Upsampling property of the Z Transform

Let $f[n]$ and $g[n]$ be two sequences with z-transforms $F(z)$ and $G(z)$.

Observe that if

$$f[n] = \begin{cases} g[n/2] & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

then

$$F(z) = \sum_{n=-\infty}^{\infty} f[n] z^{-n} = \sum_{k=-\infty}^{\infty} f[2k] z^{-2k} = \sum_{n=-\infty}^{\infty} g[n] (z^2)^{-n} = G(z^2)$$

In general, if

$$f[n] = \begin{cases} g[n/m] & \text{if } n \text{ is a multiple of } m \\ 0 & \text{otherwise} \end{cases}$$

then

$$F(z) = \sum_{n=-\infty}^{\infty} f[n] z^{-n} = \sum_{k=-\infty}^{\infty} f[mk] z^{-mk} = \sum_{n=-\infty}^{\infty} g[n] (z^m)^{-n} = G(z^m)$$