Finite Automata

Part Two

Recap from Last Time

Old MacDonald Had a Symbol,
♫ **Σ**-eye-**ε**-ey**∈**, Oh! ♫

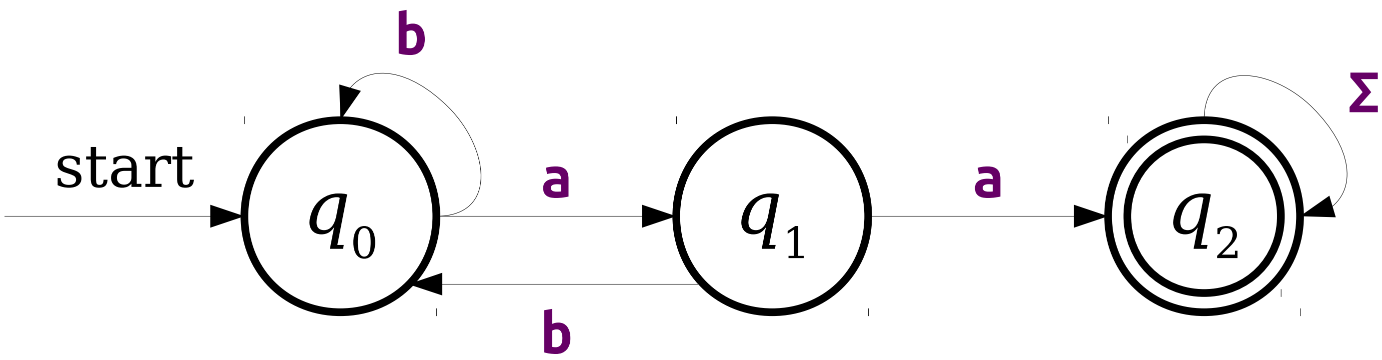
- You may have noticed that we have several letter-E-ish symbols in CS103, which can get confusing!
- Here's a quick guide to remembering which is which:
  - Typically, we use the symbol **Σ** to refer to an ***alphabet***.
  - The ***empty string*** is length 0 and is denoted **ε**.
  - In set theory, use **∈** to say "is an ***element of***."
  - In set theory, use **⊆** to say "is a ***subset of***."

DFAs

- A ***DFA*** is a
  - ***D***eterministic
  - ***F***inite
  - ***A***utomaton
- DFAs are the simplest type of automaton that we will see in this course.

Recognizing Languages with DFAs

$L = \{ w \in \{\texttt{a}, \texttt{b}\}^* \mid w \text{ contains } \texttt{aa} \text{ as a substring} \}$

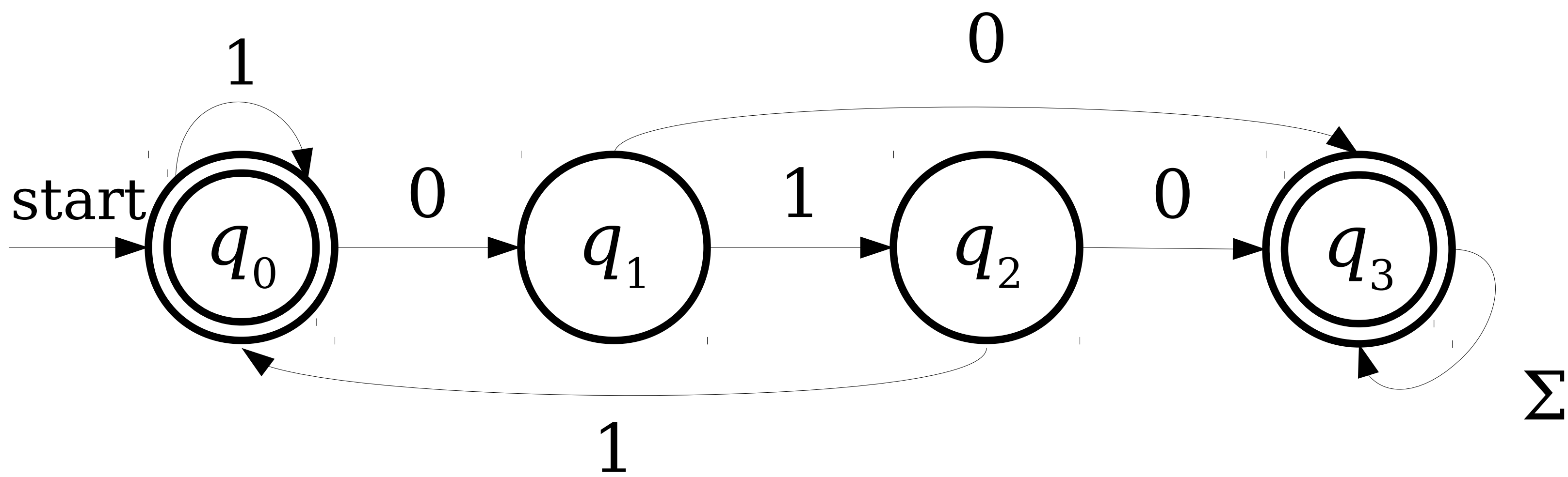


DFAs

- A DFA is defined relative to some alphabet Σ.
- For each state in the DFA, there must be ***exactly one*** transition defined for each symbol in Σ.
  - This is the "deterministic" part of DFA.
- There is a unique start state.
- There are zero or more accepting states.

New Stuff!

Which table best represents the transitions for the DFA shown below?



(A)

	0	1
q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
q_3	q_3	q_3

(B)

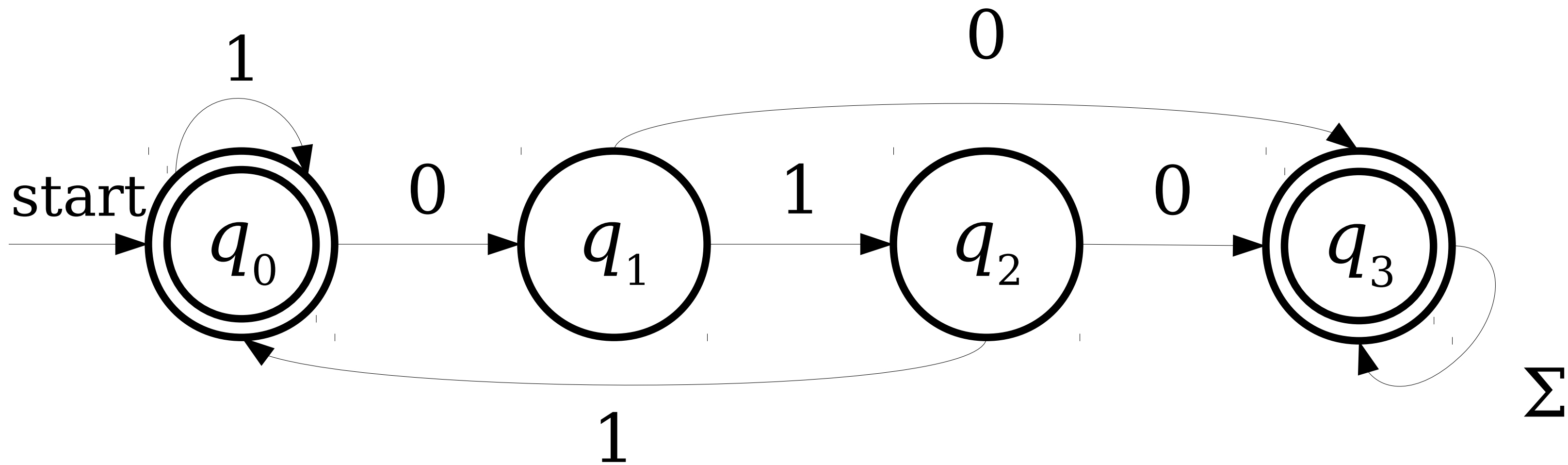
	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_0	q_3
q_3	q_3	q_3

(C)

	0	1	Σ
q_0	q_1	q_0	/
q_1	q_3	q_2	/
q_2	q_3	q_0	/
q_3	/	/	q_3

Answer at **PollEv.com/cs103** or text **CS103** to **22333** once to join, then **A**, **B**, **C**, or **D (none of the above)**.

