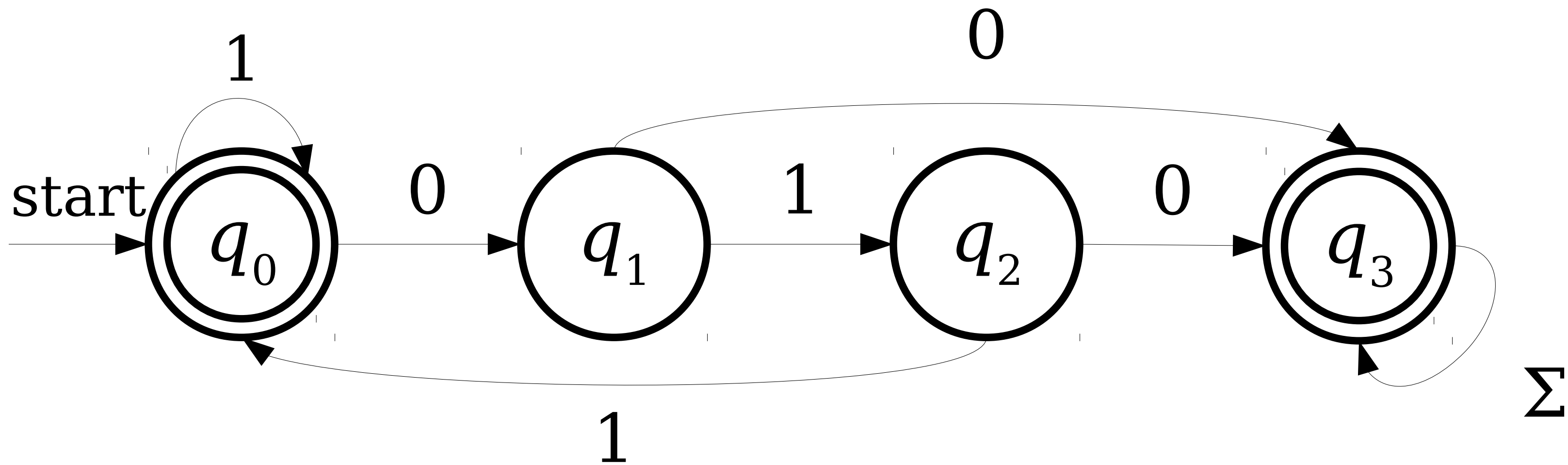
Finite Automata

Part Three

Recap from Last Time

Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

These stars indicate accepting states.

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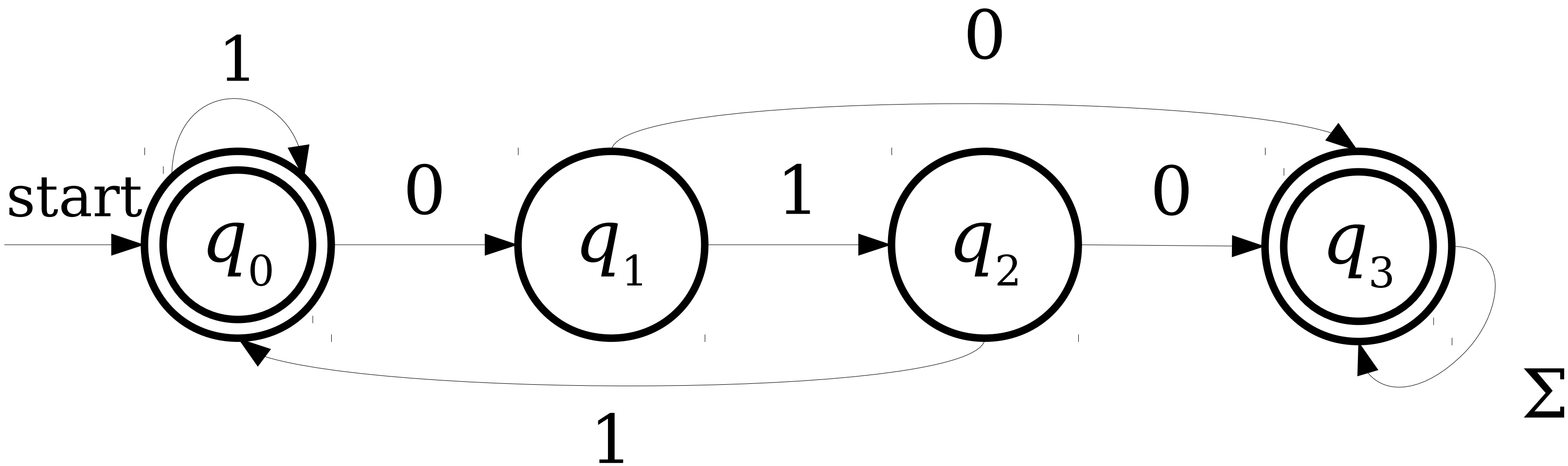

