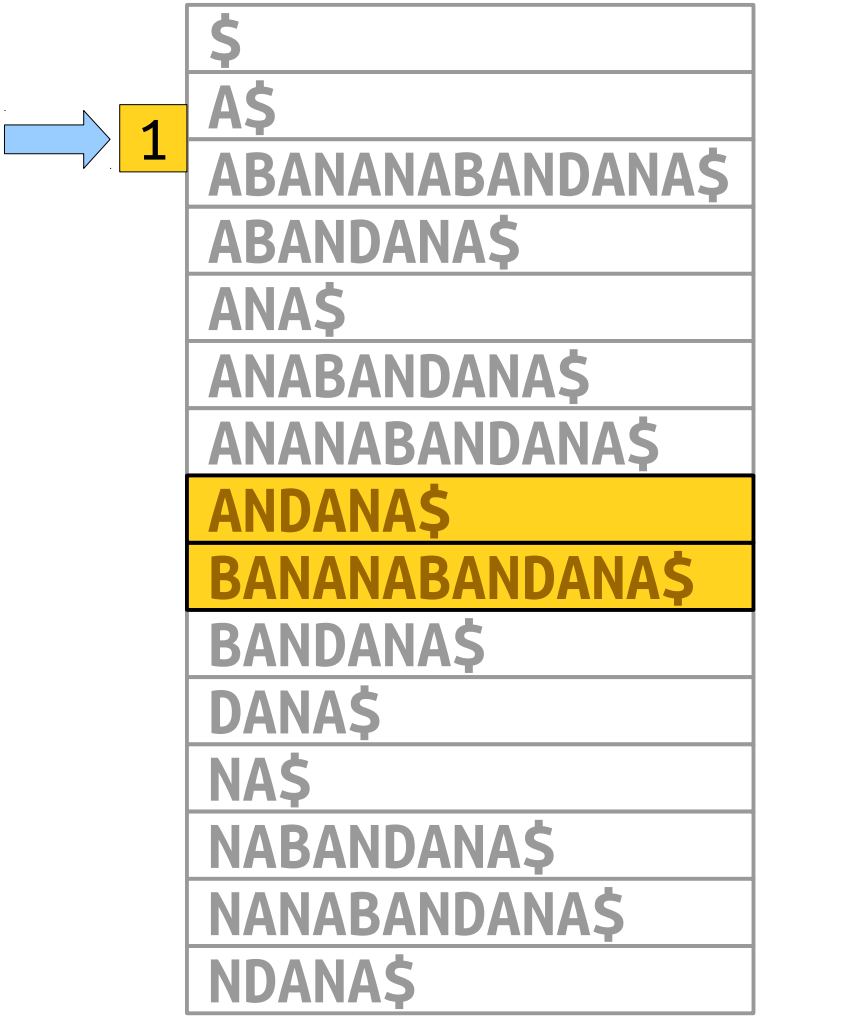

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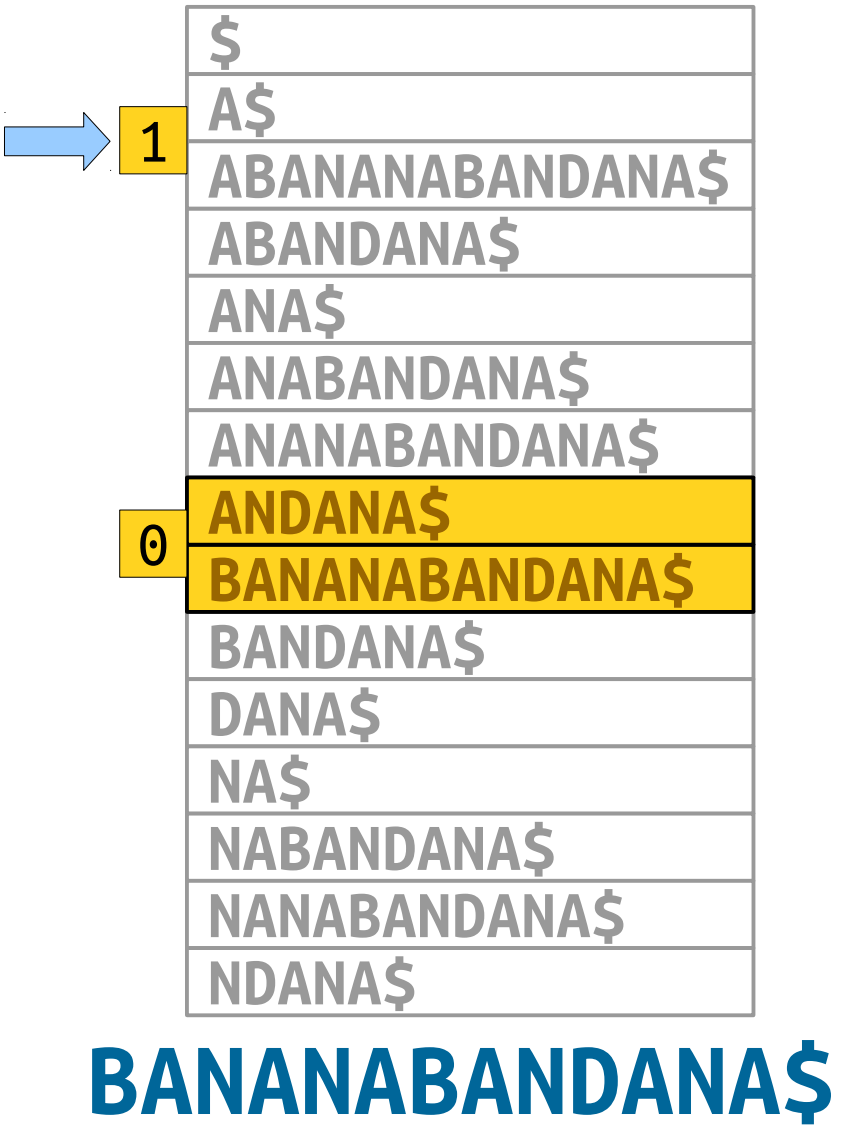

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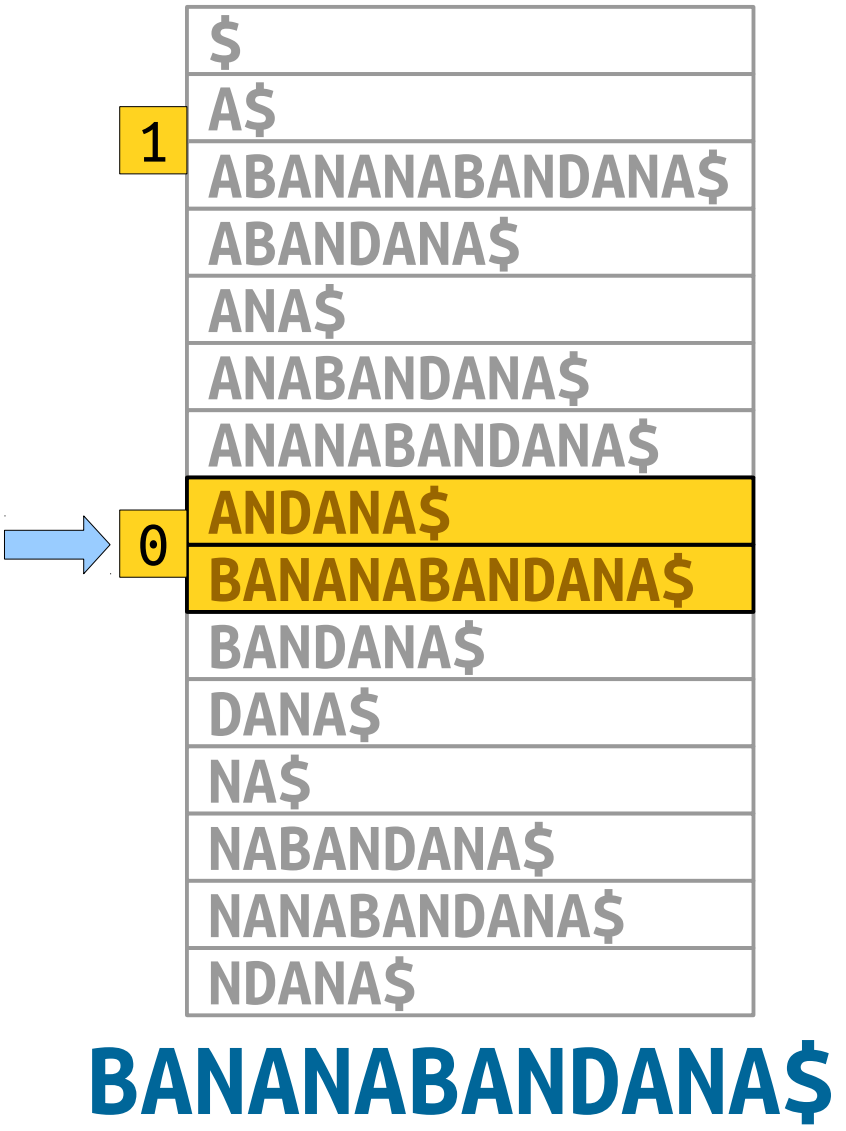

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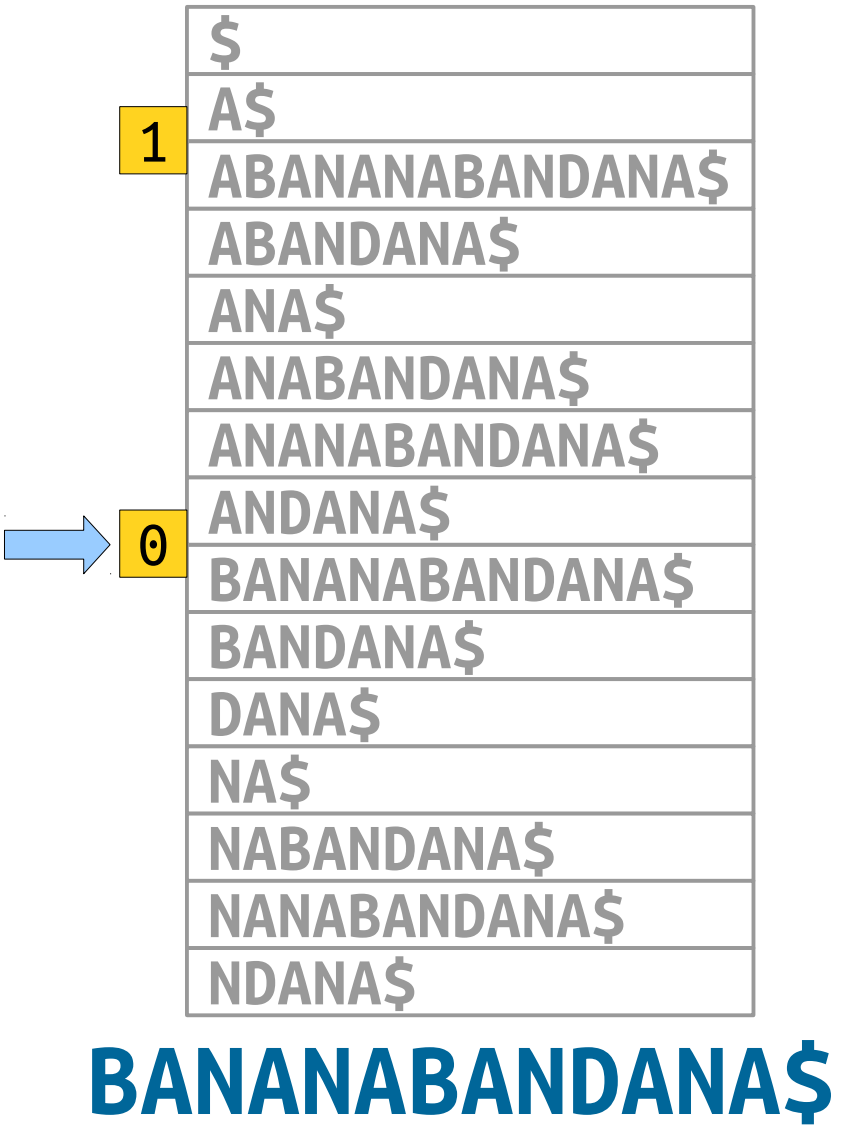

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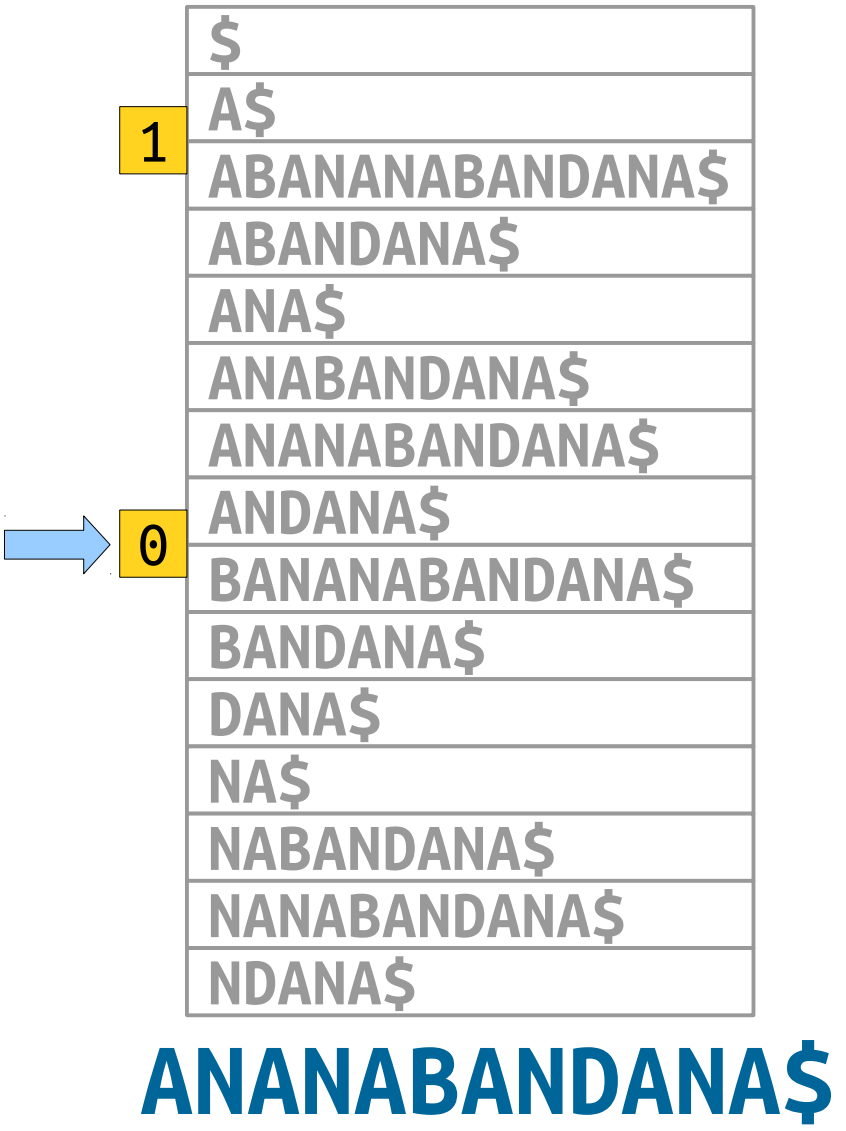

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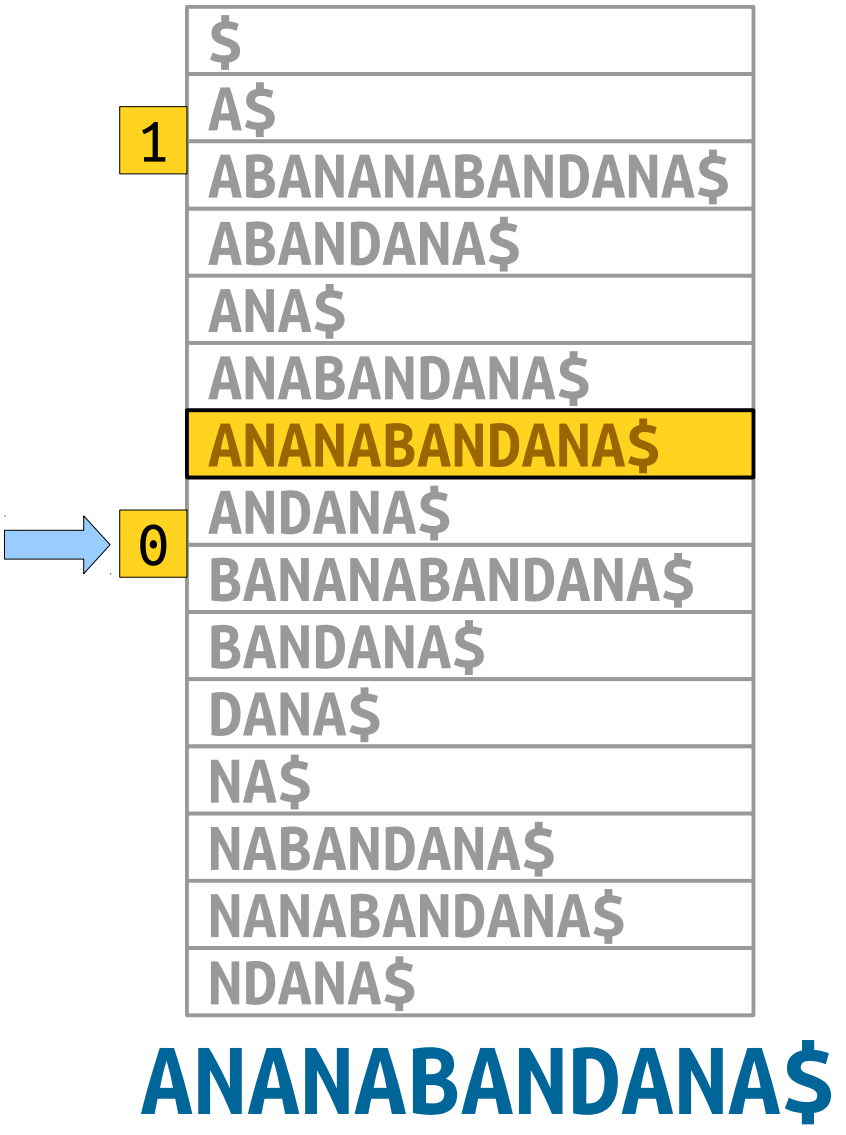

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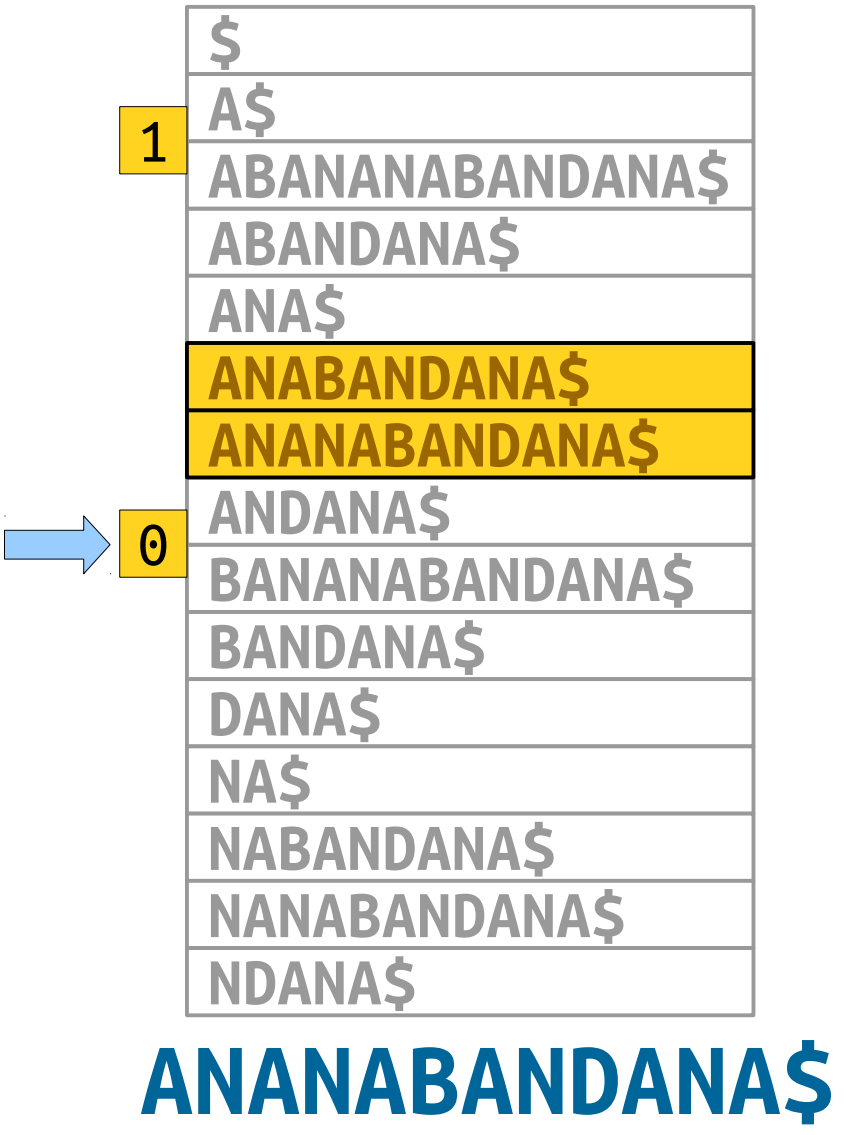

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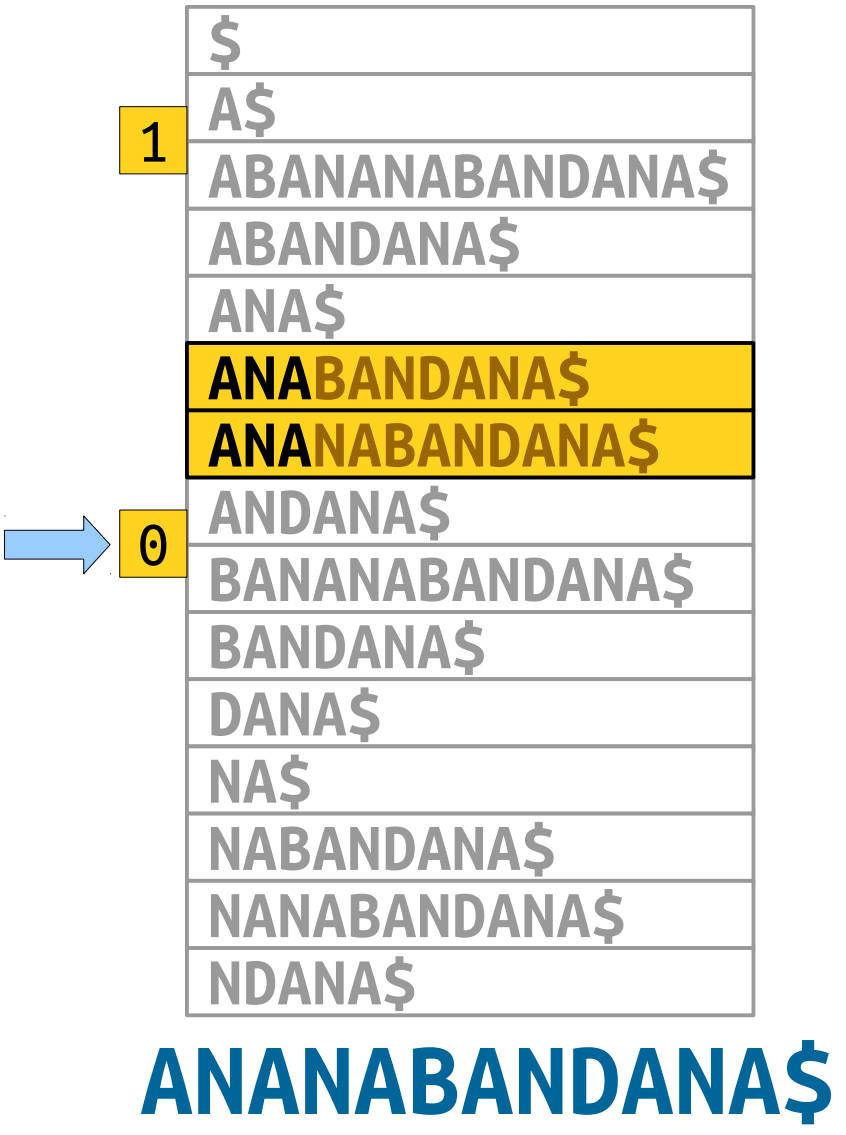

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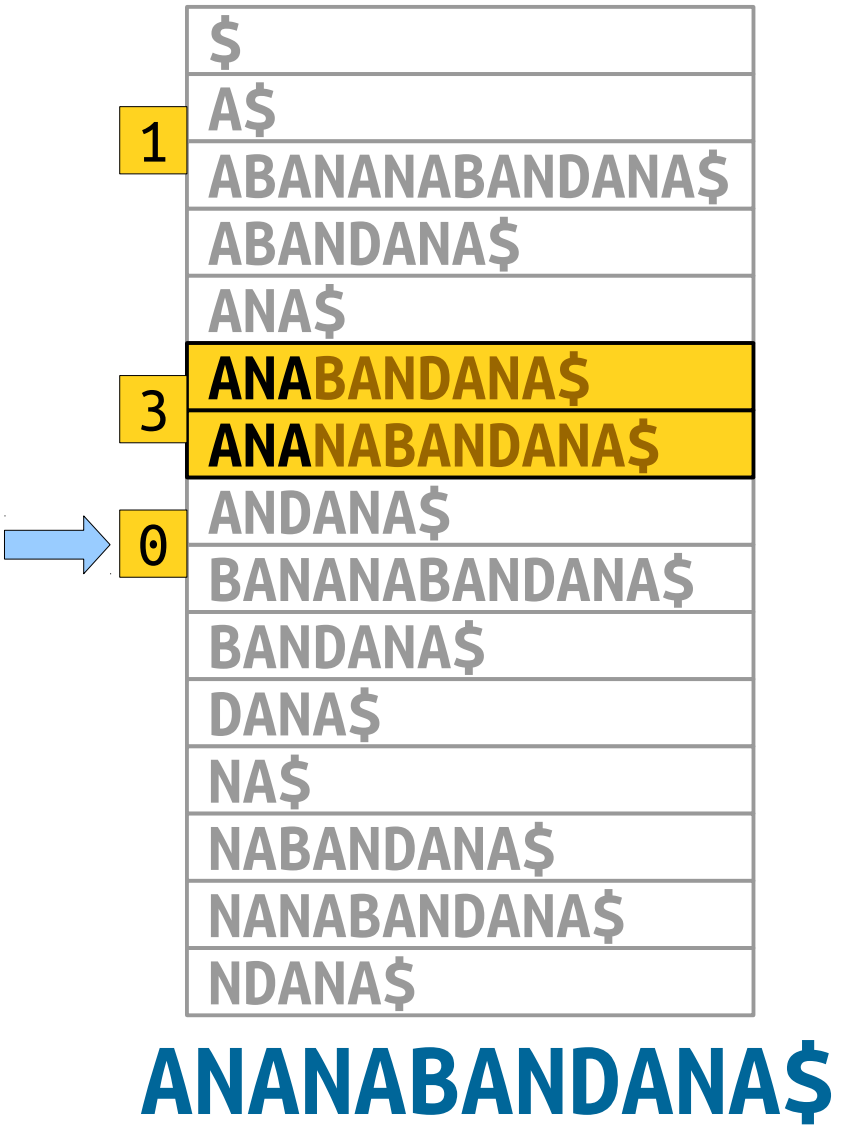

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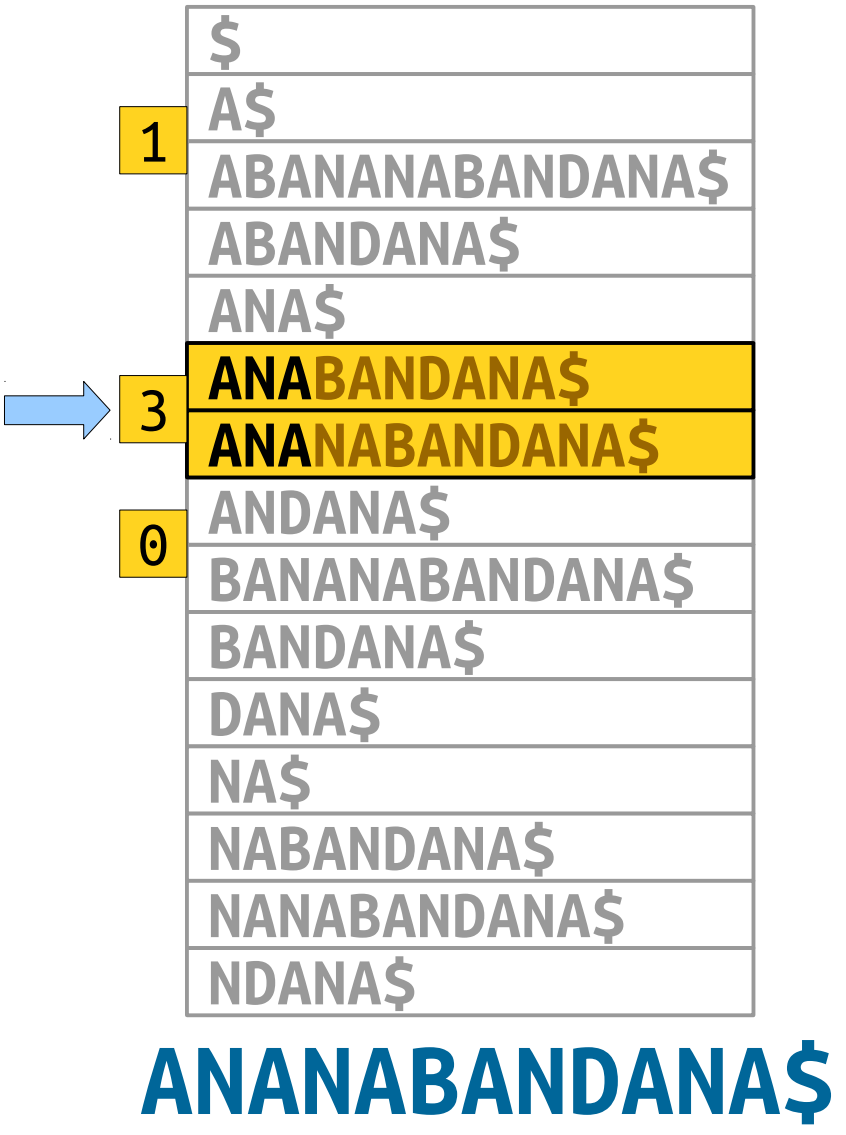

