Finite Automata

Part Two

Recap from Last Time

Strings

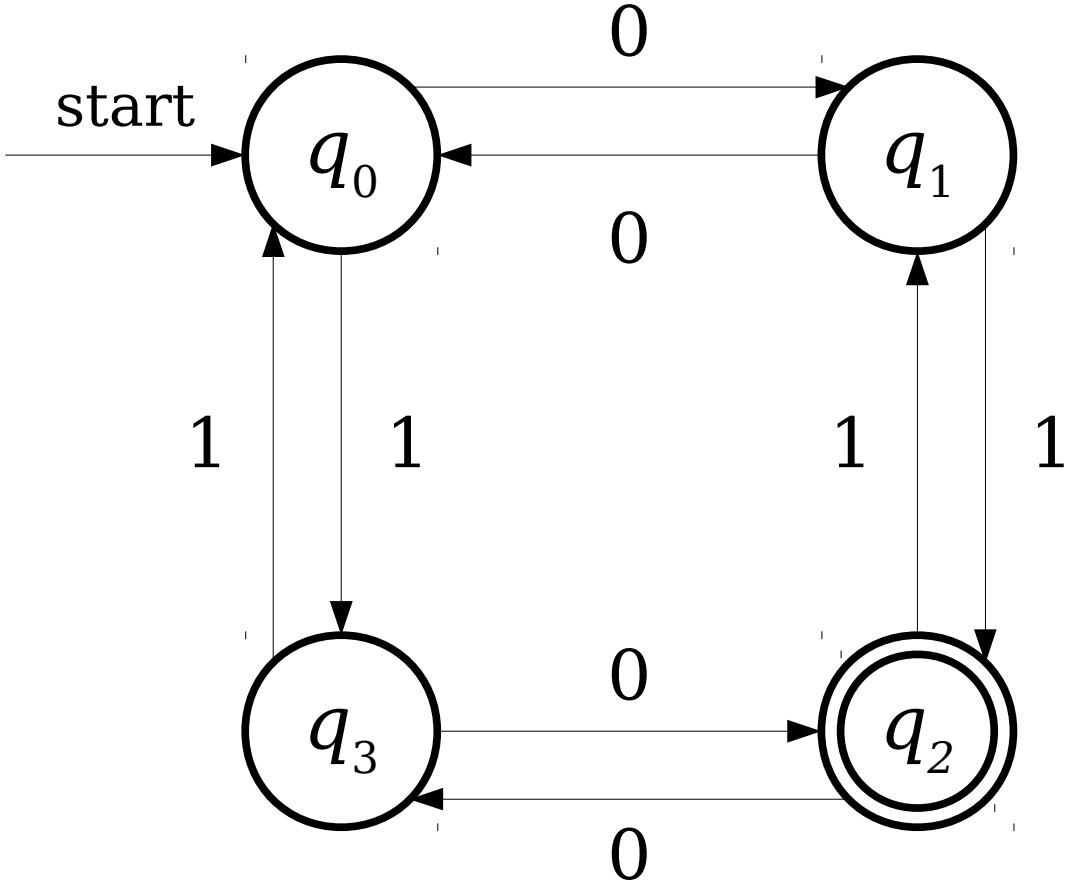
- An ***alphabet*** is a finite, nonempty set of symbols called ***characters***.
  - Typically, we use the symbol **Σ** to refer to an alphabet.
- A ***string over an alphabet Σ*** is a finite sequence of characters drawn from Σ.
- Example: If Σ = {**a**, **b**}, here are some valid strings over Σ:

  a **aabaaabbabaaabaaaabbb** **abbababba**
- The ***empty string*** has no characters and is denoted **ε**.
- Calling attention to an earlier point: since all strings are finite sequences of characters from Σ, you cannot have a string of infinite length.

Languages

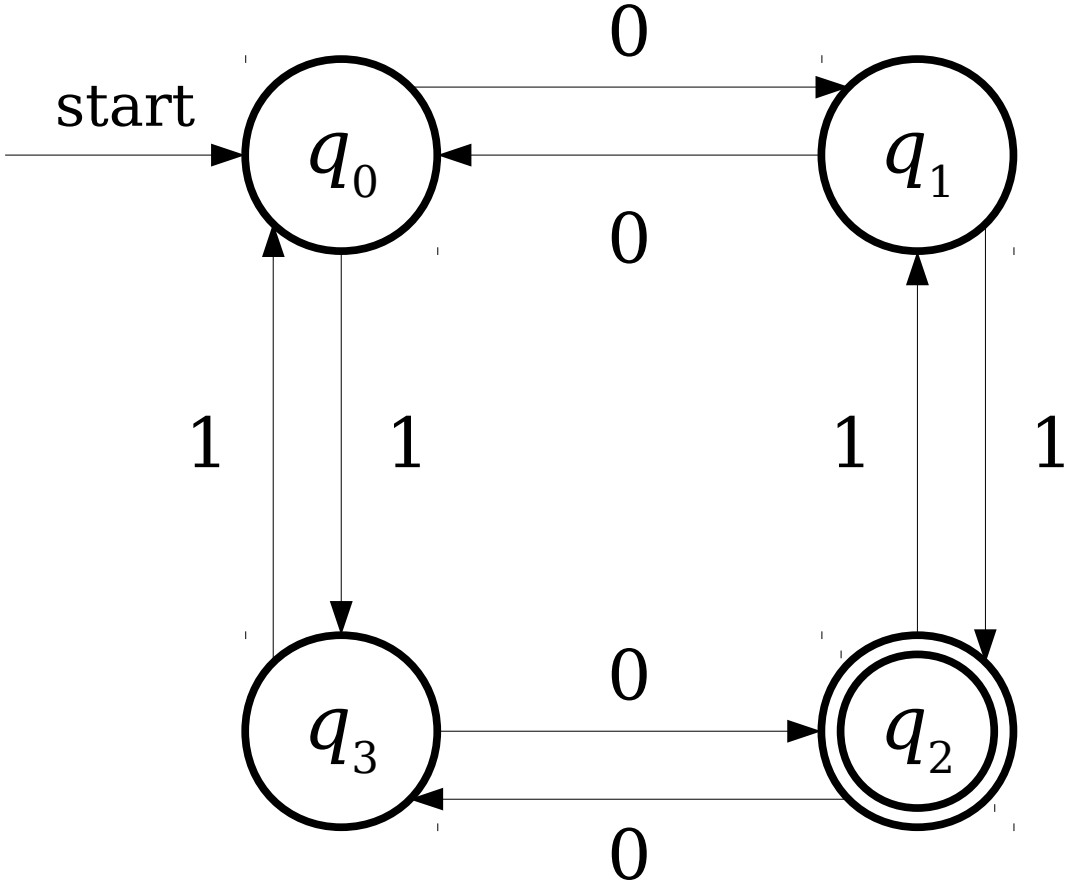
- A ***formal language*** is a set of strings.
- We say that L is a ***language over Σ*** if it is a set of strings over Σ.
- Example: The language of palindromes over Σ = {**a**, **b**, **c**} is the set
  - {ε, **a**, **b**, **c**, **aa**, **bb**, **cc**, **aaa**, **aba**, **aca**, **bab**, … }
- The set of all strings composed from letters in Σ is denoted **Σ***.
- Formally, we say that L is a language over Σ if $L \subseteq \Sigma^*$.

A Simple Finite Automaton



0 1 0 1 1 0

A Simple Finite Automaton



1 0 1 0 0 0

The ***language of an automaton*** is the set of strings that it accepts.

If D is an automaton, we denote the language of D as $\mathscr{L}(D)$.

$$\mathscr{L}(D) = \{\ w \in \Sigma^* \mid D \text{ accepts } w\ \}$$

DFAs

- A ***DFA*** is a
  - ***D***eterministic
  - ***F***inite
  - ***A***utomaton
- DFAs are the simplest type of automaton that we will see in this course.