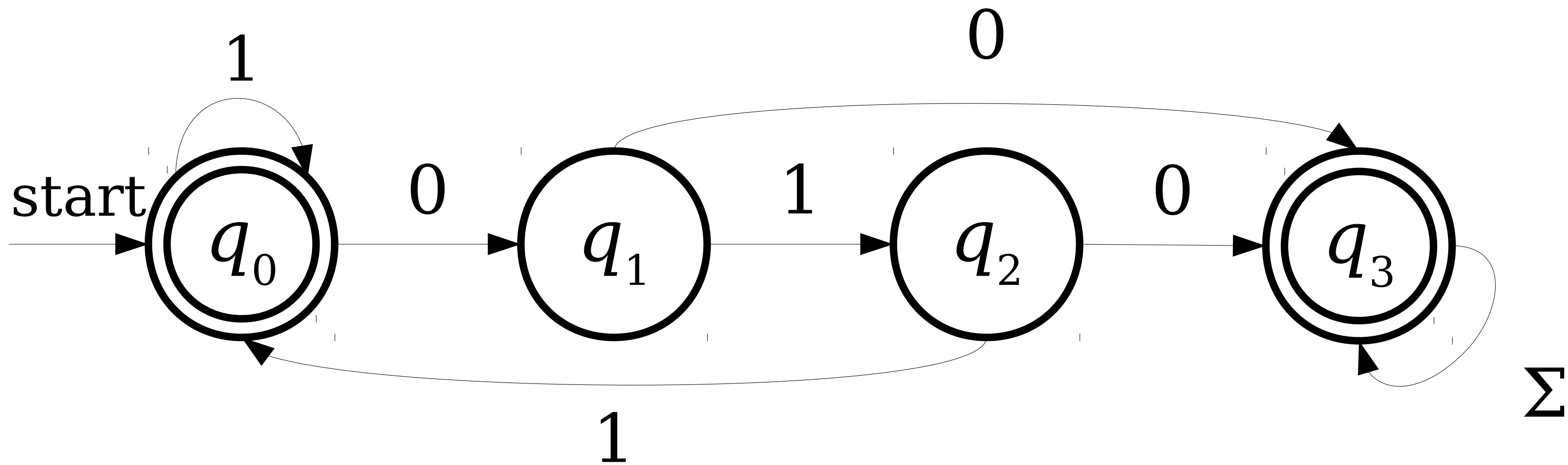
Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

These stars indicate accepting states.

Tabular DFAs



	0	1
*q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
*q_3	q_3	q_3

Since this is the first row, it's the start state.

My Turn to Code Things Up!

```
int kTransitionTable[kNumStates][kNumSymbols] = {
     {0, 0, 1, 3, 7, 1, …},
      …
};
bool kAcceptTable[kNumStates] = {
    false,
    true,
    true,
    …
};
bool SimulateDFA(string input) {
    int state = 0;
    for (char ch: input) {
        state = kTransitionTable[state][ch];
    }
    return kAcceptTable[state];
}
```

The Regular Languages

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

If L is a language and $\mathscr{L}(D) = L$, we say that D ***recognizes*** the language L.

The Complement of a Language

- Given a language $L \subseteq \Sigma^*$, the ***complement*** of that language (denoted $\overline{L}$) is the language of all strings in Σ^* that aren't in L.
- Formally:

$$\overline{L} = \Sigma^* - L$$

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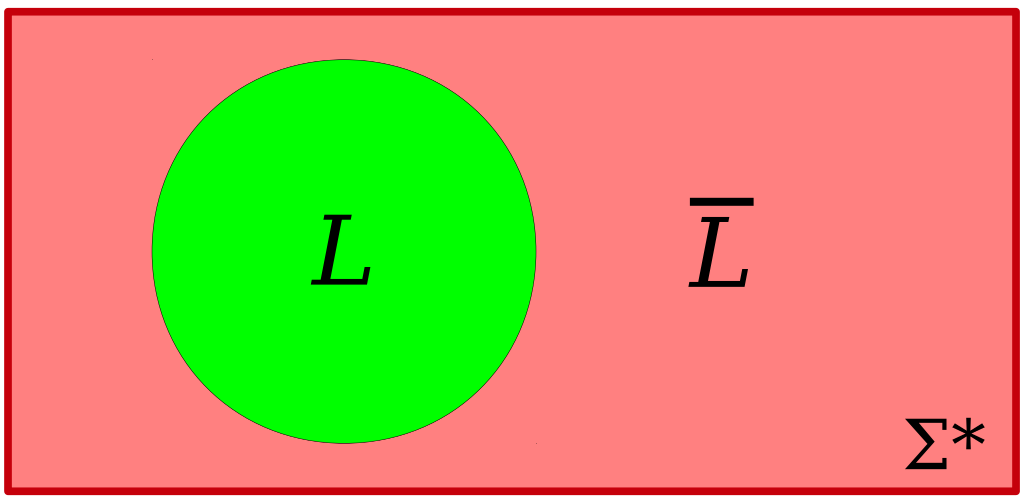
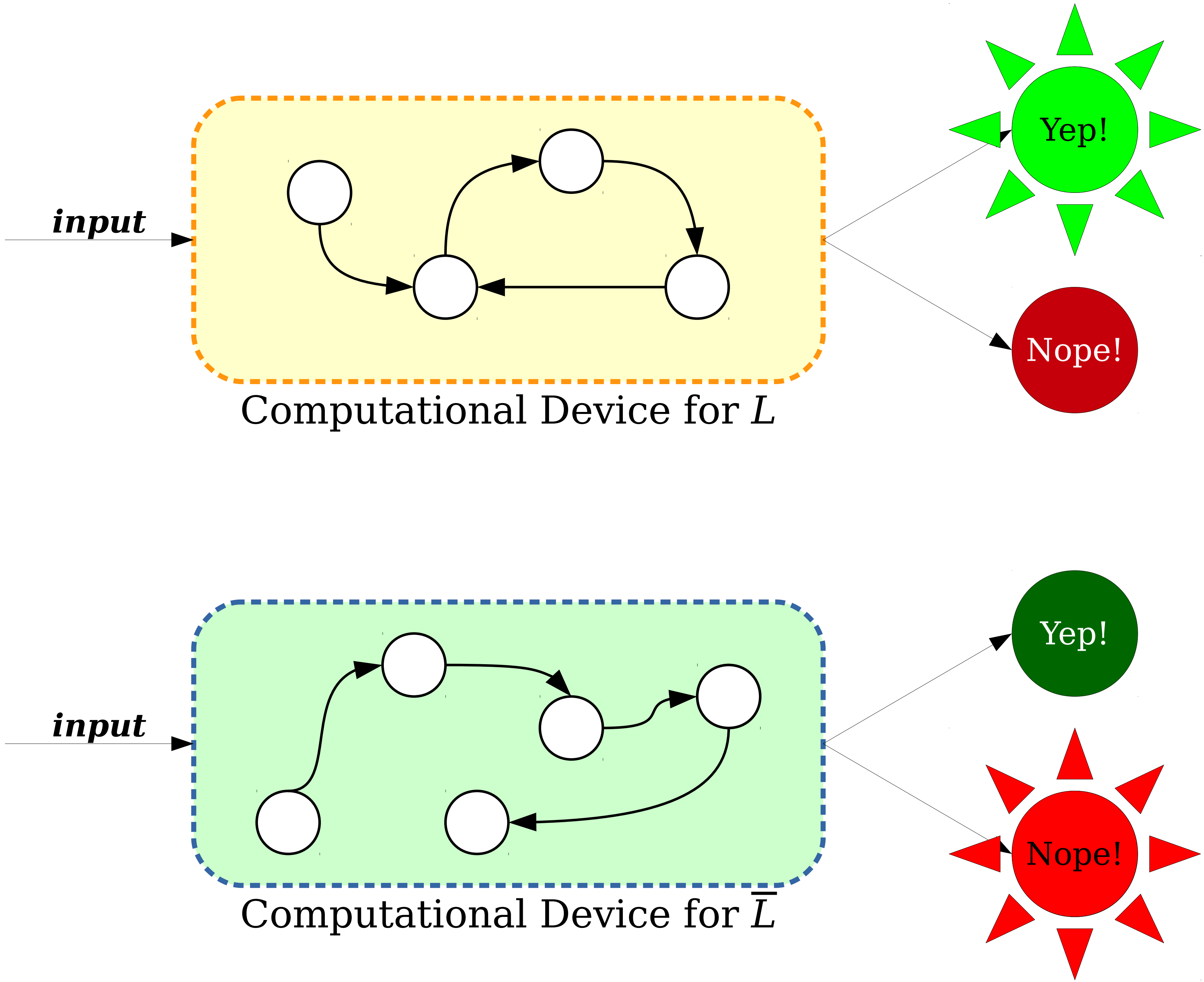