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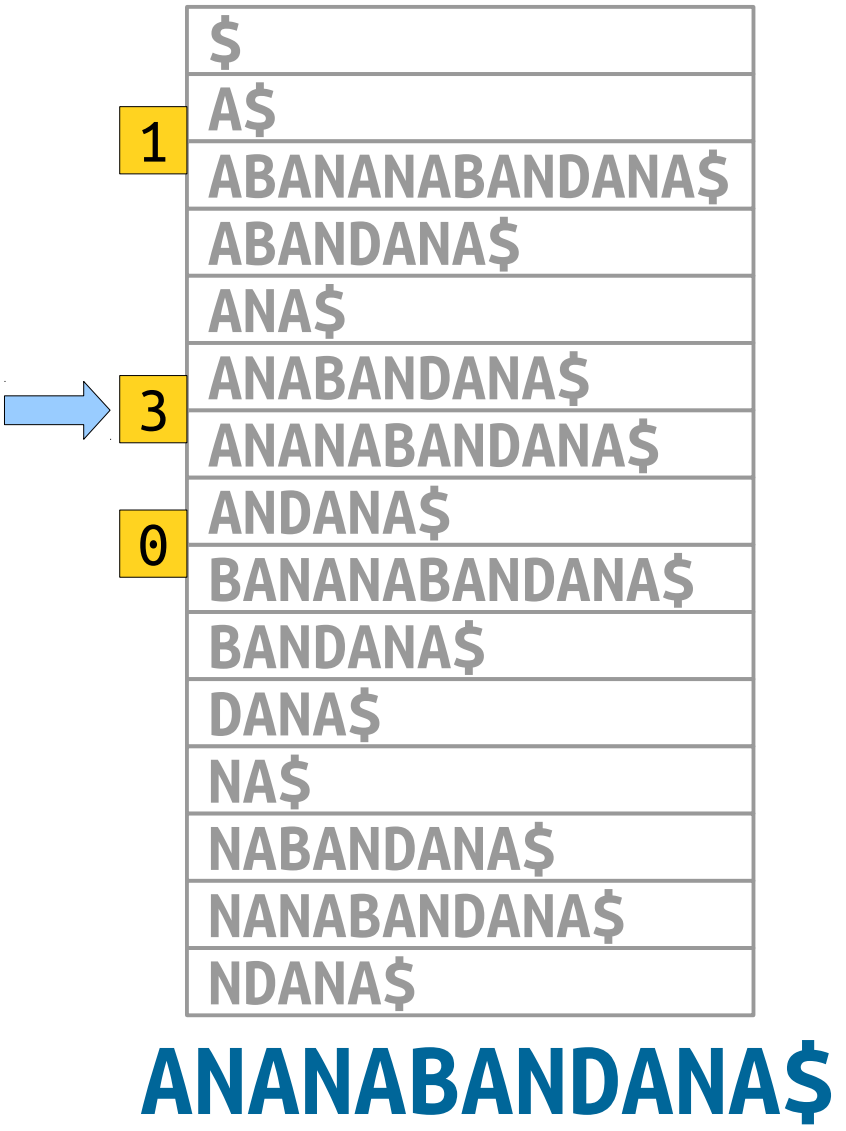

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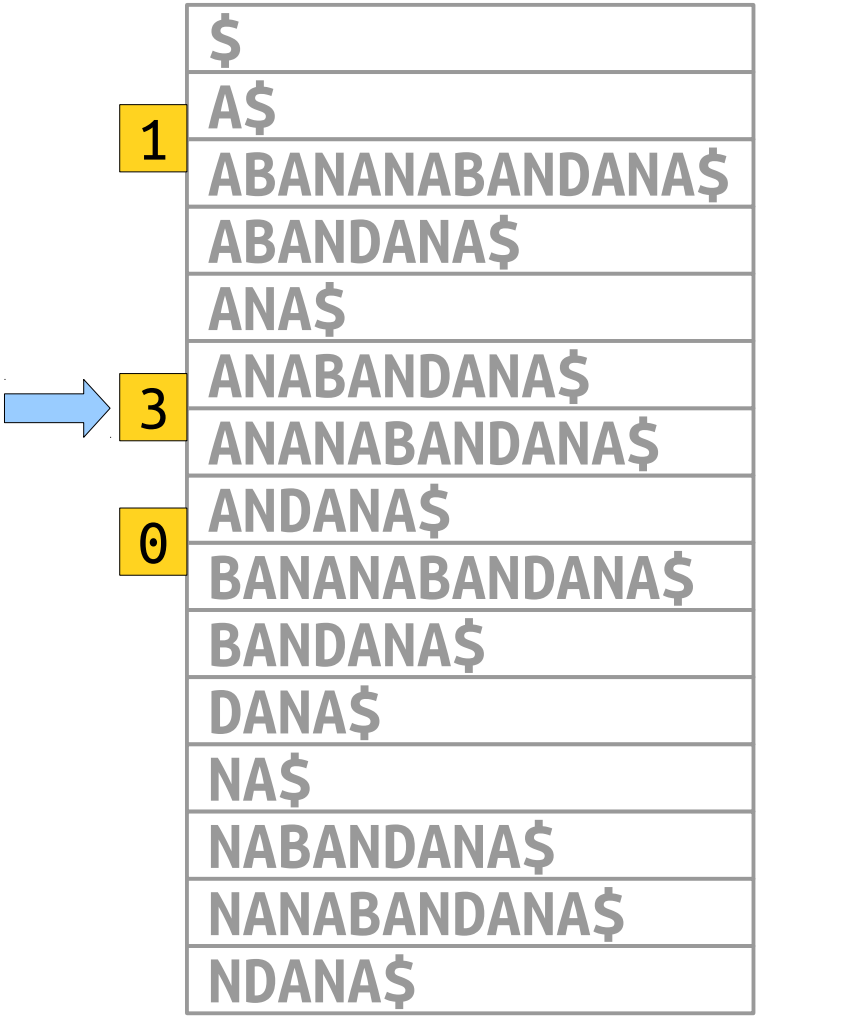

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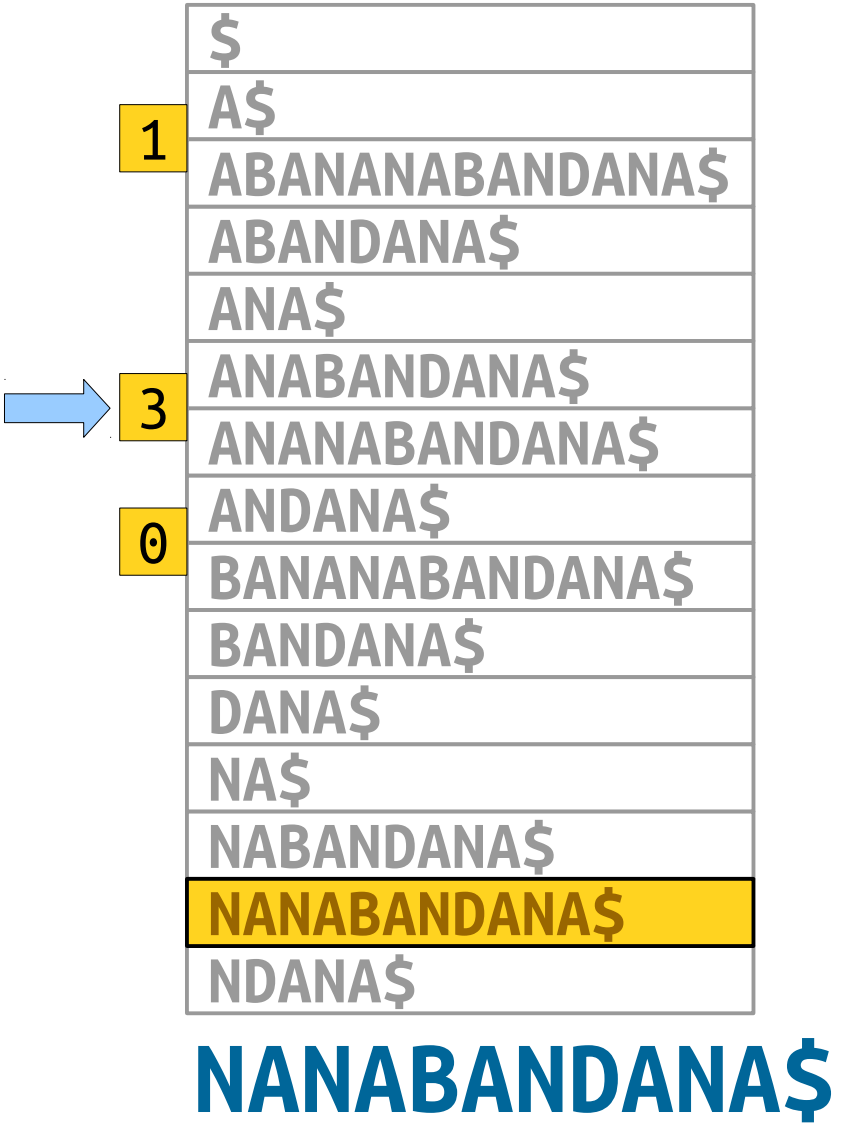

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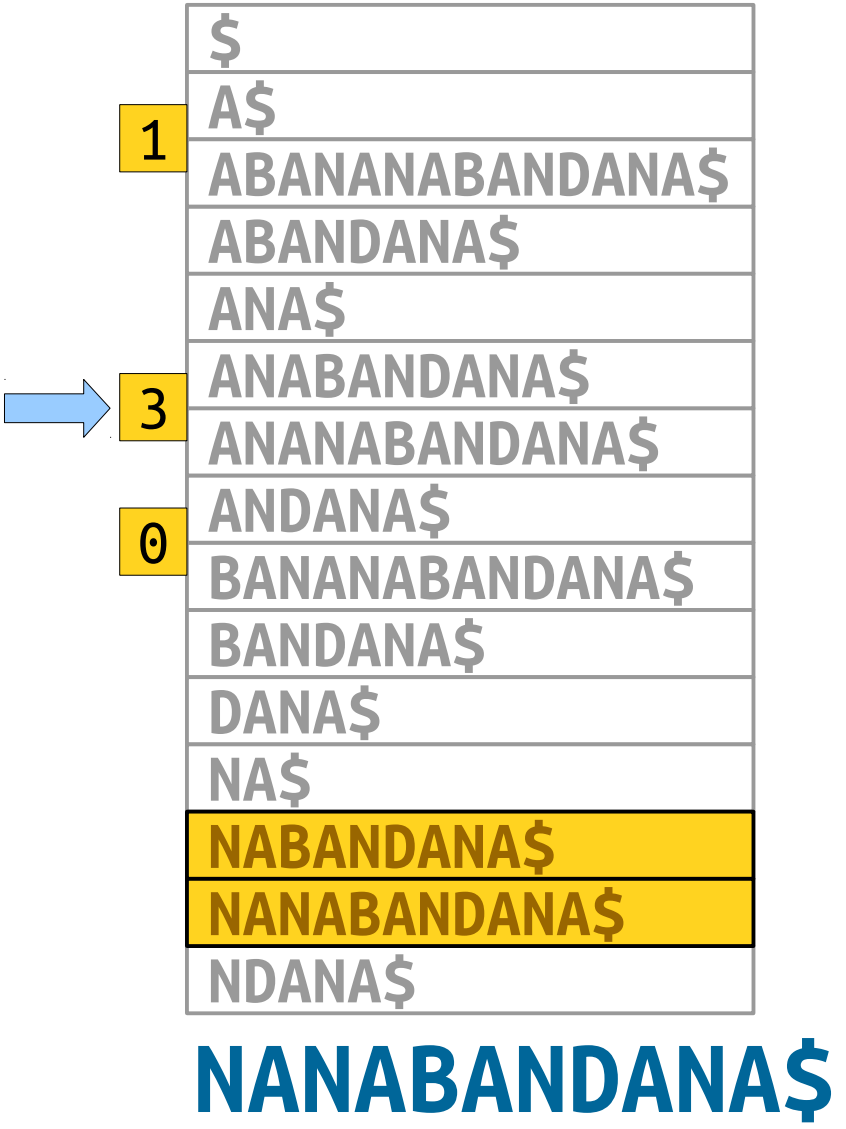

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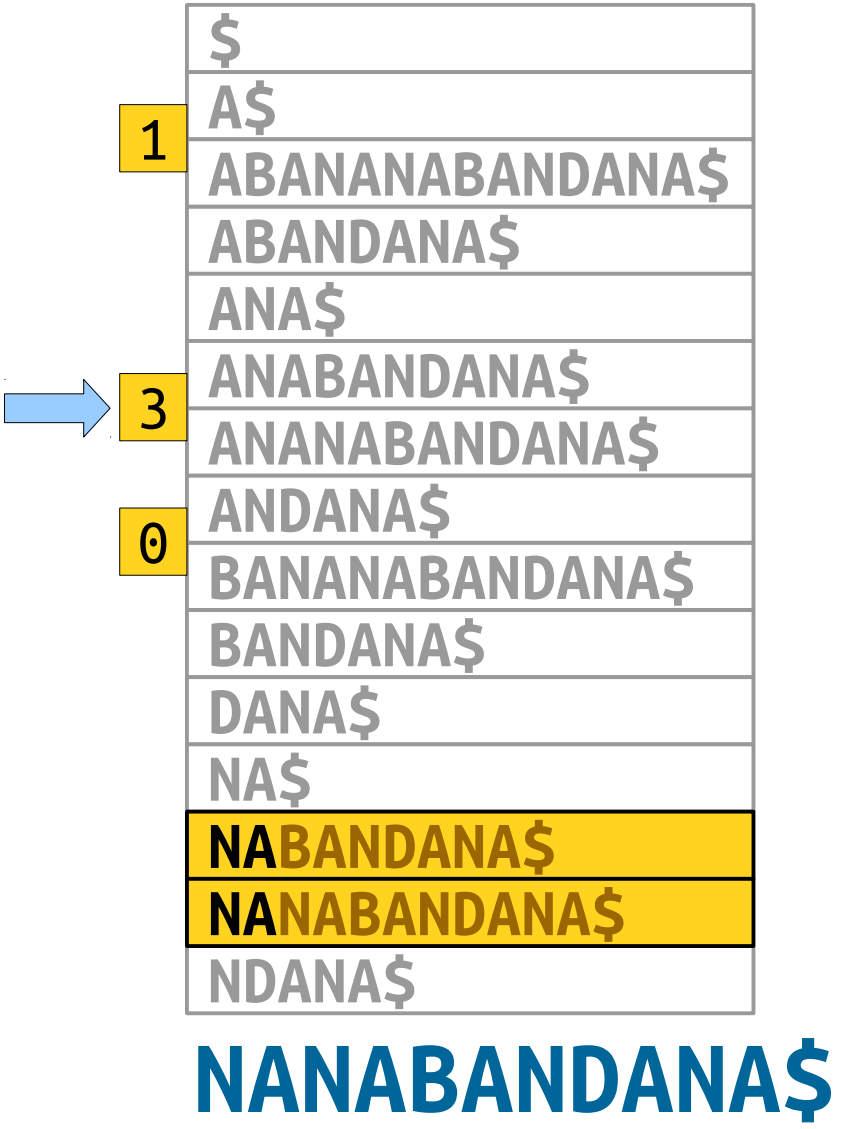

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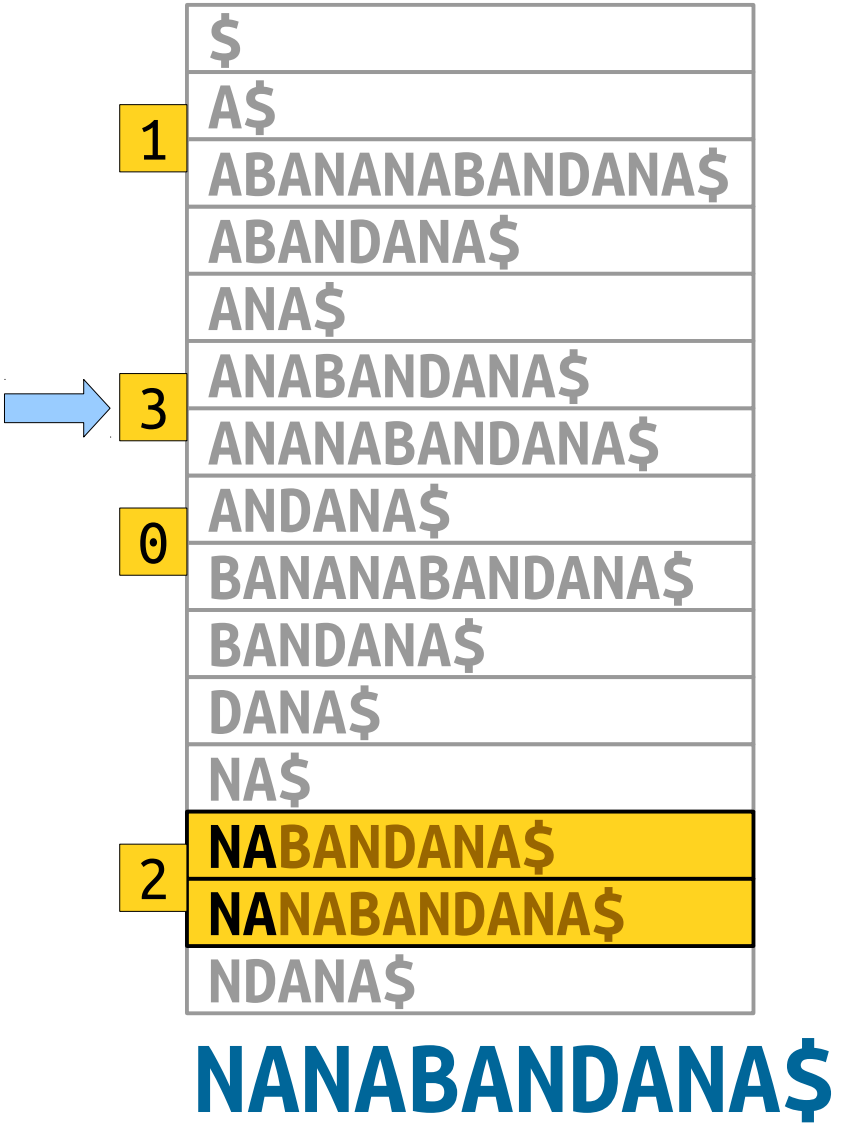

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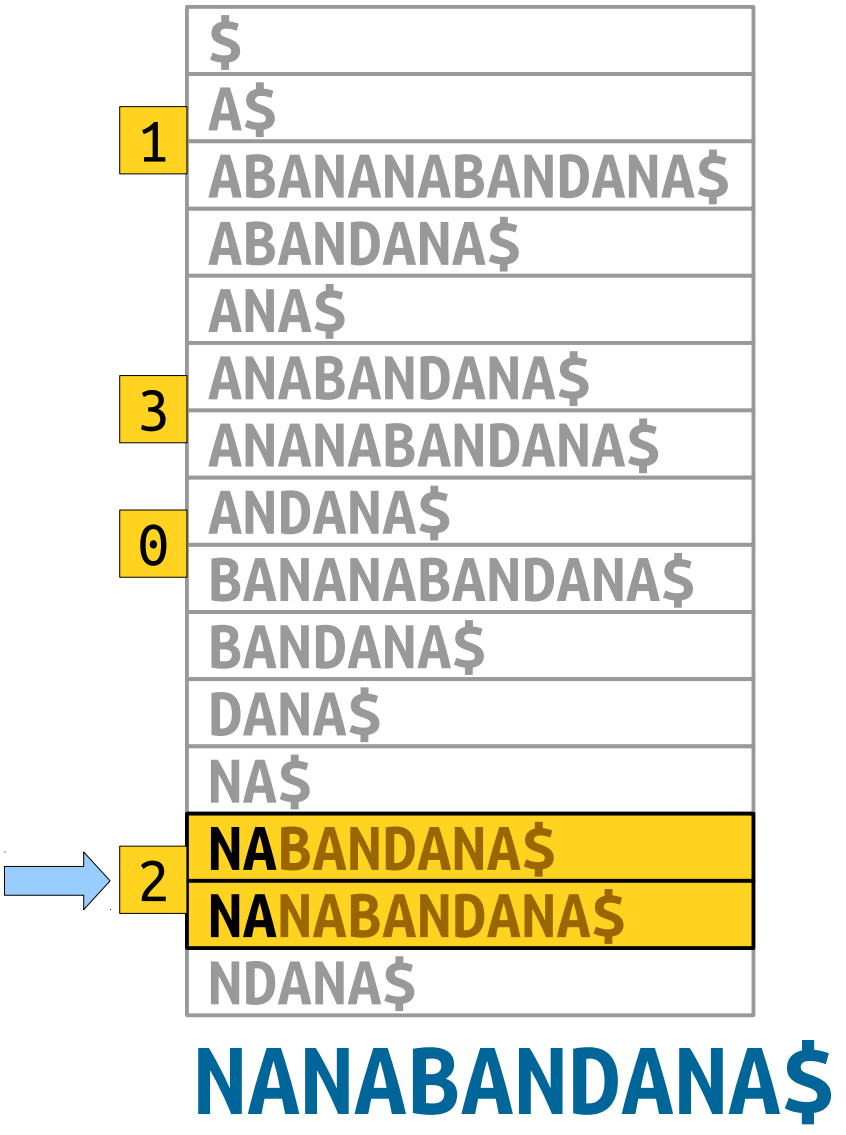

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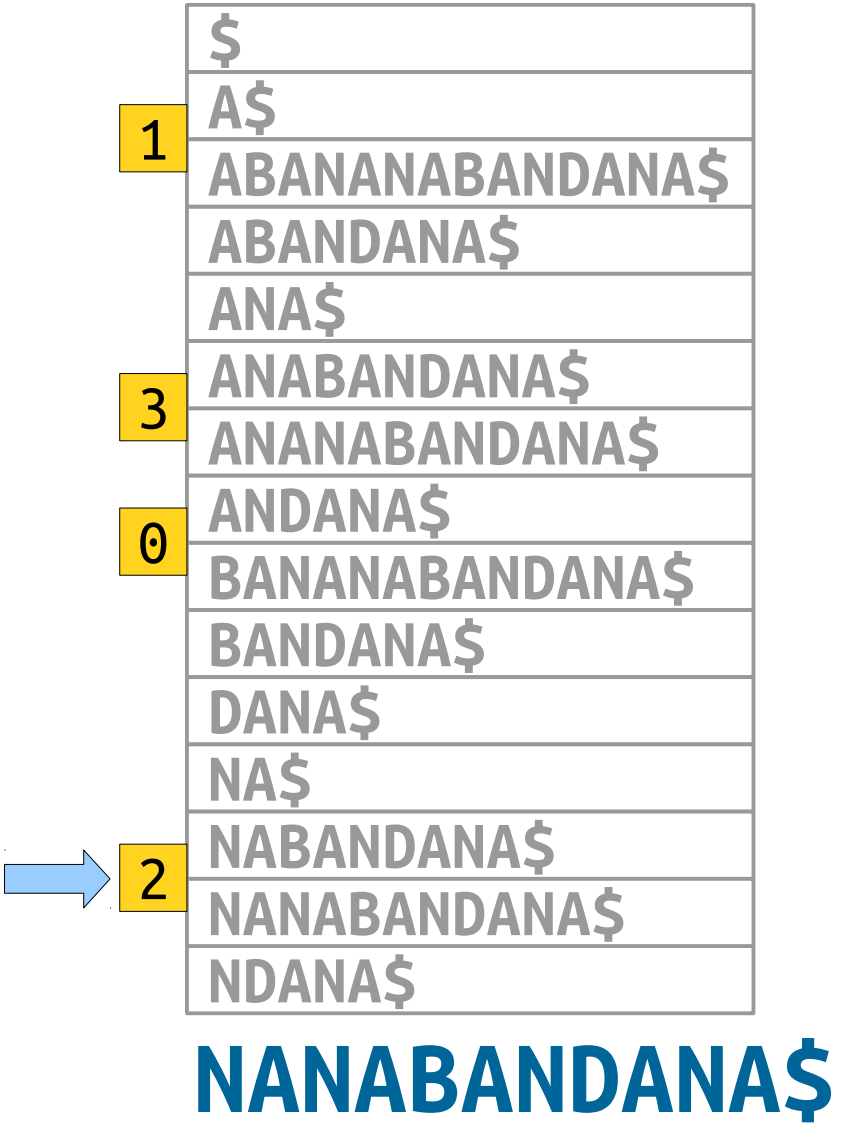

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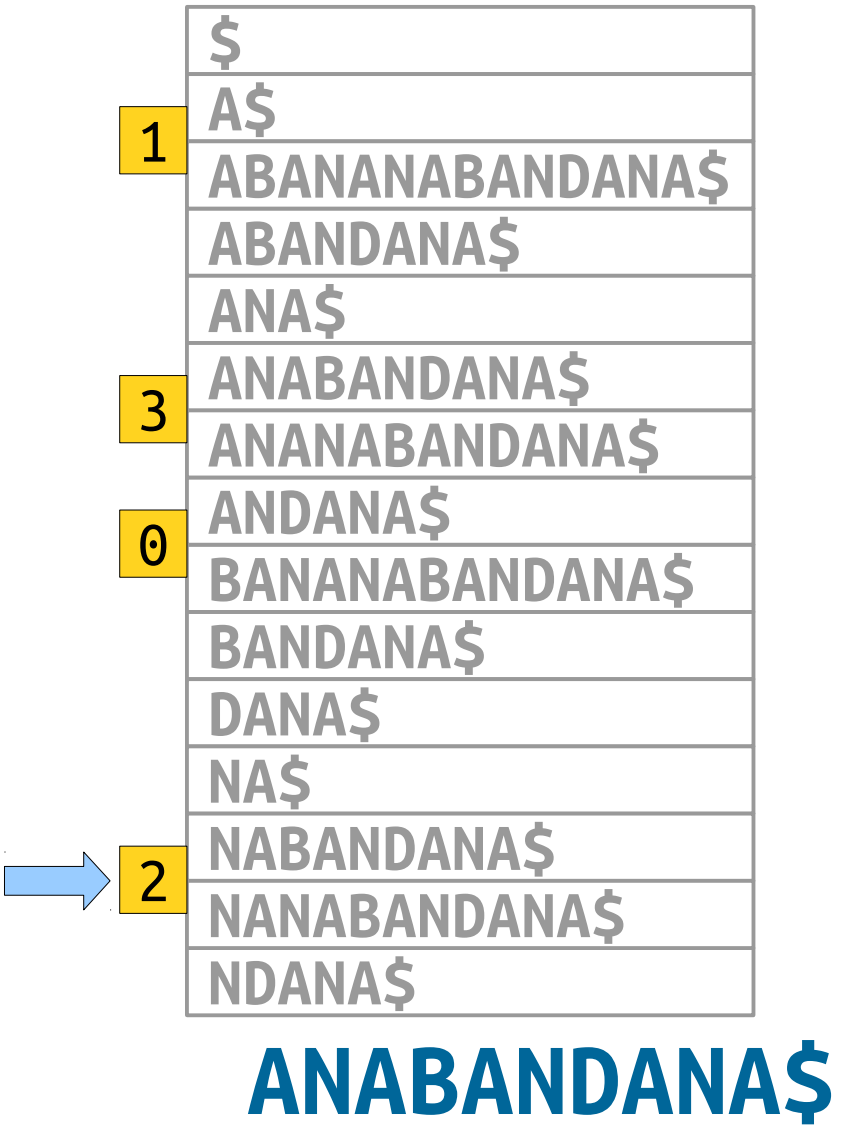

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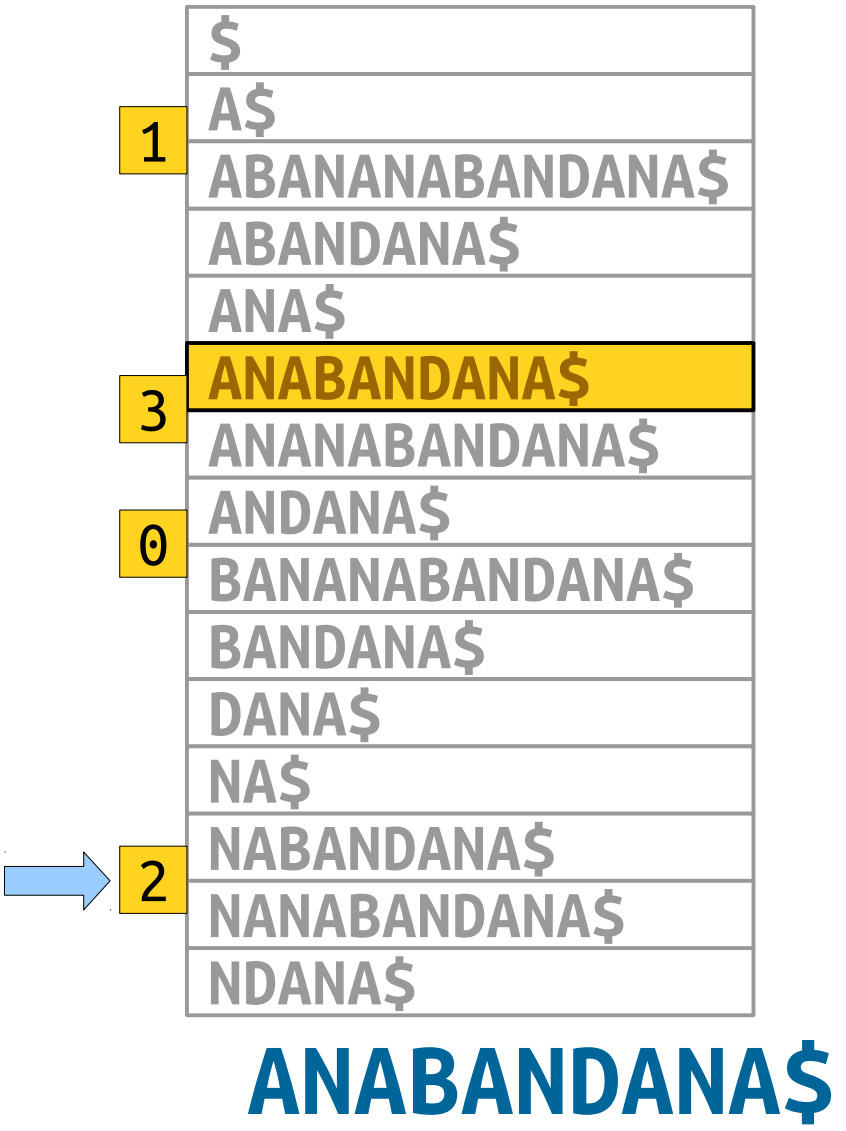