DFAs, Informally

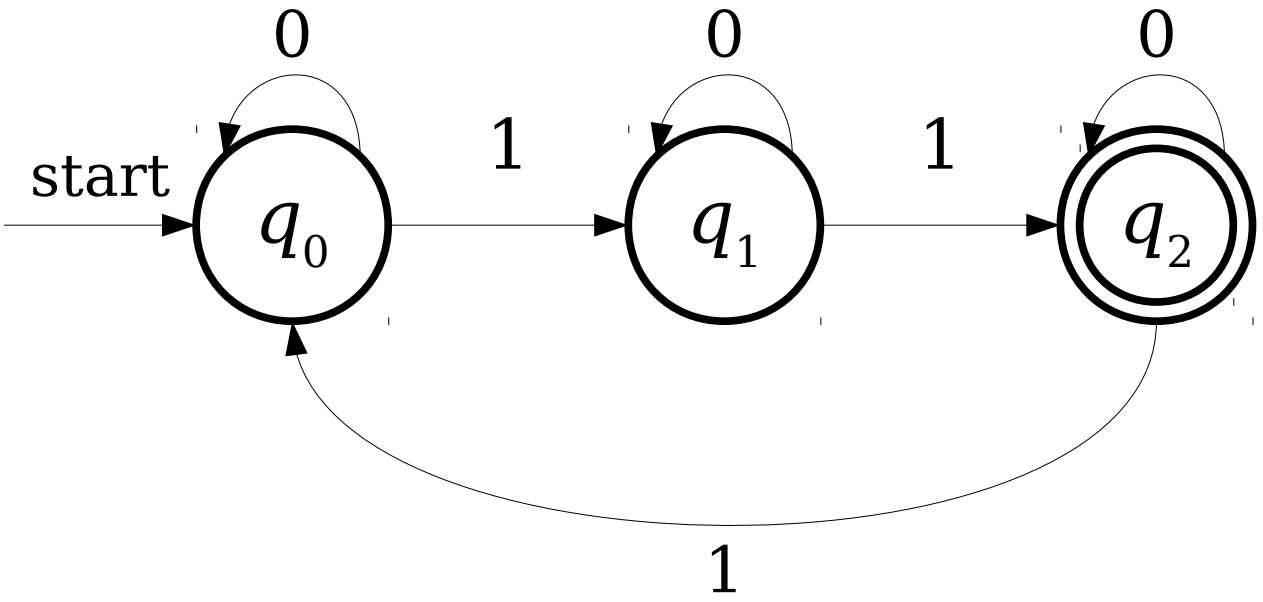
- A DFA is defined relative to some alphabet Σ.
- For each state in the DFA, there must be ***exactly one*** transition defined for each symbol in Σ.
  - This is the “deterministic” part of DFA.
- There is a unique start state.
- There are zero or more accepting states.

Designing DFAs

- At each point in its execution, the DFA can only remember what state it is in.
- ***DFA Design Tip:*** Build each state to correspond to some piece of information you need to remember.
  - Each state acts as a “memento” of what you're supposed to do next.
  - Only finitely many different states ≈ only finitely many different things the machine can remember.

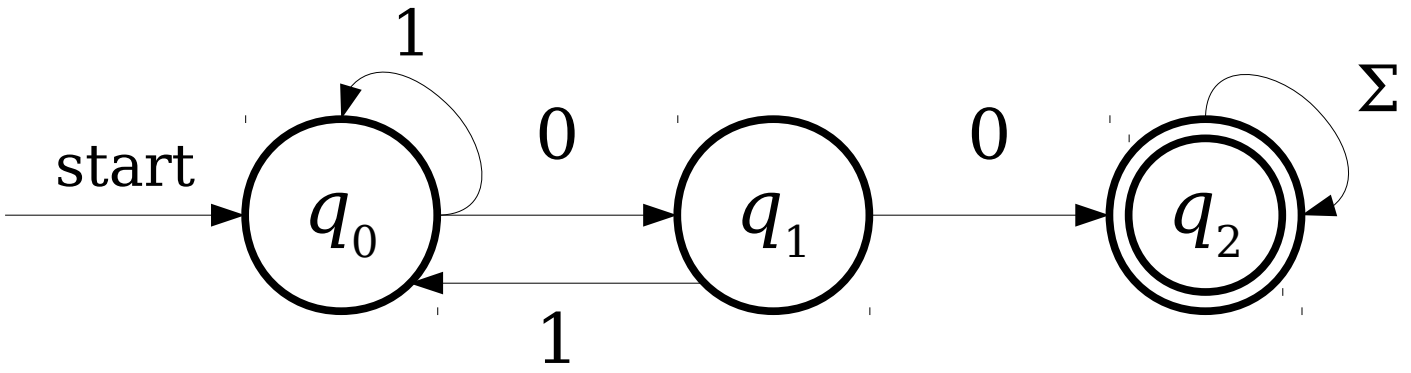
Recognizing Languages with DFAs

$L = \{ w \in \{0, 1\}^* |$ the number of 1's in w is congruent to two modulo three $\}$



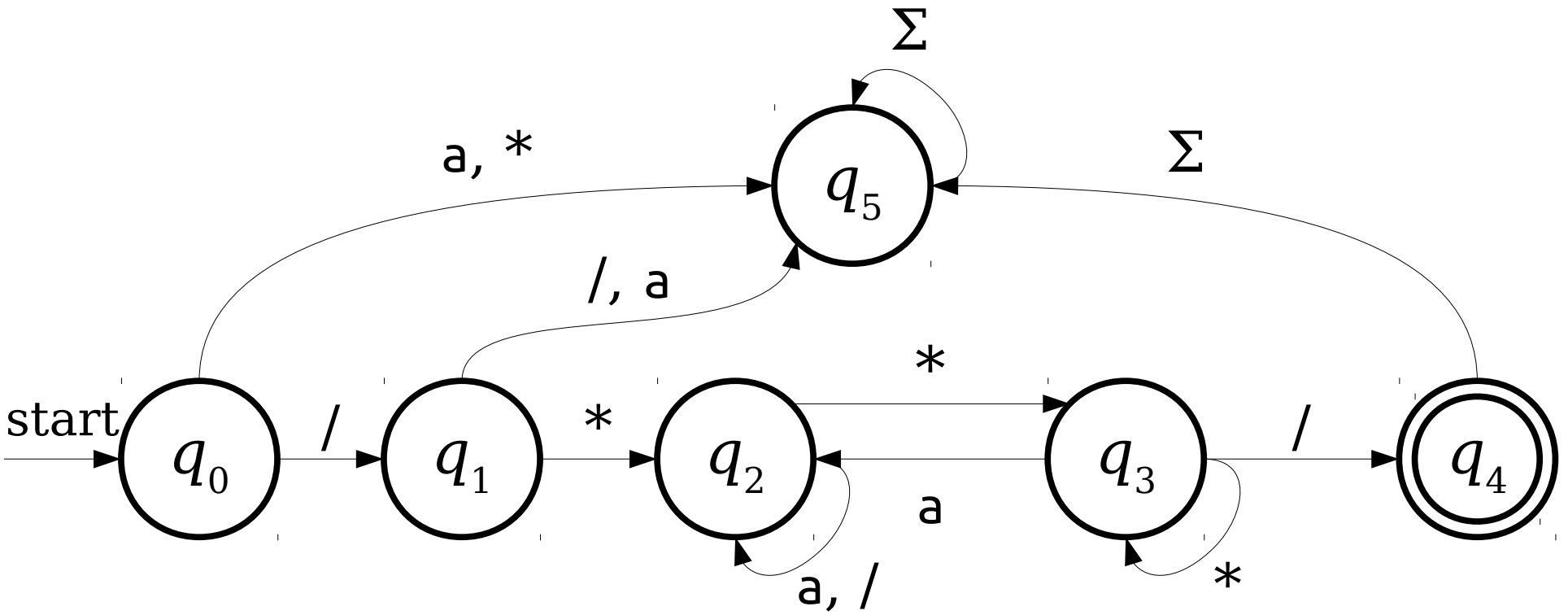
Recognizing Languages with DFAs

$L = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ contains } \texttt{00} \text{ as a substring} \}$



