

ENO-LLF

$$u_t + \left(\frac{u^2}{2}\right)_x = 0 \quad (1)$$

Consider Burgers' equation (above), discretized in a conservative way with forward-Euler:

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{\mathcal{F}_{i+1/2}^n - \mathcal{F}_{i-1/2}^n}{\Delta x} = 0. \quad (2)$$

Recall from lecture (and section) that we compute $\mathcal{F}_{i+1/2}^n$ the numerical flux via a variety of ENO schemes.

1. Recall that Lax-Friedrichs defines, for flux $\mathcal{F}_{i_0+1/2}$, $D_i^1 H^\pm = f(\phi_i) \pm \alpha_{i_0+1/2} \phi_i$. Show that this is equivalent to adding some viscosity to the solution and that this viscosity vanishes as $\Delta t, \Delta x \rightarrow 0$.

2. In Local Lax-Friedrichs (or ENO-LLF), we take $\alpha_{i_0+1/2} = \max\{|\lambda_{i_0}|, |\lambda_{i_0+1}|\}$. A common mistake (one that shows up even in the literature) is to try to compute a global divided difference table as:

$$D_i^1 H^\pm = f(u_i) \pm \alpha_{i+1/2} u_i \quad (3)$$

Which is *very* different from the correct, **local** divided difference table for $\mathcal{F}_{i_0+1/2}$,

$$D_i^1 H^\pm = f(u_i) \pm \alpha_{i_0+1/2} u_i \quad (4)$$

There are advantages to computing a global divided difference table, by saving the need to recompute quantities that've already been computed. How can you do this for ENO-LLF while preserving the correct scheme?

Discrete Conservative Form

1. Consider the following discretizations of Burger's equation (1). Show that they are both consistent, but that only one of them is in conservative form. Which one is in conservative form?

(a)

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + \frac{(u_{j+1}^n)^2 - (u_j^n)^2}{2\Delta x} = 0$$

(b)

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + u_j^n \frac{u_{j+1}^n - u_j^n}{\Delta x} = 0$$

2. The equations given above are both consistent with Burger's equation, but only one of them gives shocks traveling at the right speed. Why is this?

3. Consider the simple advection equation (5). Show that for this equation, ENO-Roe and ENO-LLF schemes will produce identical results.

$$u_t + au_x = 0 \tag{5}$$