1. Write out the symmetric matrix equation for the standard second order central difference approximation to the equation
\[
\nabla \cdot \left( \frac{1}{\rho} \nabla p \right) = \nabla \cdot \vec{u}
\]
with the following boundary conditions:
\[
\begin{align*}
p(0, y) &= 1 \\
p(1, y) &= 1 \\
p_y(x, y) &= 0 \quad \text{for } y \in \{0, 1\}
\end{align*}
\]
You should assume a MAC grid (ie. that velocities live on cell faces, and that pressure and density live in the cell centers), and you may not assume a constant density. Write the equations for the following three cells:
(a) an internal cell (something sufficiently far from the boundary, ie. \( p_{ij} \))
(b) a cell that lies along the x-axis boundary (ie. \( p_{i1} \)), and
(c) a cell that lies along the y-axis boundary (ie. \( p_{1j} \)).

2. Physically, when we have an incompressible flow with all Neumann boundary conditions, what does the compatibility condition require? Is something similar required for compressible flow?