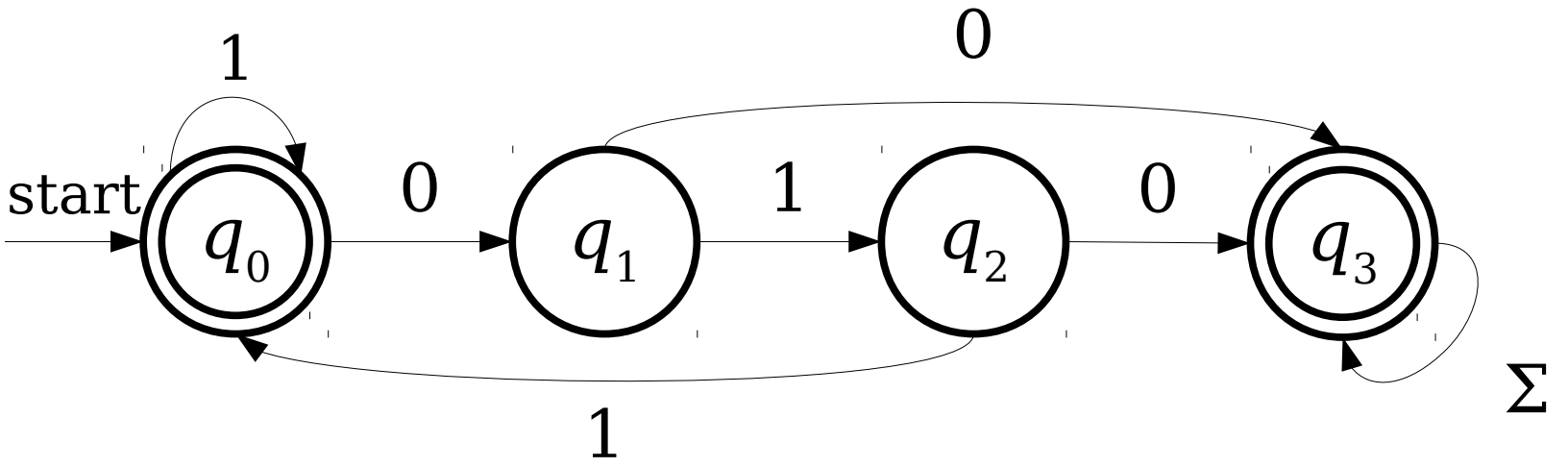
Recognizing Languages with DFAs

$L = \{\ w \in \{\texttt{a}, \texttt{*}, \texttt{/}\}^* \mid w \text{ represents a C-style comment}\ \}$



New Stuff!

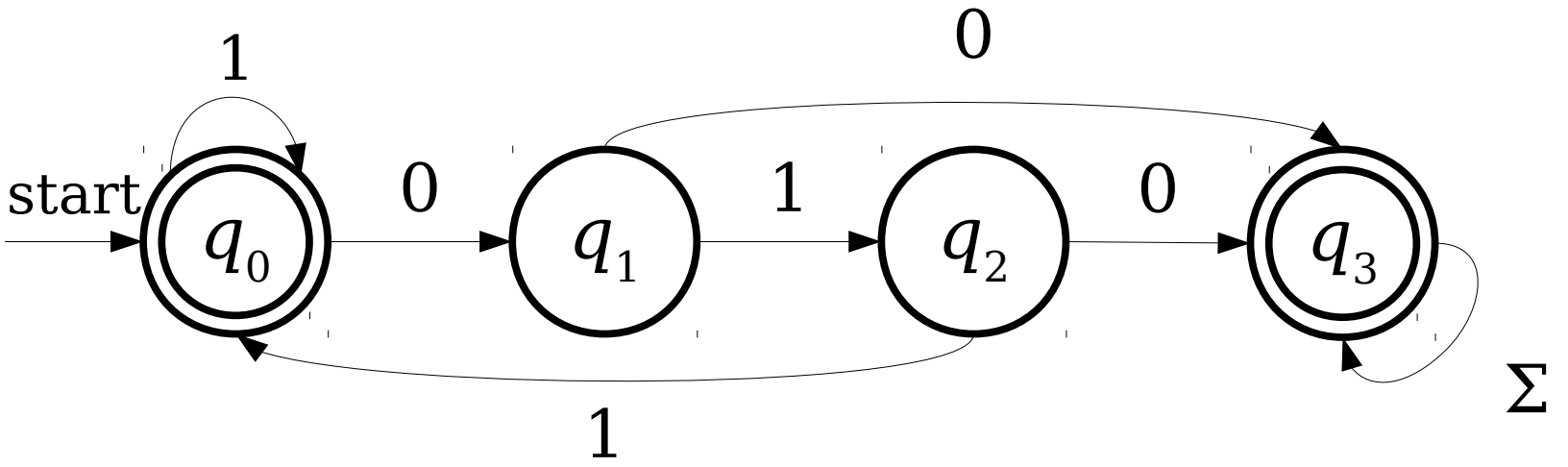
Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

These stars indicate accepting states.

Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

Since this is the first row, it's the start state.

Code? In a Theory Course?

```
int kTransitionTable[kNumStates][kNumSymbols] = {
     {0, 0, 1, 3, 7, 1, …},
      …
};
bool kAcceptTable[kNumStates] = {
    false,
    true,
    true,
    …
};
bool SimulateDFA(string input) {
    int state = 0;
    for (char ch: input)
        state = kTransitionTable[state][ch];
    return kAcceptTable[state];
}
```

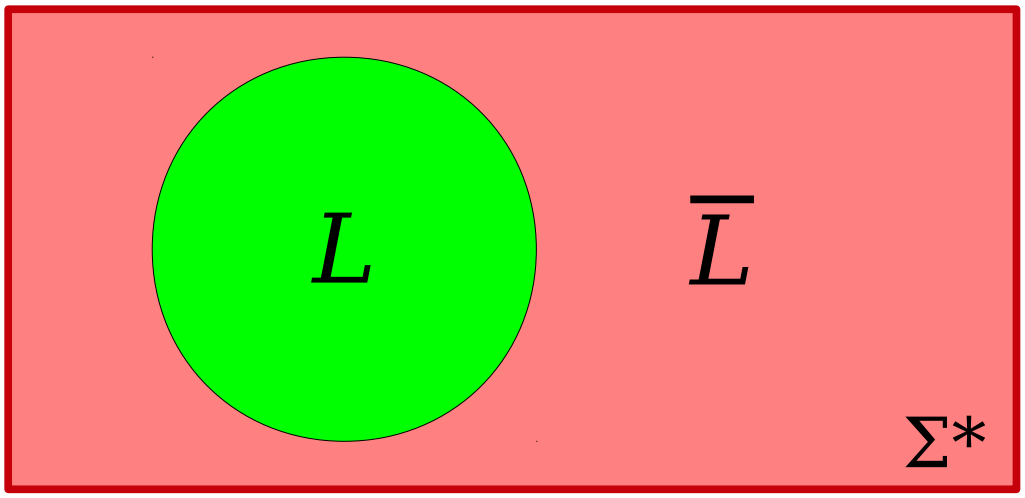
The Regular Languages

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

The Complement of a Language

- Given a language $L \subseteq \Sigma^*$, the ***complement*** of that language (denoted $\overline{L}$) is the language of all strings in Σ^* that aren't in L.
- Formally:

