### Overview of Cuts

- **Reductions**
- **NP-hard problems**

#### Global min cut

- \( \binom{n}{2} \) different cuts that realize the global min cut (NP-hard)

#### S-t min cut

- For fixed s, t, there could exponentially many s-t min cuts
- Min cut (at most) \( \binom{n-1}{2} \) different values

#### Max cuts (very hard)

- \( \frac{1}{2} \) approximation (randomized)
- \( \frac{1}{2} \) approx. (deterministic)
- 0.878 approx. (SFP relaxation 1995)
- Generalize to submodular functions

### Linear Time, Polynomial time?

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### NP-hard problems

- Decision problem: a problem that has a yes/no answer.
- All problems in NP are decision problems.

### P and NP

- **P**: The set of decision problems whose "yes" answers can be verified in polynomial time.
- **NP**: The set of decision problems whose "yes" answers can be verified in time bounded by a polynomial function.

### Problems that have polynomial time solvers are in P

- **P \subseteq NP**

### Tautology

- Problem that is not in NP.

### Reductions

- Independent set: Is there an induced subgraph (bundle of nodes with at least \( k \) nodes) with no edge between nodes?
3-SAT: Given a satisfying assignment to a boolean of 3-SAT form:

\[ (x_1 \lor x_2 \lor x_3) \land (x_2 \lor x_1 \lor x_3) \land \ldots \]

\[ \rightarrow \text{3-Sat can be reduced to } \leq^P \text{ Independent-set.} \]

\[ \rightarrow \text{Given an instance of 3-Sat, and a Blackbox for independent-set, solve the instance of 3-Sat in polynomial-time addition work and polynomial number of calls to the Blackbox.} \]

- We create a gadget for each clause.

\[ \text{Call Independent-set on this graph, return "yes" if } \exists \text{ independent set of size } k. \]
Proof: \( I \leq VC \Rightarrow \checkmark \text{ exercise} \)

you are left with just node.

If we have independent

set \( s \rightarrow v - s \) is a cover.

exercise.

\( VC \leq \text{ independent set} \)

Independent set \( \leq VC \).