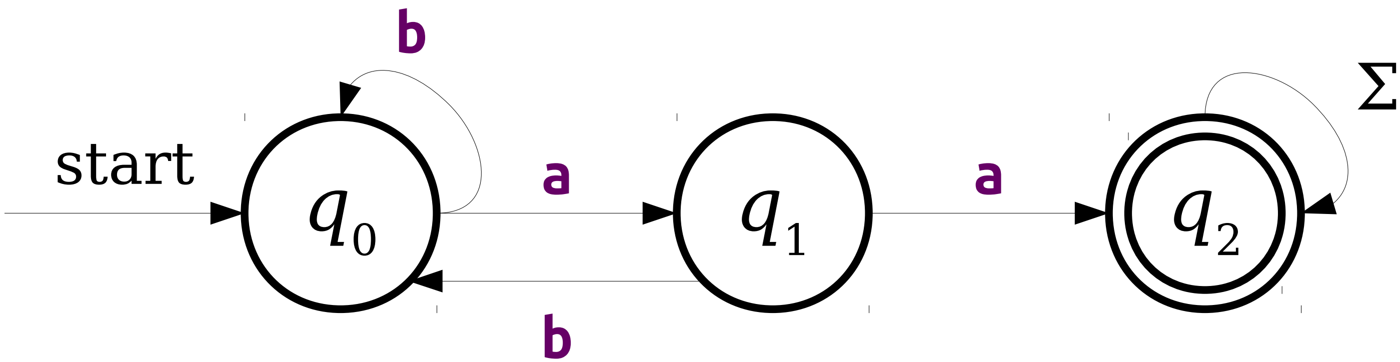
Complements of Regular Languages

- As we saw a few minutes ago, a ***regular language*** is a language accepted by some DFA.
- ***Question:*** If L is a regular language, is $\overline{L}$ necessarily a regular language?
- If the answer is "yes," then if there is a way to construct a DFA for L, there must be some way to construct a DFA for $\overline{L}$.
- If the answer is "no," then some language L can be accepted by some DFA, but $\overline{L}$ cannot be accepted by any DFA.

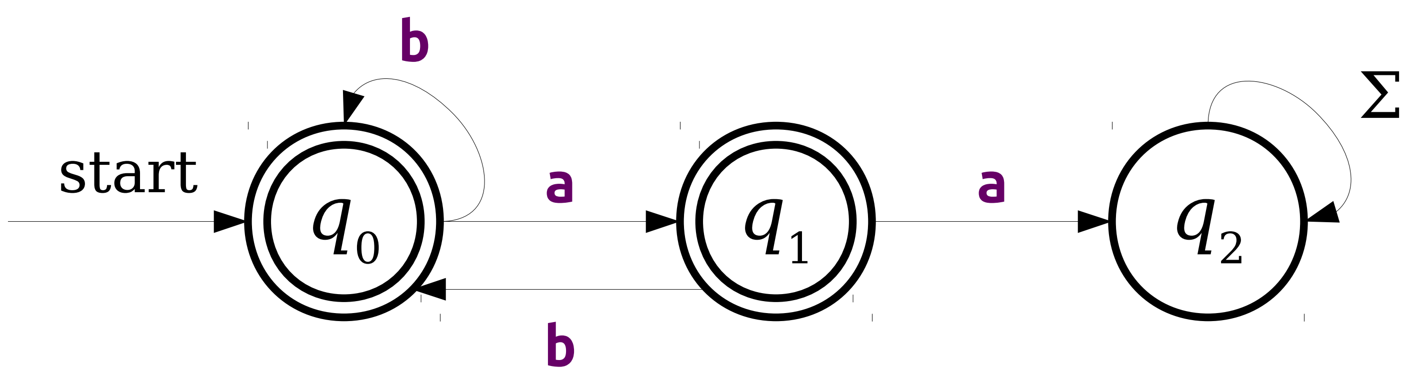

input
Computational Device for L
Yep!
Nope!
input
Computational Device for $\overline{L}$
Yep!
Nope!

Complementing Regular Languages

$L = \{ w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ contains } \mathtt{aa} \text{ as a substring } \}$

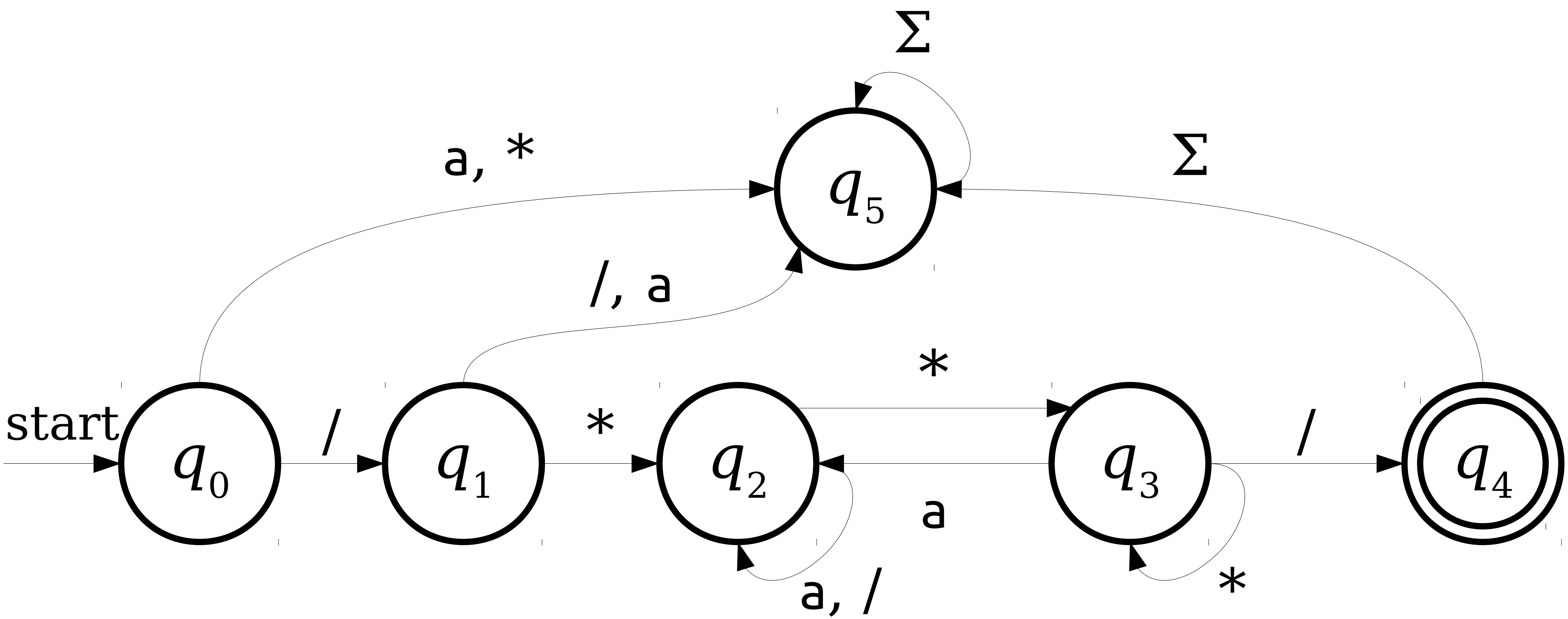


$\overline{L} = \{ w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w$ ***does not*** $\text{contain } \mathtt{aa} \text{ as a substring } \}$