$$\overline{L} = \Sigma^* - L$$

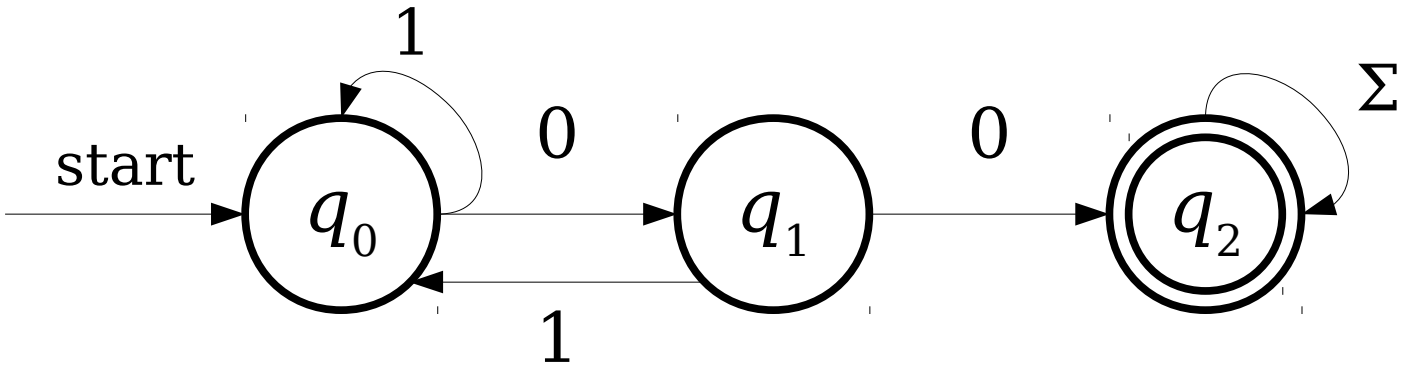


Complements of Regular Languages

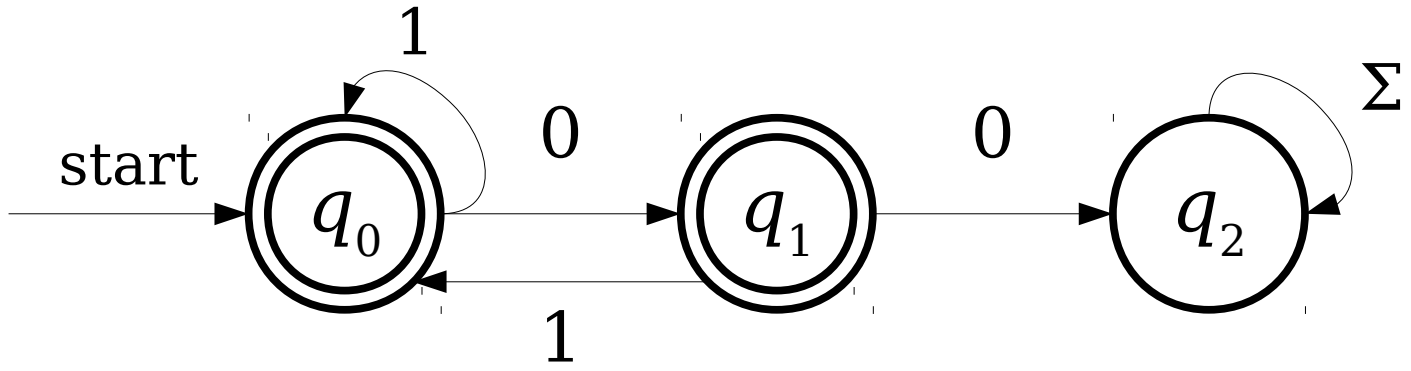
- As we saw a few minutes ago, a ***regular language*** is a language accepted by some DFA.
- ***Question:*** If L is a regular language, is $\overline{L}$ necessarily a regular language?
- If the answer is "yes," then if there is a way to construct a DFA for L, there must be some way to construct a DFA for $\overline{L}$.
- If the answer is "no," then some language L can be accepted by some DFA, but $\overline{L}$ cannot be accepted by any DFA.

Complementing Regular Languages

$L = \{ w \in \{0, 1\}^* \mid w \text{ contains } 00 \text{ as a substring} \}$

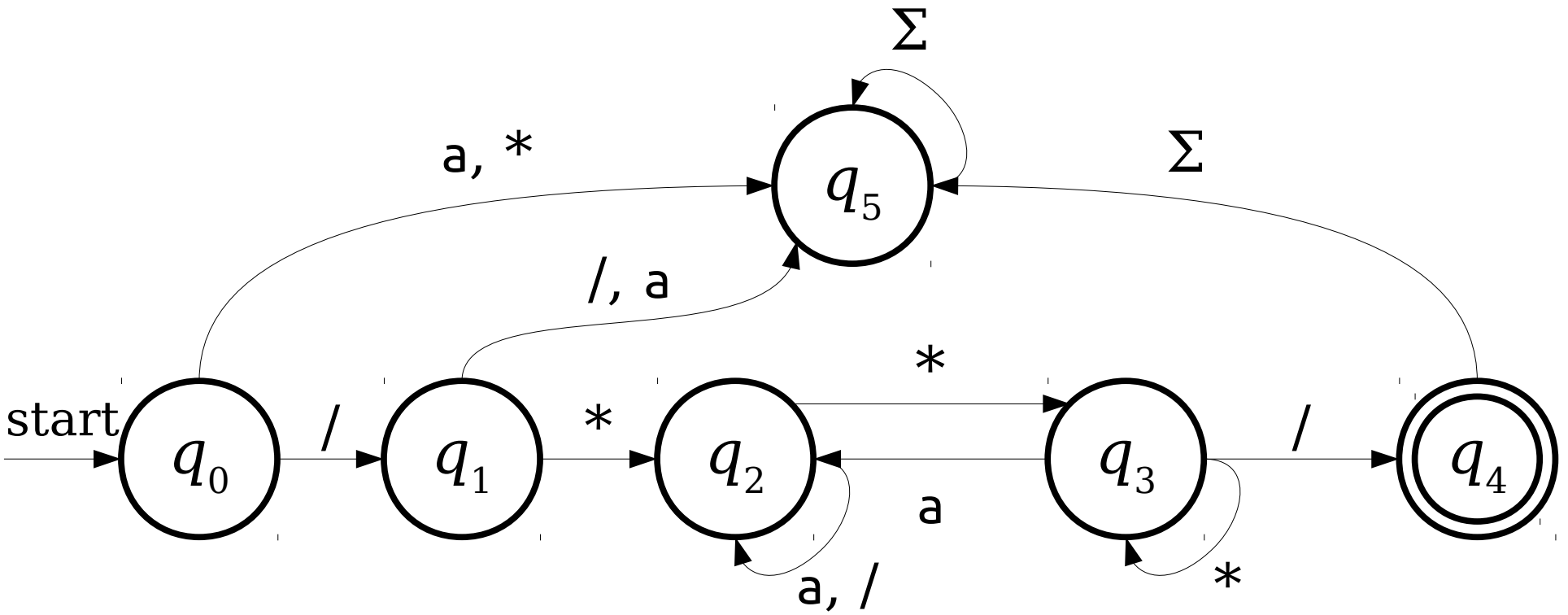


$\overline{L} = \{ w \in \{0, 1\}^* \mid w \textbf{\textit{ does not }} \text{contain } 00 \text{ as a substring} \}$



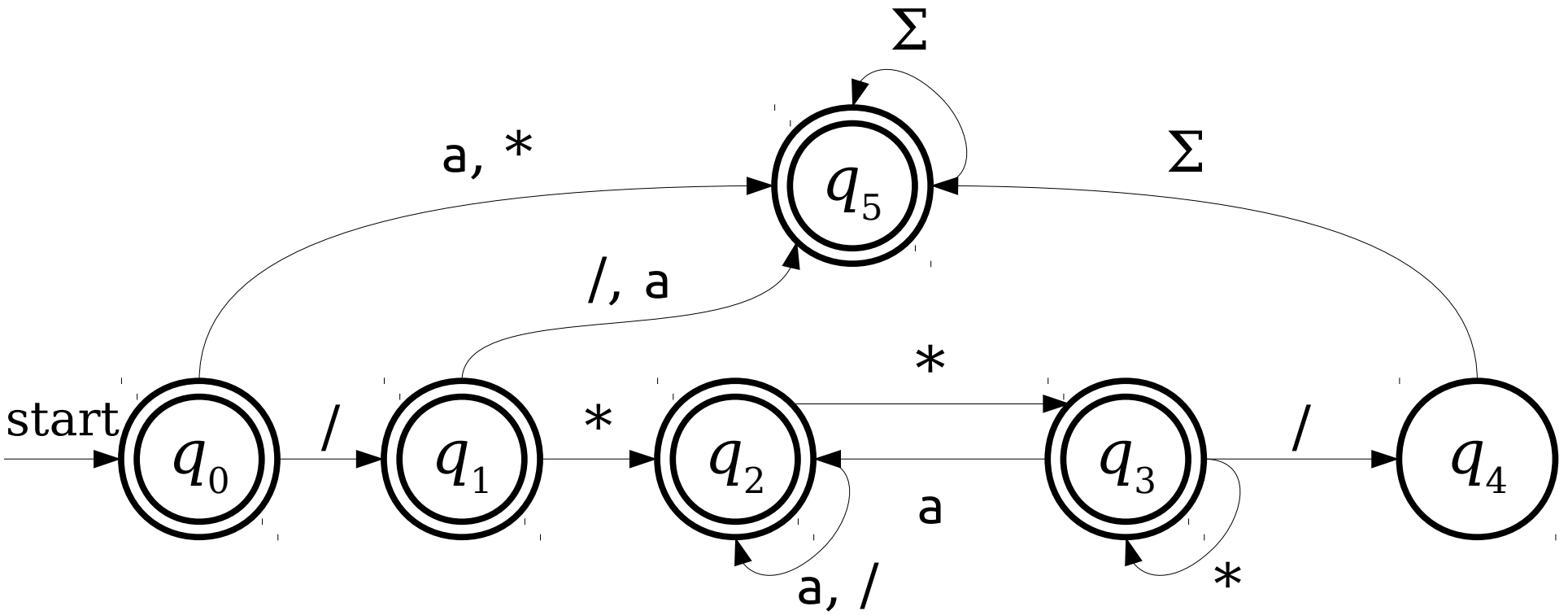
More Elaborate DFAs

$L = \{\ w \in \{\texttt{a}, \texttt{*}, \texttt{/}\}^* \mid w \text{ represents a C-style comment}\ \}$