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q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

Since this is the first row, it's the start state.

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

NFAs

- An ***NFA*** is a
  - ***N***ondeterministic
  - ***F***inite
  - ***A***utomaton
- Can have missing transitions or multiple transitions defined on the same input symbol.
- Accepts if *any possible series of choices* leads to an accepting state.

ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.

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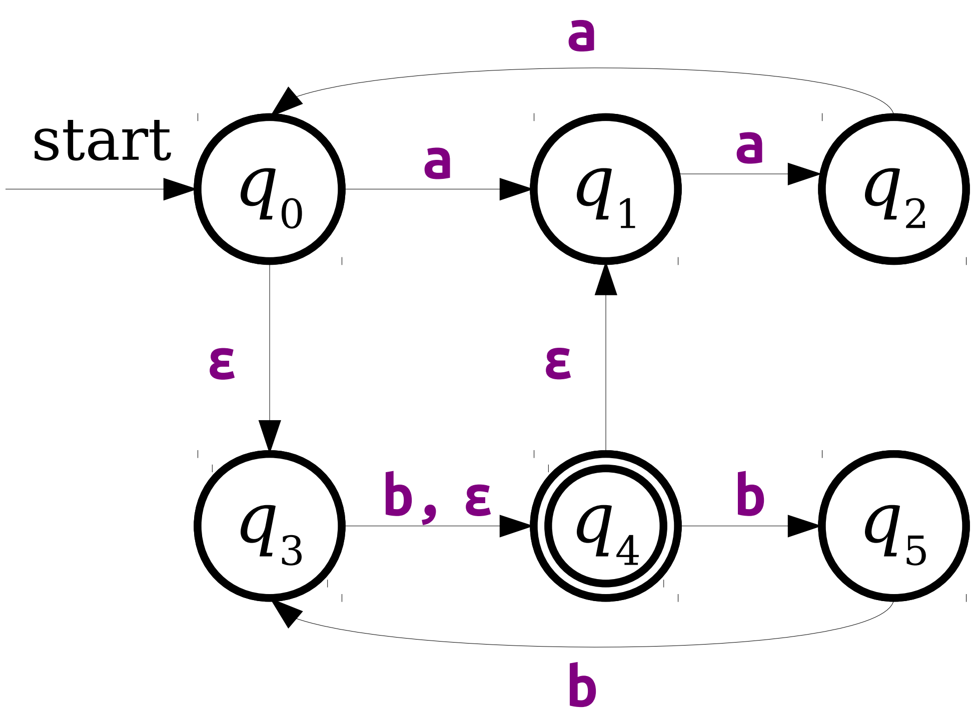

