GLOBAL MIN CUT (Karger's algorithm).

This class:

$s$--$t$ min cut, max flow

$\rightarrow$ focus on directed graphs.

For undirected graphs, will follow by adding

and $\leftrightarrow$ (add both directions, with same capacities)

Greedy doesn't work.

Min cut problem

We can add edge.

$S$--$V$--$U$--$B$

Key: The max flow from $s$ to $t$ is equal to the minimum cut size.

→ **FORD FULKERSON ALGORITHM**
  finds a flow whose value equals some cut size (and is thus optimal)
  that cut is also the minimum cut.

Directed cut?

 Tight duality?

 Linear program

Define residual Graph: $R_f$

Let $f$ be a legal flow
  (flows are non-negative
    only leave $s$
    only created at $s$
    only destroy at $t$
    flow in = flow out everywhere else)

→ For directed edge $(u,v) \in E$
  if $f_{uv} + \delta_{uv} \leq c_{uv}$
    add edge $(v,u)$ with weight $c_{uv} - f_{uv}$ "forward"
> flow upon termination is equal to cutsize of $B$.

> Time is the instance when termination already happened.

> only rational weights

(Kleinberg-Tardos reading)

**RUN TIME ANALYSIS:**

\[
\begin{align*}
0(m) & \text{ Time to create } R_f \\
0(n) & \text{ Time to find path in } R_f
\end{align*}
\]

Repeat $C$ times where $C$ is max-flow

= no. of edges $m + |E|$ size of edge set.

$O(mn)$ \[\approx \begin{array}{c}
(1956) \text{ FF}\end{array}
\]

\[\text{BFS, DFS ?}
\]

In 1972, Edmunds-Karp: $O(nm^2)$ (1972 & Karp shortest path)


**Companion to Global min cut**

Run it $n-1$ time

$\Rightarrow$ global min-cut $O(n^3m)$

Karger: $O(n^2 \log^3(n))$

In HW

\[\binom{n}{2} \text{ distinct cuts.}\]

there are only $(n-1)$ distinct cuts.

induction: $n$ smaller, using max-flow

$\Rightarrow$ global min cuts $nC_2$

**BIPARTITE GRAPH** (Undirected): $G$ is one in which nodes are split in two sets $A, B$.

$\Rightarrow$ all edges go between $A \times B$.

$\Rightarrow$ No edges on inside $A, B$.

edges that become saturated are optimal soln.

Nodo are heterogeneous.

Bipartite matching problem: (Max flow to solve).

job 1 person $\rightarrow$ jobs.