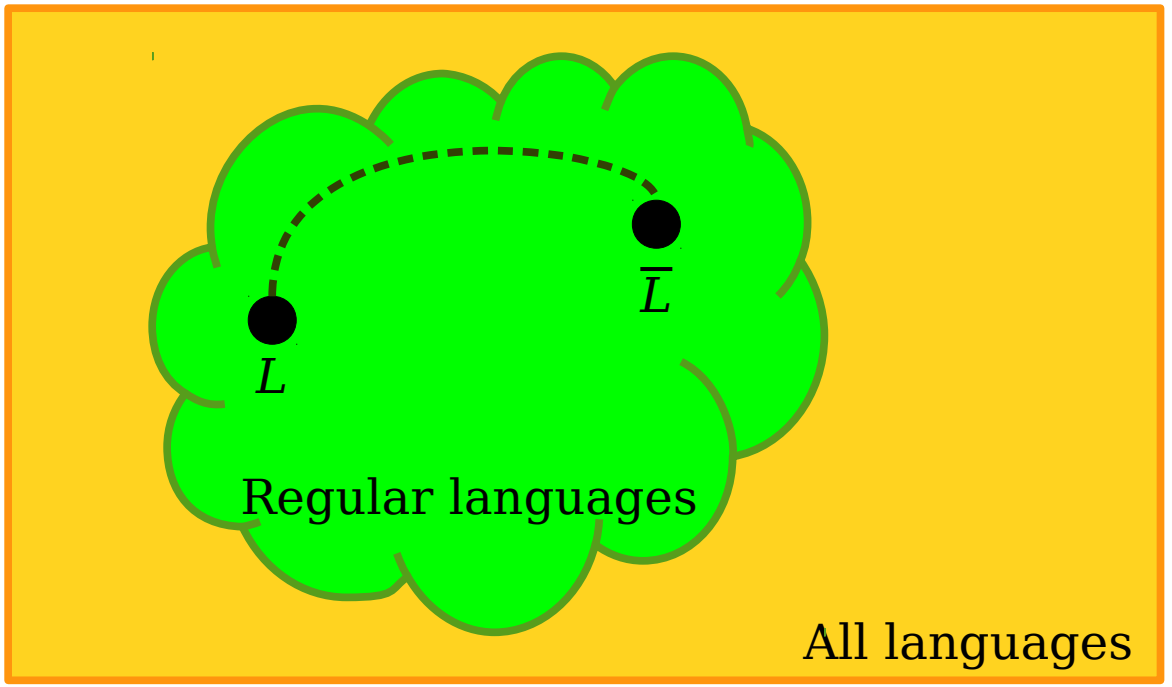
More Elaborate DFAs

$\overline{L}$ = { $w \in$ {a, *, /}* | w *doesn't* represent a C-style comment }



Closure Properties

- ***Theorem:*** If L is a regular language, then $\overline{L}$ is also a regular language.
- As a result, we say that the regular languages are ***closed under complementation***.



Time-Out For Announcements!

Problem Sets

- As a reminder, PS5 is due this Friday.
- Have questions?
  - Ask them on Piazza!
  - Stop by office hours!
- We recommend that you work through at least one or two of the problems by the end of the evening.

Midterms Graded

- We've graded the first midterm exam! We'll hand back graded midterms at the end of class today.
- Check the solution sets for statistics, information on how to estimate your grade, common mistakes, and techniques for improving going forward.

WiCS Board Applications 2017-2018

Interested in joining Stanford Women in Computer Science (WiCS) next year? Apply for the **WiCS Board** today!

Applications **are live** at
https://goo.gl/forms/2PFLVbwfS6HkO2Dk1

Find a list of the teams **here**
https://quip.com/j4DYAbnFeMTC

The Deadline is **Friday, May 12th** at **11:59 PM**

Contact Anvita (avgupta) or Nancy (xnancy) with any questions!

Your Questions

“Why do you delete questions you don't want to answer?”

Typically, I only delete questions if they aren't constructive or they're shibboleth questions. Please feel free to reach out to me via email if you'd like to chat about things!

“What do you think when people say ‘college should be the best 4 years of your life?’”

“Do you think that Stanford is too pre-professional?”

It's complicated. On the one hand, I think it is important to think about career prospects post-graduation. On the other hand, I'm concerned that people worry way too much about this way too early on.

I'll speak a bit more candidly in class.

Back to CS103!