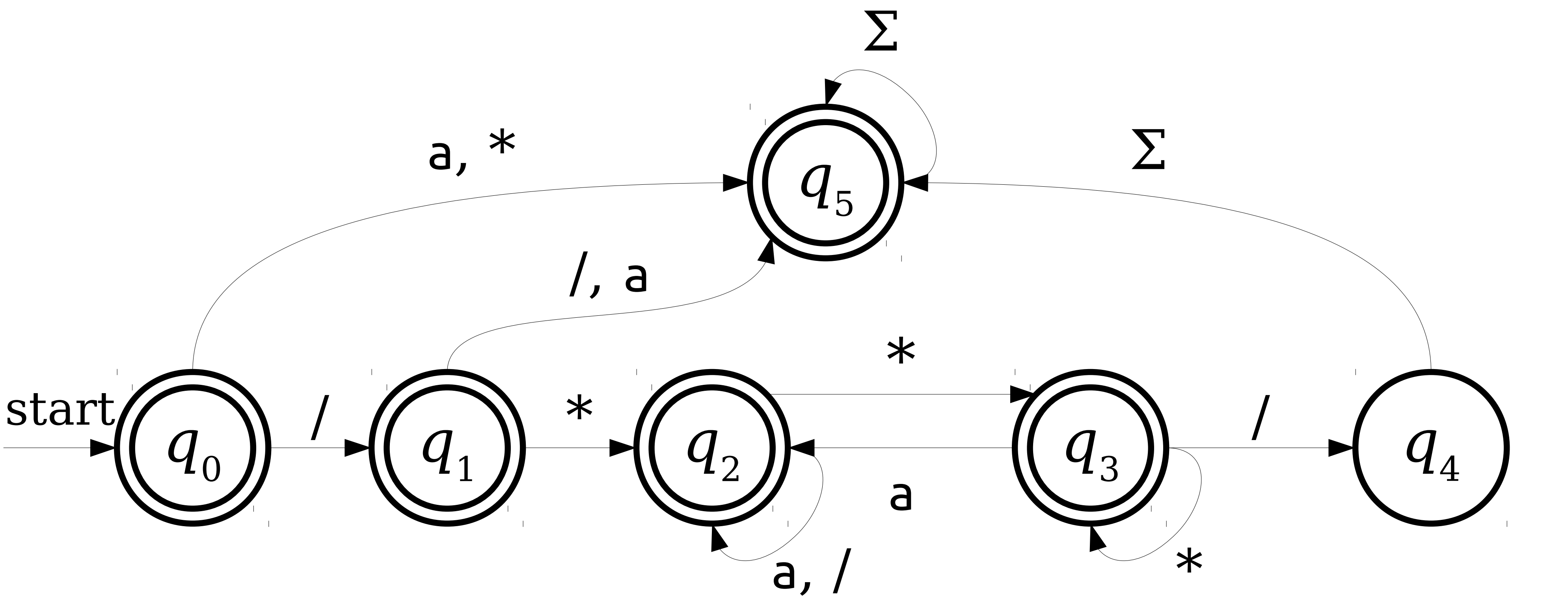
More Elaborate DFAs

$L = \{ w \in \{$**a**, *****, **/**$\}^* \mid w$ represents a C-style comment $\}$



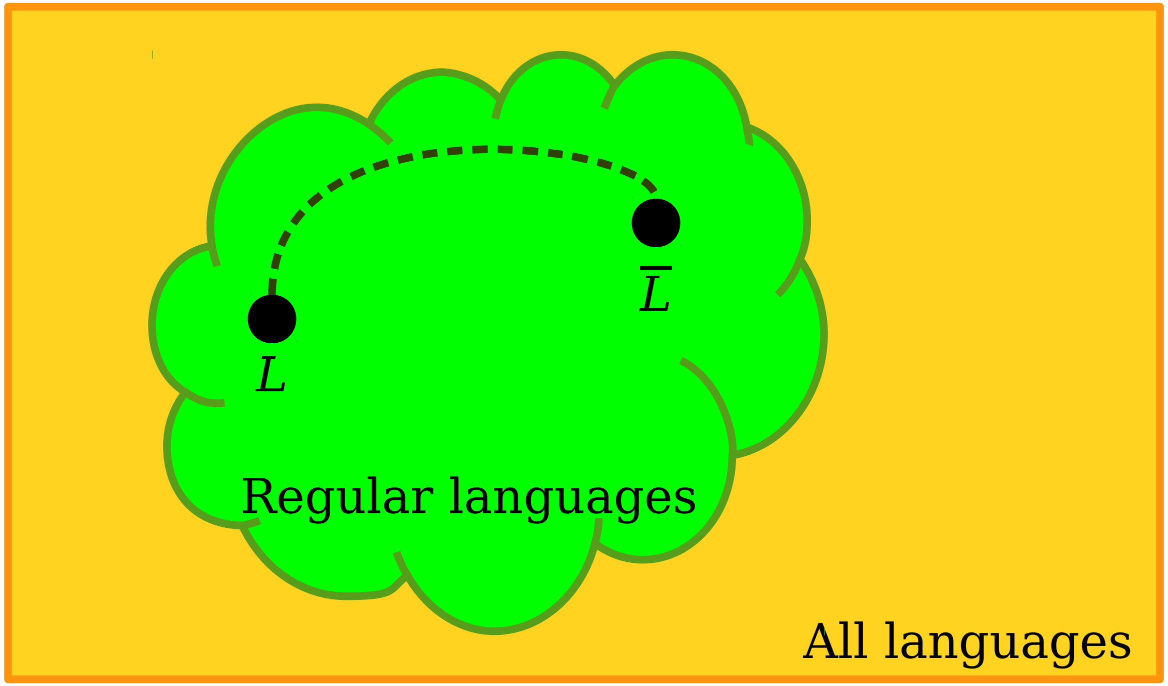
More Elaborate DFAs

$\overline{L}$ = { w ∈ {**a**, *****, **/**}* | *w doesn't* represent a C-style comment }



Closure Properties

- ***Theorem:*** If L is a regular language, then $\overline{L}$ is also a regular language.
- As a result, we say that the regular languages are ***closed under complementation***.



Time-Out For Announcements!

Additional Practice

- Looking to improve your performance in CS103? We've released two handouts:
  - Handout 31: How to Improve in CS103
  - Handout 32: Extra Practice Problems 2
- Set aside a few minutes each day to get some additional practice. That will add up extremely quickly!

treehacks

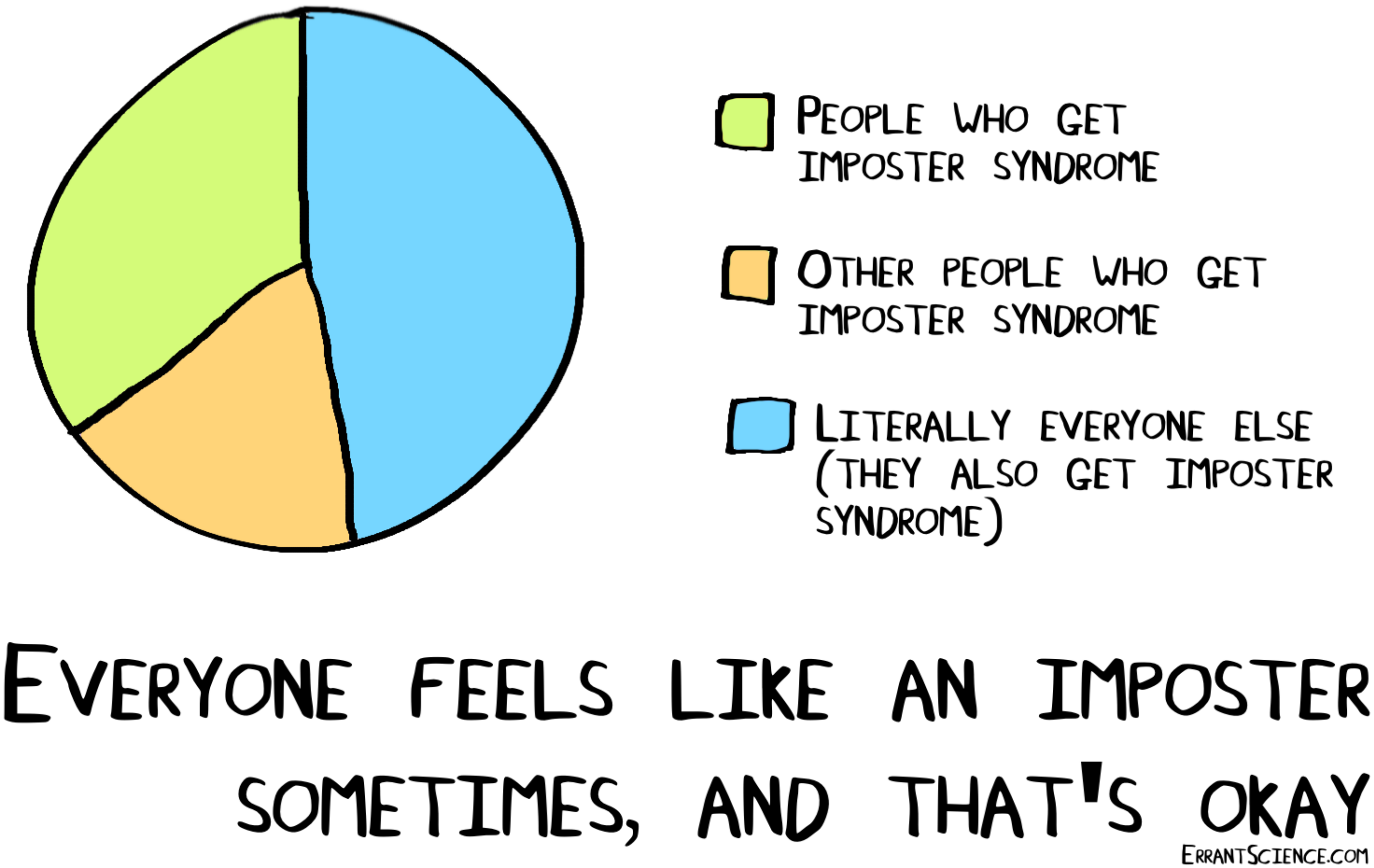
STANFORD'S NATIONAL HACKATHON

THIS WEEKEND FEB 16-18

HUANG ENGINEERING CENTER

BEGINNERS WELCOME!