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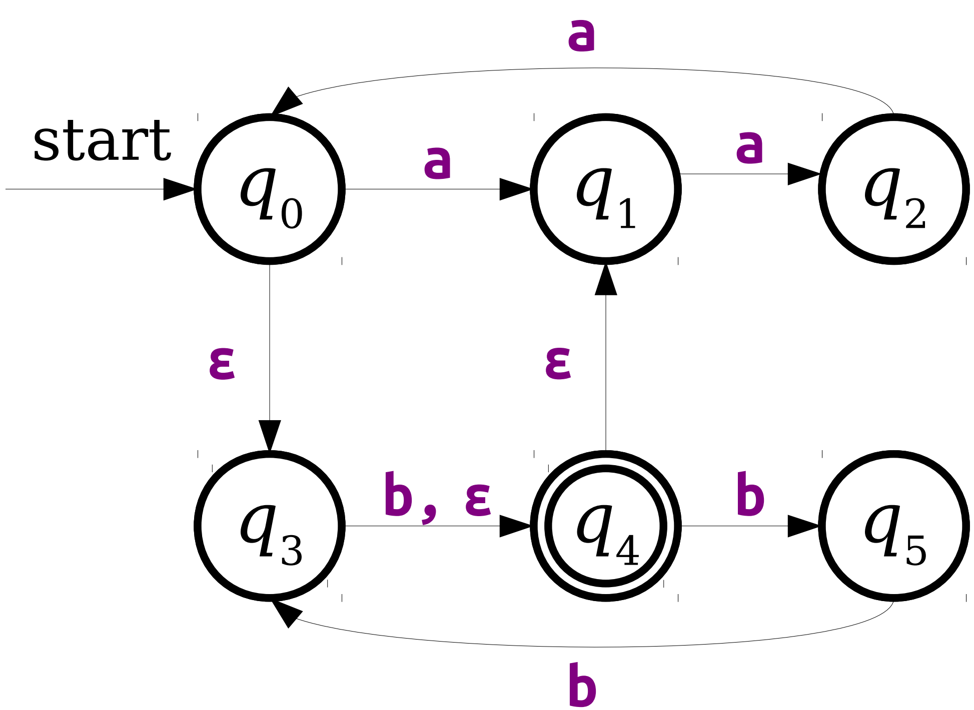

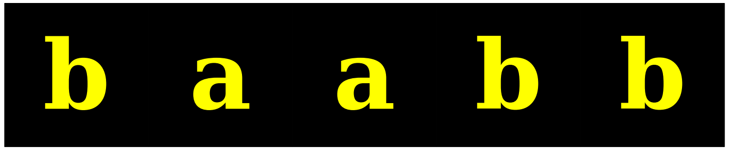

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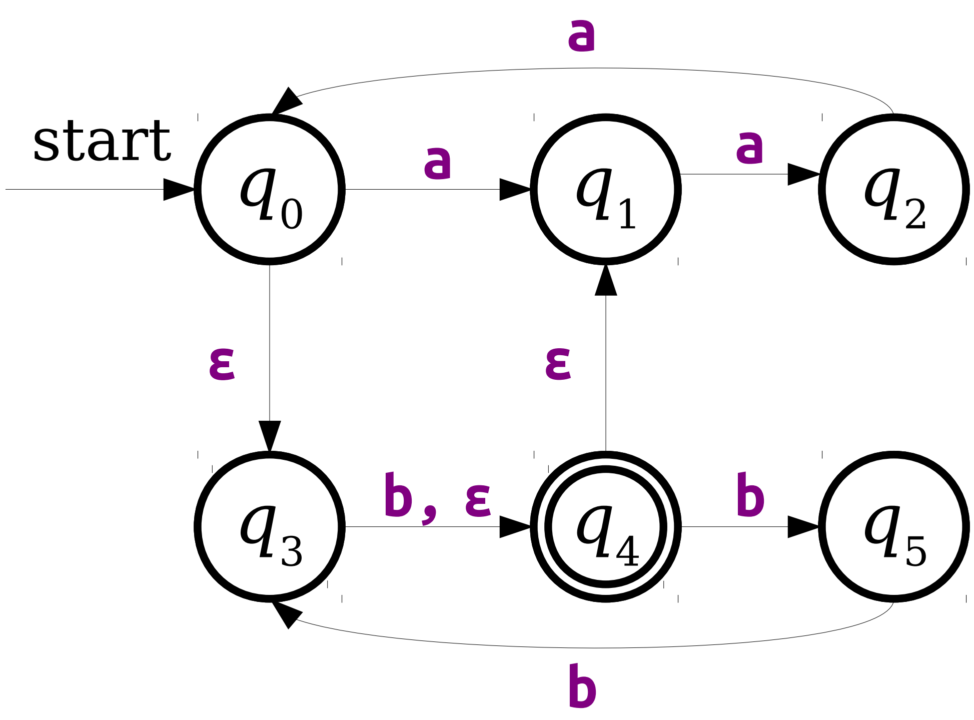