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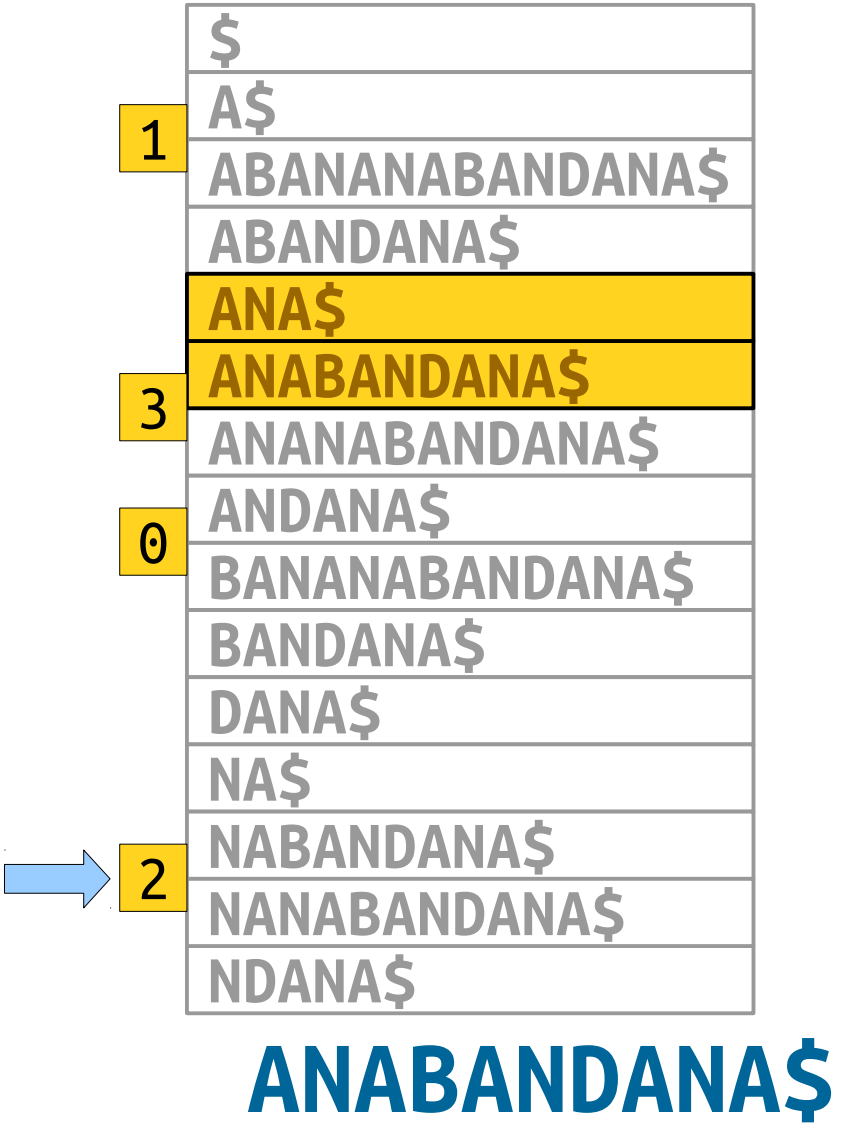

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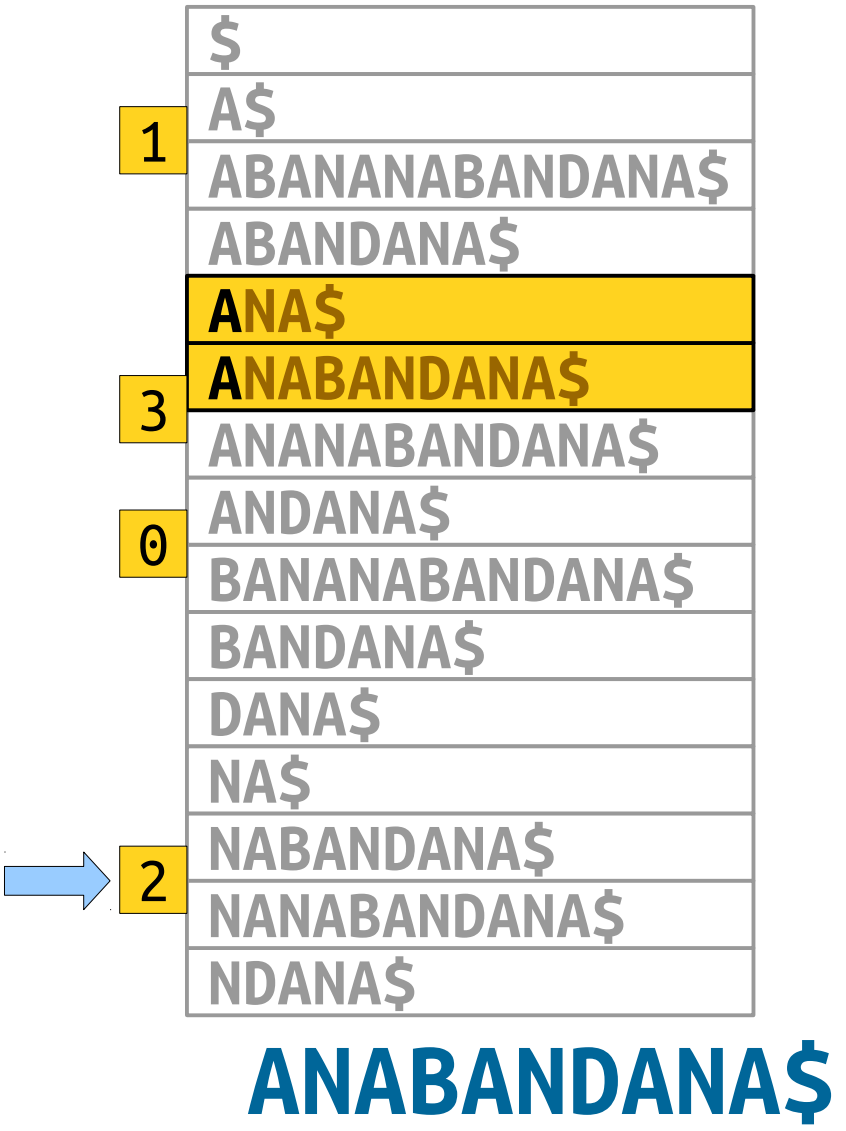

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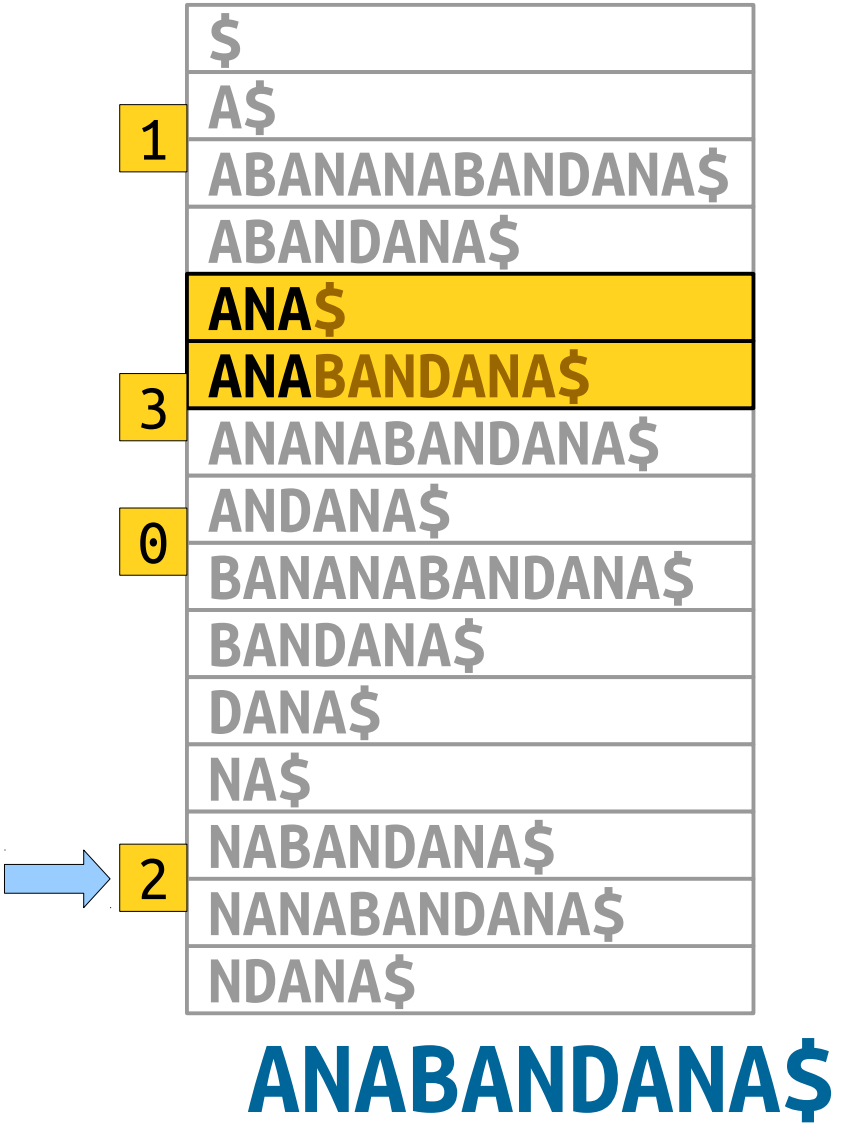

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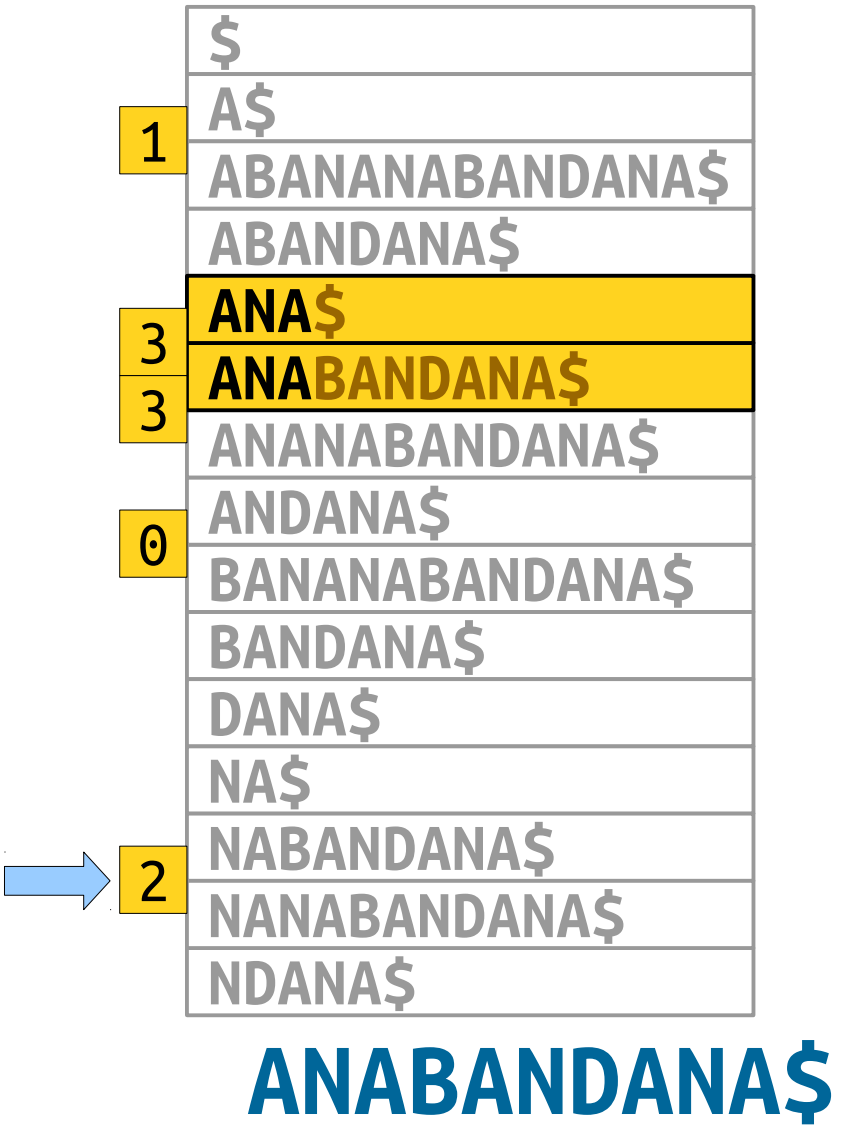

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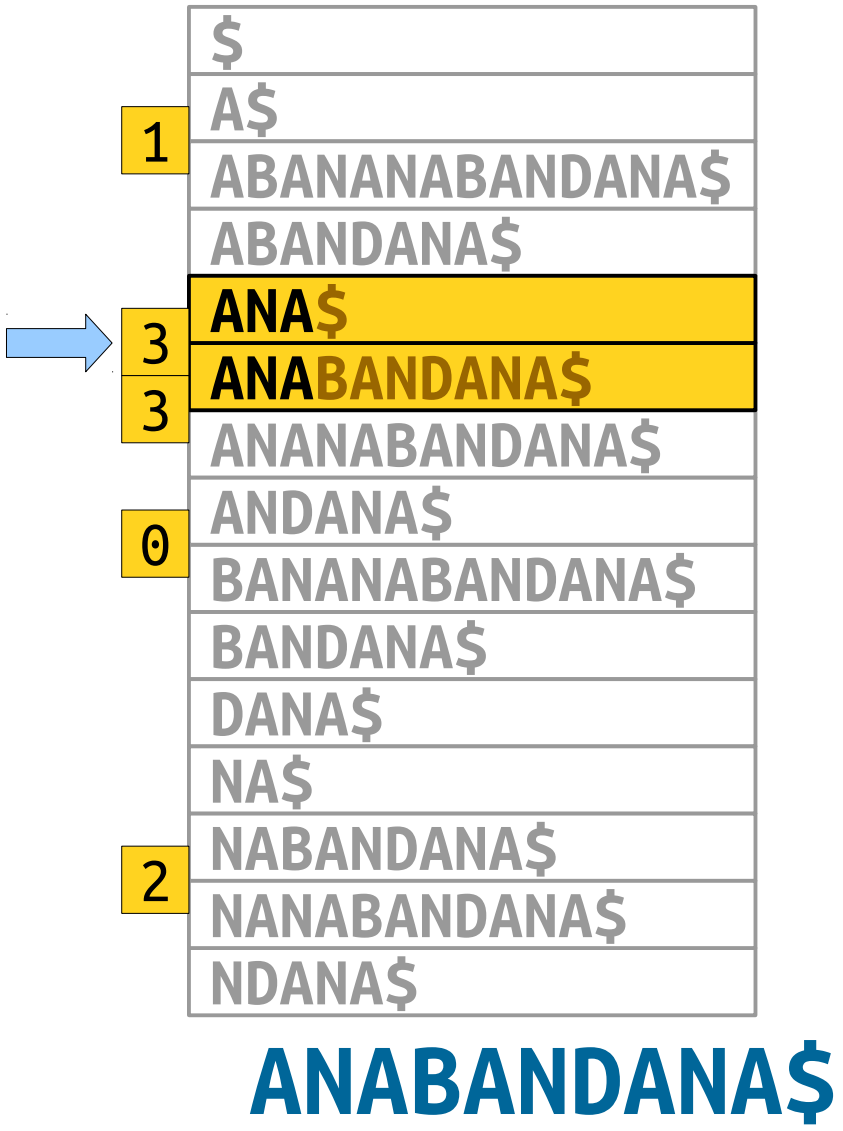

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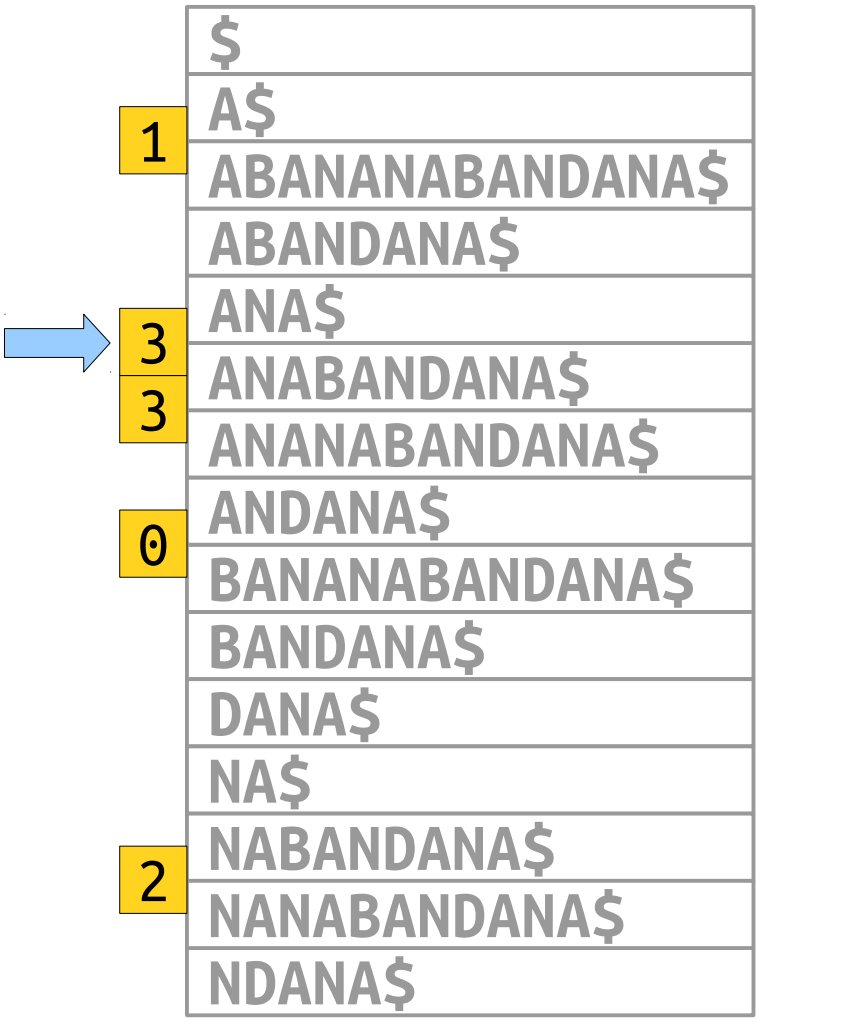

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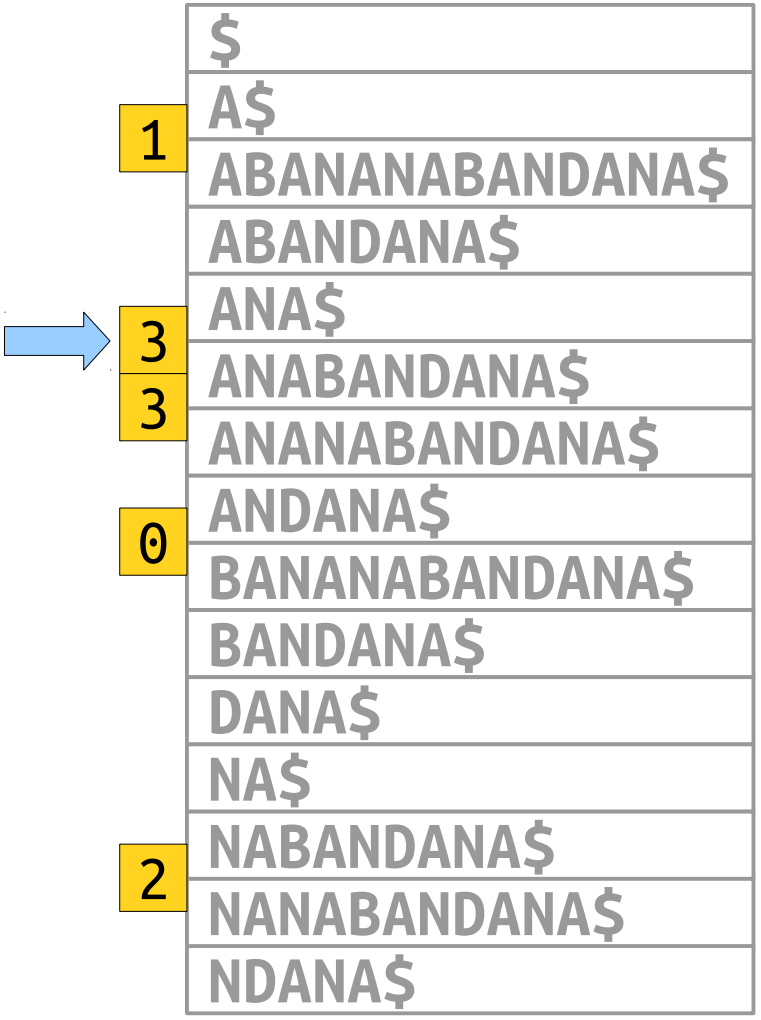

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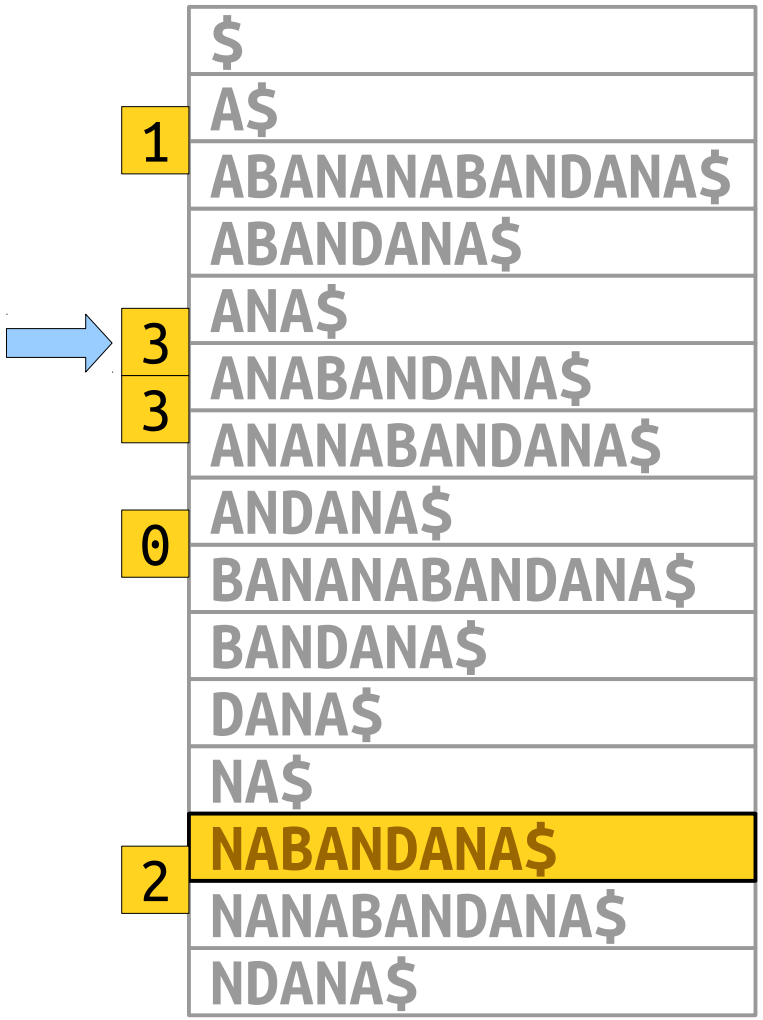

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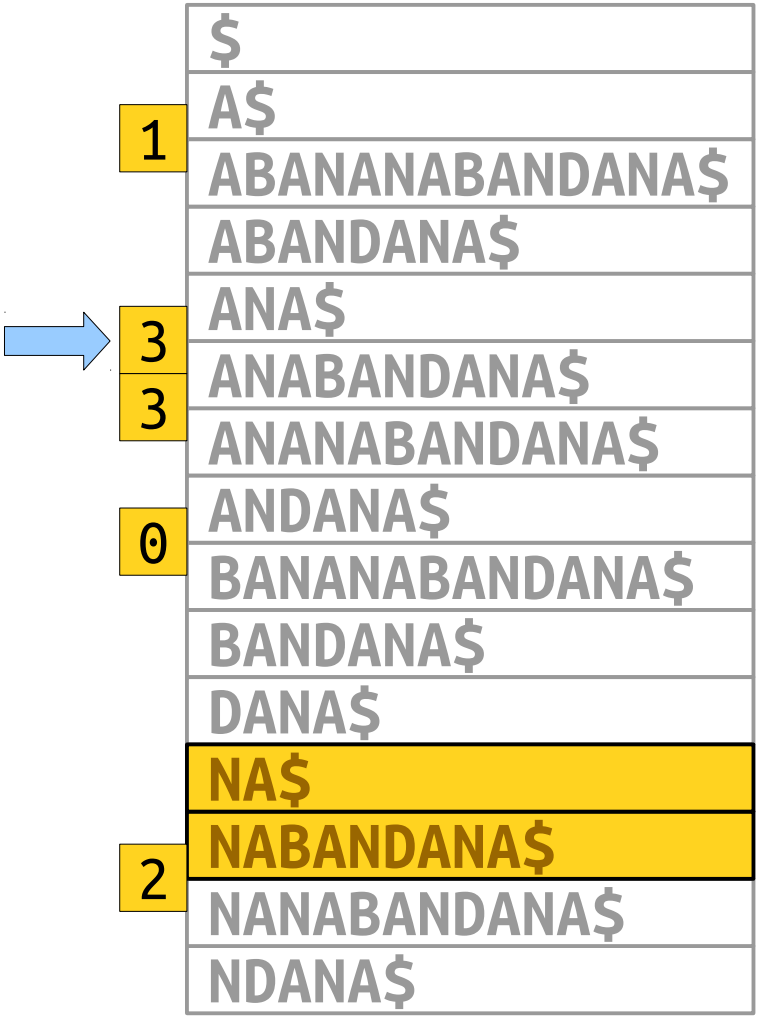

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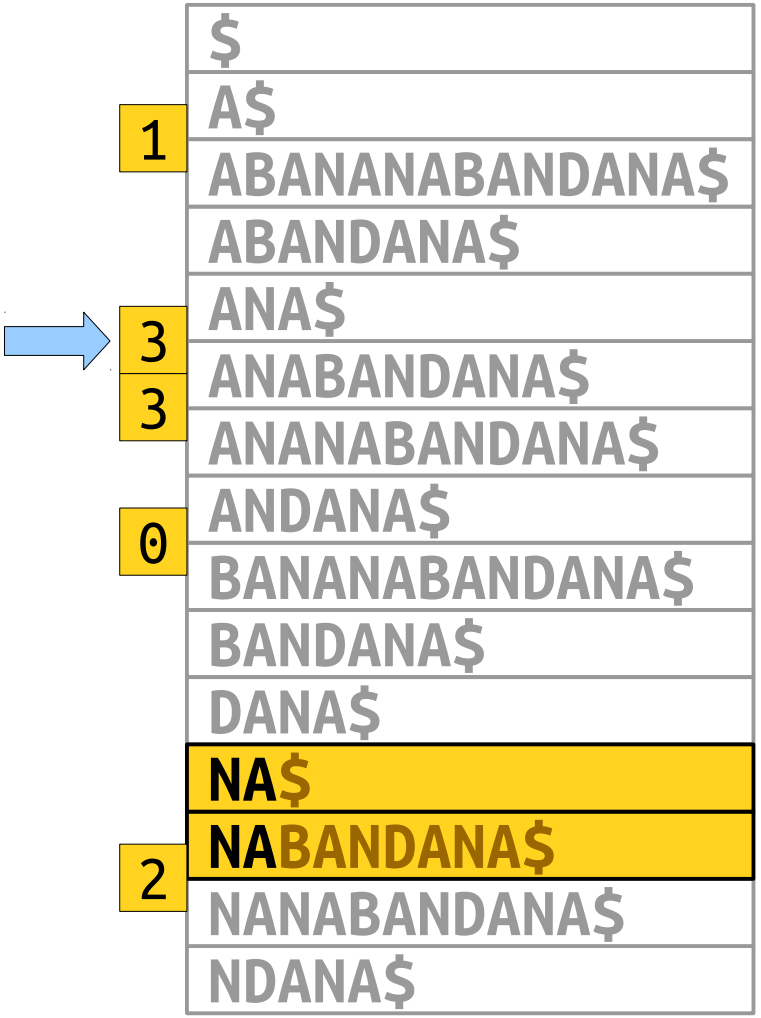

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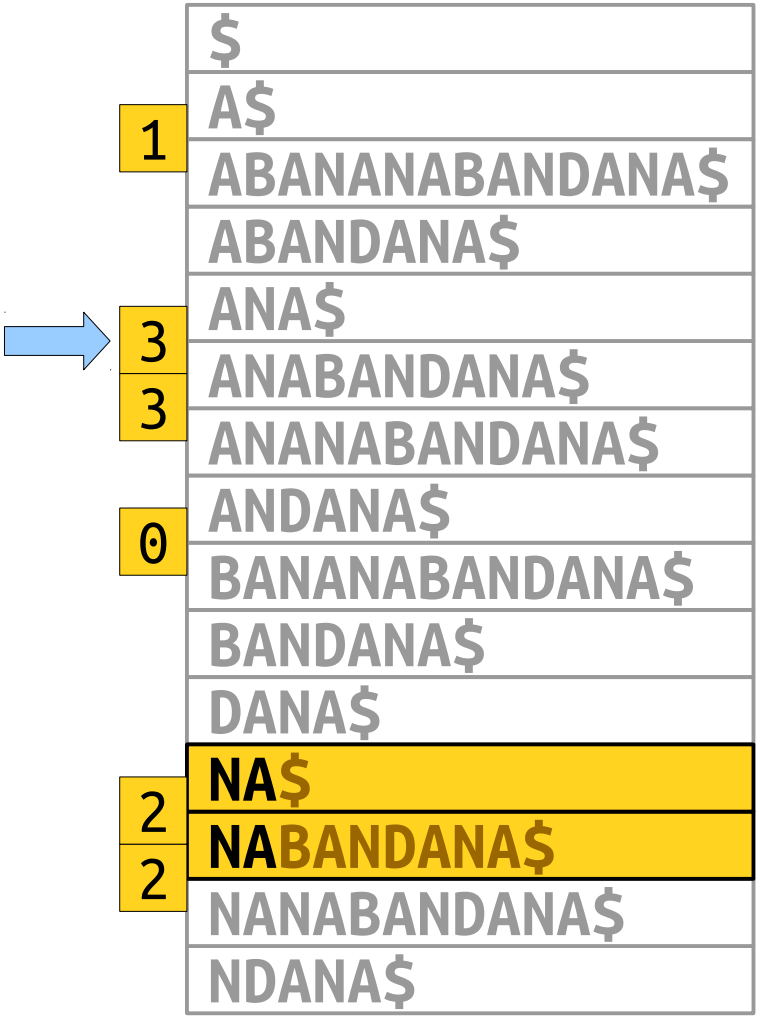

