Finite Automata
Part Three
Recap from Last Time
Tabular DFAs

These stars indicate accepting states.
Since this is the first row, it's the start state.
A language $L$ is called a **regular language** if there exists a DFA $D$ such that $\mathcal{L}(D) = L$. 
NFAs

• An **NFA** is a
  • **N**ondeterministic
  • **F**inite
  • **A**utomaton

• Can have missing transitions or multiple transitions defined on the same input symbol.

• Accepts if *any possible series of choices* leads to an accepting state.
ε-Transitions

- NFAs have a special type of transition called the \textit{ε-transition}.
- An NFA may follow any number of ε-transitions at any time without consuming any input.
**ε-Transitions**

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.
Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- The NFA accepts if any of the states that are active at the end are accepting states. It rejects otherwise.
Just how powerful are NFAs?
New Stuff!
NFAs and DFAs

- Any language that can be accepted by a DFA can be accepted by an NFA.
- Why?
  - Every DFA essentially already is an NFA!
- **Question**: Can any language accepted by an NFA also be accepted by a DFA?
- Surprisingly, the answer is **yes**!
Thought Experiment:
How would you simulate an NFA in software?
$q_0 \xrightarrow{\text{a}} q_1 \xrightarrow{\text{b}} q_2 \xrightarrow{\text{a}} q_3$

$\Sigma$

Start

$\begin{array}{cccccc}
& a & b & a & b & a & a \\
\end{array}$
\[ \sum \]

- Start: \( q_0 \)
- \( q_1 \) with input \( a \)
- \( q_2 \) with input \( b \)
- \( q_3 \) with input \( a \)

Input: \( a \) \( b \) \( a \) \( b \) \( a \) \( b \) \( a \)
\[
\begin{align*}
\sum & \\
\text{start} & \rightarrow q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_3
\end{align*}
\]

\[
\begin{array}{cccccc}
a & b & a & b & a & a \\
\end{array}
\]
\begin{align*}
&
\begin{array}{c}
\text{start} \\
\rightarrow \\
q_0
\end{array}
\xrightarrow{a} \quad \begin{array}{c}
q_1
\xrightarrow{b} \quad q_2
\xrightarrow{a} \quad q_3
\end{array}
\end{align*}

\[ \Sigma \]

\[ a \quad b \quad a \quad b \quad a \]
\[
q_3 \rightarrow q_2 \rightarrow q_1 \rightarrow q_0 \rightarrow \Sigma
\]
\[ \sum \]\n
Start state: \( q_0 \)
- Transitions:
  - \( q_0 \) to \( q_1 \) on input \( a \)
  - \( q_1 \) to \( q_2 \) on input \( b \)
  - \( q_2 \) to \( q_3 \) on input \( a \)

Input string: \( \ldots \quad ? \quad ? \quad ? \quad ? \quad ? \quad a \quad ? \quad ? \quad ? \quad ? \quad ? \quad \ldots \)
**Transition Table**

<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Diagram**

- **Start State:** \(q_0\)
- **Transition on a:** \(q_0 \rightarrow q_1\)
- **Transition on b:** \(q_1 \rightarrow q_2\)
- **Transition on a:** \(q_2 \rightarrow q_3\)

**Symbol:** \(\Sigma\)
\[ \Sigma \]

\[ \begin{array}{c|c|c}
      & a & b \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\end{array} \]
\[
\begin{array}{ccc}
\Sigma & a & b \\
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\end{array}
\]
### Transition Table

<table>
<thead>
<tr>
<th>States</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The given automaton has the following transitions:

- From state $q_0$, on input $a$, it transitions to state $q_1$.
- From state $q_1$, on input $b$, it transitions to state $q_2$.
- From state $q_2$, on input $a$, it transitions to state $q_3$.
- The start state is $q_0$.