NFAs

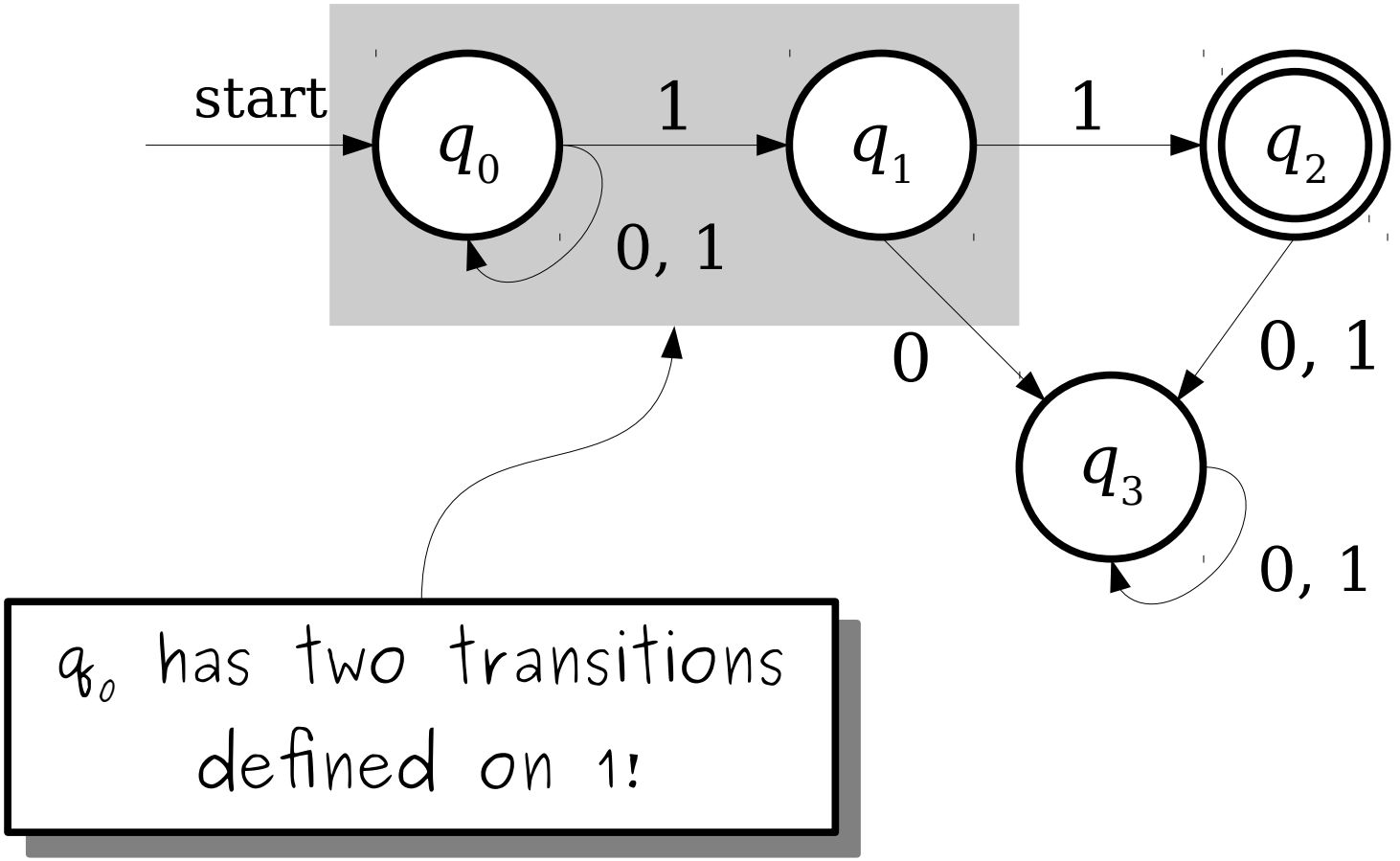
NFAs

- An ***NFA*** is a
  - ***N***ondeterministic
  - ***F***inite
  - ***A***utomaton
- Structurally similar to a DFA, but represents a fundamental shift in how we'll think about computation.

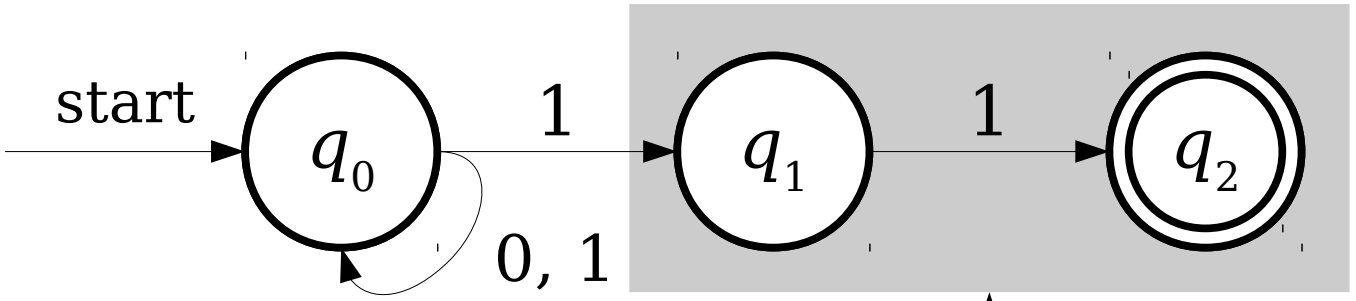
(Non)determinism

- A model of computation is ***deterministic*** if at every point in the computation, there is exactly one choice that can make.
- The machine accepts if that series of choices leads to an accepting state.
- A model of computation is ***nondeterministic*** if the computing machine may have multiple decisions that it can make at one point.
- The machine accepts if ***any*** series of choices leads to an accepting state.

A Simple NFA

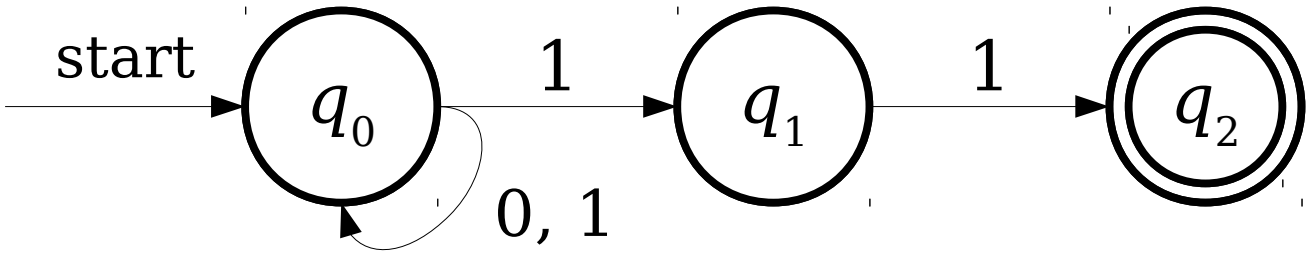

start
q_0
q_1
q_2
q_3
1
1
0, 1
0
0, 1
0, 1
q_0 has two transitions defined on 1!

A More Complex NFA



If a NFA needs to make a transition when no transition exists, the automaton dies and that particular path rejects.

A More Complex NFA



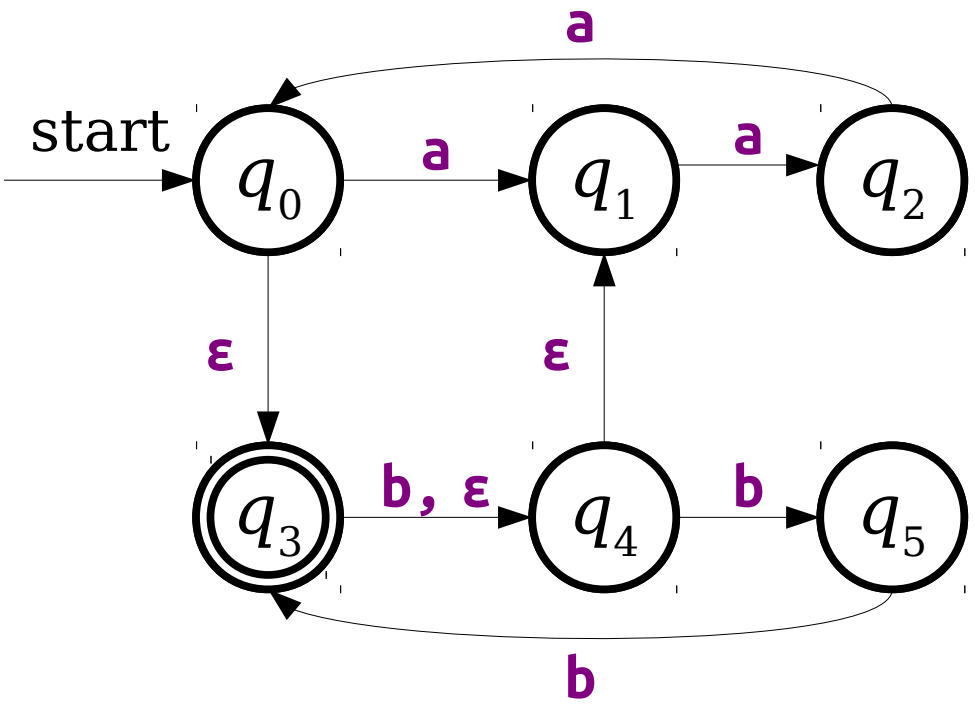
Question to ponder:
What does this NFA accept?

NFA Acceptance

- An NFA *N* accepts a string *w* if there is some series of choices that lead to an accepting state.
- Consequently, an NFA *N* rejects a string *w* if *no possible* series of choices lead it into an accepting state.
- It's easier to show that an NFA does accept something than to show that it doesn't

ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.



ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.
- NFAs are not *required* to follow ε-transitions. It's simply another option at the machine's disposal.

Intuiting Nondeterminism

- Nondeterministic machines are a serious departure from physical computers. How can we build up an intuition for them?
- There are two particularly useful frameworks for interpreting nondeterminism:
  - ***Perfect guessing***
  - ***Massive parallelism***

Perfect Guessing

- We can view nondeterministic machines as having ***Magic Superpowers*** that enable them to guess choices that lead to an accepting state.
  - If there is at least one choice that leads to an accepting state, the machine will guess it.
  - If there are no choices, the machine guesses any one of the wrong guesses.
- No known physical analog for this style of computation – this is totally new!

Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- (Here's a rigorous explanation about how this works; read this on your own time).
  - Start off in the set of all states formed by taking the start state and including each state that can be reached by zero or more ε-transitions.
  - When you read a symbol **a** in a set of states *S*:
    - Form the set *S'* of states that can be reached by following a single **a** transition from some state in *S*.
    - Your new set of states is the set of states in *S'*, plus the states reachable from *S'* by following zero or more ε-transitions.

So What?