Ever felt you weren't good enough to be in STEM? Afraid of being "found out" because you don't think you belong?



Learn how to combat Imposter Phenomenon at a very special workshop led by Dr. Nicole Cabrera Salazar! Lunch will be served.

~Wed. 2/14, 11:30 am - 1:30 pm, PAB 232~

Back to CS103!

NFAs

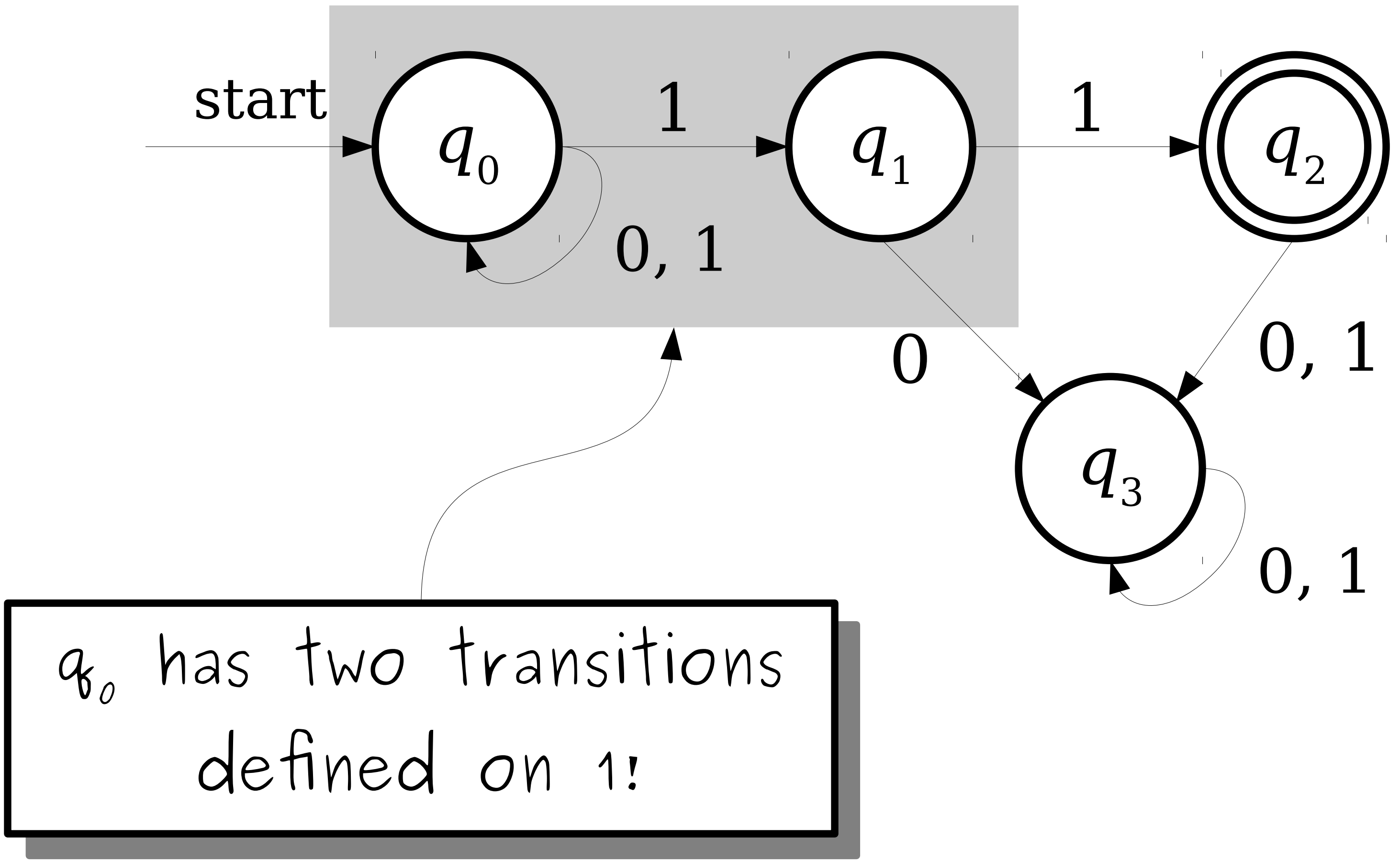
NFAs

- An ***NFA*** is a
  - ***N***ondeterministic
  - ***F***inite
  - ***A***utomaton
- Structurally similar to a DFA, but represents a fundamental shift in how we'll think about computation.

(Non)determinism

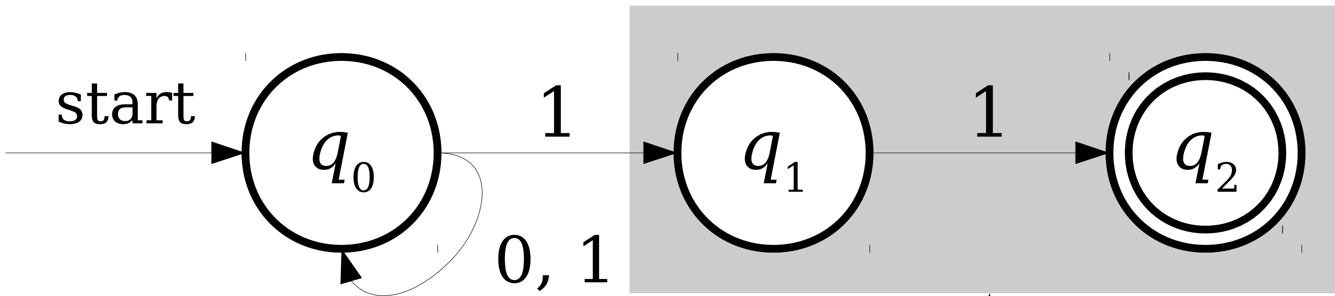
- A model of computation is ***deterministic*** if at every point in the computation, there is exactly one choice that can make.
- The machine accepts if that series of choices leads to an accepting state.
- A model of computation is ***nondeterministic*** if the computing machine may have multiple decisions that it can make at one point.
- The machine accepts if ***any*** series of choices leads to an accepting state.
  - (This sort of nondeterminism is technically called ***existential nondeterminism***, the most philosophical-sounding term we'll introduce all quarter.)

A Simple NFA

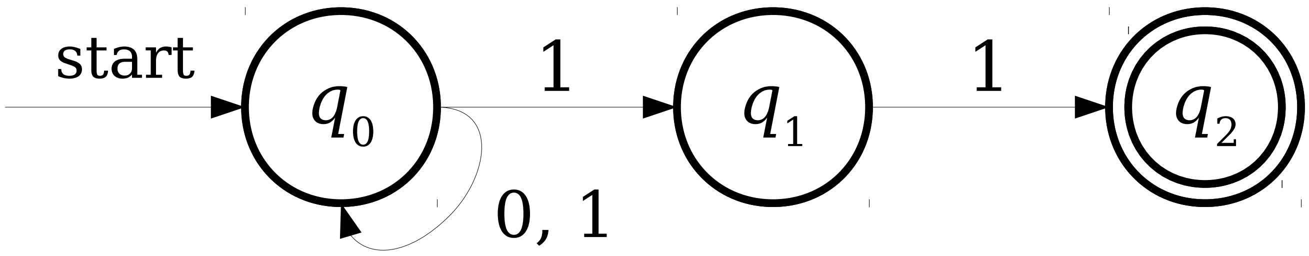


start
q_0
1
q_1
1
q_2
0, 1
0
0, 1
q_3
0, 1
q_0 has two transitions defined on 1!