The corresponding table for the transition function is:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0}$</td>
<td>${q_0, q_1}$</td>
<td>${q_0}$</td>
</tr>
<tr>
<td>${q_0, q_1}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ q_0 \quad \text{start} \quad a \quad q_1 \quad b \quad q_2 \quad a \quad q_3 \]

\[
\begin{array}{c|cc}
& a & b \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & & \\
\end{array}
\]
<table>
<thead>
<tr>
<th>State Set</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State Set</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{q_0}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{q_0}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Diagram:
- Start state: \(q_0\)
- First transition: \(q_0 \to q_1\) on input \(a\)
- Second transition: \(q_1 \to q_2\) on input \(b\)
- Third transition: \(q_2 \to q_3\) on input \(a\)
- Loop transition: \(q_3 \to q_0\) on input \(\sum\)
\[ q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_3 \]

Transition table:

<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ \Sigma \]

**Diagram:**

- Start state: \( q_0 \)
- Transitions:
  - From \( q_0 \) to \( q_1 \) on input \( a \)
  - From \( q_1 \) to \( q_2 \) on input \( b \)
  - From \( q_2 \) to \( q_3 \) on input \( a \)
- \( q_3 \) is a loop

**Table:**

<table>
<thead>
<tr>
<th>State</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {q_0} )</td>
<td>( {q_0, q_1} )</td>
<td>( {q_0} )</td>
</tr>
<tr>
<td>( {q_0, q_1} )</td>
<td>( {q_0, q_1} )</td>
<td>( {q_0, q_1} )</td>
</tr>
<tr>
<td>( {q_0, q_1} )</td>
<td>( {q_0, q_1} )</td>
<td>( {q_0, q_1} )</td>
</tr>
<tr>
<td>( {q_0, q_1} )</td>
<td>( {q_0, q_1} )</td>
<td>( {q_0, q_1} )</td>
</tr>
<tr>
<td>( {q_3} )</td>
<td>( {q_3} )</td>
<td>( {q_3} )</td>
</tr>
</tbody>
</table>
\[
\begin{array}{c}
\sum \\
\{ \{ q_0 \} \}
\end{array}
\]
\[ \Sigma \]

```
<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td></td>
</tr>
<tr>
<td>{q_0}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Diagram:
- Start state: \( q_0 \)
- Transitions:
  - \( a \) from \( q_0 \) to \( q_1 \)
  - \( b \) from \( q_1 \) to \( q_2 \)
  - \( a \) from \( q_2 \) to \( q_3 \)
  - Loop from \( q_3 \) to itself
\[
\begin{array}{c}
\Sigma \\
\downarrow \\
\text{start} \\
\text{a} \\
\text{b} \\
\text{a} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
& a & b \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\end{array}
\]
\[ \Sigma \]

Start

\[ q_0 \quad \xrightarrow{a} \quad q_1 \quad \xrightarrow{b} \quad q_2 \quad \xrightarrow{a} \quad q_3 \]

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( { q_0 } )</td>
<td>( { q_0, q_1 } )</td>
<td>( { q_0 } )</td>
</tr>
<tr>
<td>( { q_0, q_1 } )</td>
<td>( { q_0, q_1 } )</td>
<td>( { q_0, q_2 } )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ \Sigma \]

\[ \begin{array}{|c|c|c|} 
\hline
& a & b \\
\hline
\{ q_0 \} & \{ q_0, q_1 \} & \{ q_0 \} \\
\{ q_0, q_1 \} & \{ q_0, q_1 \} & \{ q_0, q_2 \} \\
\{ q_0, q_2 \} & & \\
\hline
\end{array} \]
<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The diagram shows a finite automaton with states $q_0, q_1, q_2, q_3$.

- The start state is $q_0$.
- The transitions are:
  - $a$ from $q_0$ to $q_1$
  - $b$ from $q_1$ to $q_2$
  - $a$ from $q_2$ to $q_3$

The table represents the transition function:

<table>
<thead>
<tr>
<th>Current State</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0}$</td>
<td>${q_0, q_1}$</td>
<td>${q_0}$</td>
</tr>
<tr>
<td>${q_0, q_1}$</td>
<td>${q_0, q_1}$</td>
<td>${q_0, q_2}$</td>
</tr>
<tr>
<td>${q_0, q_2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What should this row look like?