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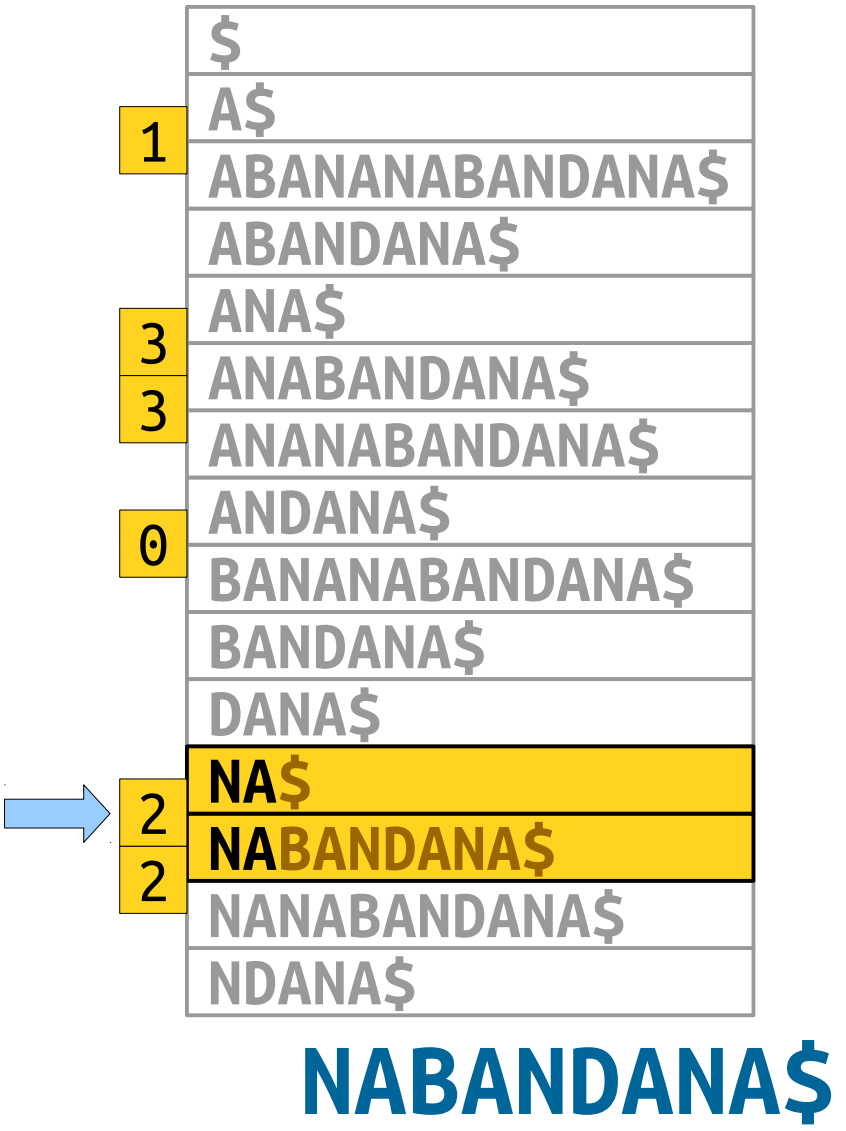

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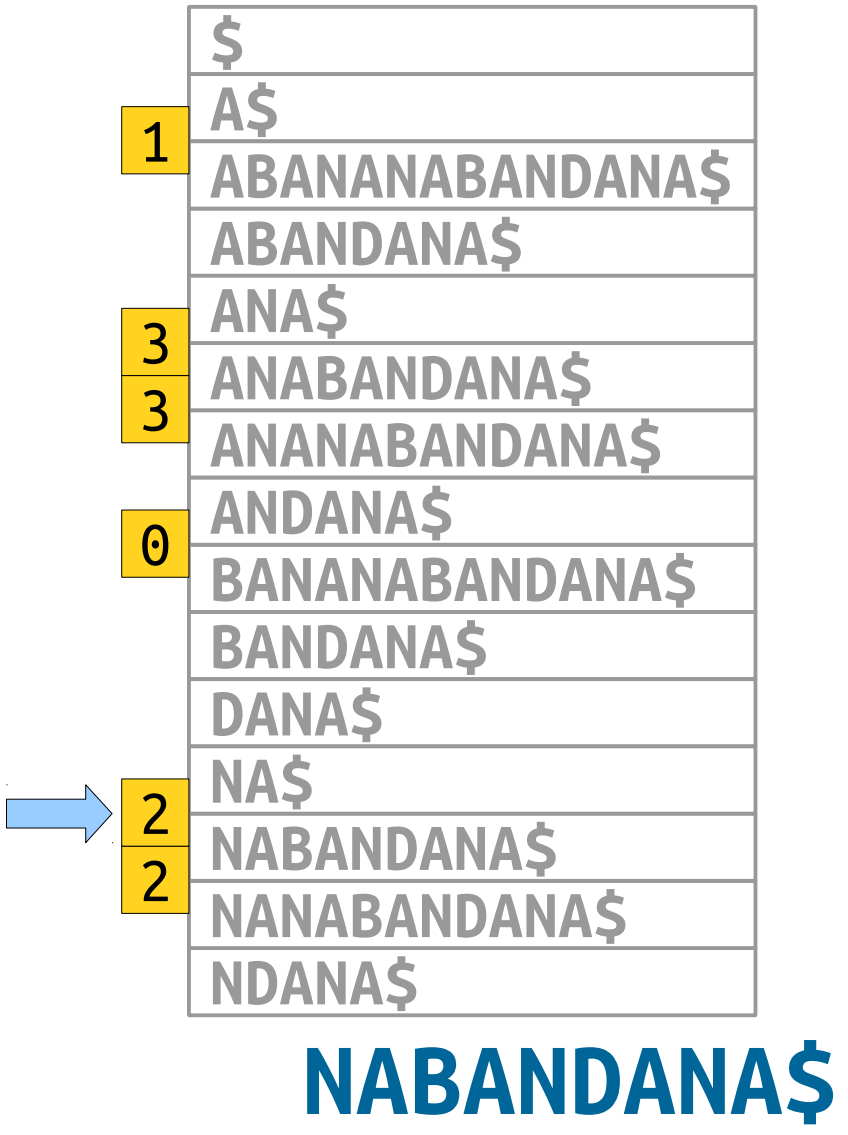

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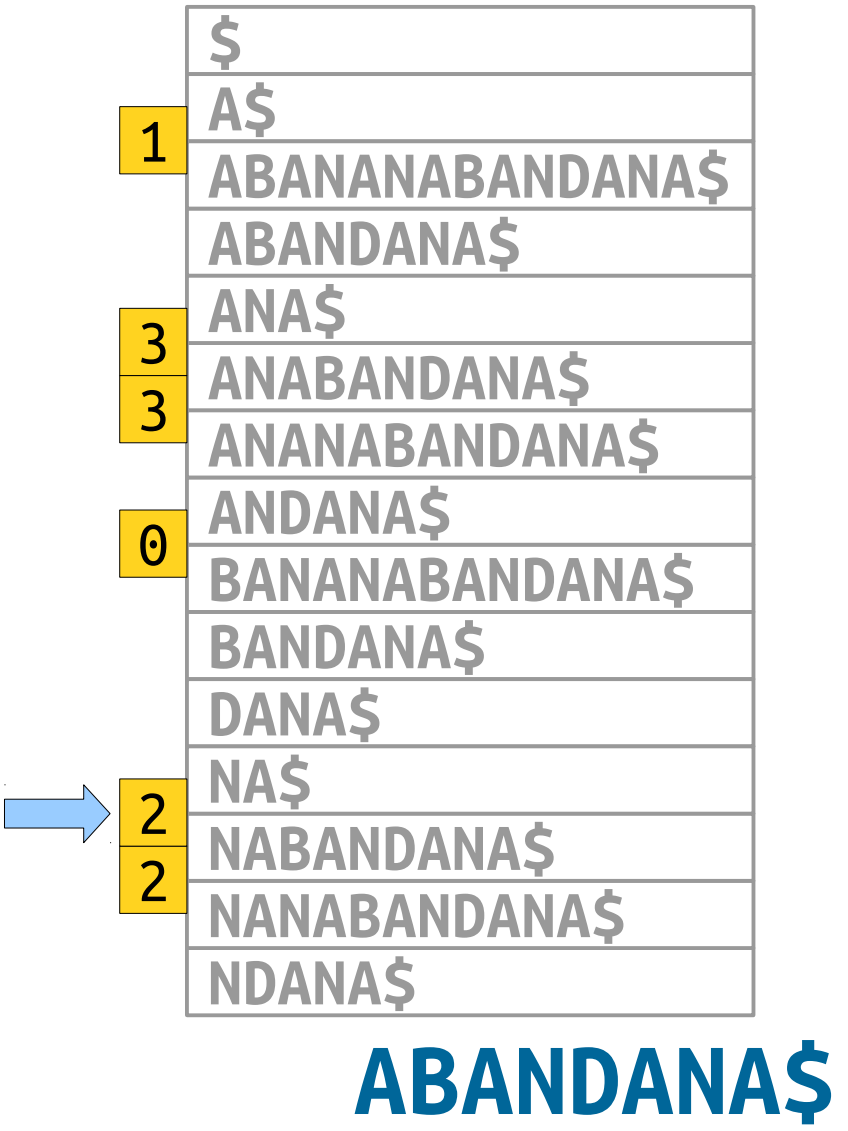

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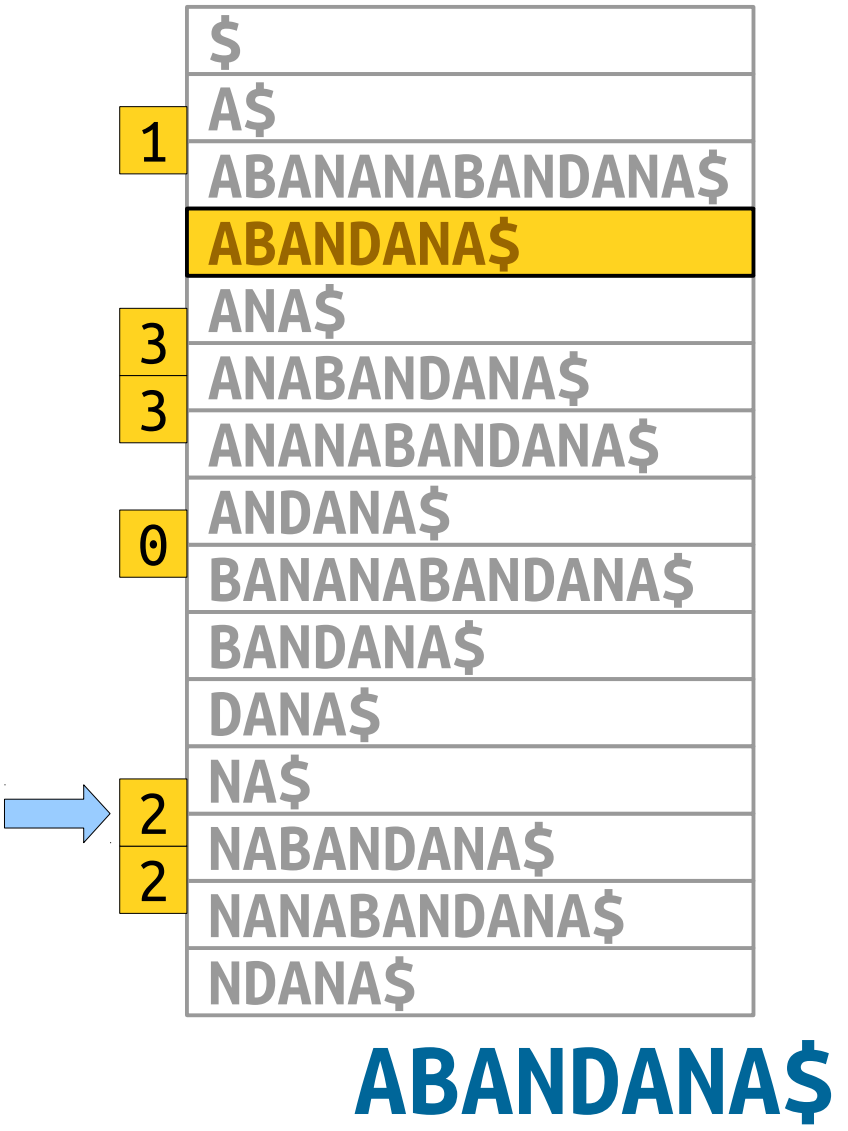

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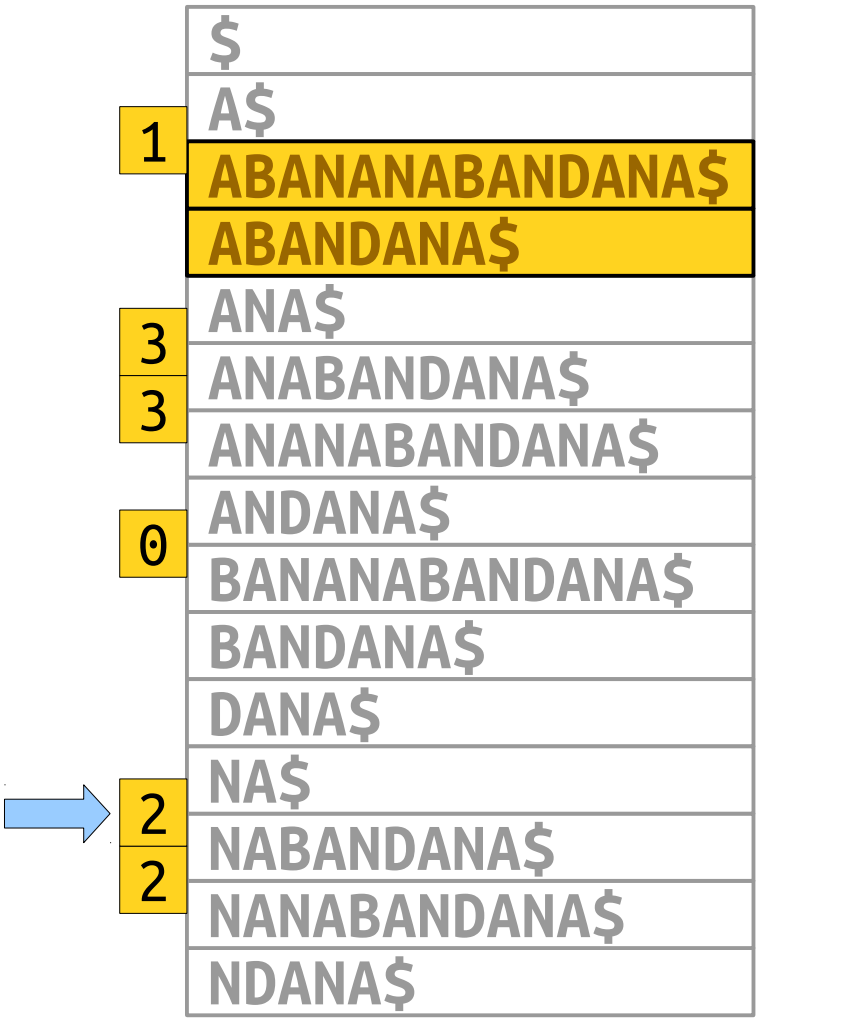

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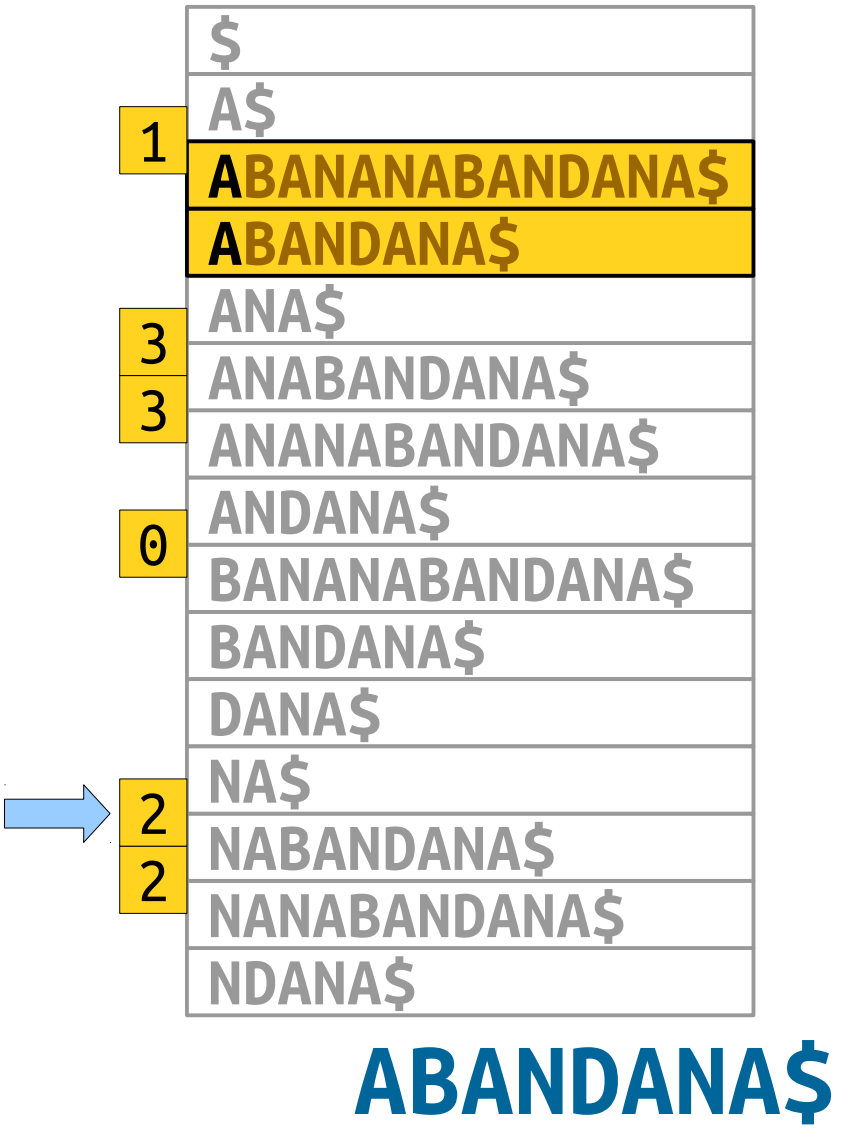

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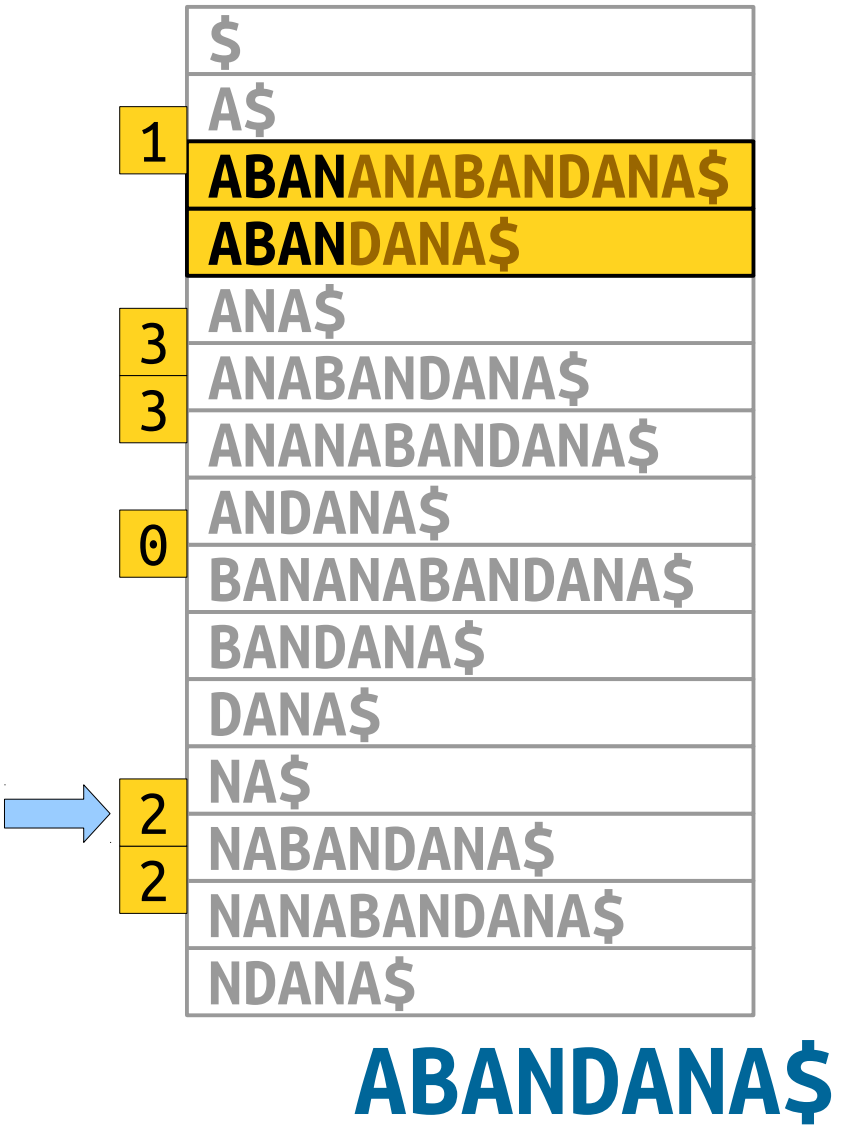

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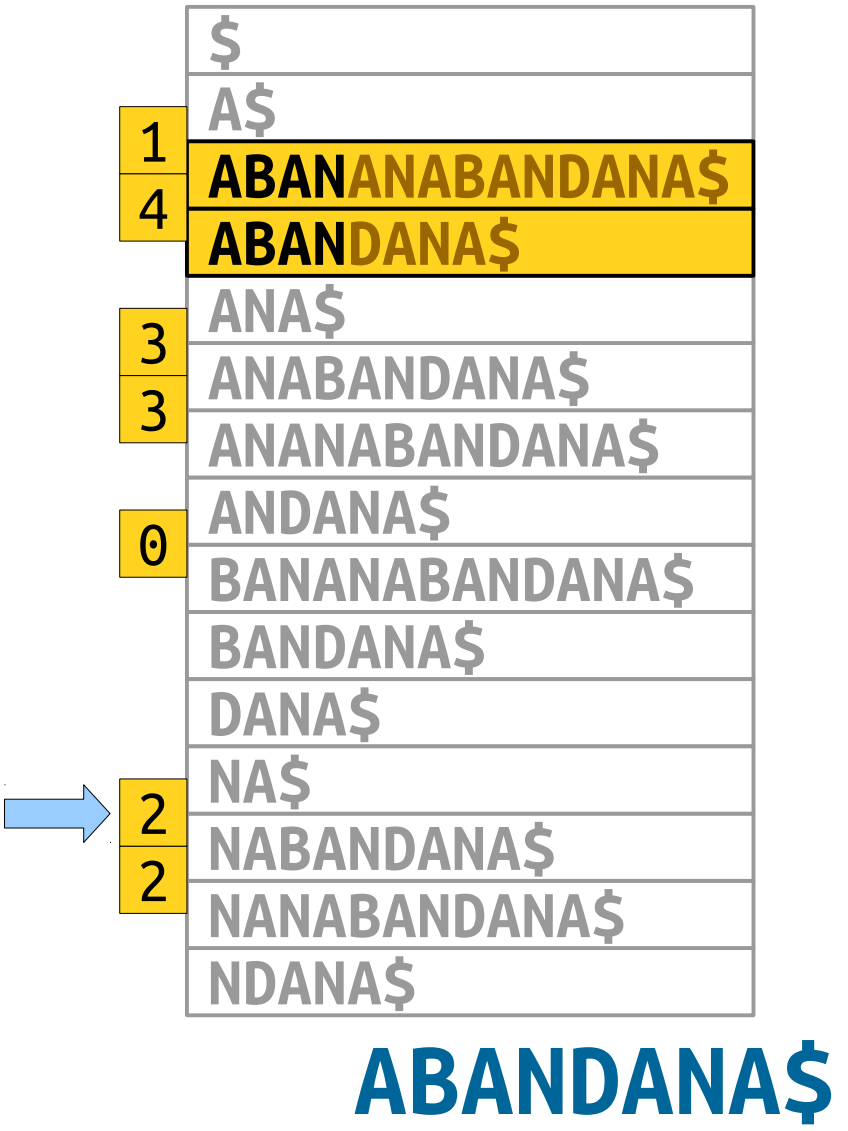

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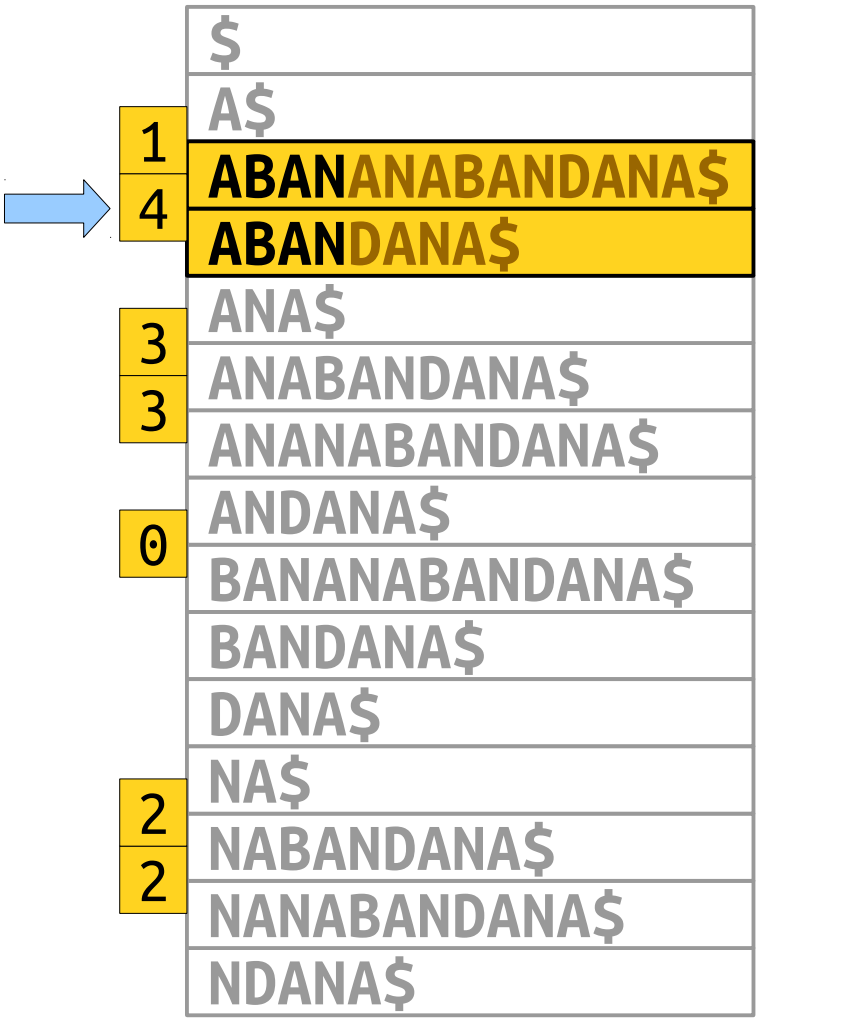

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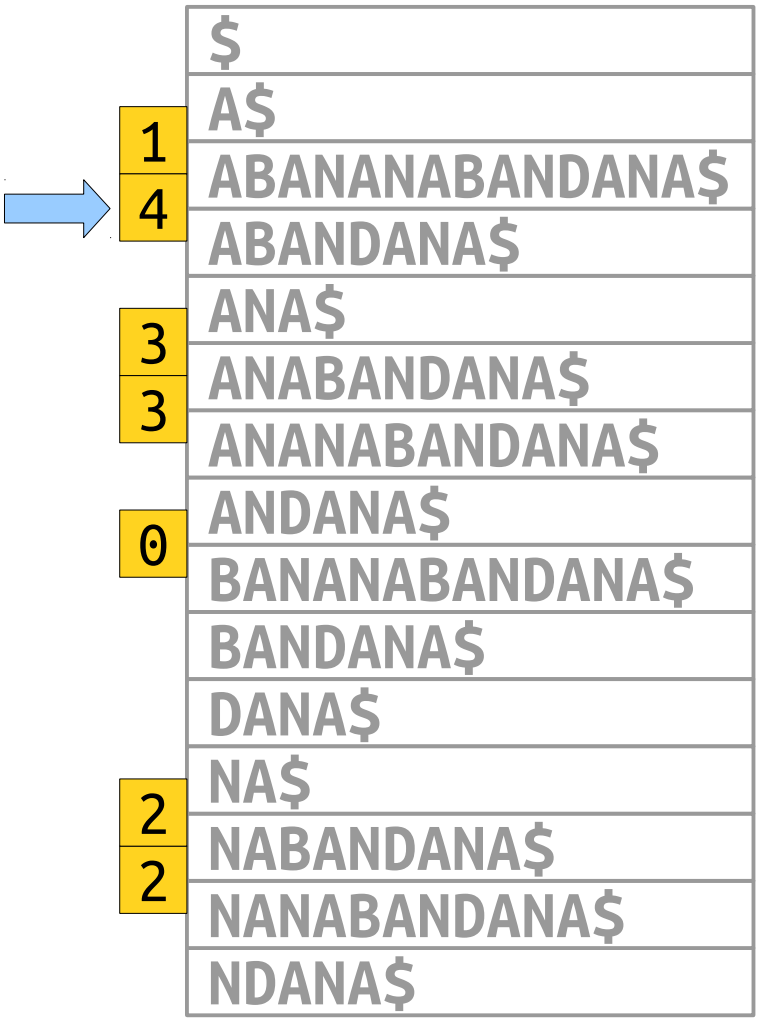

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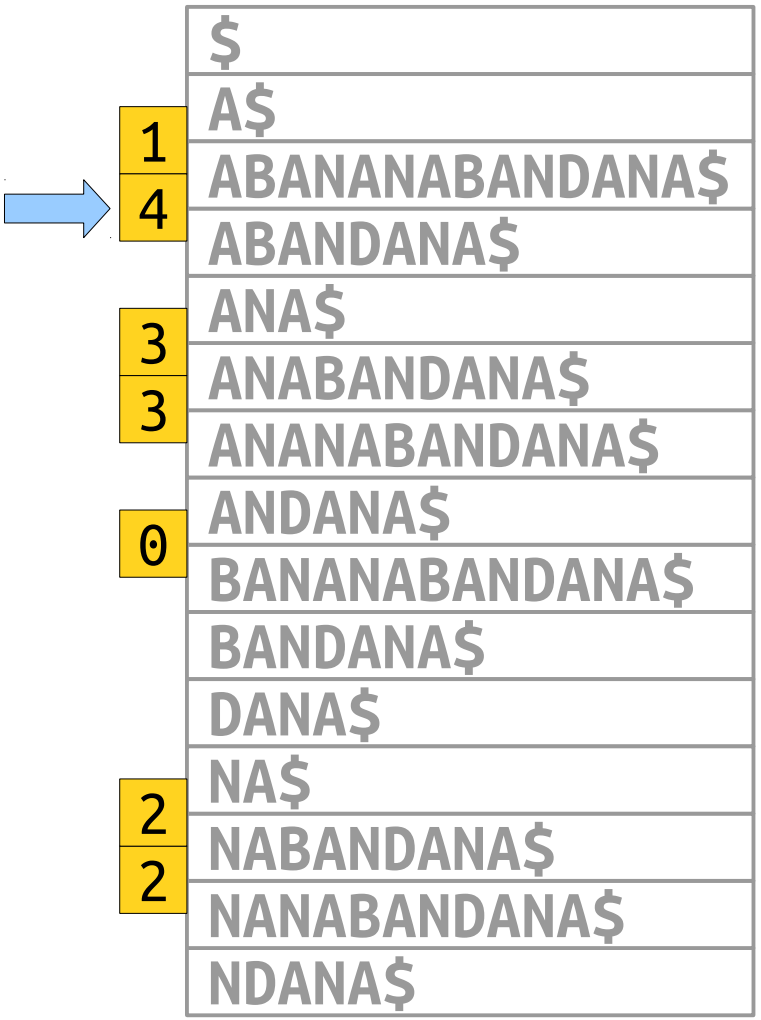