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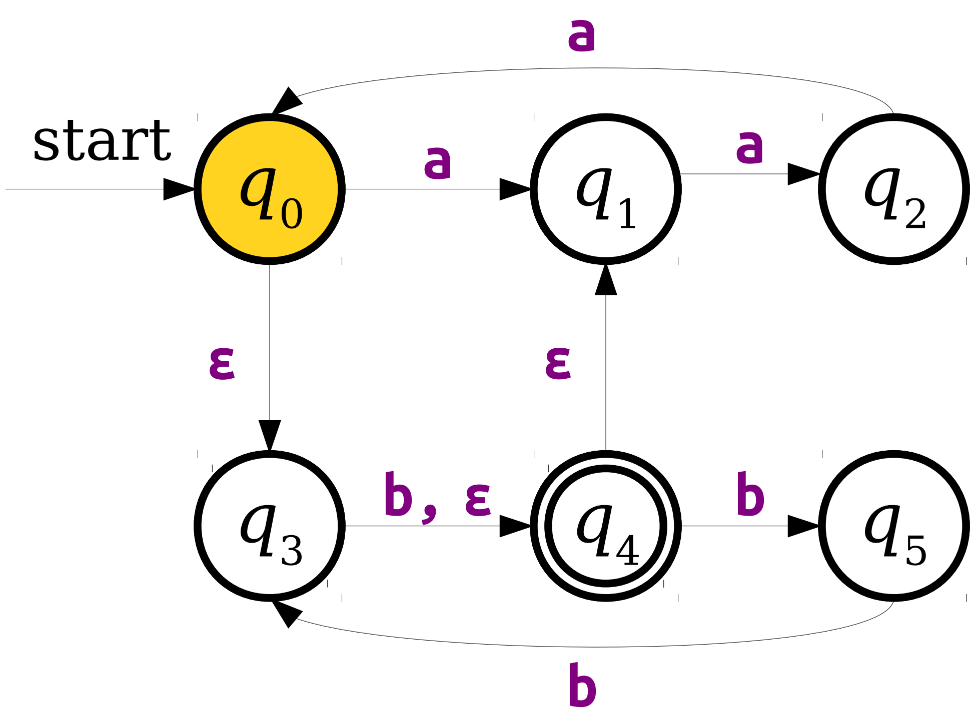

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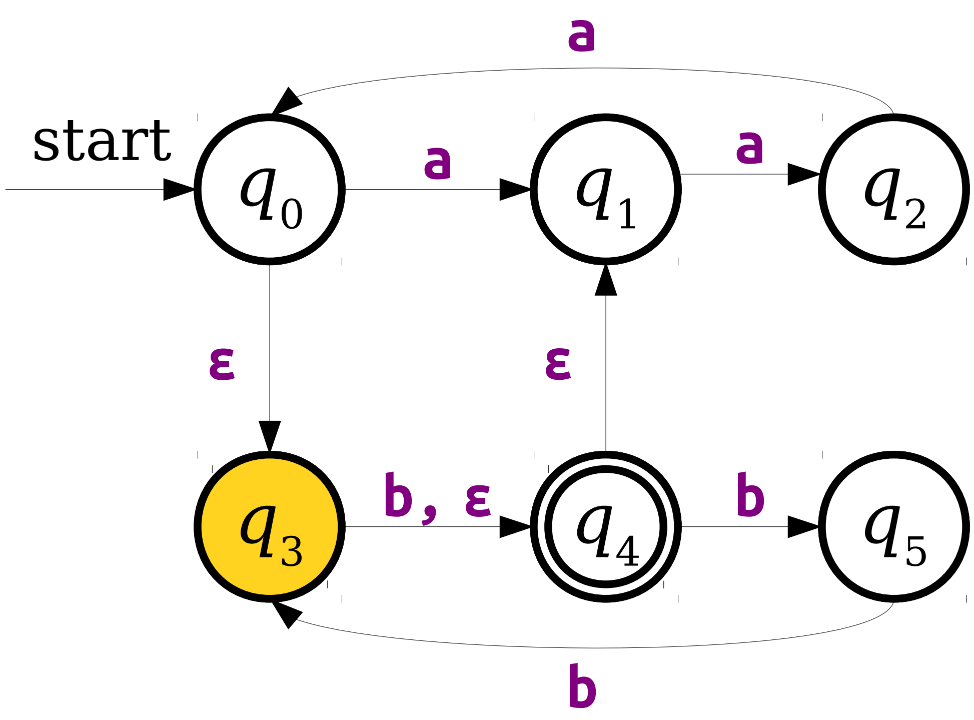