A More Complex NFA



If a NFA needs to make a transition when no transition exists, the automaton **dies** and that particular path does not accept.



As with DFAs, the language of an NFA N is the set of strings that N accepts:

$$\mathscr{L}(N) = \{ w \in \Sigma^* \mid N \text{ accepts } w \}.$$

What is the language of the NFA shown above?

A. { **01011** }
B. $\{ w \in \{$**0**, **1**$\}^* \mid w$ contains at least two **1**s $\}$
C. $\{ w \in \{$**0**, **1**$\}^* \mid w$ ends with **11** $\}$
D. $\{ w \in \{$**0**, **1**$\}^* \mid w$ ends with **1** $\}$
E. None of these, or two or more of these.

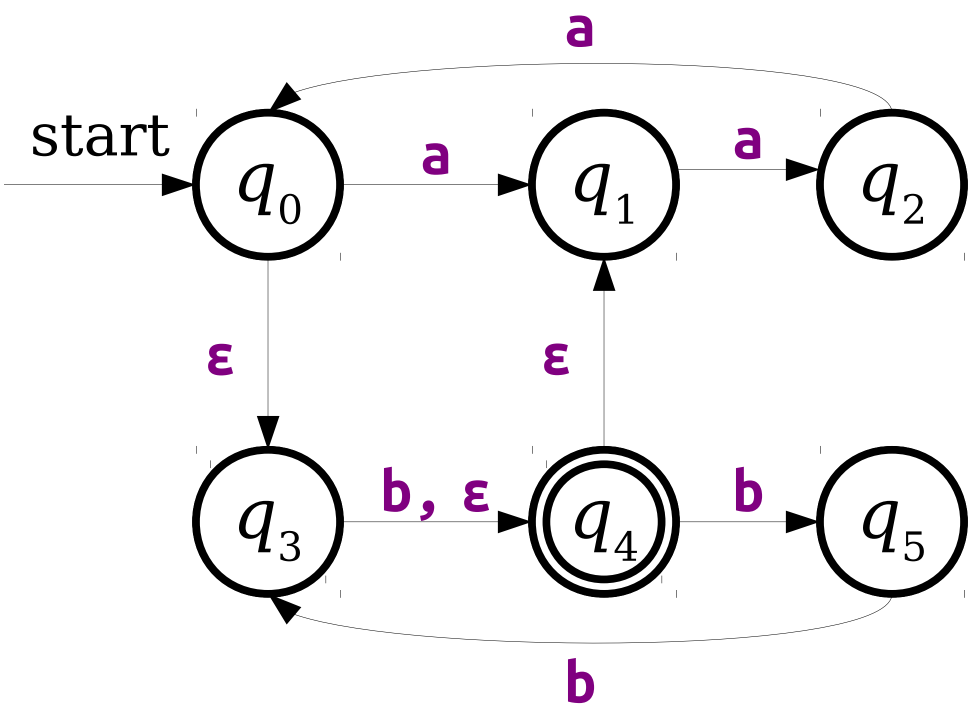
Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then ***A***, …, or ***E***.

NFA Acceptance

- An NFA *N* accepts a string *w* if there is some series of choices that lead to an accepting state.
- Consequently, an NFA *N* rejects a string *w* if *no possible* series of choices lead it into an accepting state.
- It's easier to show that an NFA does accept something than to show that it doesn't.

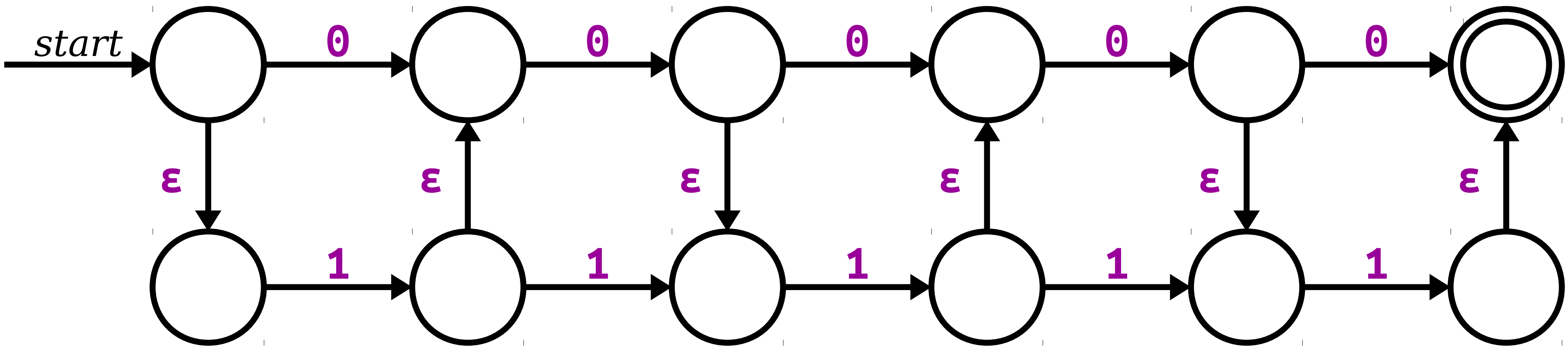
ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.



ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.
- NFAs are not *required* to follow ε-transitions. It's simply another option at the machine's disposal.



Suppose we run the above NFA on the string **10110**. How many of the following statements are true?

- There is at least one computation that finishes in an accepting state.
- There is at least one computation that finishes in a rejecting state.
- There is at least one computation that dies.
- This NFA accepts **10110**.
- This NFA rejects **10110**.

Answer at **PollEv.com/cs103** or text **CS103** to **22333** once to join, then **a number**.

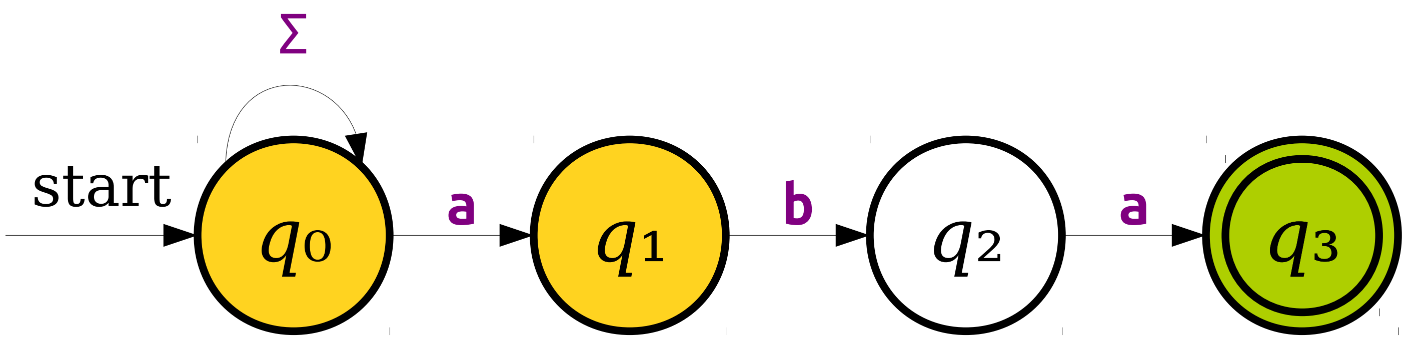
Intuiting Nondeterminism

- Nondeterministic machines are a serious departure from physical computers. How can we build up an intuition for them?
- There are two particularly useful frameworks for interpreting nondeterminism:
  - ***Perfect guessing***
  - ***Massive parallelism***

Perfect Guessing

- We can view nondeterministic machines as having ***Magic Superpowers*** that enable them to guess choices that lead to an accepting state.
  - If there is at least one choice that leads to an accepting state, the machine will guess it.
  - If there are no choices, the machine guesses any one of the wrong guesses.
- No known physical analog for this style of computation – this is totally new!

Massive Parallelism



a	b	a	b	a

We're in at least one accepting state, so there's some path that gets us to an accepting state.

Therefore, we accept!

Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- (Here's a rigorous explanation about how this works; read this on your own time).
  - Start off in the set of all states formed by taking the start state and including each state that can be reached by zero or more ε-transitions.
  - When you read a symbol **a** in a set of states *S*:
    - Form the set *S'* of states that can be reached by following a single **a** transition from some state in *S*.
    - Your new set of states is the set of states in *S'*, plus the states reachable from *S'* by following zero or more ε-transitions.

So What?