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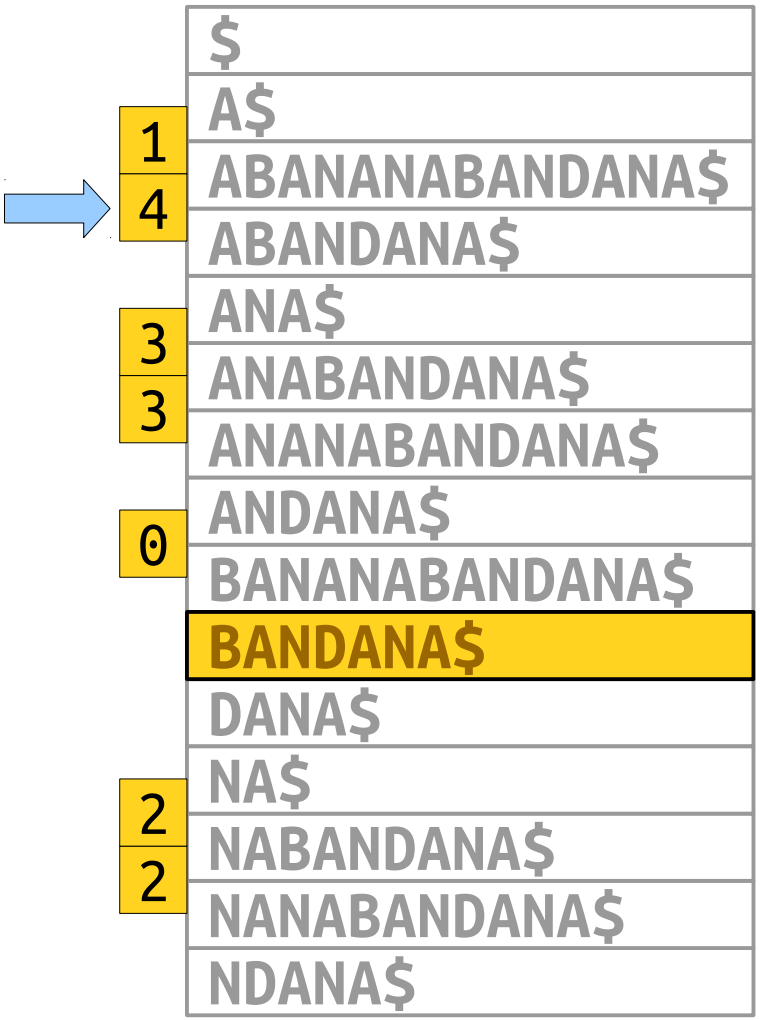

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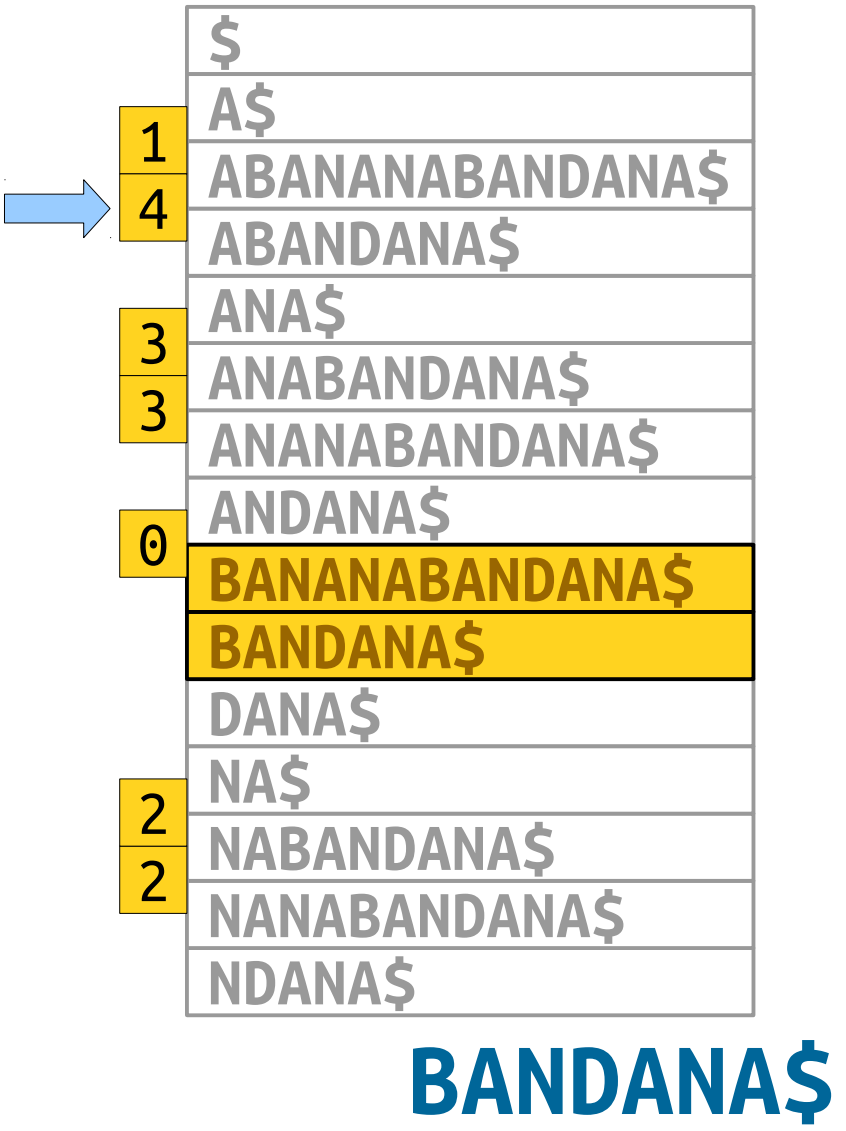

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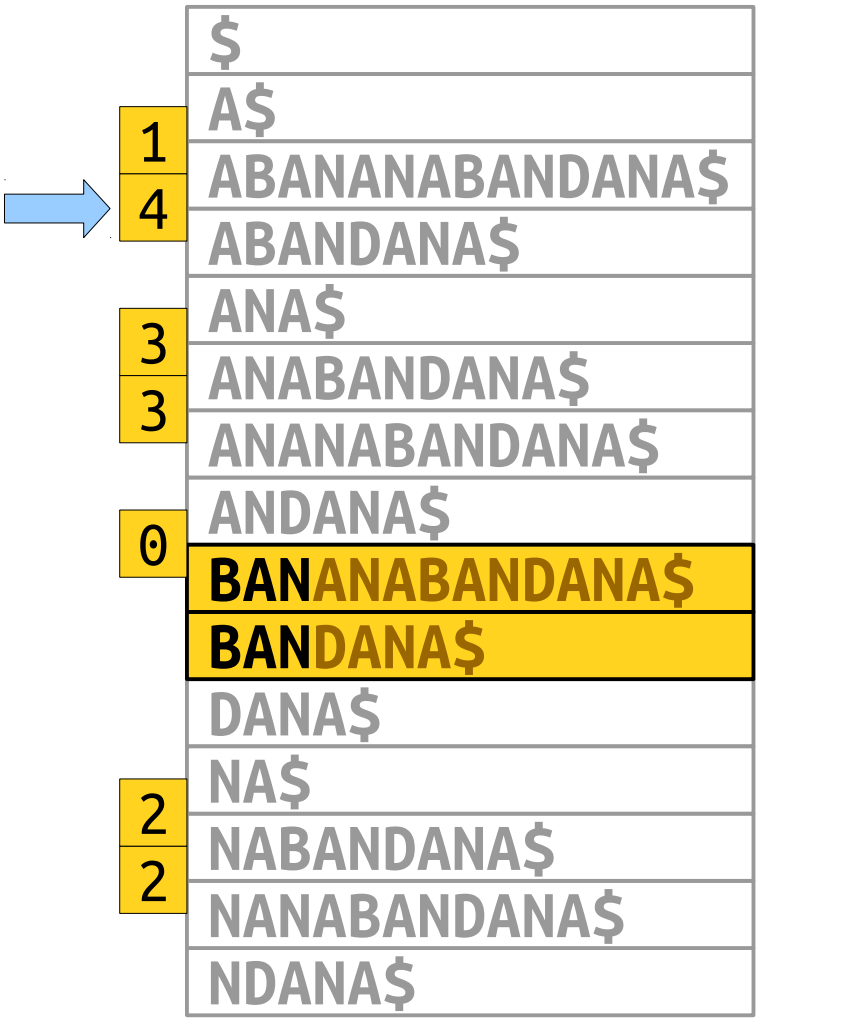

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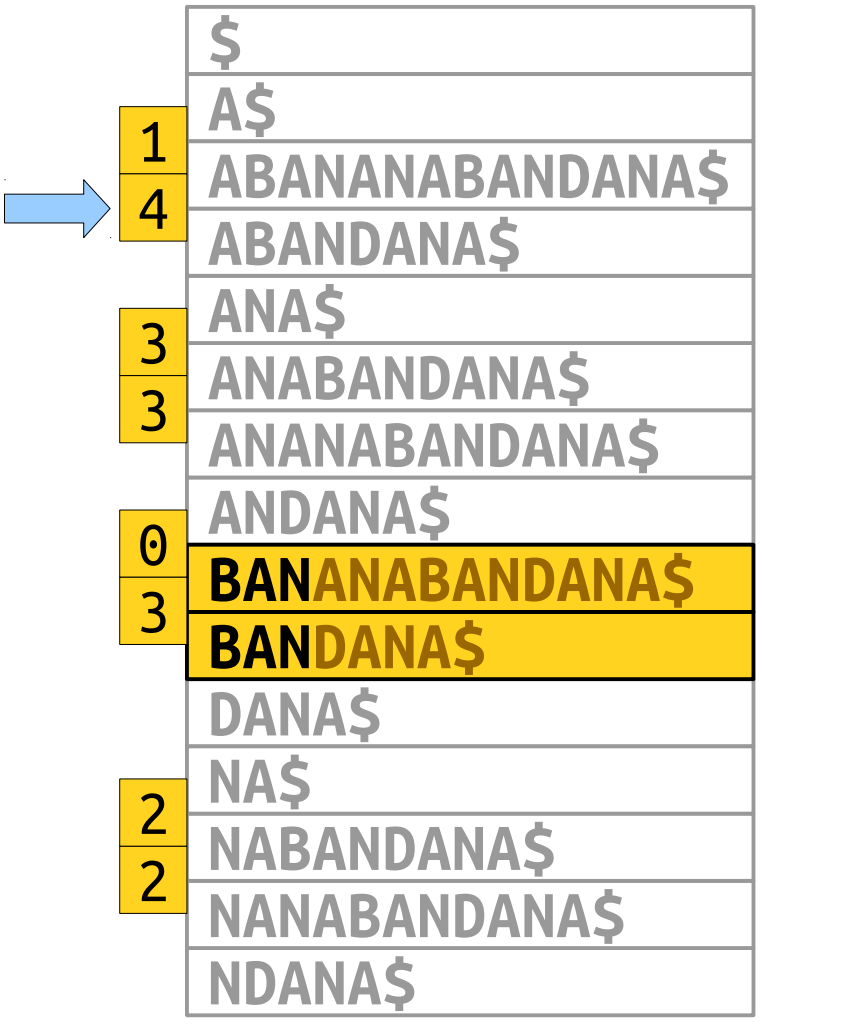

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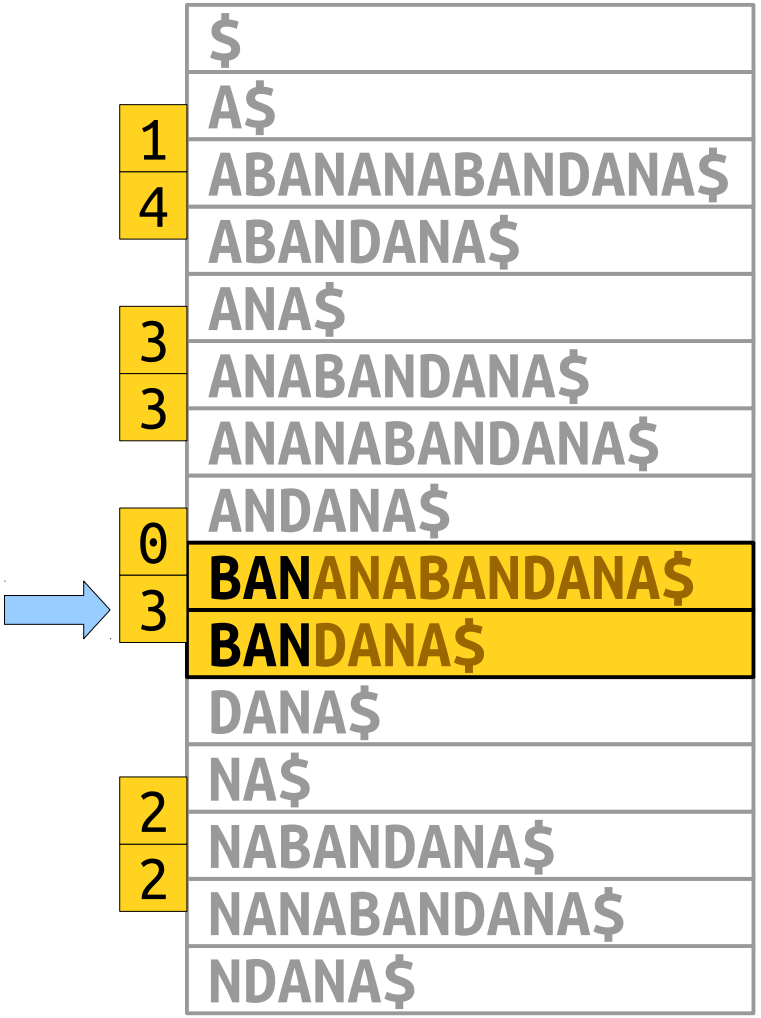

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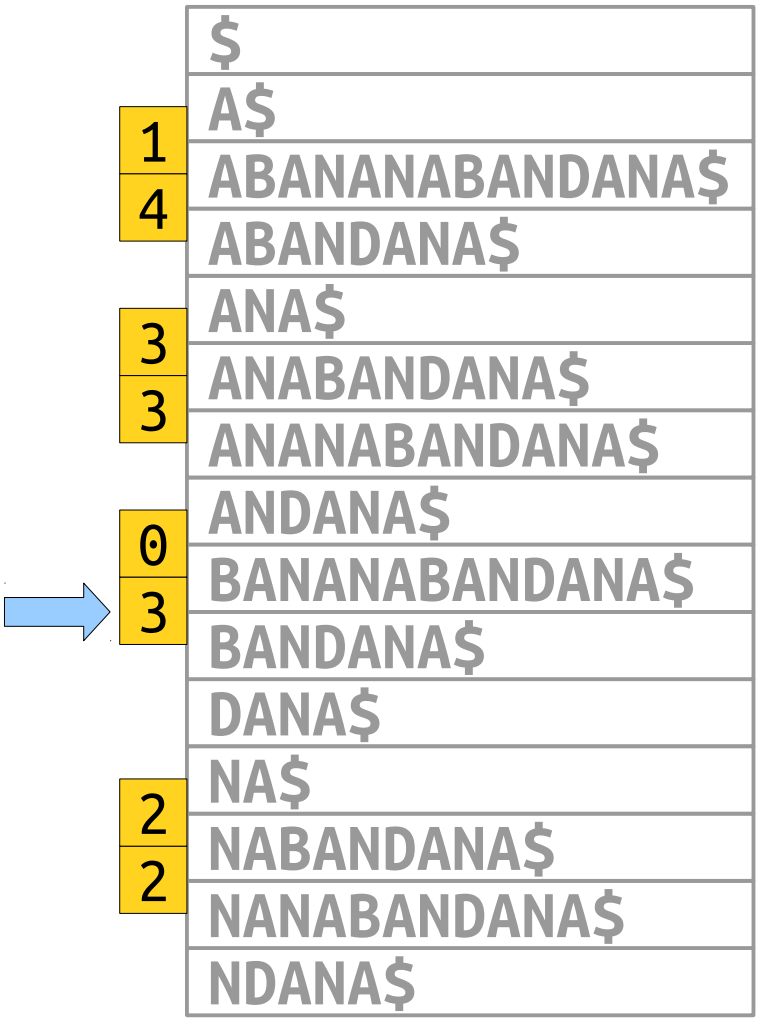

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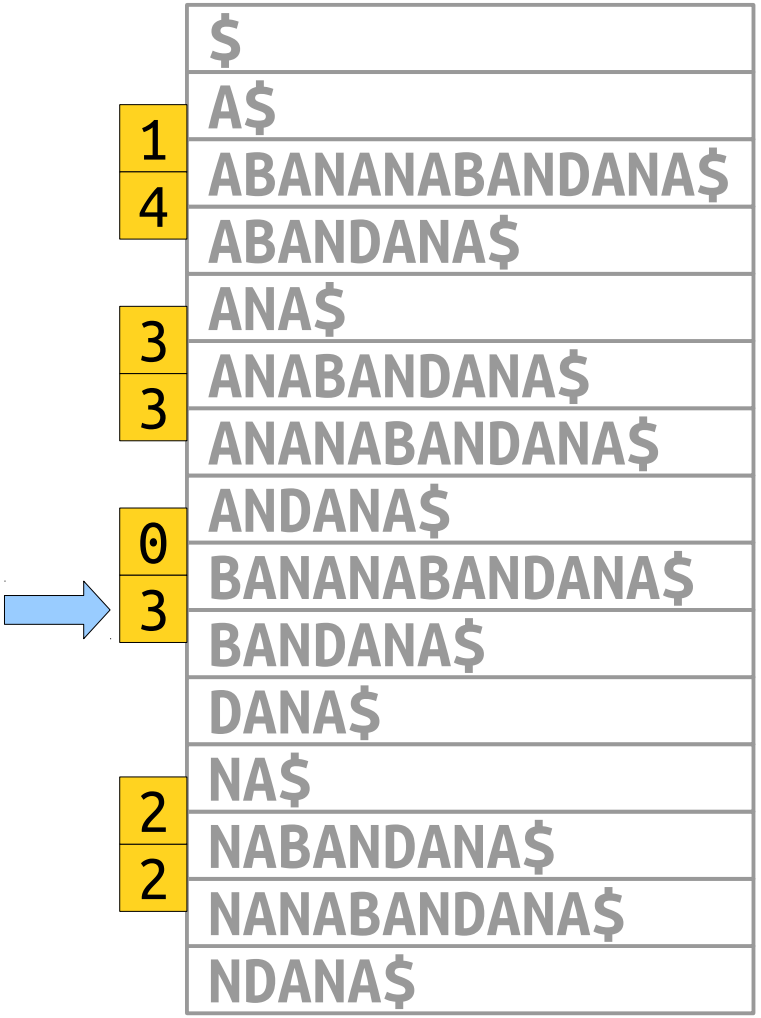

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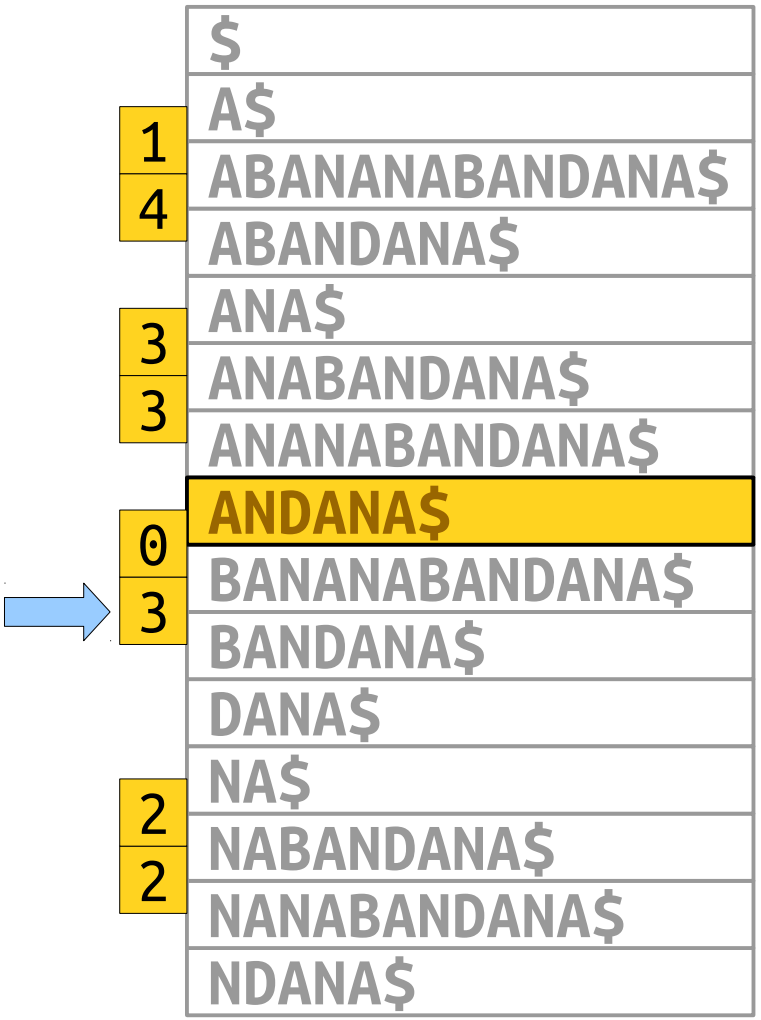

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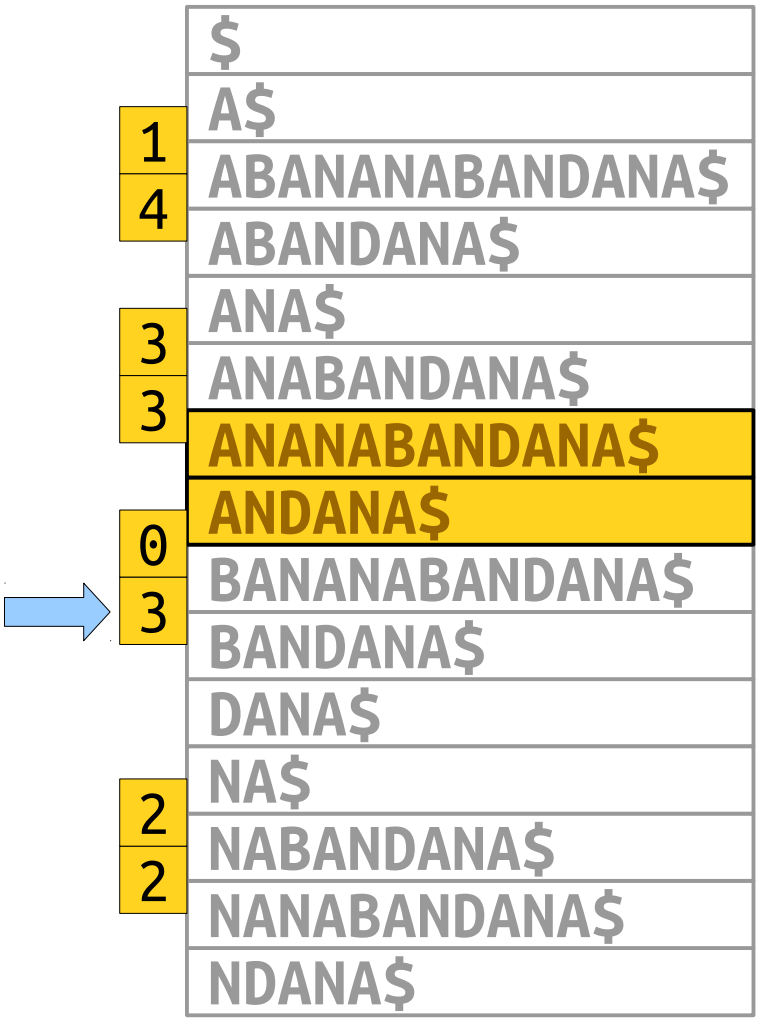

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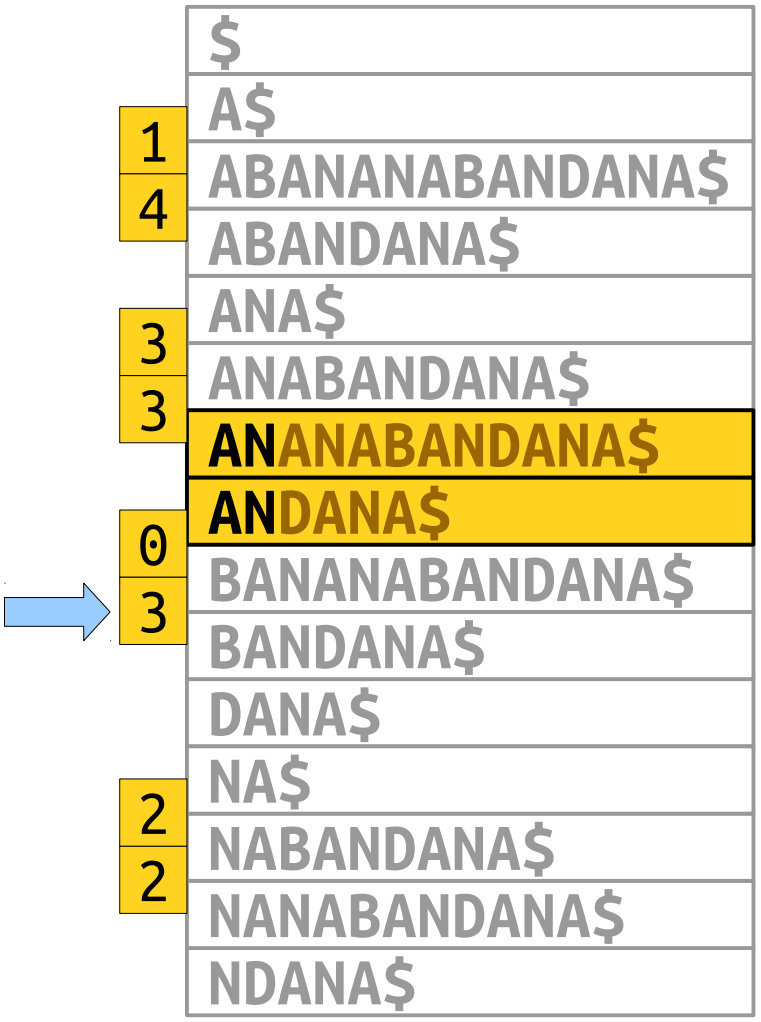

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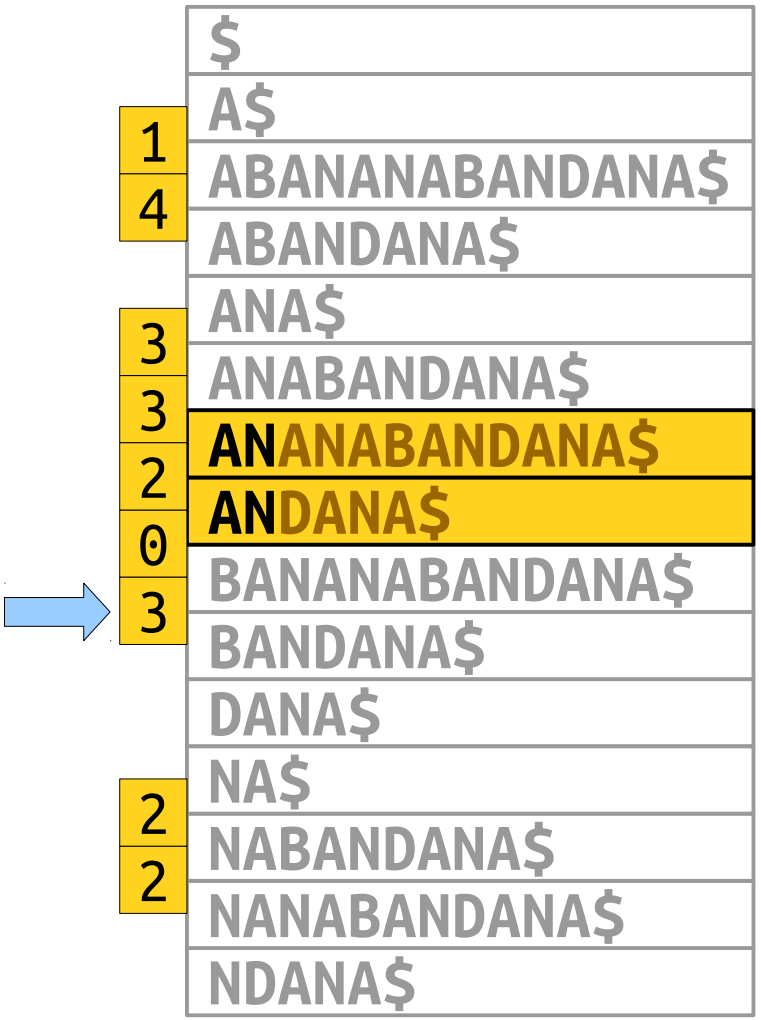

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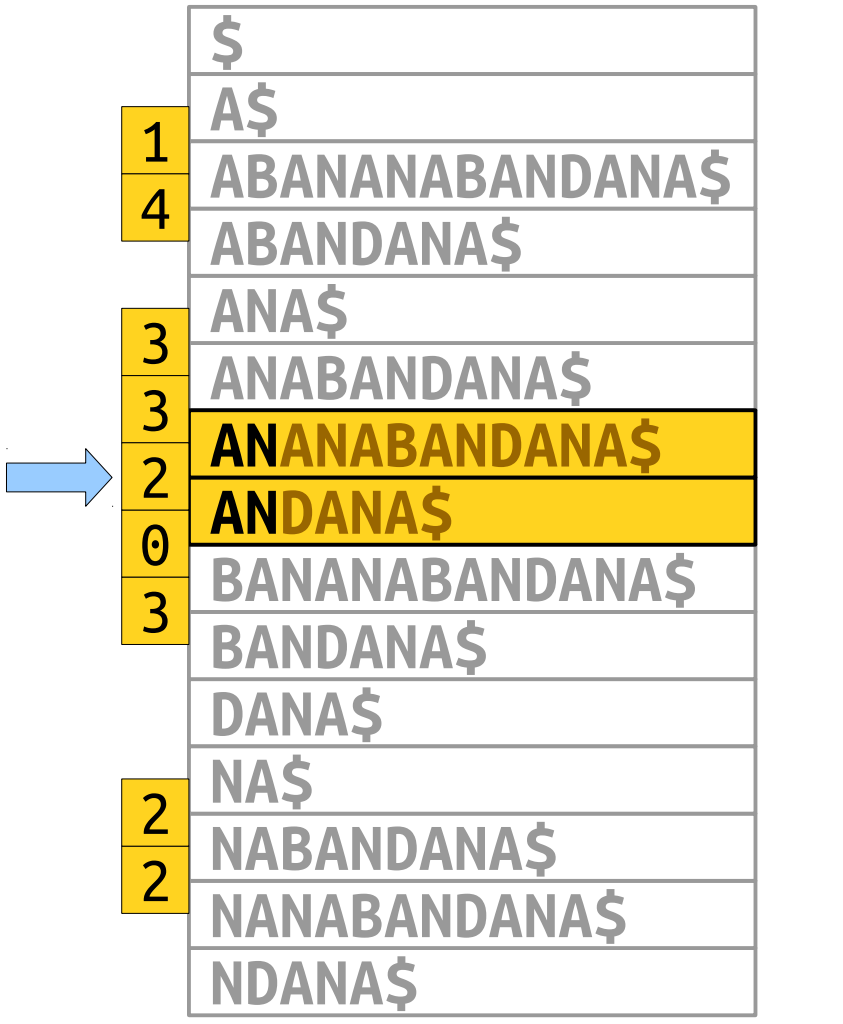

