Approximation Algorithms

1. Bin Packing
2. Asymmetric TSP

Given n items in [0,1], pack into smallest number of unit size bins.

Gabriel's algorithm (Any-Fit algorithm)

1. Order items arbitrarily

   while (the current item fits into any already opened bin, put item into that bin).

   otherwise open a new bin.

At any point during the algorithm, there is at most one bin that is not strictly more than half full.

Proof: Consider an item. 2 cases:

1. Heavy \((\geq \frac{1}{2})\)
2. Light \((< \frac{1}{2})\)

\[ \text{# bins} \leq 2 \sum_{i=1}^{n} \frac{a_i}{2^i} + 1 \leq 2 \sum_{i=1}^{n} \frac{a_i}{2^i} \]

OPT \(\geq \left\lceil \sum_{i=1}^{n} \frac{a_i}{2^i} \right\rceil \]
Asymptotic polynomial time approx scheme.

\[ 0 \leq \varepsilon \leq 1 \] 
\[ \frac{\text{time}}{\text{OPT}} \leq 2(1+\varepsilon)\text{OPT} + 1 \]

\[ \# \text{time} \leq \frac{2\bar{Z}_S}{Z_{a_1}} \leq 2\bar{Z}_a = 2 \]

→ Complete Graph
→ Triangle Inequality
→ No symmetry (not necessarily)

Let's construct a graph. Consider a directed graph \( G \) which is a cycle - a short path from \( i \) to \( j \) in an undirected graph \( H \).

Let \( G \) be complete.

Why does \( G \) have a inequality?

This is a family:

\[ d_j = 1 \]
\[ d_j = n - 1 \]

Cycle covers:

HW9: A collection of (directed) cycles that cover all nodes (each node is part of exactly one cycle).

For the rest of the lecture, graph is directed.

Arborescence: we can compute in polynomial time, minimum spanning.

Arborescence:

\[ \frac{\text{time}}{\text{OPT}} < 300 \]
Key 1: Hamiltonian cycles are cycle covers.

- Minimum length cycle cover can be computed in polynomial time.

> Every time we add a cycle cover, we group together at least half the nodes.

$\implies$ at most $\log_2(n)$ cycle covers are needed.

$\rightarrow$ All other operations (short cutting) reduces cost.

$\text{Cost} \leq \log_2(n) \cdot MEC \leq \log(n) \cdot \text{OPT}$

$\rightarrow$ Held-Karp 82.

- LP's with exponentially many constraints can be solved as long as we have a black box to find violated constraints (or to correctly claim there are none).

- Out-degree of $S \geq 1$ $\forall S \in V$

Out-degree out of every single cut is at least 1.

$\rightarrow$ How do you write a linear program for TSP?

Indicator variable per edge $X_e$

$\min \sum_{e \in E} X_e$ $c_{ij}$

How to write it with linear constraints?

$\forall i \quad \text{indegree}(i) = 1$

$\forall i \quad \text{outdegree}(i) = 1$ + $i$

$\rightarrow$ only constraint for cycle cover

2012. Amin Saberi $\frac{\log(n)}{\log(\log(n))}$