
Lecture Outline

- Signal Envelope Distributions: Rayleigh, Rician, Nakagami
- Level Crossing Rate and Average Fade Duration
- Wideband Channel Models
- Scattering Function

1. Signal Envelope Distributions

- CLT approximation leads to Rayleigh distribution (in-phase and quadrature zero mean and jointly Gaussian):
  \[ p_Z(z) = \sqrt{\frac{2}{\pi \sigma^2}} \exp\left[-\frac{z^2}{2\sigma^2}\right], \quad z \geq 0. \]
- A LOS component leads to a received signal with non-zero mean. The Rician distribution models signal envelope in this case, with \( K \) factor dictating the relative power of the LOS component:
  \[ p_Z(z) = \sqrt{\frac{\rho}{\pi \sigma^2}} \exp\left[-\frac{(z^2 + \sigma^2)}{2\sigma^2}\right] I_0\left(\frac{z\rho}{\sigma^2}\right), \quad z \geq 0. \]
- Experimental results support a Nakagami distribution for some environments. Similar to Rician, but can model “worse than Rayleigh.” Model generally leads to closed-form expressions in BER and diversity analysis:
  \[ p_Z(z) = \frac{2^m m^{m-1}}{\Gamma(m)} \exp\left[-\frac{m z^2}{\rho_r}\right], \quad m \geq .5. \]

2. Level Crossing Rate and Average Fade Duration

- Level crossing rate \( L_R \) is the rate at which a signal envelope crosses the threshold \( R \).
- In Rayleigh fading the level crossing rate is \( L_R = \sqrt{\frac{2}{\pi}} f_D \rho e^{-\rho^2} \), where \( \rho = R/\sqrt{\rho_r} \).
- Average Fade Duration (AFD) measures how long a signal’s envelope or power stays below a given target threshold: derived from the level crossing rate.
- For Rayleigh fading \( T_r = \rho e^{\rho^2-1}/(\rho f_D \sqrt{2\pi}) \), where \( \rho \) is the ratio of target envelope level to average envelope level (or square root of ratio of target power level to average power level).

3. Wideband Channel Models

- In wideband multipath channels the individual multipath components can be resolved by the receiver. True if \( T_m > 1/B \).
- If the components can be resolved then they can be combined for diversity gain (e.g. using an equalizer).

4. Channel Scattering Function:

- For deterministic channels, the scattering function is defined as the Fourier transform of \( c(\tau, t) \) with respect to \( t \).
- Typically \( c(\tau, t) \) is unknown, so it must be characterized statistically.
- Since the underlying process \( c(\tau, t) \) is Gaussian, we only need to characterize its mean and correlation. We assume \( c(\tau, t) \) has mean zero.
• Autocorrelation of $c(\tau, t)$ is $A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1, \tau_2; \Delta t)\delta(\tau_1 - \tau_2) = A_c(\tau; \Delta t)$ since we assume channel response associated with different scatterers is uncorrelated.

• Statistical scattering function defined as $S(\tau, \rho) = \mathcal{F}_{\Delta t}[A_c(\tau, \Delta t)]$.

• This function measures the average channel gain as a function of both delay $\tau$ and Doppler $\rho$.

• $S(\tau, \rho)$ easy to measure empirically and is used to get average delay spread $T_M$, rms delay spread $\sigma_\tau$, and Doppler spread $B_d$ for empirical channel measurements.

Main Points

• The signal envelope under narrowband fading with uniform AOA is Rayleigh. Other common distribution are Ricean (when a LOS component exists) and Nakagami.

• Average Fade Duration used to determine how long a user is in continuous outage (e.g. for coding design).

• Wideband models characterized by scattering function, which measures average channel gain relative to delay and Doppler.

• Scattering function used to obtain key channel characteristics of rms delay spread and Doppler spread, which are important for system design.