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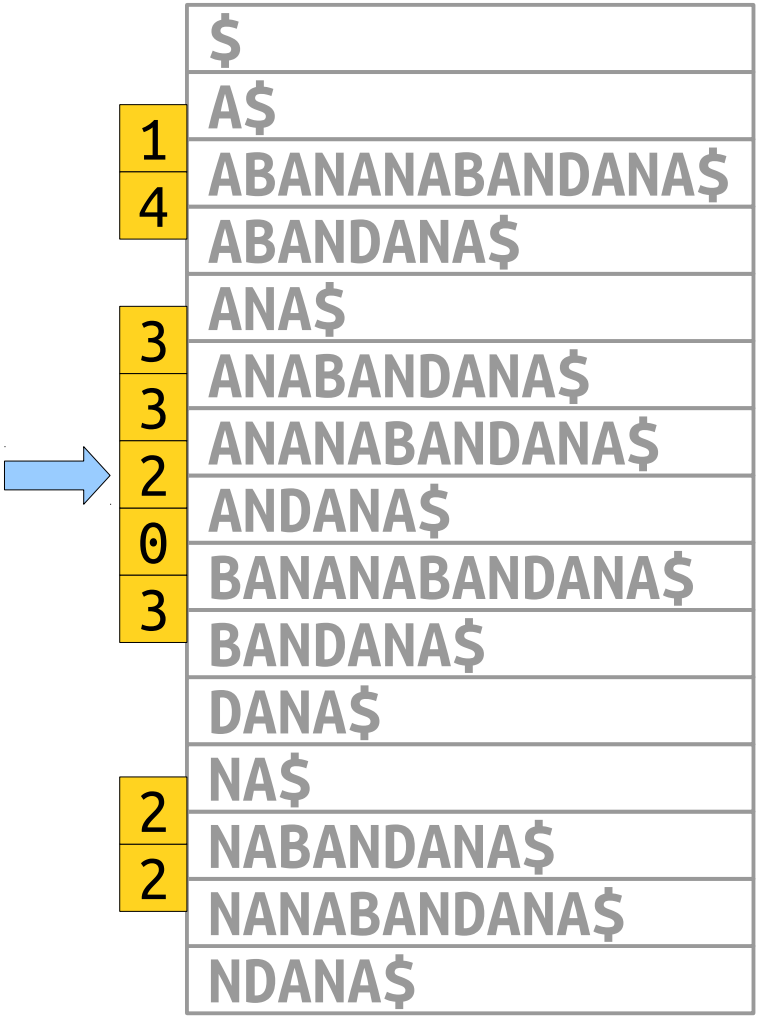

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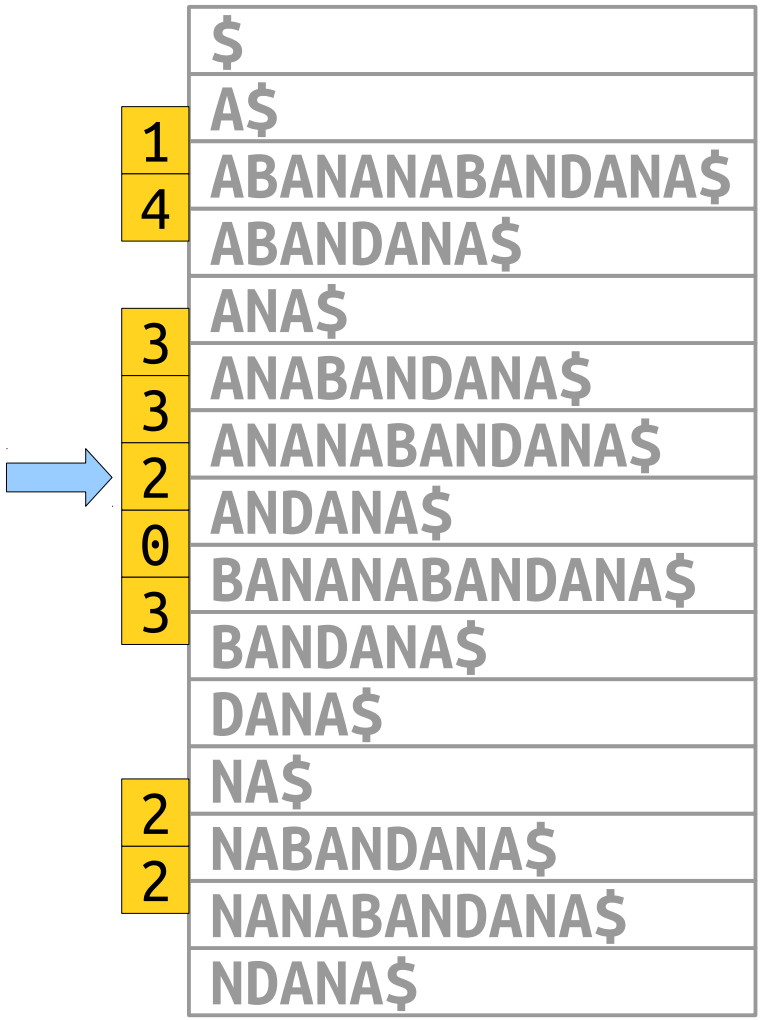

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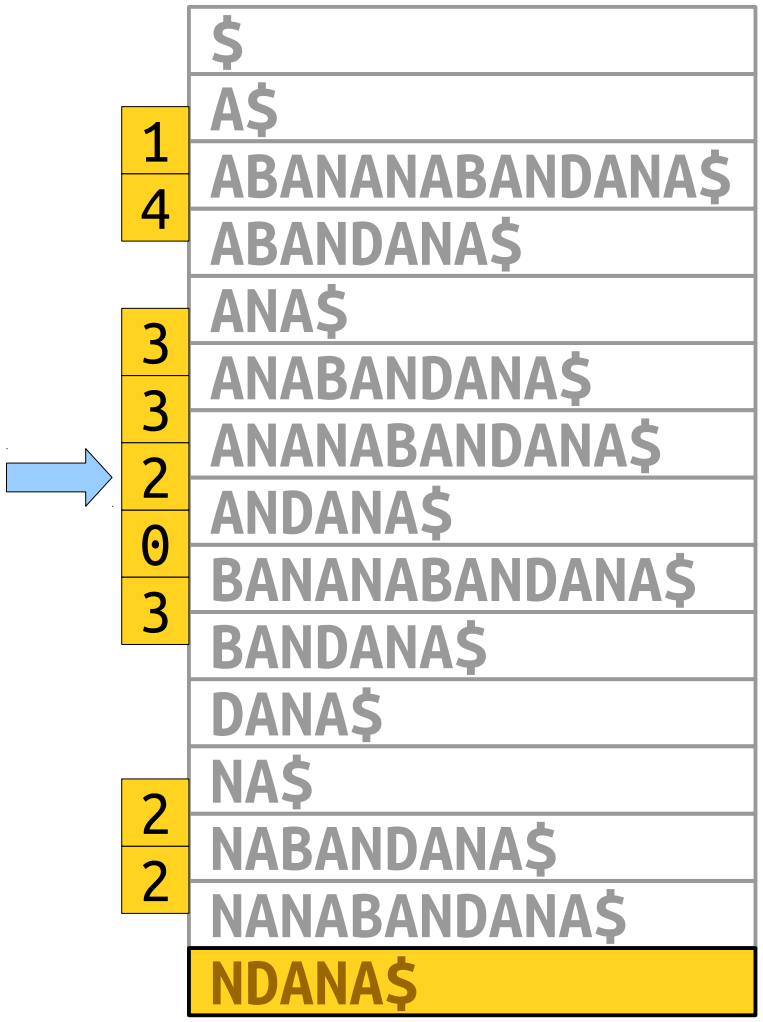

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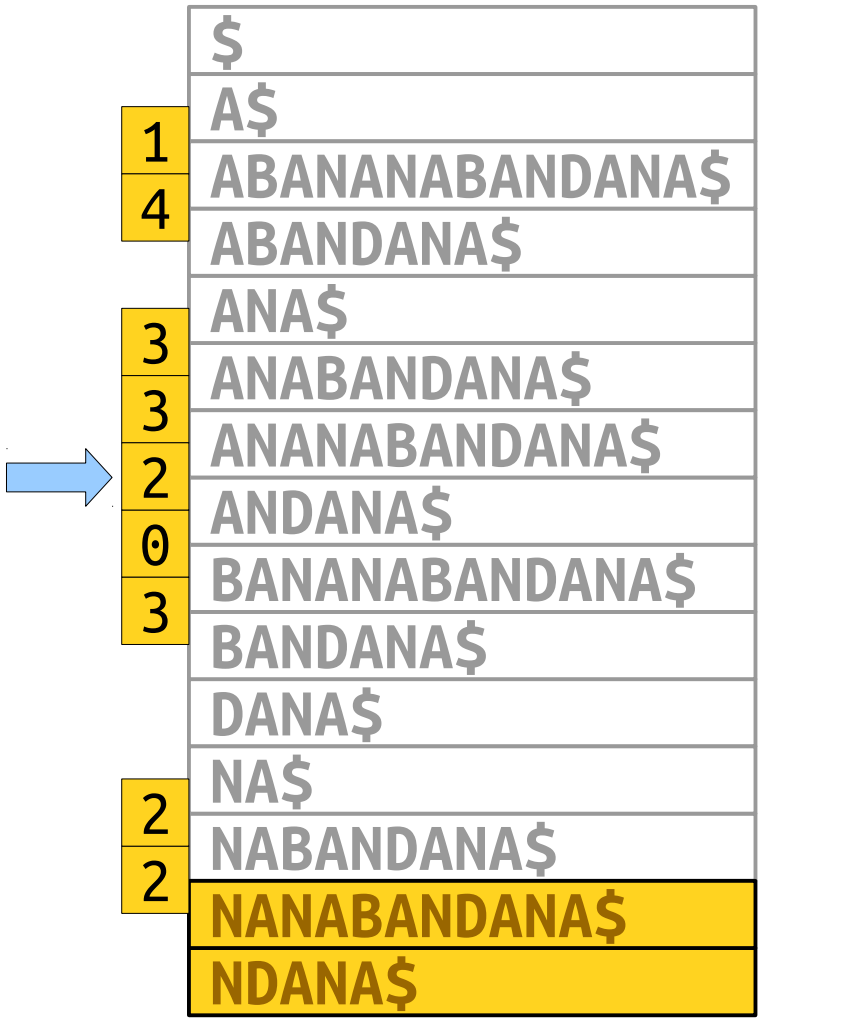

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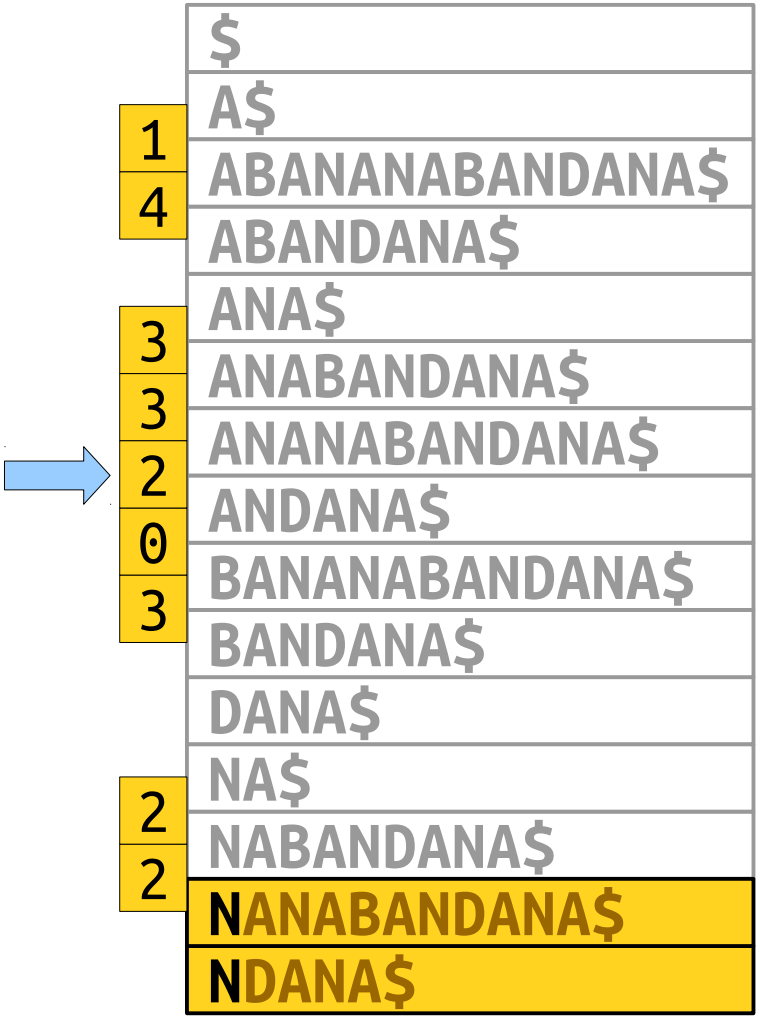

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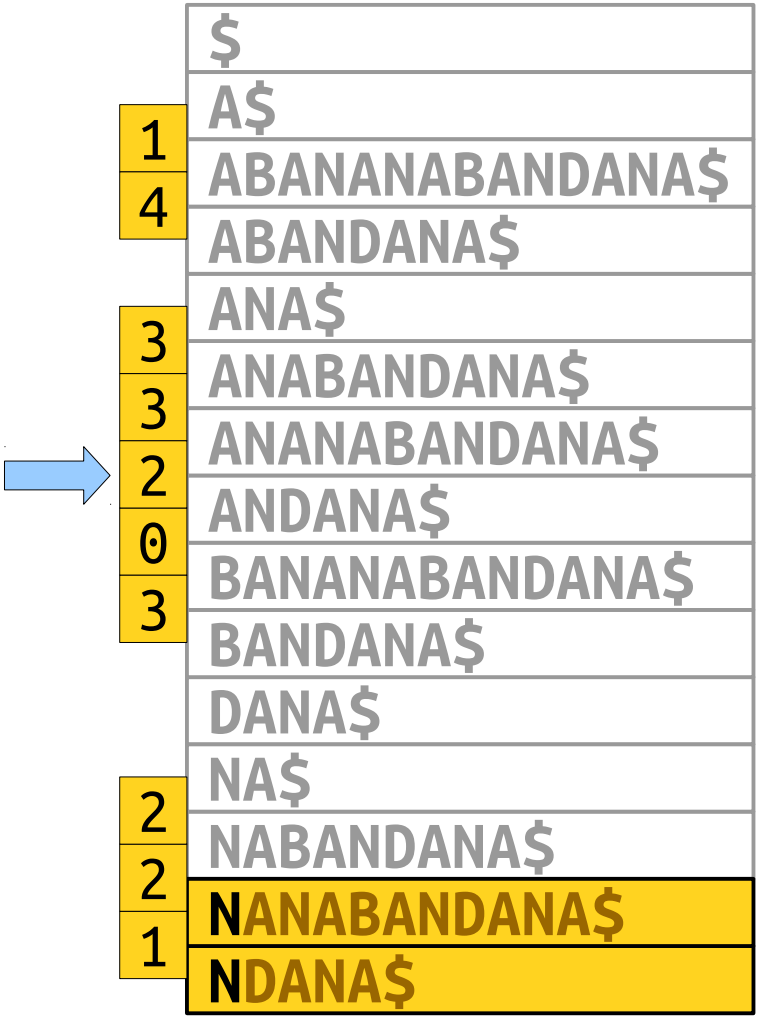

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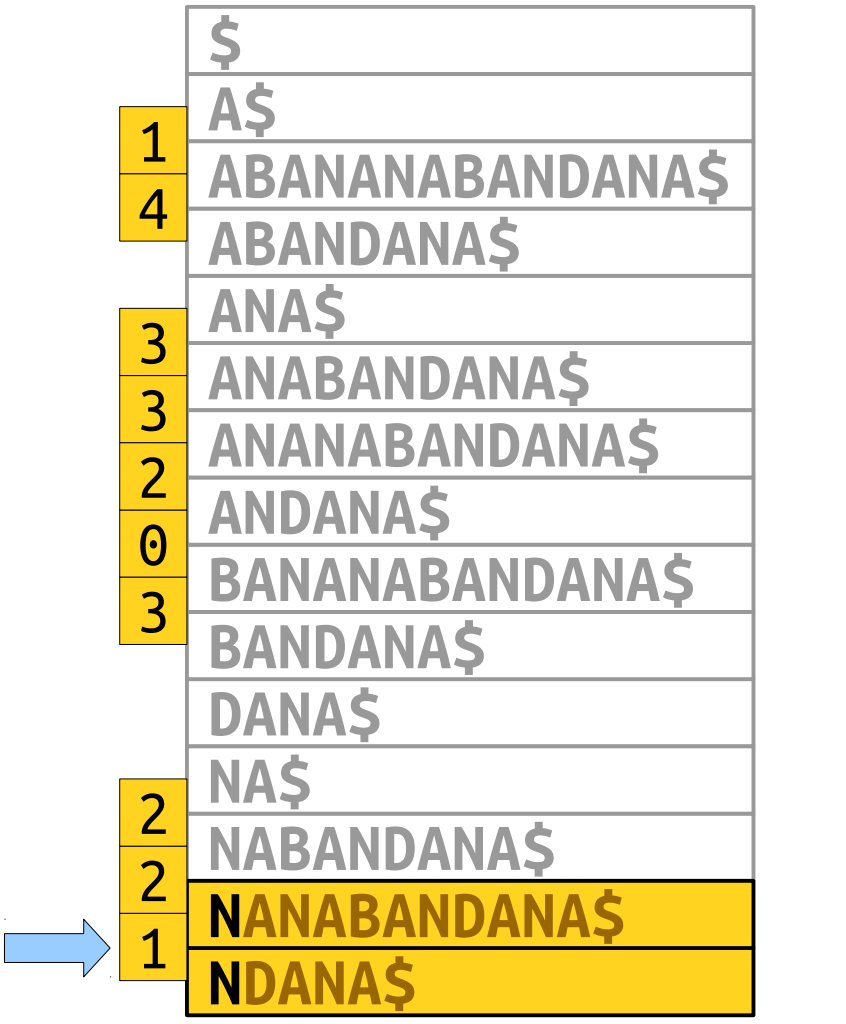

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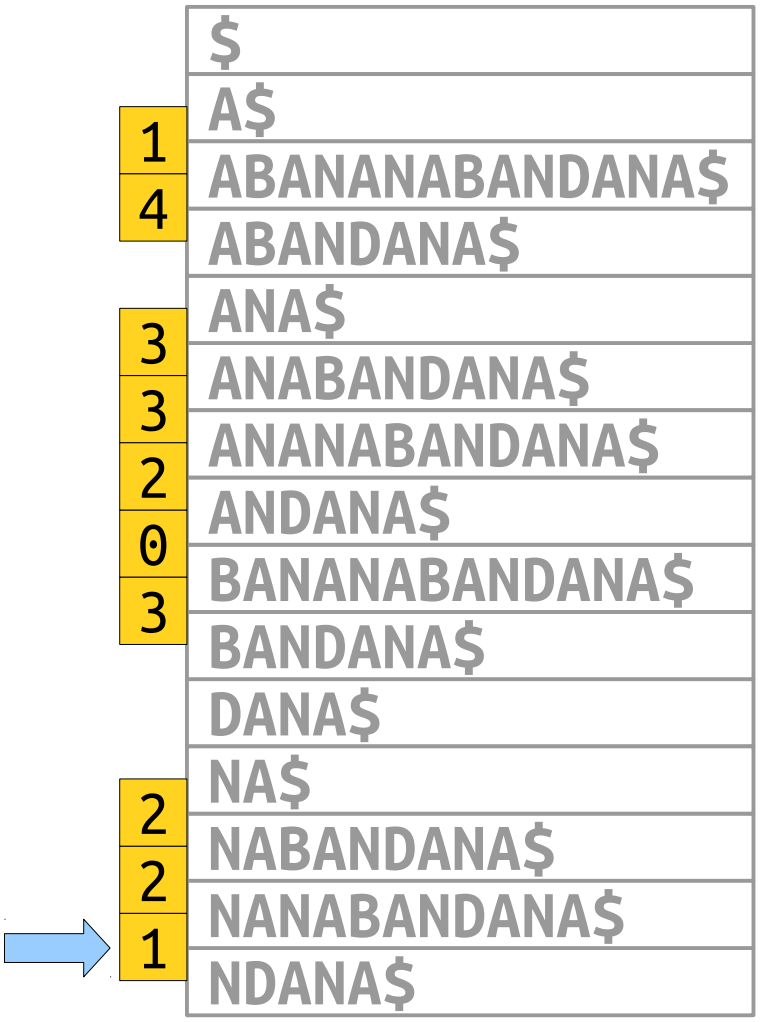

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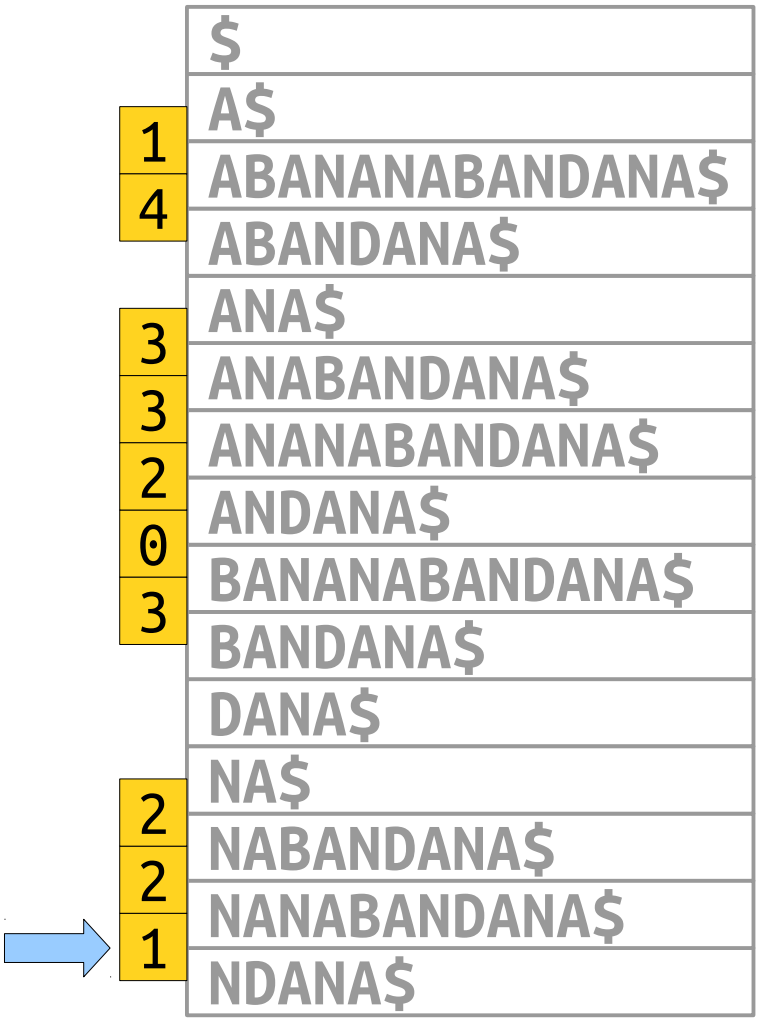

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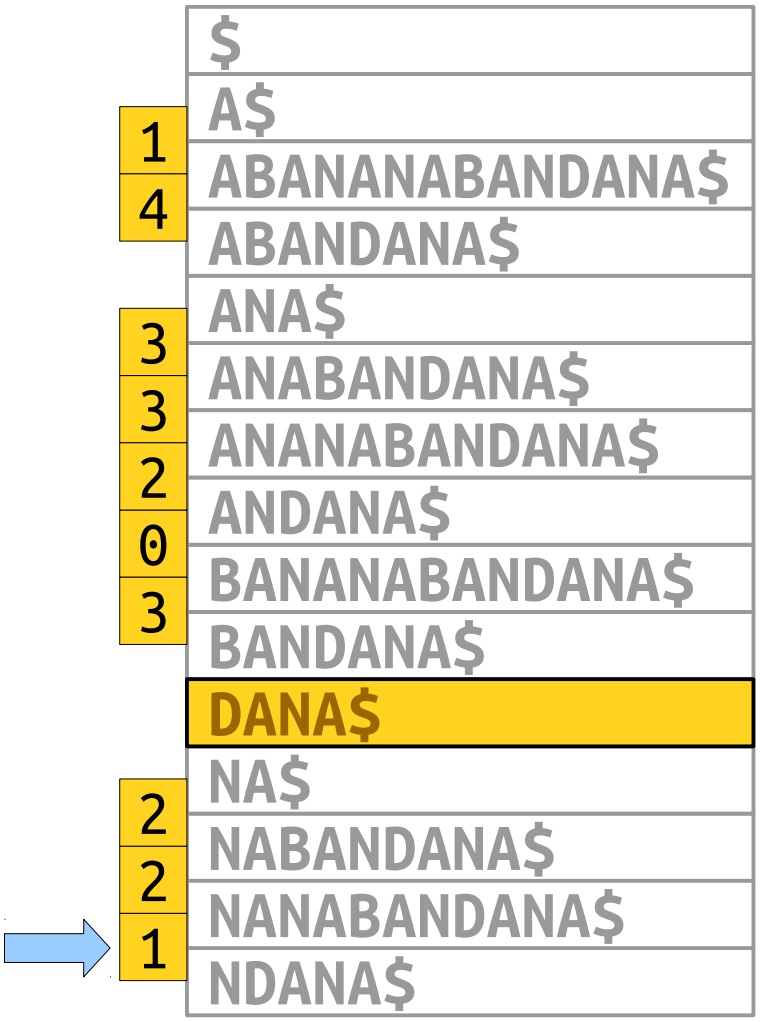

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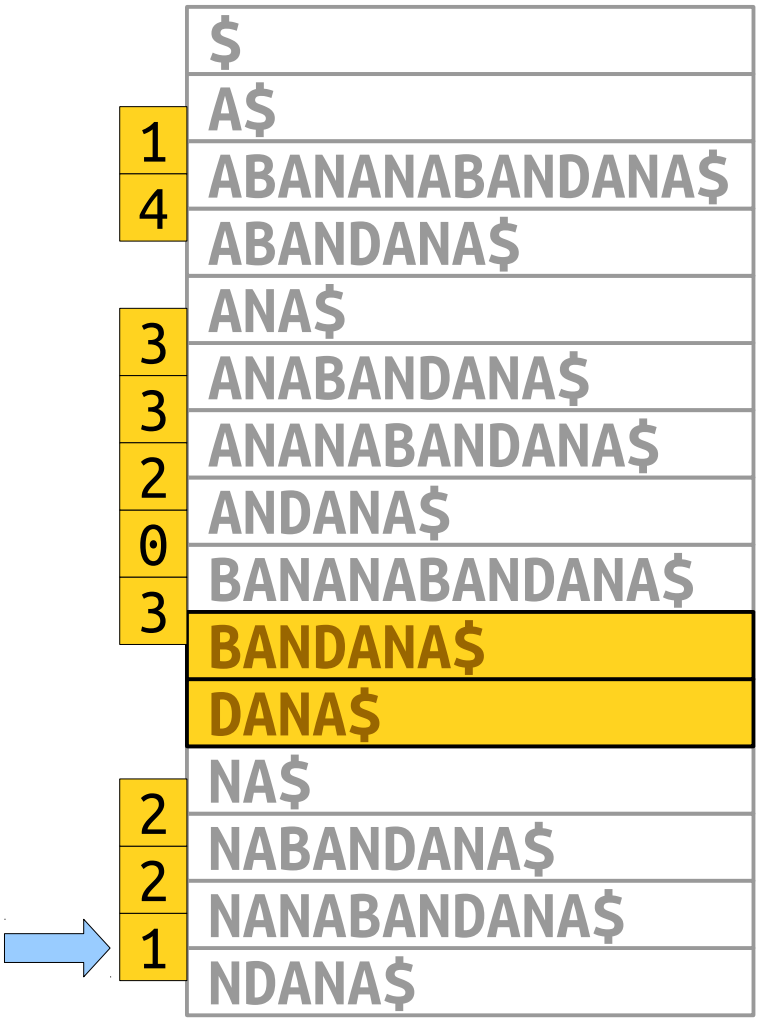