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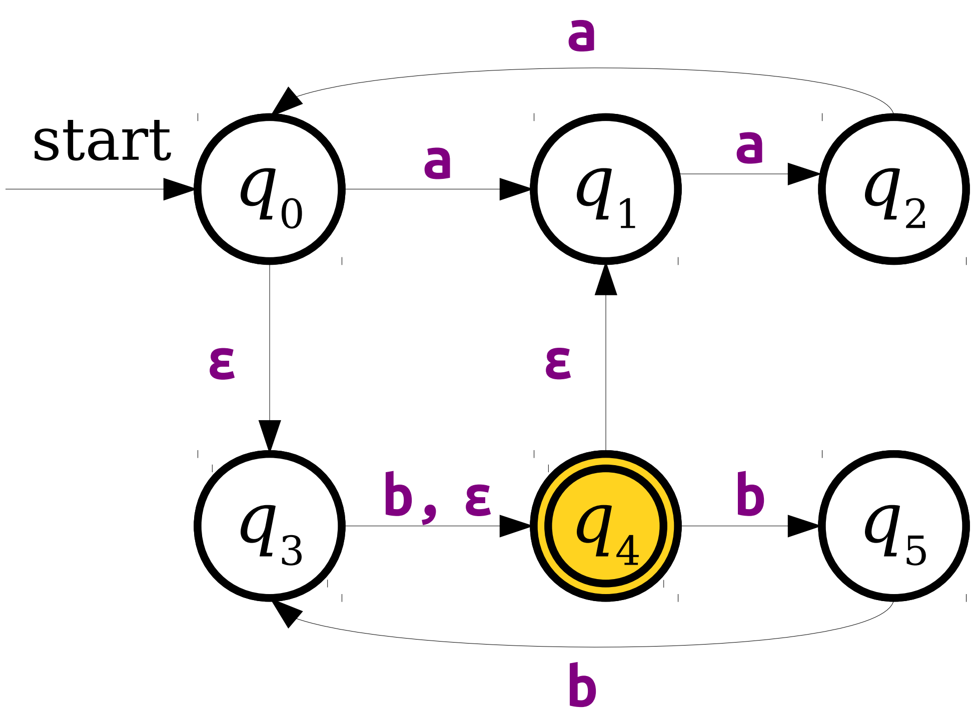

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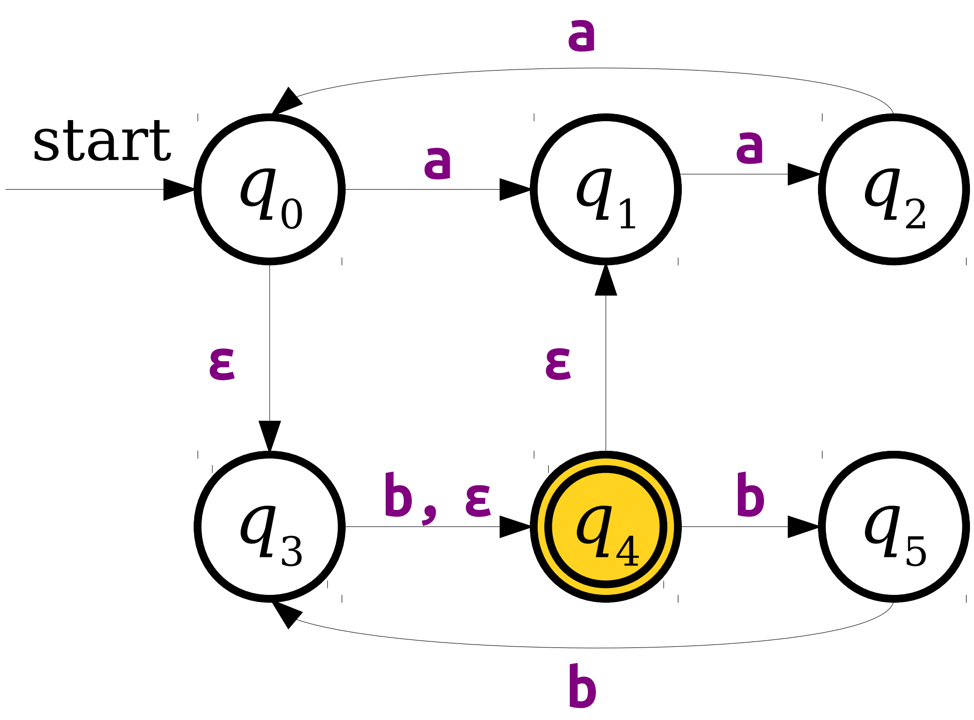

