

Extra Practice Problems 3

Here's one final set of practice problems for next week's midterm exam. We've released solutions to these problems along with this packet of problems, but we strongly recommend that you not look over them until you've attempted and completed these problems.

Problem One: First-Order Logic

(From the Fall 2015 midterm exam)

Let's imagine that you're *really* hungry and want to build an infinitely tall cheese sandwich. Your sandwich will consist an infinite alternating sequence of slices of bread and slices of cheese.

Using the predicates

- Bread*(b), which states that b is a piece of bread;
- Cheese*(c), which states that c is a piece of cheese; and
- Atop*(x, y), which says that x is directly on top of y ,

write a statement in first-order logic that says “every piece of bread has a piece of cheese directly on top of it, every piece of cheese has a piece of bread directly on top of it, and there's at least one piece of bread.”

Problem Two: Propositional Logic

Below are a series of English descriptions of relations among propositional variables. For each description, write a propositional formula that precisely encodes that relation. Then, briefly explain the intuition behind your formula. You may find the online truth table tool useful here.

- i. For the variables $a, b, c,$ and d : the variables, written out in alphabetical order, alternate between true and false.
- ii. For the variables $a, b, c,$ and d : the variables, written out in alphabetical order, alternate between true and false, except that your formula cannot use the \vee connective.

Problem Three: More Modular Arithmetic

Here's a few more properties of the modular congruence relation to explore! In this problem, assume all variables represent integers.

- i. Prove that if $w \equiv_k y$ and $x \equiv_k z$, then $w + x \equiv_k y + z$.
- ii. Prove that if $w \equiv_k y$ and $x \equiv_k z$, then $wx \equiv_k yz$.

Problem Four: Transitive Sets

(From the Spring 2015 midterm exam.)

A **transitive set** is a set S where $S \subseteq \wp(S)$.

- i. Find two different examples of transitive sets and two examples of sets that aren't transitive.

In set theory, an urelement is a non-set object contained inside a set. For example, consider the set $A = \{1, 2, \{3, 4\}\}$. Here, 1 and 2 are urelements of A because 1 and 2 are elements of A but are not themselves sets. The set $\{3, 4\}$ is an element of A but not an urelement of A because $\{3, 4\}$ is a set. The numbers 3 and 4 are not urelements of A because neither is an element of A , but they are urelements of the set $\{3, 4\}$.

- ii. Let S be a transitive set. Prove that if T is any set that contains at least one urelement, then $T \notin S$.