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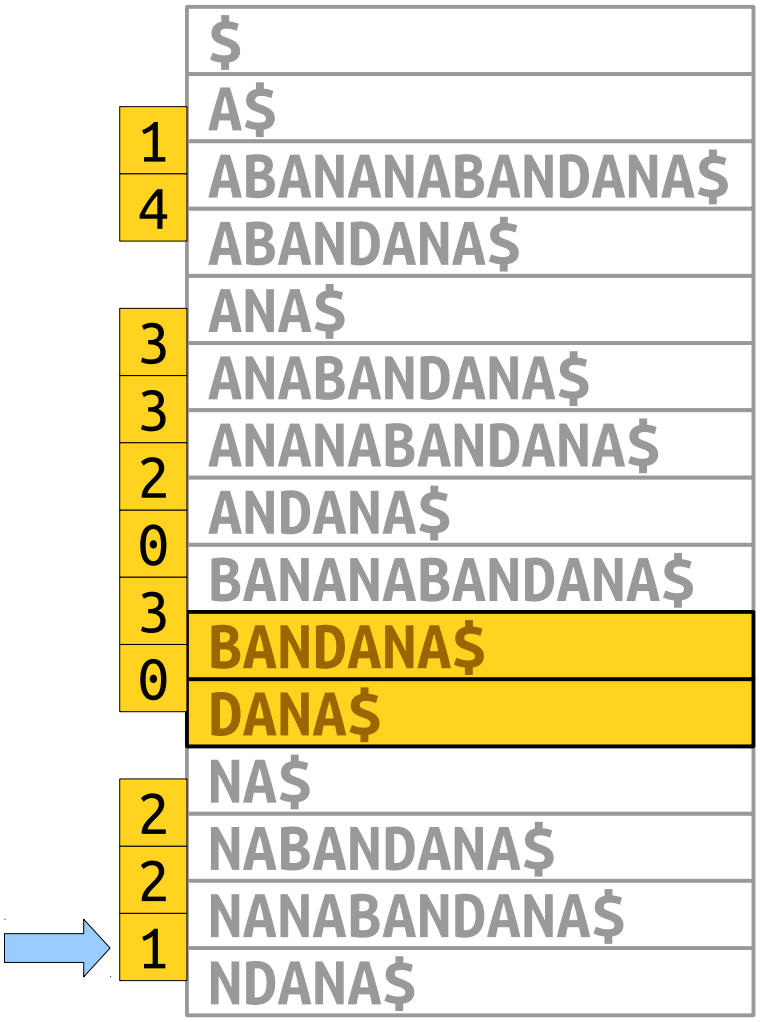

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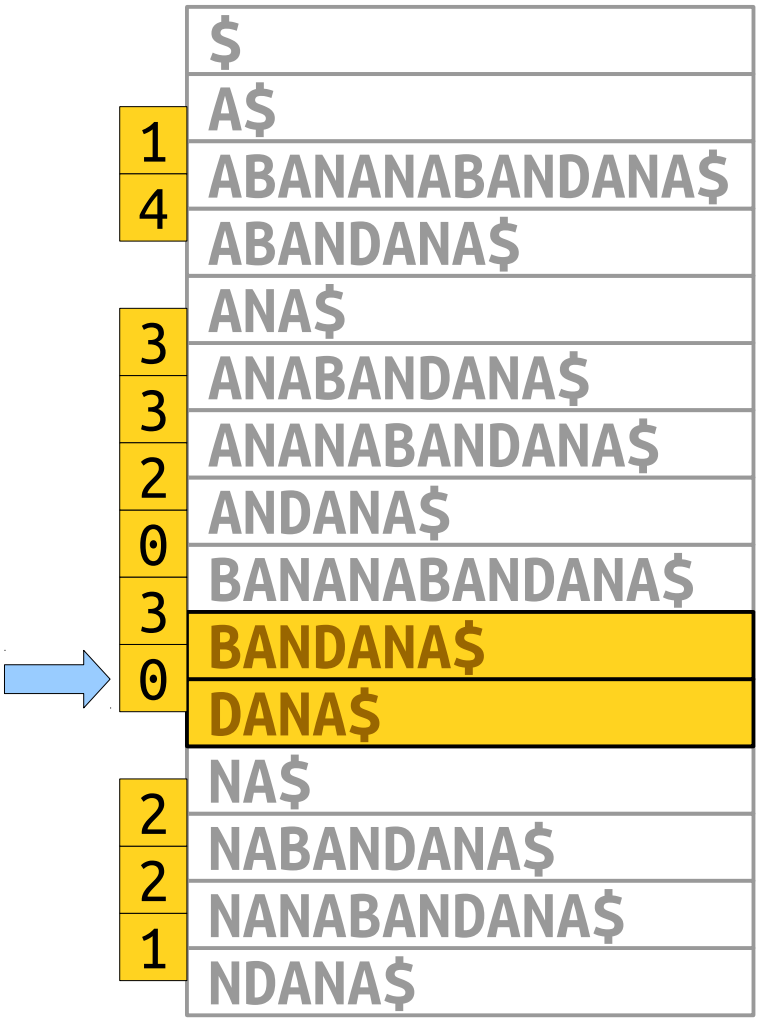

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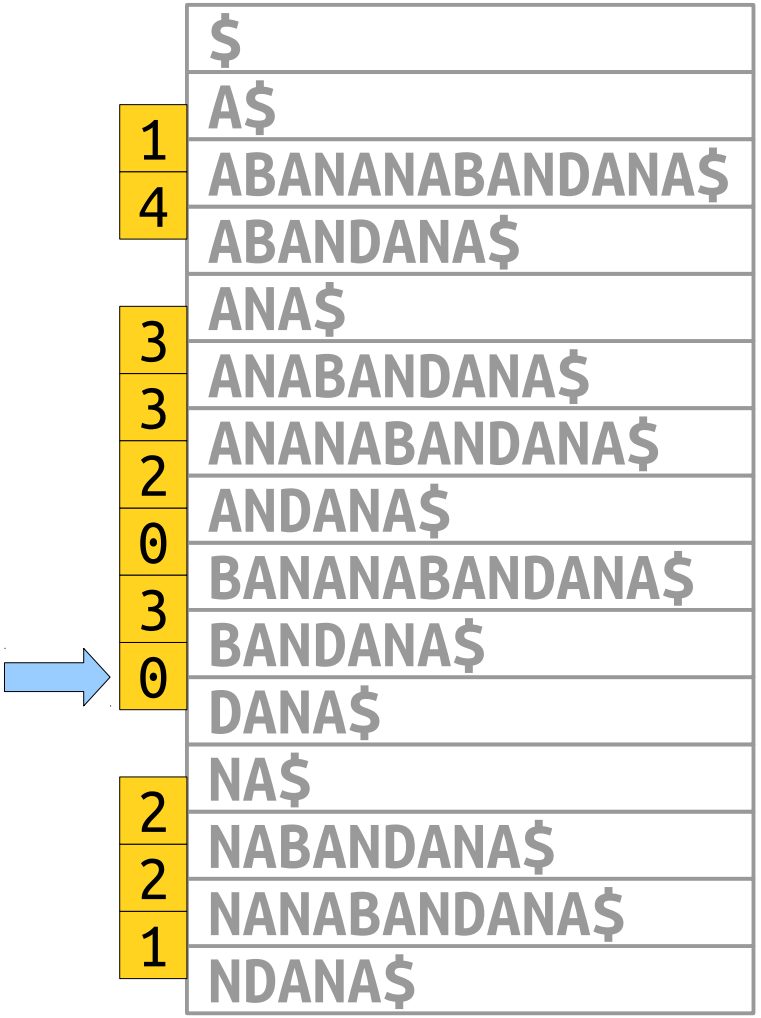

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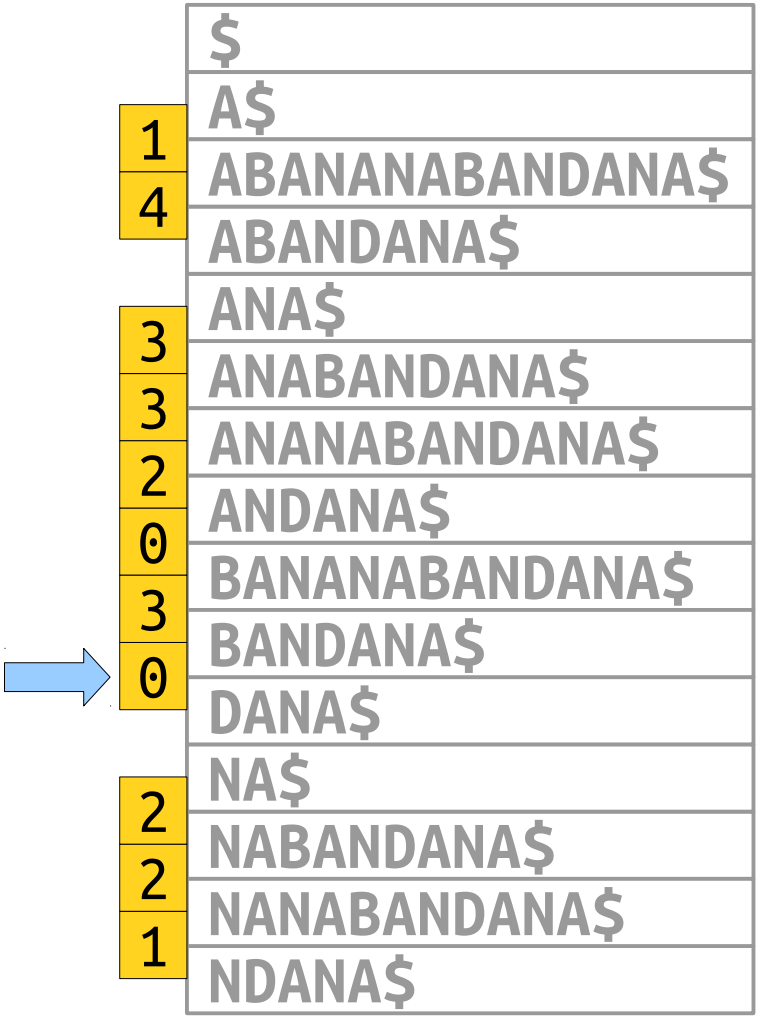

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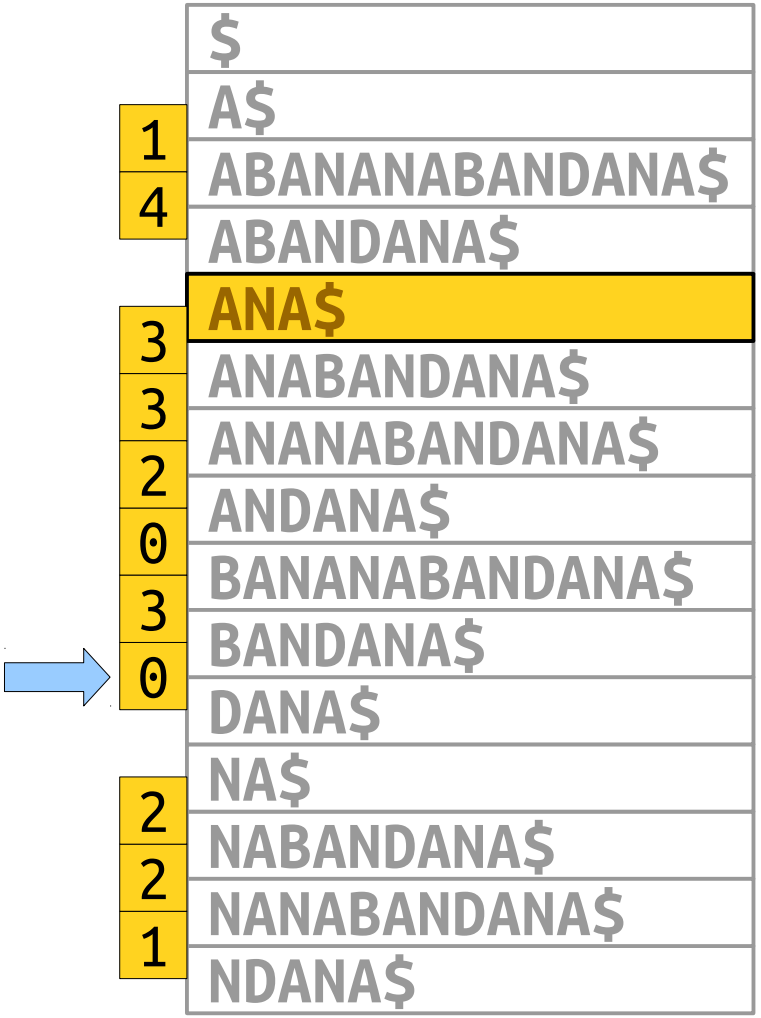

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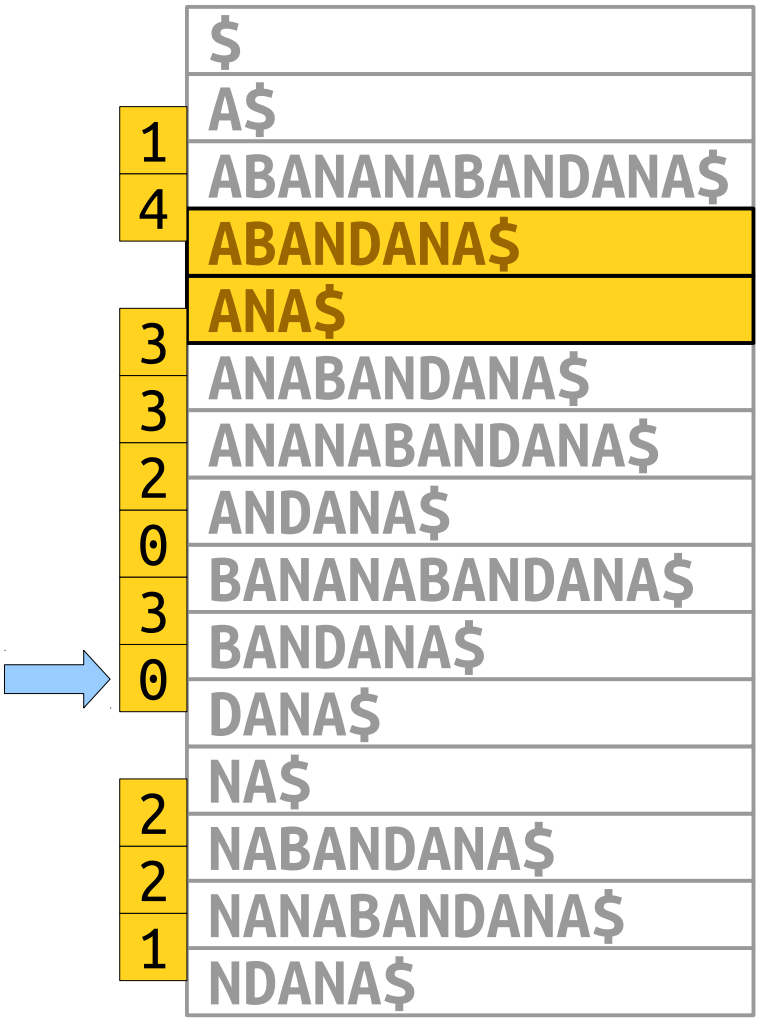

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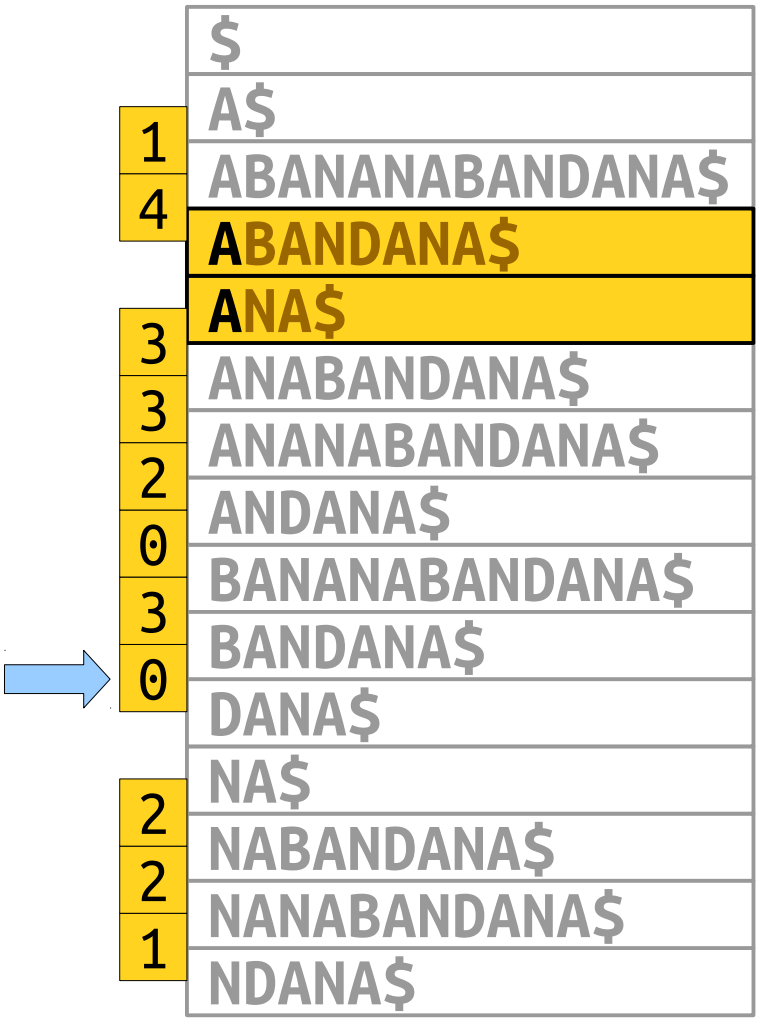

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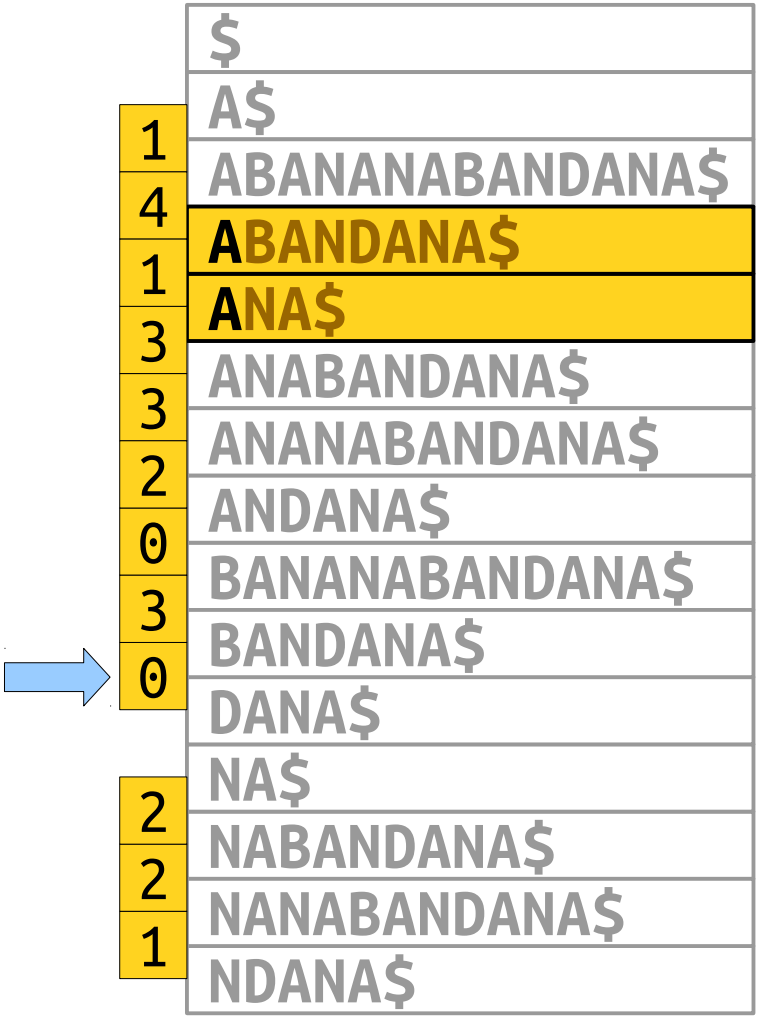

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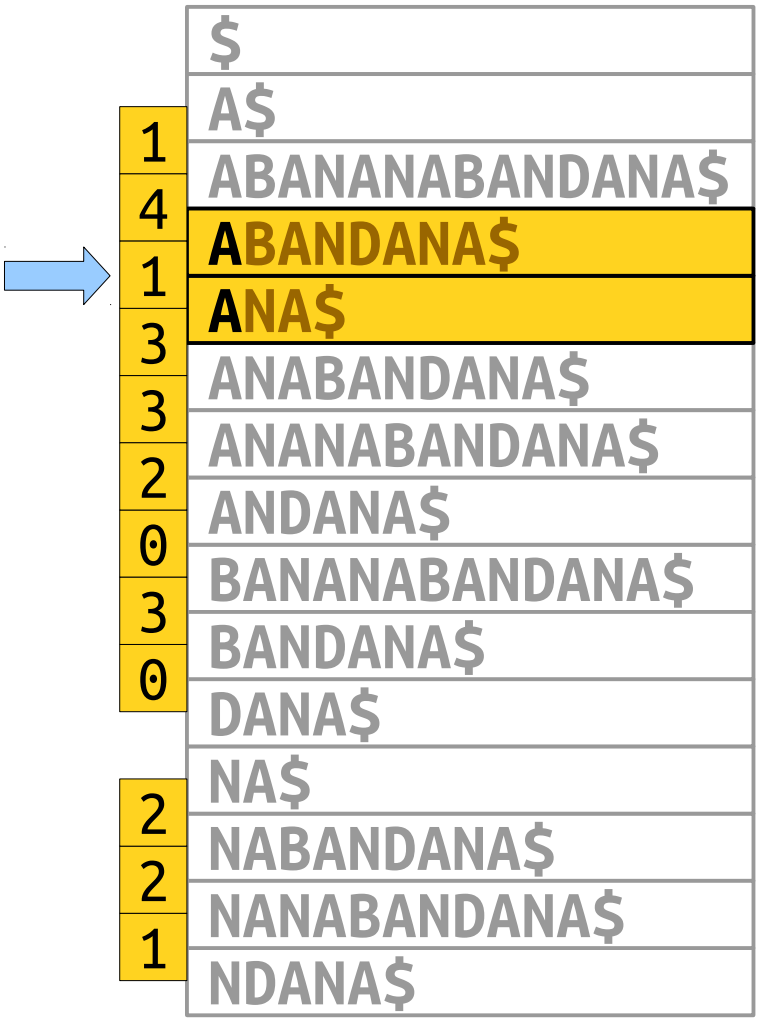

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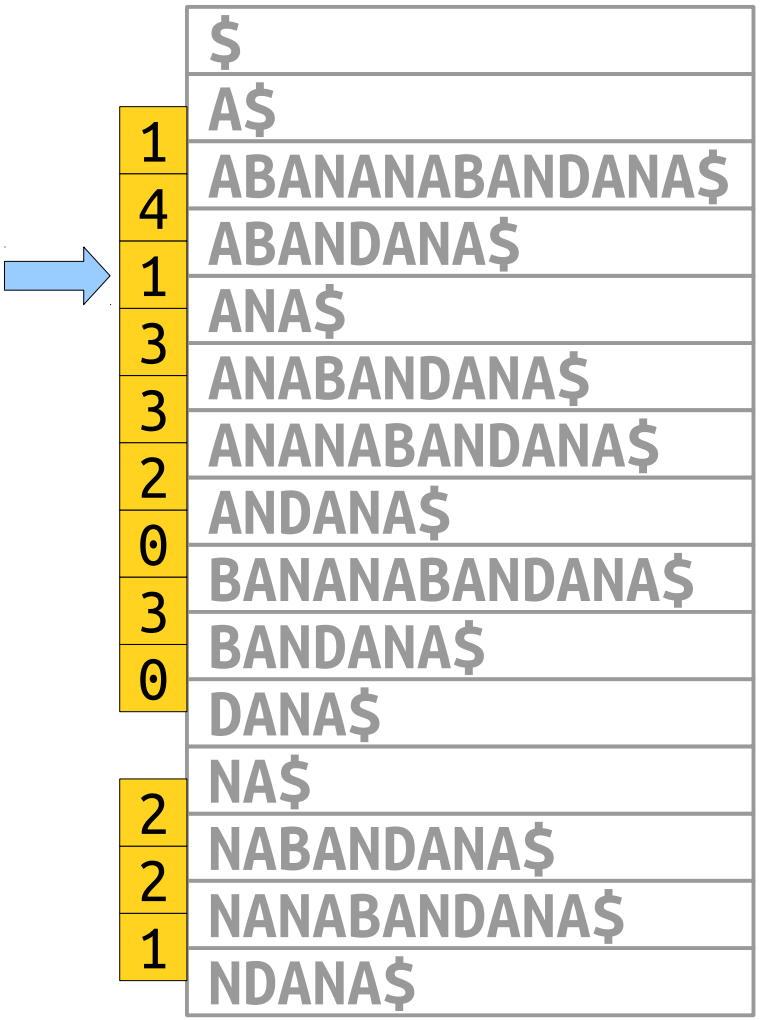

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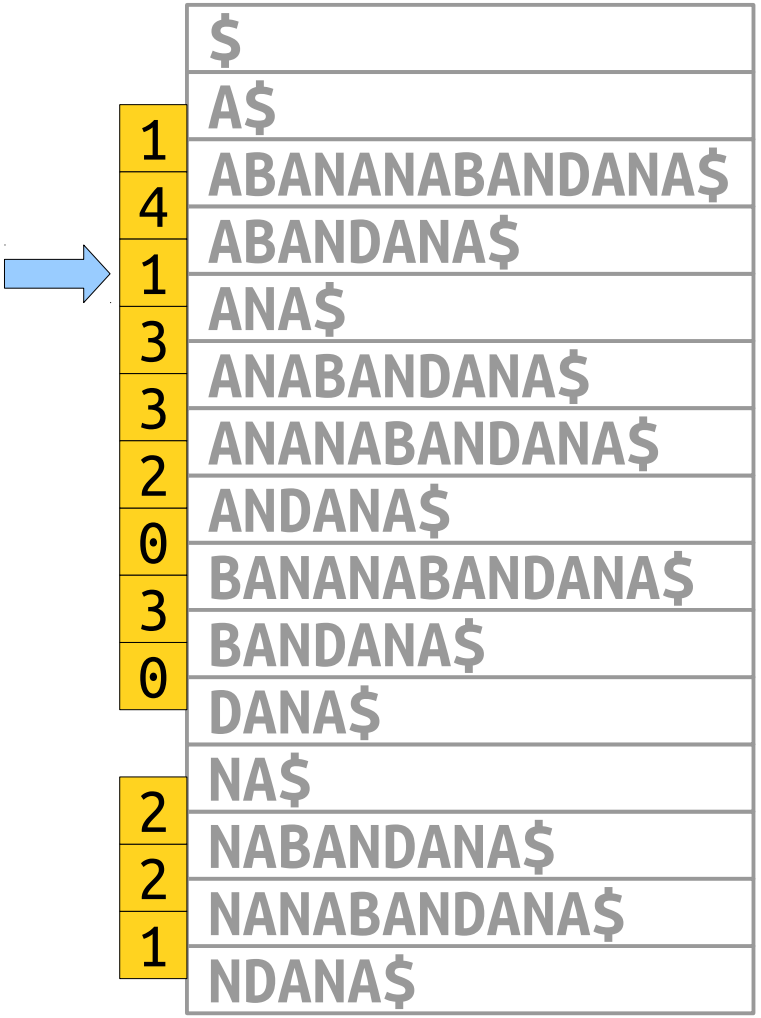

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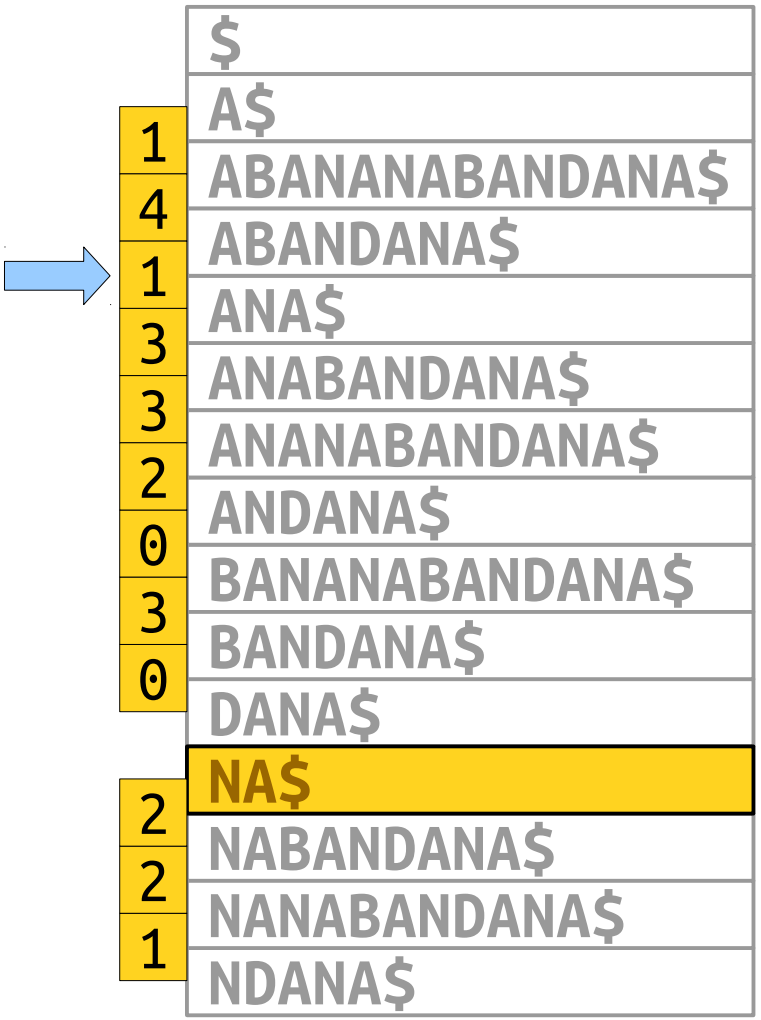

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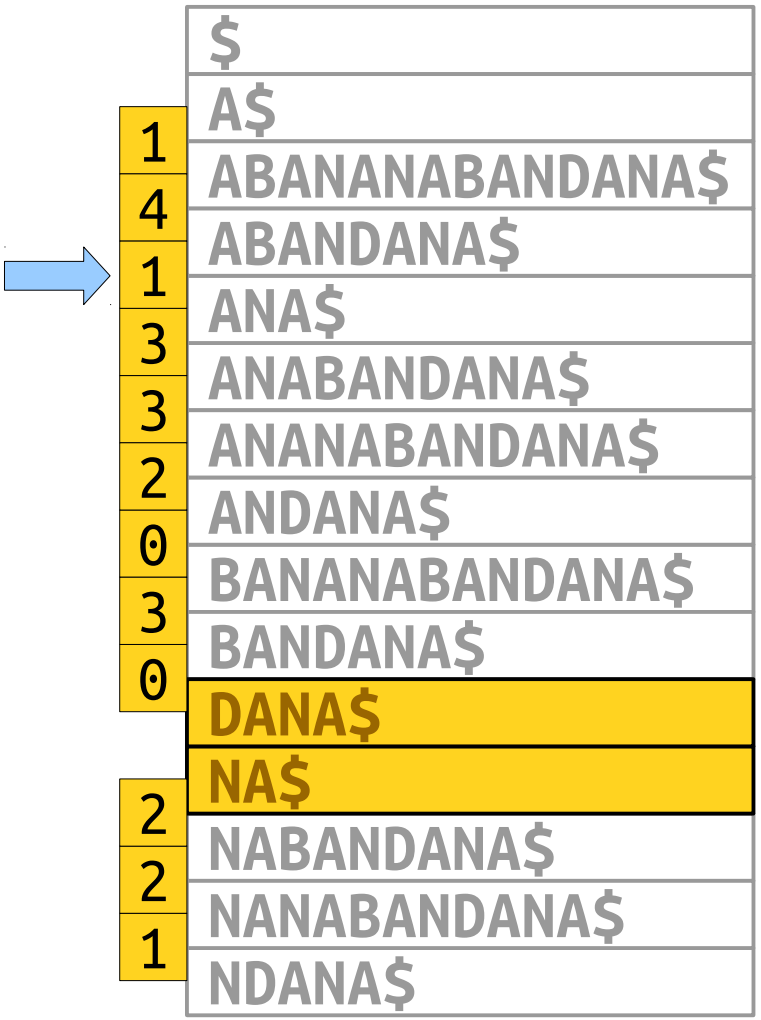