- Each intuition of nondeterminism is useful in a different setting:
  - Perfect guessing is a great way to think about how to design a machine.
  - Massive parallelism is a great way to test machines – and has nice theoretical implications.
- Nondeterministic machines may not be feasible, but they give a great basis for interesting questions:
  - Can any problem that can be solved by a nondeterministic machine be solved by a deterministic machine?
  - Can any problem that can be solved by a nondeterministic machine be solved ***efficiently*** by a deterministic machine?
- The answers vary from automaton to automaton.

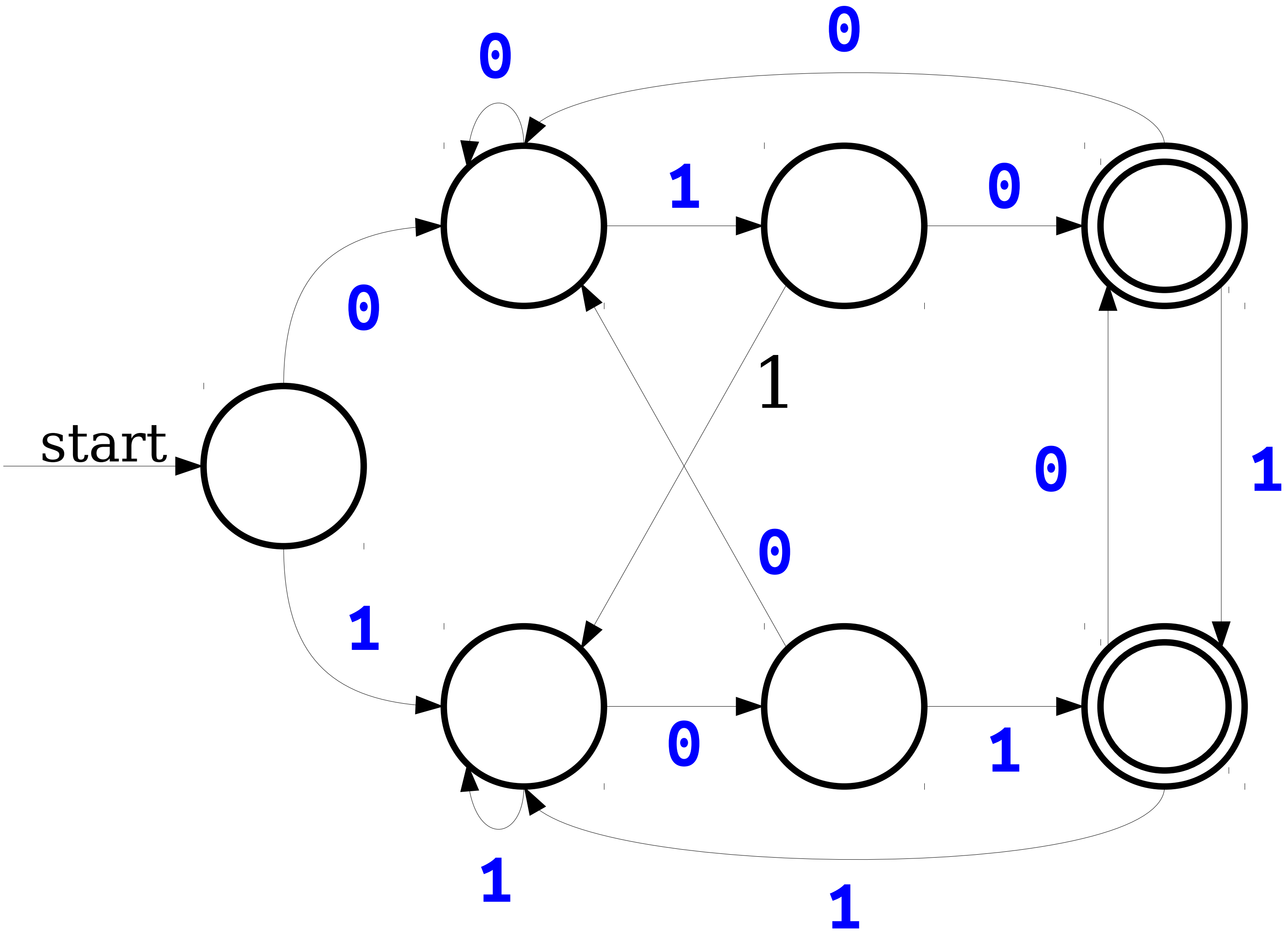
Designing NFAs

Designing NFAs

- When designing NFAs, *embrace the nondeterminism!*
- Good model: ***Guess-and-check***:
  - Is there some information that you'd really like to have? Have the machine *nondeterministically guess* that information.
  - Then, have the machine *deterministically check* that the choice was correct.
- The *guess* phase corresponds to trying lots of different options.
- The *check* phase corresponds to filtering out bad guesses or wrong options.