- Each intuition of nondeterminism is useful in a different setting:
  - Perfect guessing is a great way to think about how to design a machine.
  - Massive parallelism is a great way to test machines – and has nice theoretical implications.
- Nondeterministic machines may not be feasible, but they give a great basis for interesting questions:
  - Can any problem that can be solved by a nondeterministic machine be solved by a deterministic machine?
  - Can any problem that can be solved by a nondeterministic machine be solved ***efficiently*** by a deterministic machine?
- The answers vary from automaton to automaton.

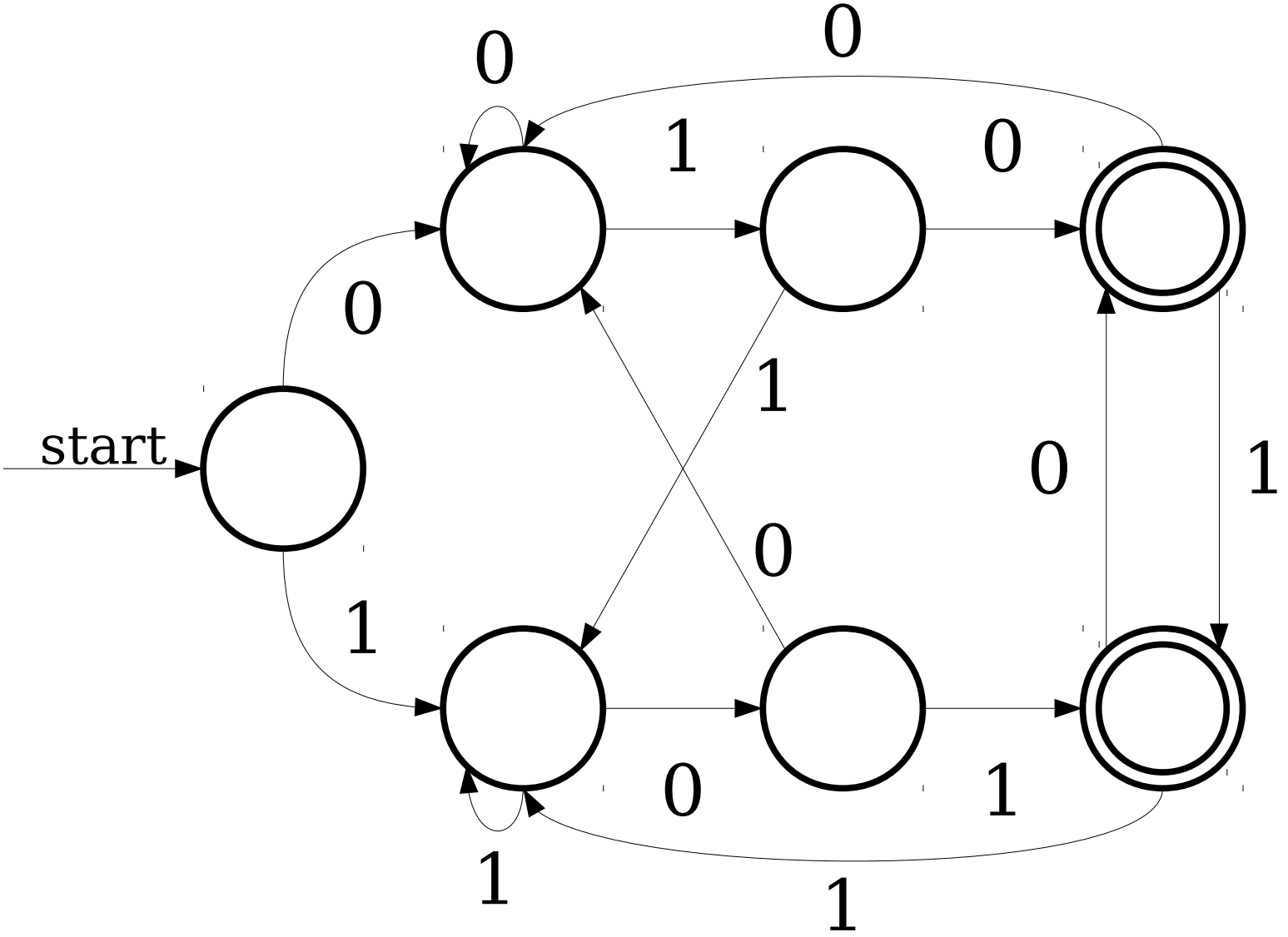
Designing NFAs

Designing NFAs

- When designing NFAs, *embrace the nondeterminism!*
- Good model: ***Guess-and-check***:
  - Is there some information that you'd really like to have? Have the machine *nondeterministically guess* that information.
  - Then, have the machine *deterministically check* that the choice was correct.
- The *guess* phase corresponds to trying lots of different options.
- The *check* phase corresponds to filtering out bad guesses or wrong options.

Guess-and-Check

$L = \{ w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w \text{ ends in } \mathtt{010} \text{ or } \mathtt{101} \}$