<table>
<thead>
<tr>
<th>Current State</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0, q_2}$</td>
<td>${q_0, q_3}$</td>
<td>${q_0}$</td>
</tr>
<tr>
<td>${q_0, q_2}$</td>
<td>${q_0, q_3}$</td>
<td>${q_0, q_2}$</td>
</tr>
<tr>
<td>${q_0, q_2}$</td>
<td>${q_0, q_1, q_3}$</td>
<td>${q_0}$</td>
</tr>
<tr>
<td>${q_0, q_2}$</td>
<td>${q_0, q_1, q_3}$</td>
<td>${q_0, q_2}$</td>
</tr>
</tbody>
</table>

Answer at PollEv.com/cs103 or text CS103 to 22333 once to join, then A, B, C, or D.
A finite automaton with states $q_0, q_1, q_2, q_3$, transitions on inputs $a$ and $b$, and initial state $q_0$. The state transitions are as follows:

- From $q_0$, on input $a$, move to $q_1$.
- From $q_1$, on input $b$, move to $q_2$.
- From $q_2$, on input $a$, move to $q_3$.

The states $q_0, q_1, q_2, q_3$ are represented in a table with inputs $a$ and $b$:

<table>
<thead>
<tr>
<th>State</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0}$</td>
<td>${q_0, q_1}$</td>
<td>${q_0}$</td>
</tr>
<tr>
<td>${q_0, q_1}$</td>
<td>${q_0, q_1}$</td>
<td>${q_0, q_2}$</td>
</tr>
<tr>
<td>${q_0, q_2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{array}{c}
\Sigma \\
\end{array}
\begin{array}{ccc}
\text{start} & \xrightarrow{a} & q_1 \\
q_0 & \xrightarrow{b} & q_2 \\
& \xrightarrow{a} & q_3
\end{array}
\]

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>State</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ \Sigma \]

Start

\[
\begin{array}{ccc}
q_0 & \xrightarrow{a} & q_1 \\
\{q_0\} & \xrightarrow{a} & \{q_0, q_1\} \\
\{q_0, q_1\} & \xrightarrow{b} & \{q_0, q_1\} \\
\{q_0, q_2\} & \xrightarrow{a} & \{q_0, q_2\}
\end{array}
\]
\[ \sum \]

The diagram shows a finite automaton with the following states and transitions:

- States: \( q_0 \) (start state), \( q_1 \), \( q_2 \), \( q_3 \)
- Alphabet: \( \Sigma \)
- Transitions:
  - From \( q_0 \) on \( a \) to \( q_1 \)
  - From \( q_1 \) on \( b \) to \( q_2 \)
  - From \( q_2 \) on \( a \) to \( q_3 \)

The transition table is as follows:

<table>
<thead>
<tr>
<th></th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( { q_0 } )</td>
<td>( { q_0, q_1 } )</td>
<td>( { q_0 } )</td>
</tr>
<tr>
<td>( { q_0, q_1 } )</td>
<td>( { q_0, q_1 } )</td>
<td>( { q_0, q_2 } )</td>
</tr>
<tr>
<td>( { q_0, q_2 } )</td>
<td>( { q_0, q_1, q_3 } )</td>
<td></td>
</tr>
</tbody>
</table>
\[ \Sigma \]

<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
</tr>
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<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
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</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td>{q_0, q_1, q_3}</td>
<td></td>
</tr>
</tbody>
</table>

Diagram:

- Start state: \(q_0\)
- Transitions:
  - \(q_0\) to \(q_1\) on \(a\)
  - \(q_0\) to \(q_1\) on \(b\)
  - \(q_1\) to \(q_2\) on \(a\)
  - \(q_2\) to \(q_3\) on \(a\)
- \(q_3\) is a final state

Table:

- \(a\):
  - \(\{q_0\}\) to \(\{q_0, q_1\}\)
  - \(\{q_0, q_1\}\) to \(\{q_0, q_1\}\)
  - \(\{q_0, q_2\}\) to \(\{q_0, q_1, q_3\}\)

- \(b\):
  - \(\{q_0\}\) to \(\{q_0\}\)
  - \(\{q_0, q_2\}\) to \(\{q_0, q_2\}\)
\[ \Sigma \]

**Diagram:**
- Start state: \( q_0 \)
- Transitions:
  - \( a \) from \( q_0 \) to \( q_1 \)
  - \( b \) from \( q_1 \) to \( q_2 \)
  - \( a \) from \( q_2 \) to \( q_3 \)

**Table:**

<table>
<thead>
<tr>
<th>( q_0 )</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{ q_0 }</td>
<td>{ q_0, q_1 }</td>
<td>{ q_0 }</td>
</tr>
<tr>
<td>{ q_0, q_1 }</td>
<td>{ q_0, q_1 }</td>
<td>{ q_0, q_2 }</td>
</tr>
<tr>
<td>{ q_0, q_2 }</td>
<td>{ q_0, q_1, q_3 }</td>