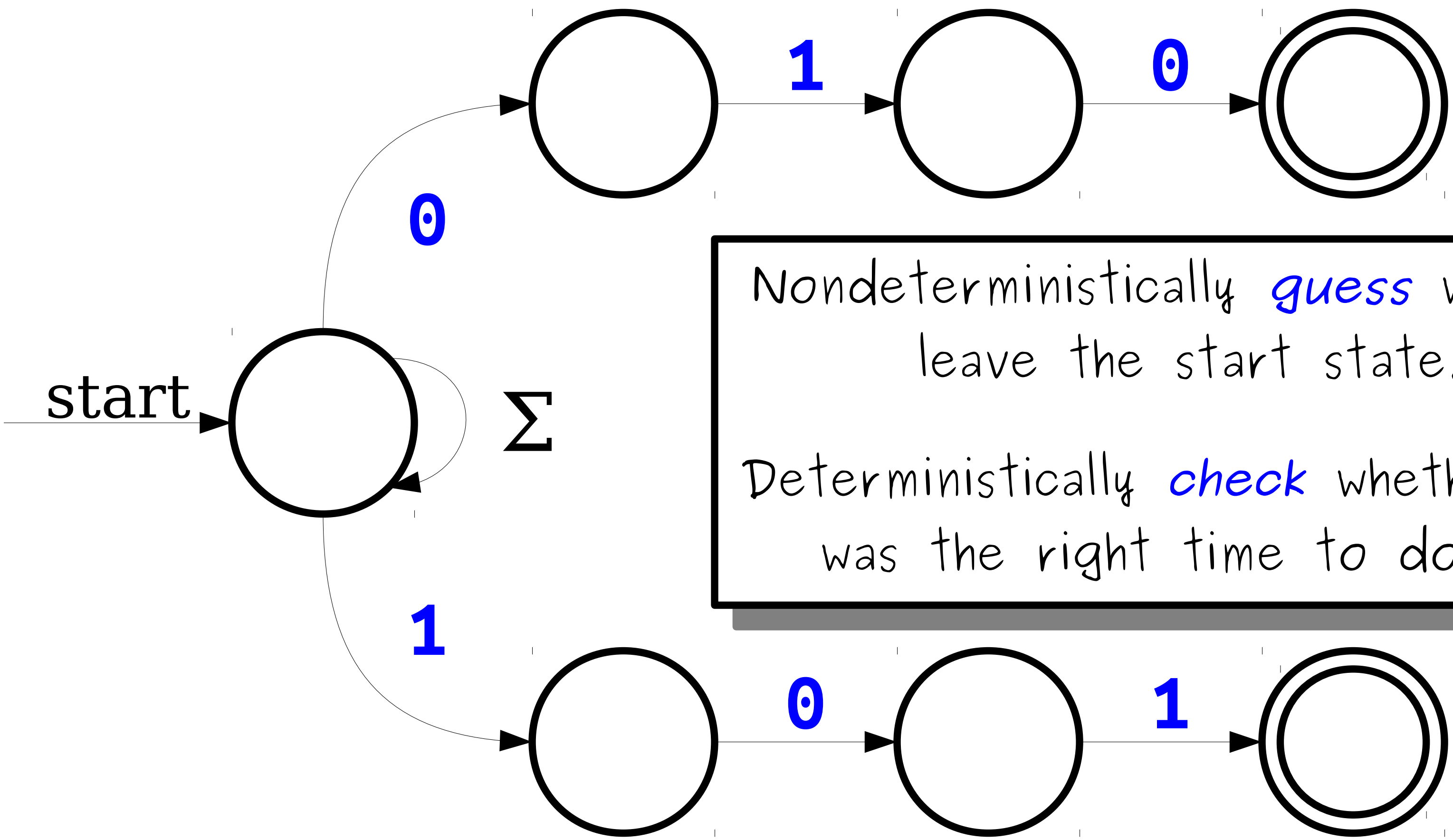
Guess-and-Check

$L = \{ w \in \{0, 1\}^* \mid w \text{ ends in } 010 \text{ or } 101 \}$



Guess-and-Check

$L = \{ w \in \{0, 1\}^* \mid w$ ends in 010 or 101 $\}$

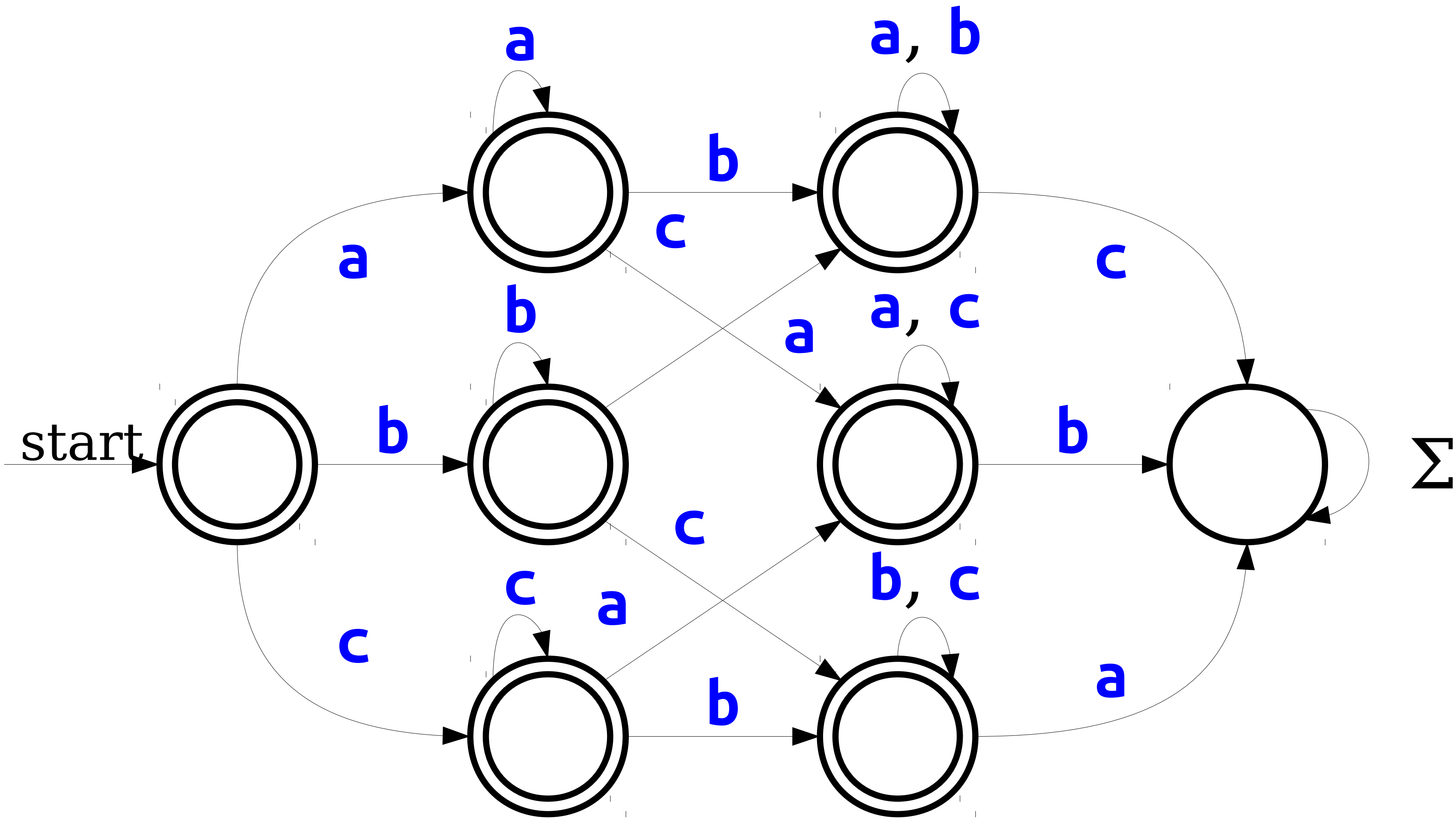


Nondeterministically *guess* when to leave the start state.

Deterministically *check* whether that was the right time to do so.

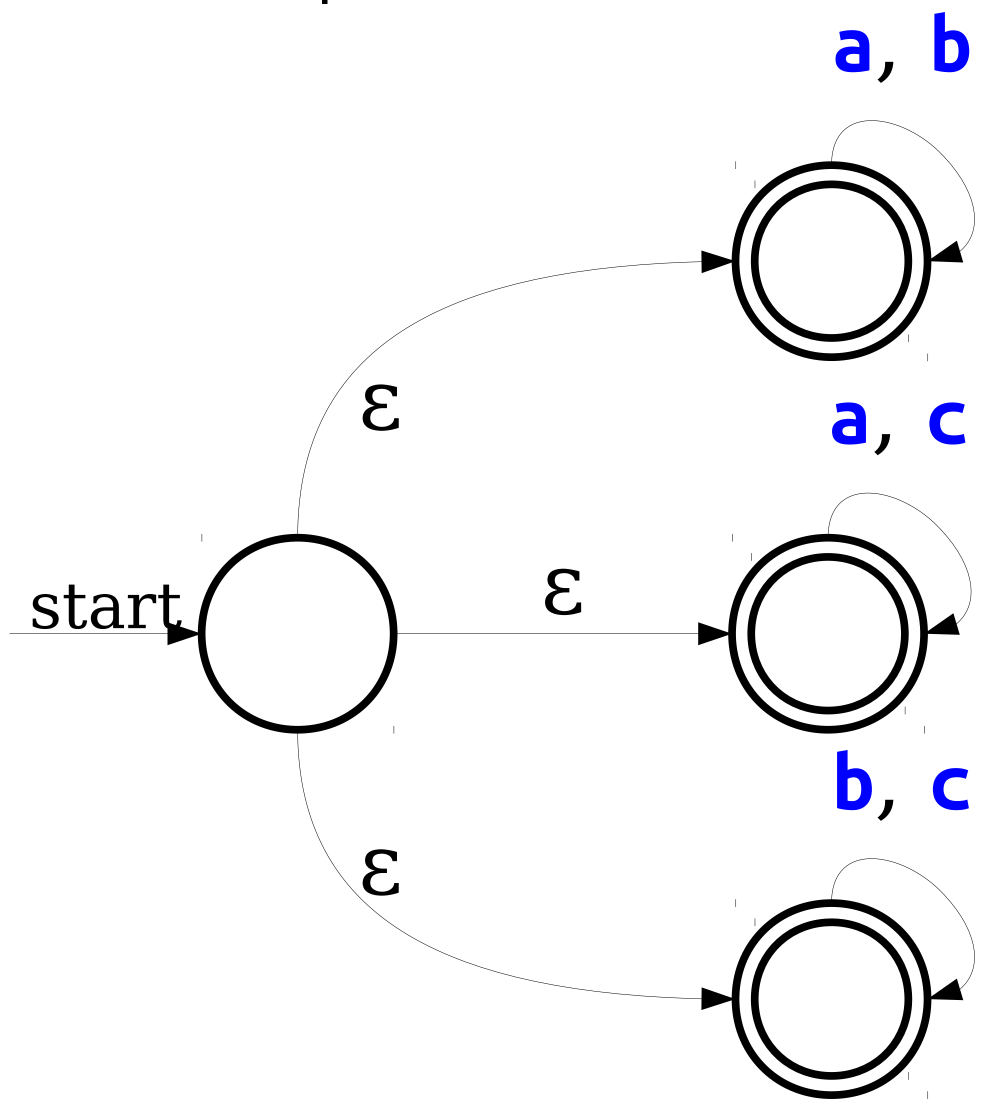
Guess-and-Check

$L = \{ w \in \{\texttt{a}, \texttt{b}, \texttt{c}\}^* \mid \text{at least one of } \texttt{a}, \texttt{b}, \text{ or } \texttt{c} \text{ is not in } w \}$



Guess-and-Check

$L = \{ w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* \mid \text{at least one of } \mathtt{a}, \mathtt{b}, \text{ or } \mathtt{c} \text{ is not in } w \}$



Nondeterministically *guess* which character is missing.

Deterministically *check* whether that character is indeed missing.

Just how powerful are NFAs?