Introduction

Informal definition

random process = randomly chosen signal:

- \{X_n; \text{ all integer } n\} (discrete time), e.g., computer data, stock market quotes, sampled speech, digitally acquired sensor measurements. Also denoted \{X(n); \text{ all integer } n\} or \{X[n]; \text{ all integer } n\}
- \{X(t); \text{ all real } t\} (continuous time), e.g., thermal noise, noisy radio signal, analog recorded speech, analog microphone output. Also denoted \{X_t; \text{ all real } t\}

“time” is an index or parameter, in some examples it is “space”

Alphabet

Fix a time \(t\) or \(n\): Process takes on values in some set \(A\) (alphabet)

Alphabet might be

- discrete, e.g., \(X_n \in \{0, 1\}, X_n \in \{1, 2, 3, 4, 5, 6\}, X_n \in \mathbb{Z}_+ = \text{nonnegative integers}\)
- continuous, e.g., \(X_n \in [0, 1), X_n \in \mathbb{R} = (-\infty, \infty)\)

(On rare occasions will consider mixed)
Random process taxonomy

So 4 basic types of random processes:

1. Discrete time, discrete alphabet (digital). E.g., coin flips, computer data, dice rolls, Dow Jones, output of digital sensor
2. Discrete time, continuous alphabet. E.g., sampled output of analog sensor/microphone/camera
3. Continuous time, discrete alphabet. E.g., random telegraph wave, Poisson counting process (number of ships/packets/customers/flies arriving by time t)
4. Continuous time, continuous alphabet. E.g., thermal noise, analog sensor/microphone output

The basic “definitions” so far are informal (not rigorous), but they help provide intuition when the precise definitions come later.

Keep coin flips in mind:

- 1 coin flip = random variable
- k coin flips = random vector
- flip a coin forever = random process

Coin flips constitute simplest nontrivial random process, and they can be used as a building block of many very interesting more complicated random processes

Note: Single output of random process at specified time $t_0$, $X_{t_0}$ or $X(t_0)$, is a random variable.

A finite collection of outputs such as $X(1), X(2)$ or $X(t_0), X(t_1), \ldots, X(t_{k-1})$ is a random vector

So random process = infinite collection of random variables, random vector = finite collection of random variables (e.g., two)

Signal Processing

Signal processing = take a signal (input) and perform an operation on it to get a new signal (output)

\[
\{X_n\} \xrightarrow{\text{Signal Processing}} \{Y_n\}
\]

E.g., filtering (linear or nonlinear), coding, quantization, A/D, D/A, regression, estimation, prediction, inference, learning, modulation, smoothing, denoising, sampling, averaging, shaping, detection, classification
EE278 Issues

  E.g., theory of probability and expectation
- How relate calculus of random processes (theory, analysis) to expected long term behavior of actual sequences/waveforms? Probabilistic/statical averages (expectations) vs. long term average (time averages, sample averages of measurements) behavior.
  E.g., expectation, law of large numbers

- Useful random processes and types of random processes.
  Specific examples of signal processing. Complicated models from simple models via signal processing
  Models: memoryless, iid, Markov, linear (autoregressive, moving average), counting, independent increment, Bernoulli, Gaussian, Poisson
  Signal processing: estimation, prediction, linear filtering, coding, modulation

A few examples of signal processing:

Coding Example

Suppose \( \{Z_n\} \) = fair coin flips process (a Bernoulli process). Code into more complicated process with memory

\[
\begin{array}{ccc}
Z_n & Z_{n-1} & Z_{n-2} \\
\end{array}
\]

\[
Y_n = g(Z_n, Z_{n-1}, Z_{n-2})
\]

<table>
<thead>
<tr>
<th>(Z_n Z_{n-1} Z_{n-2})</th>
<th>(Y_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>0.7683</td>
</tr>
<tr>
<td>001</td>
<td>-0.4233</td>
</tr>
<tr>
<td>010</td>
<td>-0.1362</td>
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<td>-1.3286</td>
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<tr>
<td>111</td>
<td>-0.7683</td>
</tr>
</tbody>
</table>
**Estimation Example**

\[ Y_n = X_n + W_n \]

\[ \hat{X}_n = \sum_k h_{n-k}Y_k = \sum_k h_kY_{n-k} \]

Want average \((X_n - \hat{X}_n)^2\) small

**Binary Detection Example**

\[ Y_n = X_n + W_n \]

\[ \hat{X}_n = \begin{cases} +1 & \text{if } Y_n \geq 0 \\ -1 & \text{otherwise} \end{cases} \]

Want average \(\Pr(X_n \neq \hat{X}_n)\) small

**Prediction Example**

\[ \hat{X}_{n+1} = f(\ldots, X_{n-1}, X_n) \]

Predict next value from past. Want average \((X_n - \hat{X}_n)^2\) small

**Modulation Example**

Amplitude modulation: Input process \(\{X(t)\}\), output

\[ Y(t) = (a_0 + a_1X(t)) \cos(2\pi f_0 t + \Theta) \]

Important in modeling (speech processing, seismology, medical signals)
Course Summary

Topics/Lecture notes follow Gray & Davisson *Introduction to Statistical Signal Processing* (2010 Edition), will have additional examples in lecture notes.

- Introduction: Chapter 1
- Probability: Chapter 2
- Random variables, vectors, and processes: Chapter 3
- Expectation and averages: Chapter 4
- Second-order theory: Chapter 5
- Lots of example processes: Chapter 6

Often topics are introduced in a limited way in an earlier chapter/set of notes

E.g., random variables and expectation are both introduced in Chapter 2.

Appendix A gathers prerequisites, especially set theory (needed immediately) and linear systems (needed for Chapter 5)