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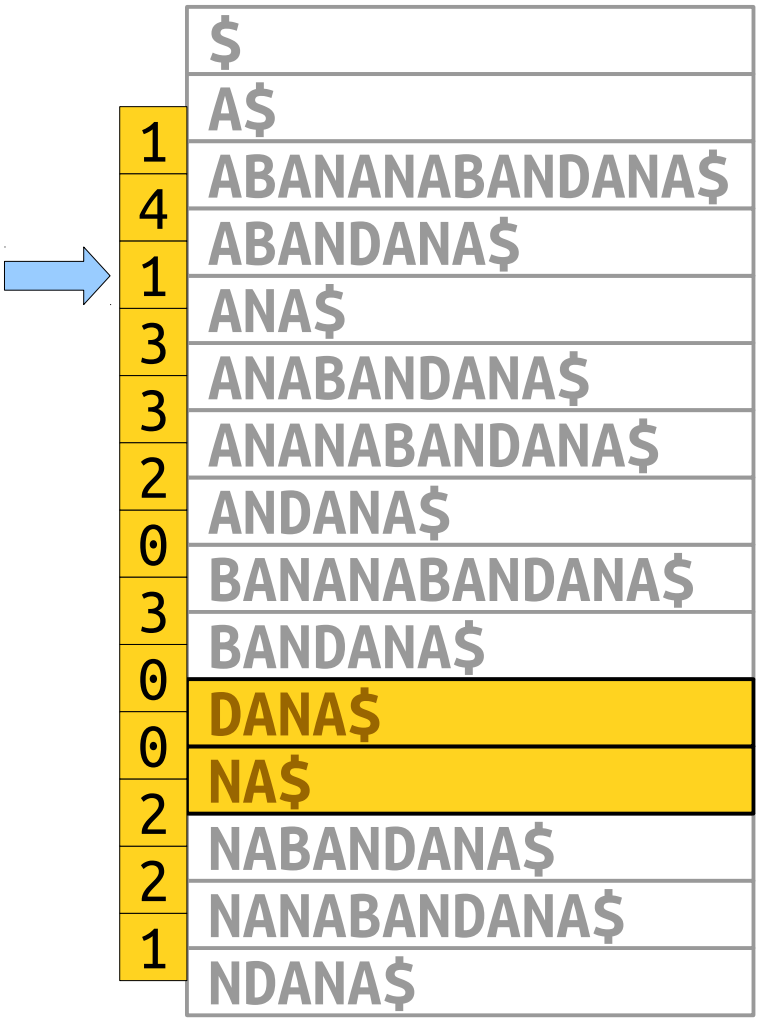

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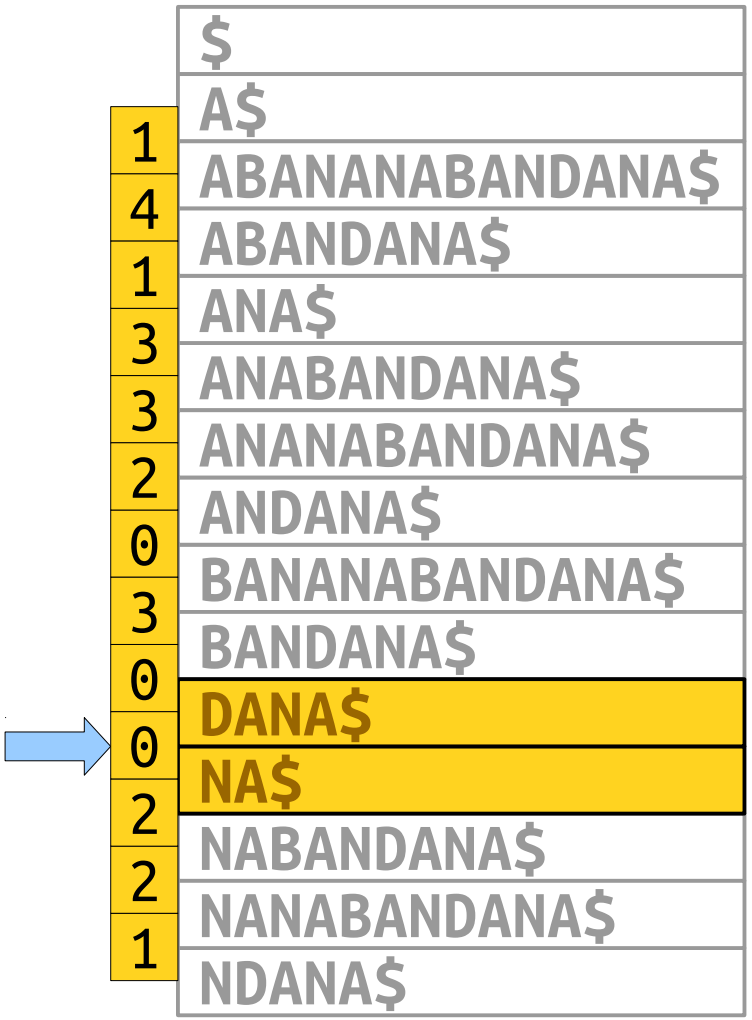

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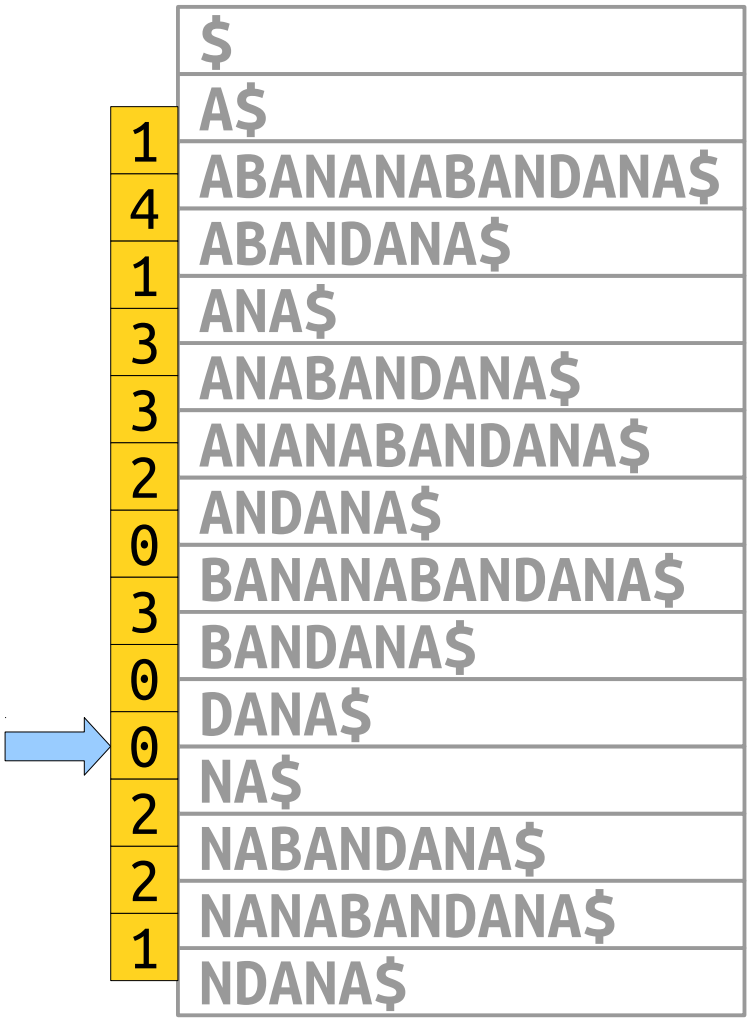

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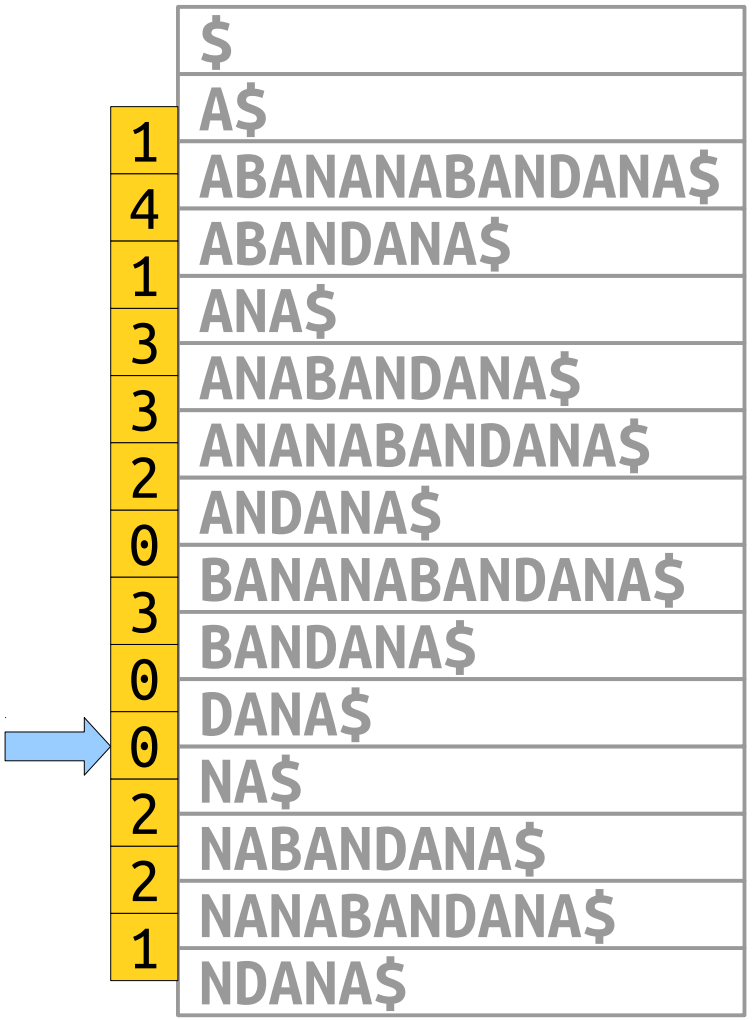

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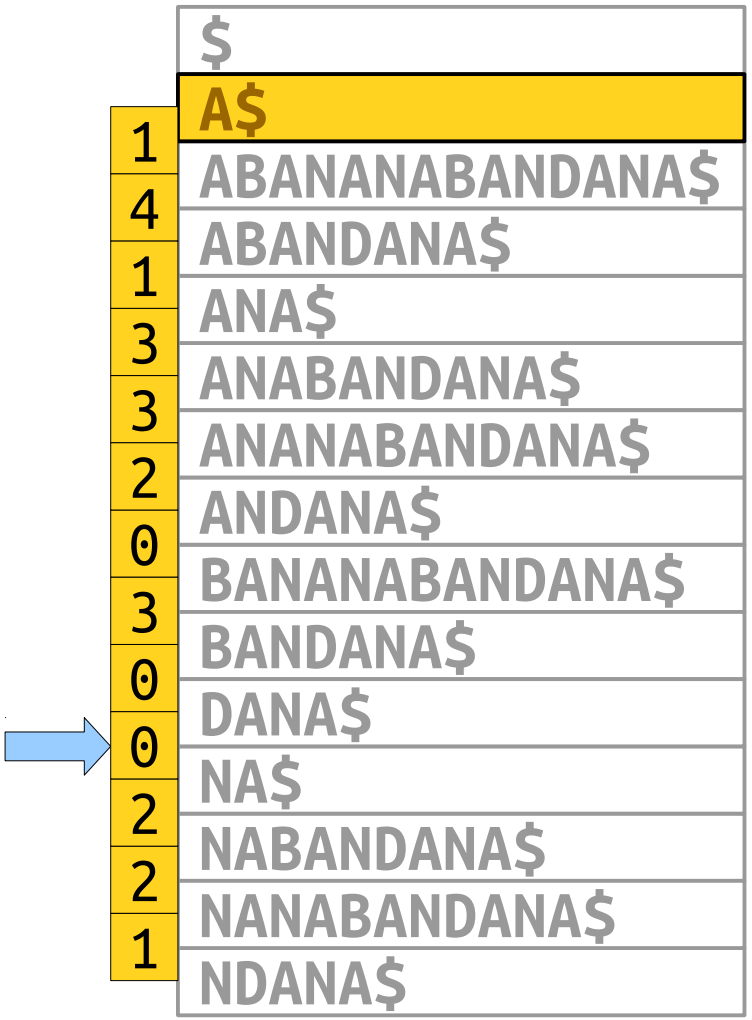

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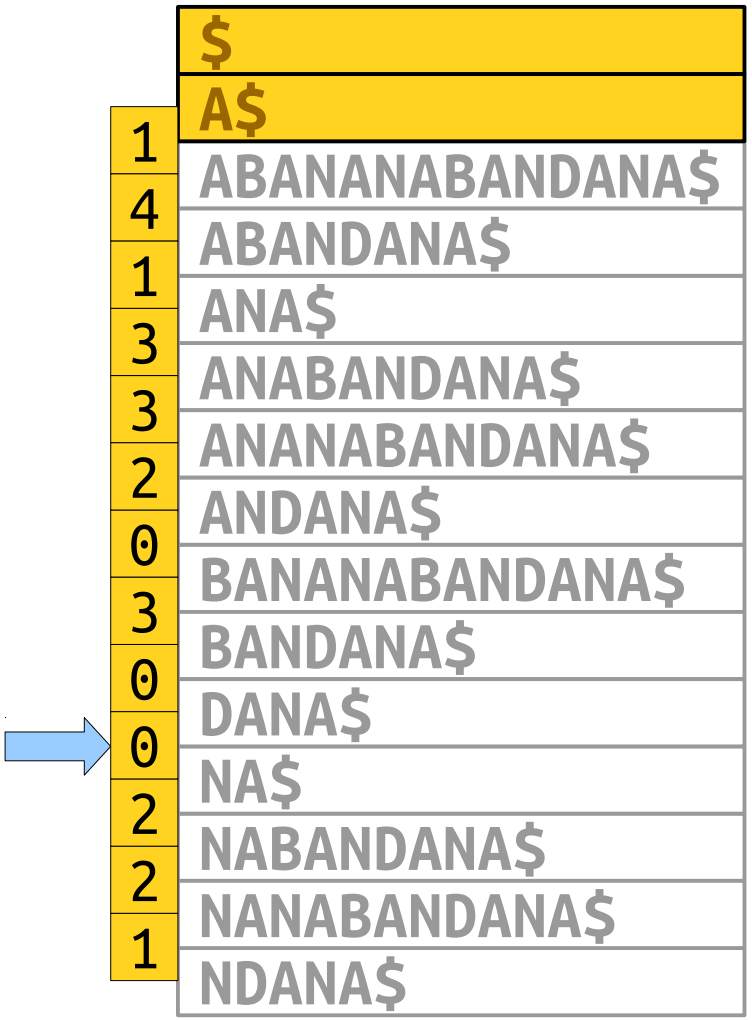

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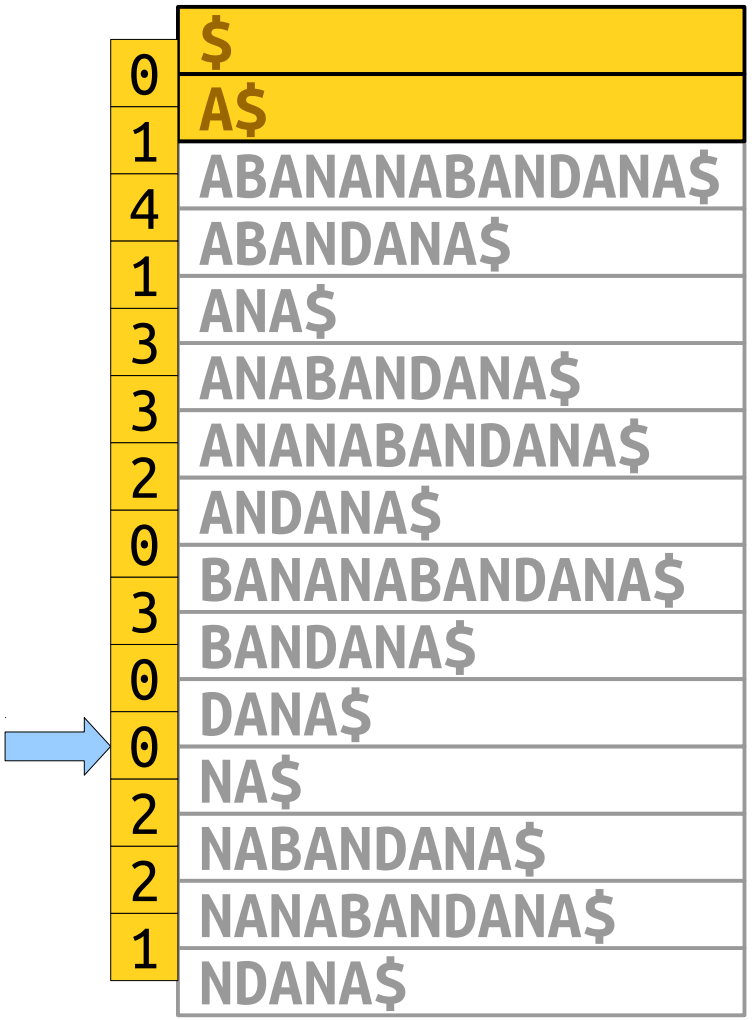

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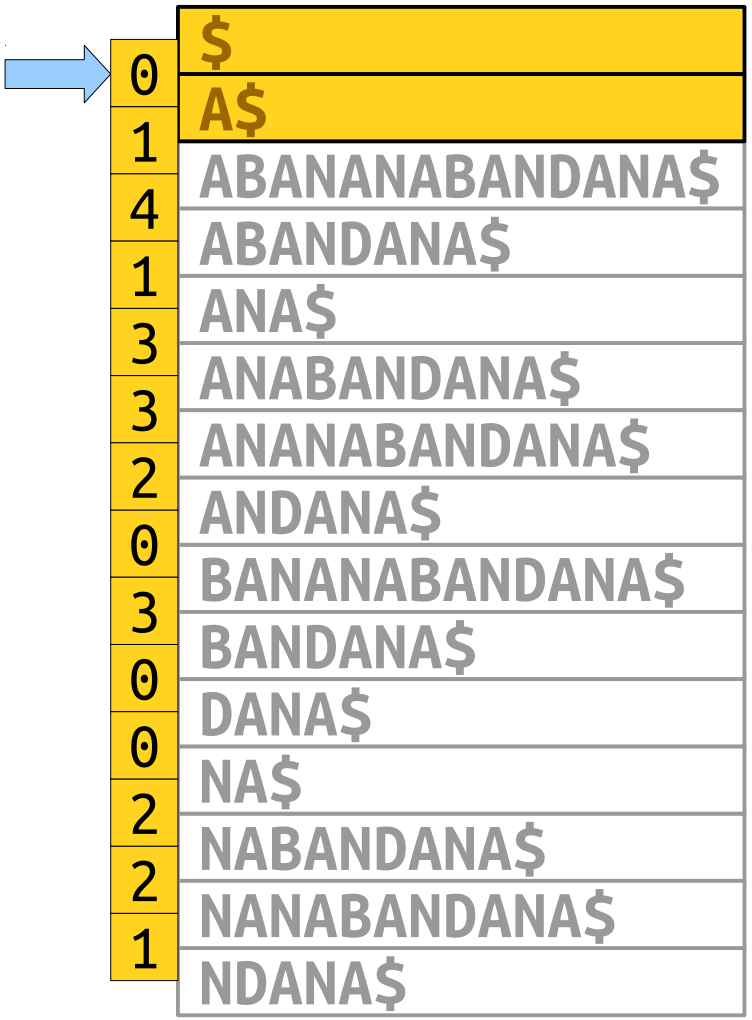

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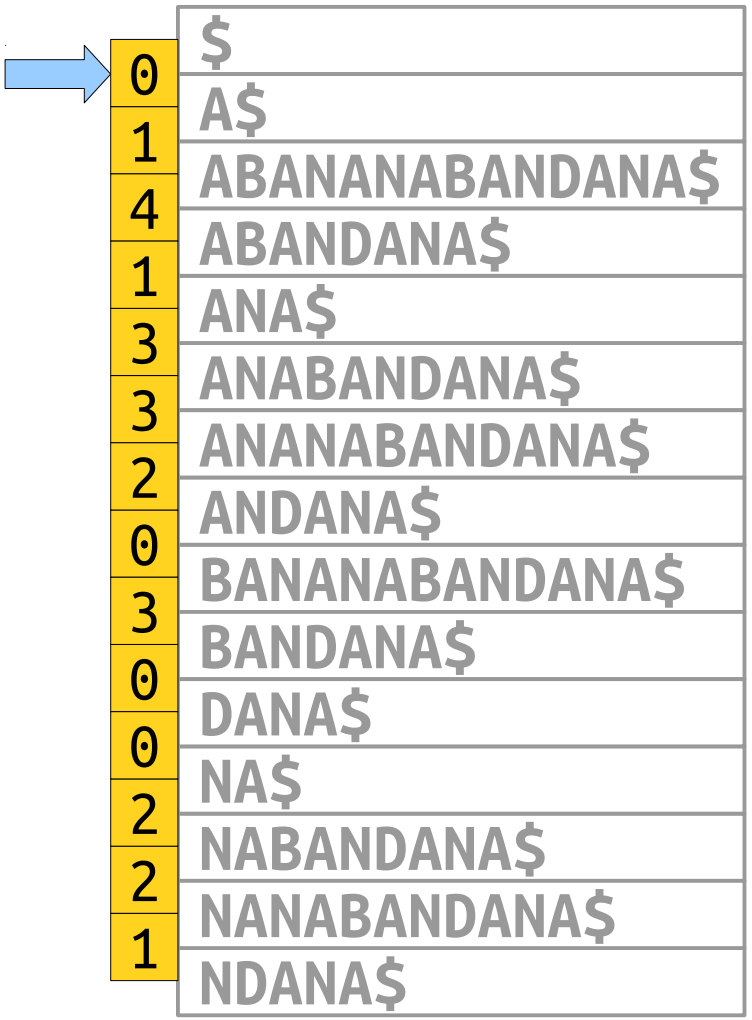

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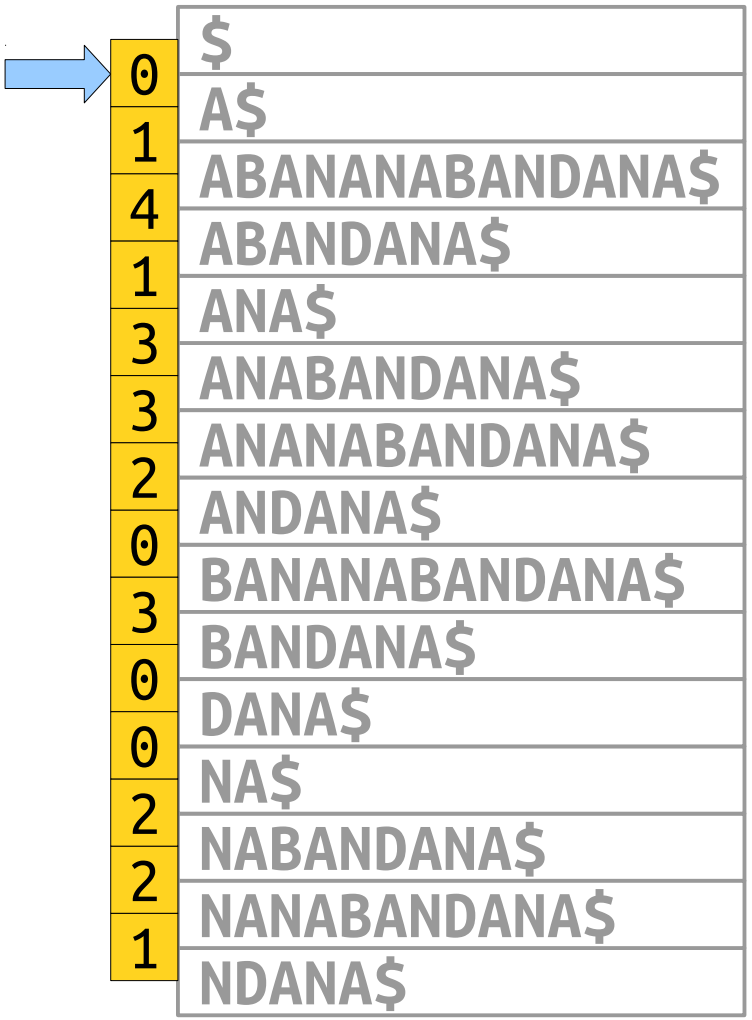

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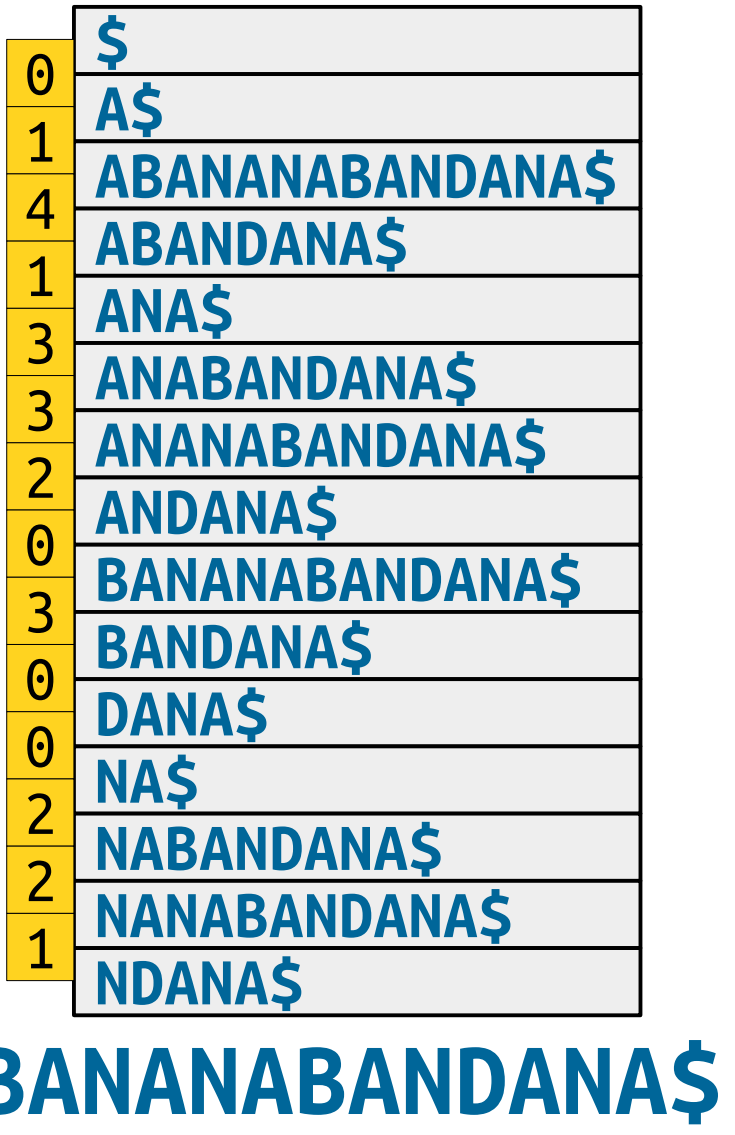

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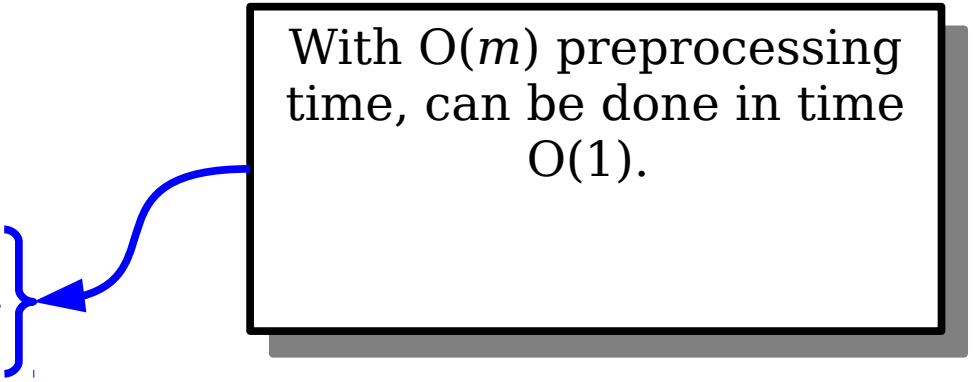

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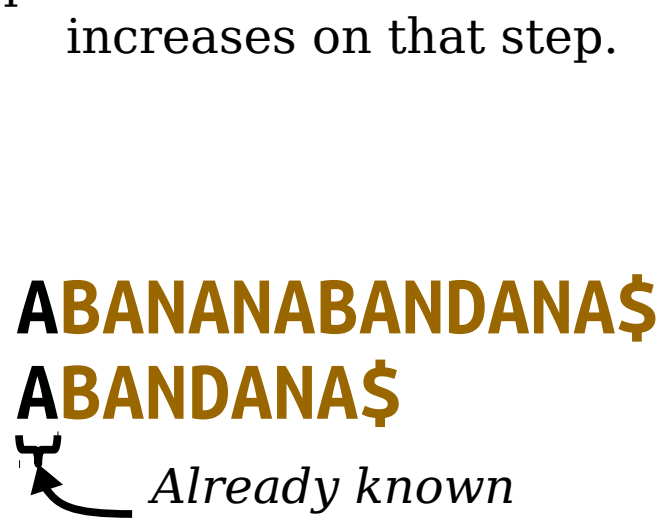

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Claim: Across all iterations, this step takes a total of O(*m*) time.

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Total runtime: O(***m***).

More to Explore

- We could easily spend a whole quarter talking about suffix arrays. Here's what we didn't cover:
  - ***Bottom-up tree simulations:*** Using LCP arrays, you can simulate any O(***m***)-time suffix tree algorithm that works with a bottom-up DFS in time O(***m***).
  - ***Faster substring searching:*** Using LCP arrays, plus RMQ, you can improve the cost of a substring search to O(***n*** + ***z*** + log ***m***).
  - ***Burrows-Wheeler transform:*** Suffix arrays, plus LCP arrays, can be used to significantly improve the performance of text compressors.
- Find this stuff interesting? These could make for really interesting final project topics!

Next Time

- ***Building Suffix Trees***
  - … is not that bad once you have a suffix array.
- ***Building Suffix Arrays***
  - … with the mother of all divide-and-conquer algorithms!