Guess-and-Check

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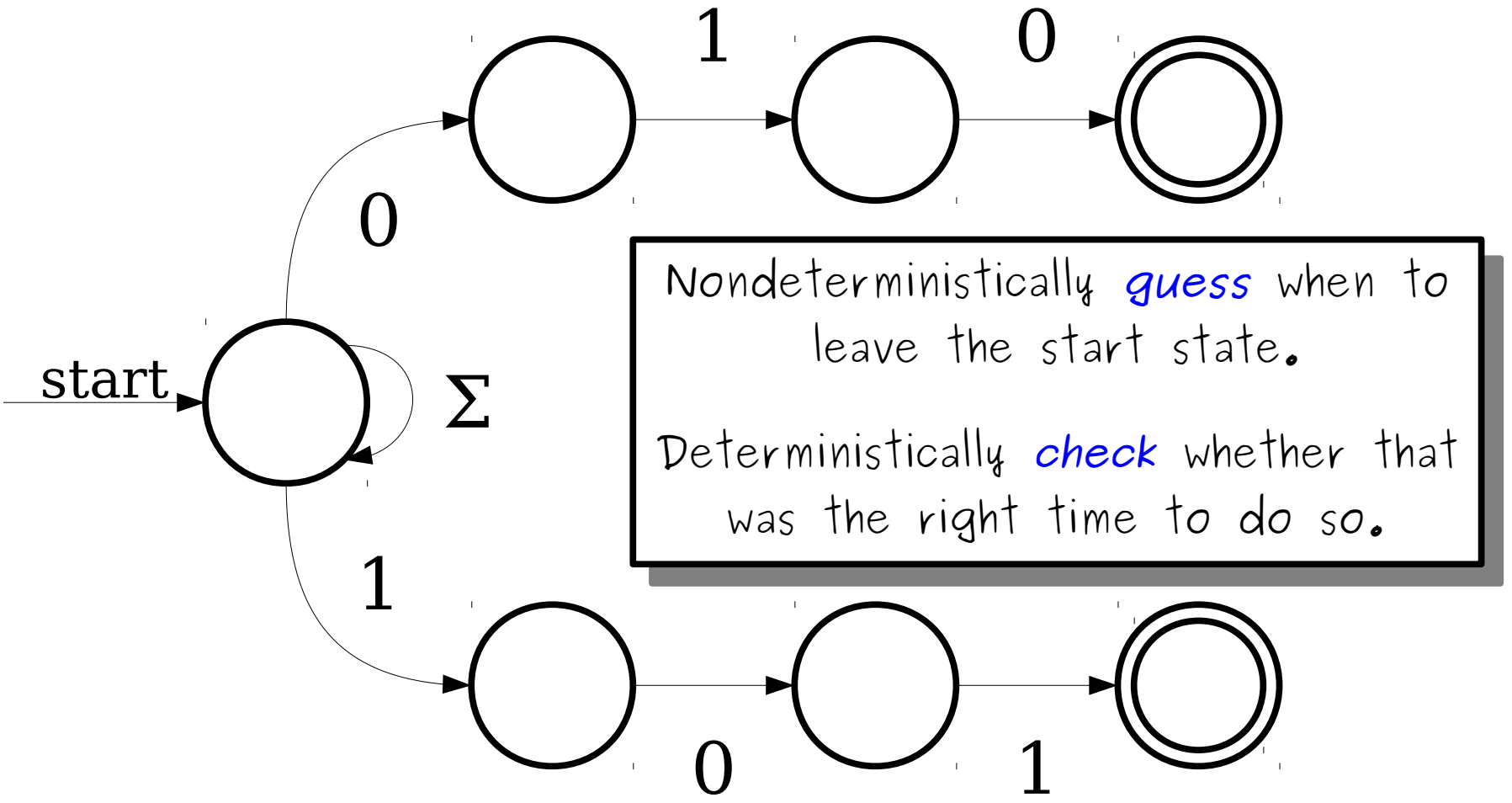
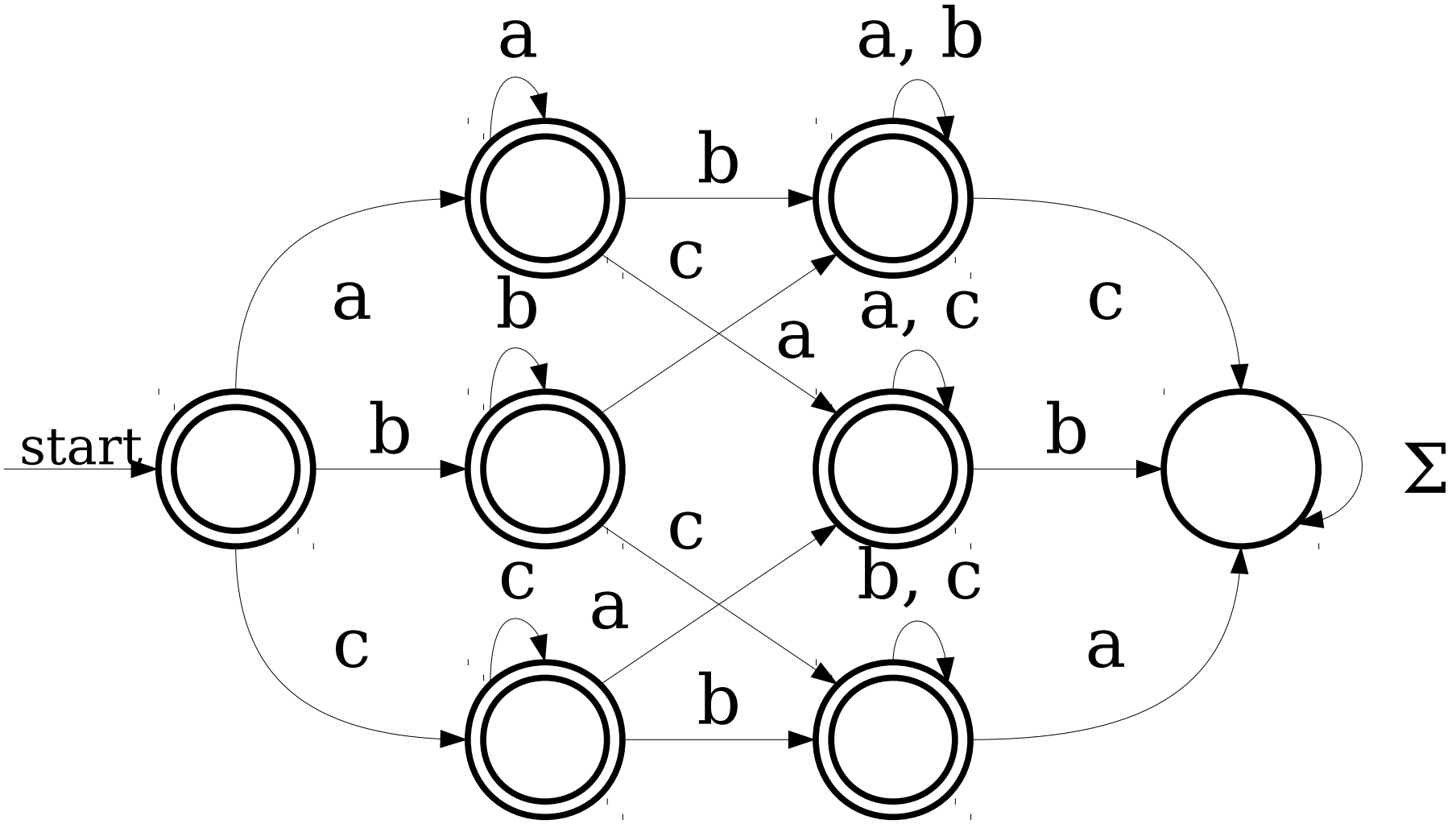


Guess-and-Check

$L = \{ w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* \mid$ at least one of **a**, **b**, or **c** is not in $w \}$



Guess-and-Check

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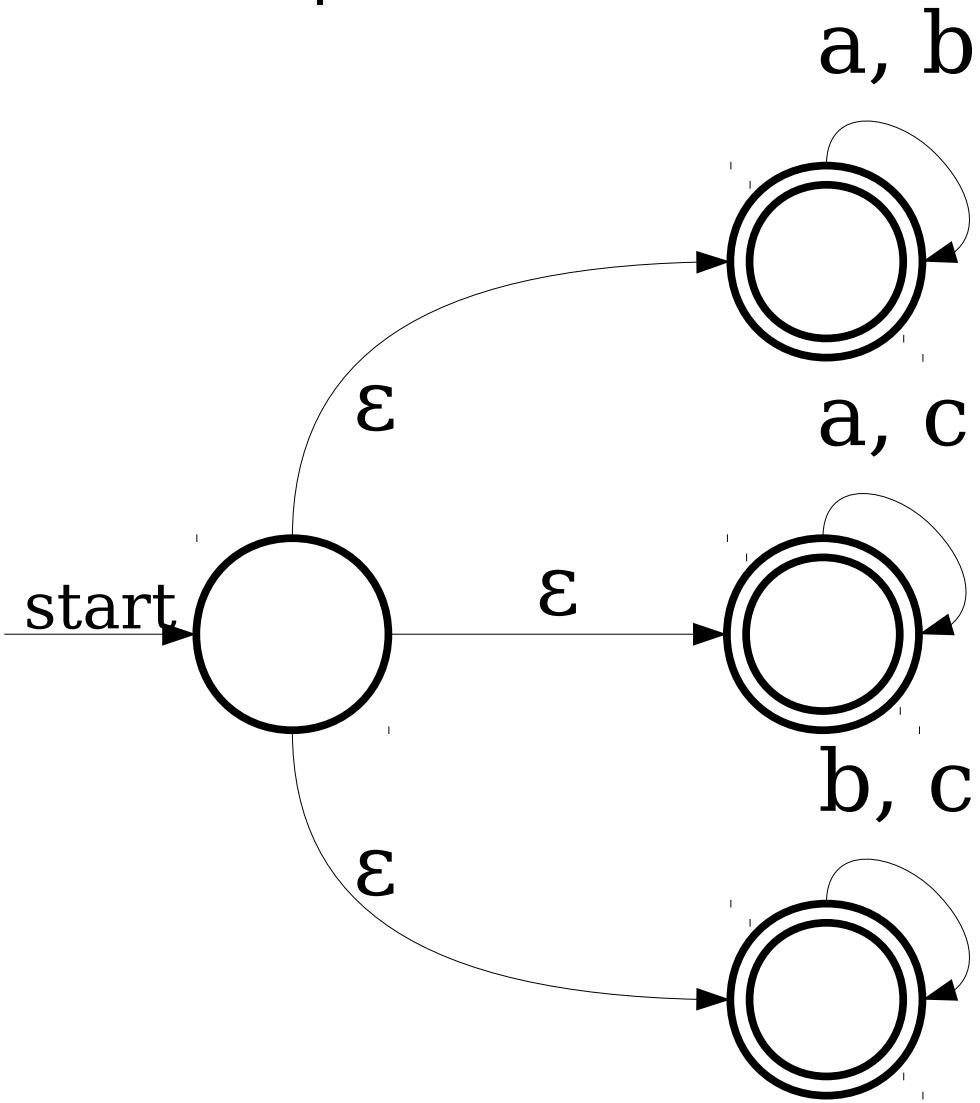

Nondeterministically *guess* which character is missing.

Deterministically *check* whether that character is indeed missing.

Just how powerful are NFAs?