<td></td>
</tr>
</tbody>
</table>

**Legend:**
- \( q_0 \): Start state
- \( q_1 \), \( q_2 \), \( q_3 \): States
- \( \Sigma \): Alphabet
- \( a \), \( b \): Input symbols
\[
\begin{array}{ccc}
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \\
\end{array}
\]
<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
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<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td>{q_0, q_1, q_3}</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{array}{c c c}
\Sigma & a & b \\
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \{q_0\} \\
\end{array}
\]
\[\Sigma\]

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>({q_0})</td>
<td>({q_0, q_1})</td>
<td>({q_0})</td>
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<td>({q_0, q_1})</td>
<td>({q_0, q_1})</td>
<td>({q_0, q_2})</td>
</tr>
<tr>
<td>({q_0, q_2})</td>
<td>({q_0, q_1, q_3})</td>
<td>({q_0})</td>
</tr>
<tr>
<td>({q_0, q_1, q_3})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Diagram**: 
- Start state: \(q_0\)
- Transitions: 
  - \(q_0 \xrightarrow{a} q_1\)
  - \(q_1 \xrightarrow{b} q_2\)
  - \(q_2 \xrightarrow{a} q_3\)
  - \(q_3\) (accept state)

**Table**: 
- \(a\): \(q_0\) transitions to \(q_1\), \(q_0\) and \(q_1\) to \(q_0\), \(q_0\) to \(q_2\), \(q_0\), \(q_1\), \(q_2\) to \(q_0\)
- \(b\): \(q_0\) to \(q_0\)

**Note**: The table represents the state transitions of the automaton for each input symbol.
<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td>{q_0, q_1, q_3}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1, q_3}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{array}{|c|c|c|}
\hline
 & a & b \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \{q_0\} \\
\{q_0, q_1, q_3\} & & \\
\hline
\end{array}
\]
<table>
<thead>
<tr>
<th>States</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td>{q_0, q_1, q_3}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1, q_3}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{tabular}{|c|c|c|}
\hline
State & a & b \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \{q_0\} \\
\{q_0, q_1, q_3\} & & \\
\hline
\end{tabular}
\]
<table>
<thead>
<tr>
<th>State Set</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td>{q_0, q_1, q_3}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1, q_3}</td>
<td>{q_0, q_1}</td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{array}{c|c|c}
\text{a} & \text{b} \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \{q_0\} \\
\{q_0, q_1, q_3\} & \{q_0, q_1\} & \\
\end{array}
\]
\[
q_3 \quad q_3 \quad q_2 \quad q_1 \quad q_0
\]

\[
\text{start} \quad a \quad b \quad a
\]

\[
\Sigma
\]

\[
\begin{array}{|c|c|c|}
\hline
 & a & b \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \{q_0\} \\
\{q_0, q_1, q_3\} & \{q_0, q_1\} & \text{---} \\
\hline
\end{array}
\]
\[
\begin{array}{c|cc}
\Sigma & a & b \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \{q_0\} \\
\{q_0, q_1, q_3\} & \{q_0, q_1\} & \\
\end{array}
\]
\[ \begin{array}{c|cc}
\Sigma & a & b \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \{q_0\} \\
\{q_0, q_1, q_3\} & \{q_0, q_1\} & \end{array} \]
\begin{align*}
\Sigma & \\
\text{start} & \rightarrow q_0 \quad \xrightarrow{a} q_1 \quad \xrightarrow{b} q_2 \quad \xrightarrow{a} q_3
\end{align*}

\begin{tabular}{|c|c|c|}
\hline
& \text{a} & \text{b} \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \{q_0\} \\
\{q_0, q_1, q_3\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\hline
\end{tabular}
<table>
<thead>
<tr>
<th>States</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td>{q_0, q_1, q_3}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1, q_3}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
