CME306 Qualifying Exam

Part I - Multiple Choice (1 point each)

1. If we have a spring with drag coefficient k_d and spring constant k_s, which of the following are sufficient to have a well-posed system?

   (a) $k_d > 0$

   (b) $k_s > 0$, $k_d > 0$

   (c) $k_s k_d < 0$

   (d) $\left(\frac{k_d}{2m}\right)^2 - \frac{k_s}{mx_0} \geq 0$

2. Suppose that we wish to discretize the equation

$$u_t - u_x = 0.$$

   Choose the best discretization among the following choices.

   (a)
$$\frac{v_i^{n+1} - v_i^n}{\Delta t} - \frac{v_{i+1}^n - 2v_i^n + v_{i-1}^n}{\Delta x^2} = 0$$

   (b)
$$\frac{v_i^{n+1} - v_i^n}{\Delta t} - \frac{v_{i+1}^n - v_i^n}{\Delta x} = 0$$

   (c)
$$\frac{v_i^{n+1} - v_i^n}{\Delta t} - \frac{v_{i+1}^n - v_{i-1}^n}{2\Delta x} = 0$$

   (d)
$$\begin{cases} \frac{\hat{v}_i^{n+1} - v_i^n}{\Delta t} - \frac{v_{i+1}^n - v_{i-1}^n}{2\Delta x} = 0 \\ \frac{\hat{v}_i^{n+2} - \hat{v}_i^{n+1}}{\Delta t} - \frac{\hat{v}_{i+1}^{n+1} - \hat{v}_{i-1}^{n+1}}{2\Delta x} = 0 \\ v_i^{n+1} = \frac{\hat{v}_i^{n+2} + v_i^n}{2} \end{cases}$$

   (e)
$$\frac{v_i^{n+1} - v_i^n}{\Delta t} - \frac{v_i^n - v_{i-1}^n}{\Delta x} = 0$$

Part II - Short answer

1. (2 points) Please discuss briefly the advantages and disadvantages of using forward- vs. backward-Euler time-stepping.

2. (2 points) Why does Lax-Richtmyer require stability in addition to consistency (i.e. why isn't consistency sufficient)?

3. (2 points) Consider a simple equilateral triangle, with side lengths $\ell_{1_0} = \ell_{2_0} = \ell_{3_0} = 1$. In world space, the sides measure ℓ_1, ℓ_2 and ℓ_3 respectively. Write down the Green strain for this deformation (it is sufficient to write down D_m and $D_m^T G D_m$).

Part III - Long Answer

1. (4 points)

$$u_t + au_x = 0 \tag{1}$$

Show that the following discretization of the advection equation (1) with $a > 0$ is either stable or unstable, then **state** the order of accuracy (ie. there is no need to justify the order of accuracy).

$$\begin{cases} u_i^* = u_i^n - a\Delta t \frac{3u_i^n - 4u_{i-1}^n + u_{i-2}^n}{2\Delta x} \\ u_i^{**} = u_i^* - a\Delta t \frac{3u_i^* - 4u_{i-1}^* + u_{i-2}^*}{2\Delta x} \\ u_i^{n+1} = \frac{u_i^{**} + u_i^n}{2} \end{cases} \tag{2}$$