</tbody>
</table>

**Diagram:**

- **Start state**: \(q_0\)
- **Final state**: \(q_3\)
- **Transitions**:
  - From \(q_0\) to \(q_1\) on \(a\)
  - From \(q_1\) to \(q_2\) on \(b\)
  - From \(q_2\) to \(q_3\) on \(a\)
- **States**:
  - \(q_0\)
  - \(q_1\)
  - \(q_2\)
  - \(q_3\)
\[
\begin{array}{c|c|c}
\Sigma & a & b \\
\hline
\{q_0\} & \{q_0, q_1\} & \{q_0\} \\
\{q_0, q_1\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\{q_0, q_2\} & \{q_0, q_1, q_3\} & \{q_0\} \\
*\{q_0, q_1, q_3\} & \{q_0, q_1\} & \{q_0, q_2\} \\
\end{array}
\]
\[
\begin{align*}
\Sigma \\
\text{start} & \rightarrow q_0 & a \rightarrow q_1 & b \rightarrow q_2 & a \rightarrow q_3 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>State Set</th>
<th>Transition a</th>
<th>Transition b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0}</td>
<td>{q_0, q_1}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>{q_0, q_1}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
<tr>
<td>{q_0, q_2}</td>
<td>{q_0, q_1, q_3}</td>
<td>{q_0}</td>
</tr>
<tr>
<td>*{q_0, q_1, q_3}</td>
<td>{q_0, q_1}</td>
<td>{q_0, q_2}</td>
</tr>
</tbody>
</table>

\begin{align*}
\text{start} & \rightarrow \{q_0\} & a \rightarrow \{q_0, q_1\} & b \rightarrow \{q_0, q_2\} & a \rightarrow \{q_0, q_1, q_3\} \\
\end{align*}
\[ \Sigma \]

Start

- $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_3$
- $\{ q_0 \} \xrightarrow{b} \{ q_0, q_1 \} \xrightarrow{a} \{ q_0, q_2 \} \xrightarrow{b} \{ q_0, q_1, q_3 \}$

Input:

- $a b a a b b a a$
\[ q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_3 \]

\[ \Sigma \]

\[ \{ q_0 \} \xrightarrow{a} \{ q_0, q_1 \} \xrightarrow{b} \{ q_0, q_2 \} \xrightarrow{a} \{ q_0, q_1, q_3 \} \]
\[ q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_3 \]

\[ \Sigma \]

\[ \{ q_0 \} \xrightarrow{a} \{ q_0, q_1 \} \xrightarrow{a} \{ q_0, q_2 \} \xrightarrow{a} \{ q_0, q_1, q_3 \} \]

\[ a \ b \ a \ a \ b \ a \ a \]

\[ \text{start} \]

\[ \{ q_0 \} \]
\[ \sum \]

Start:
- \( q_0 \) to \( q_1 \) on 'a'
- \( q_1 \) to \( q_2 \) on 'b'
- \( q_2 \) to \( q_3 \) on 'a'

Input:
- 'a b a a a b a a a'

States:
- Start: \( \{ q_0 \} \)
- \( q_1 \): \( \{ q_0, q_1 \} \)
- \( q_2 \): \( \{ q_0, q_1, q_2 \} \)
- \( q_3 \): \( \{ q_0, q_1, q_2, q_3 \} \)
\[ \Sigma \]

\[ \begin{align*}
\text{start} &\quad \rightarrow \quad q_0 \\
&\quad \rightarrow a \quad q_1 \\
q_1 &\quad \rightarrow b \quad q_2 \\
&\quad \rightarrow a \quad q_3
\end{align*} \]

\[
\begin{array}{cccccccc}
& a & b & a & a & a & b & a & a \\
\end{array}
\]

\[
\begin{array}{cccc}
\{q_0\} & a & \{q_0, q_1\} & a \\
& b & b & a & a & b & a & a
\end{array}
\]

\[
\begin{array}{cccc}
\{q_0\} & a & \{q_0, q_1\} & a \\
& b & b & a & a & b & a & a
\end{array}
\]
\[
\Sigma
\]

Start state: \( q_0 \)

Transitions:
- \( q_0 \) on \( a \) goes to \( q_1 \)
- \( q_1 \) on \( b \) goes to \( q_2 \)
- \( q_2 \) on \( a \) goes to \( q_3 \) (loop)

Input string: \( ababaabbaba \)

States:
- \( q_0 \)
- \( q_1 \)
- \( q_2 \)
- \( q_3 \)

State transitions:
- \( q_0 \) on \( a \) goes to \( q_0 \)
- \( q_1 \) on \( a \) goes to \( q_0 \)
- \( q_1 \) on \( b \) goes to \( q_1 \)
- \( q_2 \) on \( a \) goes to \( q_2 \)
- \( q_2 \) on \( b \) goes to \( q_0 \)
- \( q_3 \) on \( a \) goes to \( q_3 \)
- \( q_3 \) on \( b \) goes to \( q_3 \)

Final states:
- \( q_0 \) on \( a \) goes to \( q_0 \)
- \( q_1 \) on \( a \) goes to \( q_0 \)
- \( q_2 \) on \( a \) goes to \( q_2 \)
- \( q_3 \) on \( a \) goes to \( q_3 \)

Language accepted: \( \{ q_0, q_1, q_3 \} \)