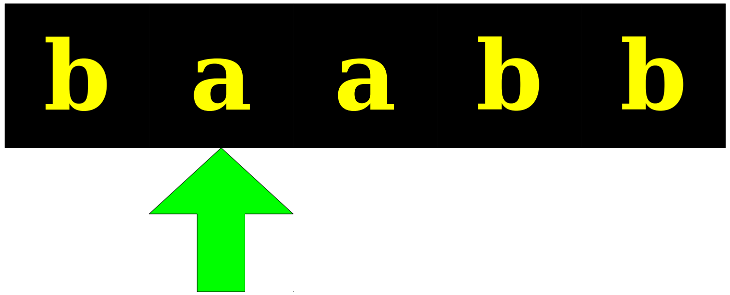

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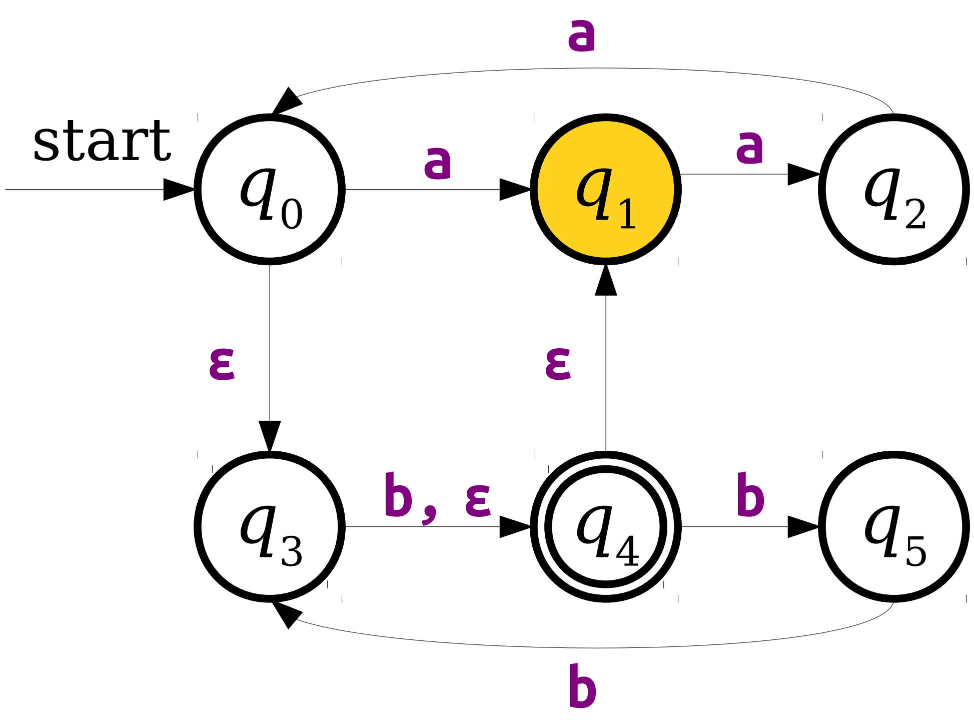

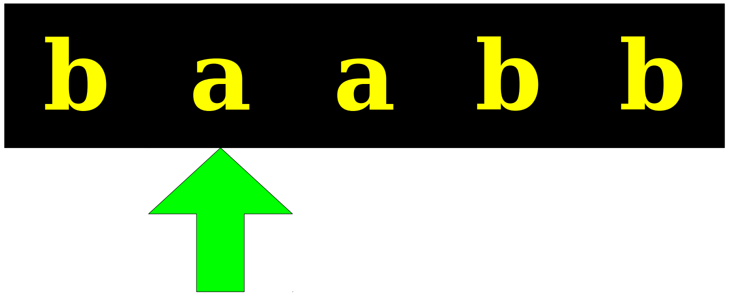

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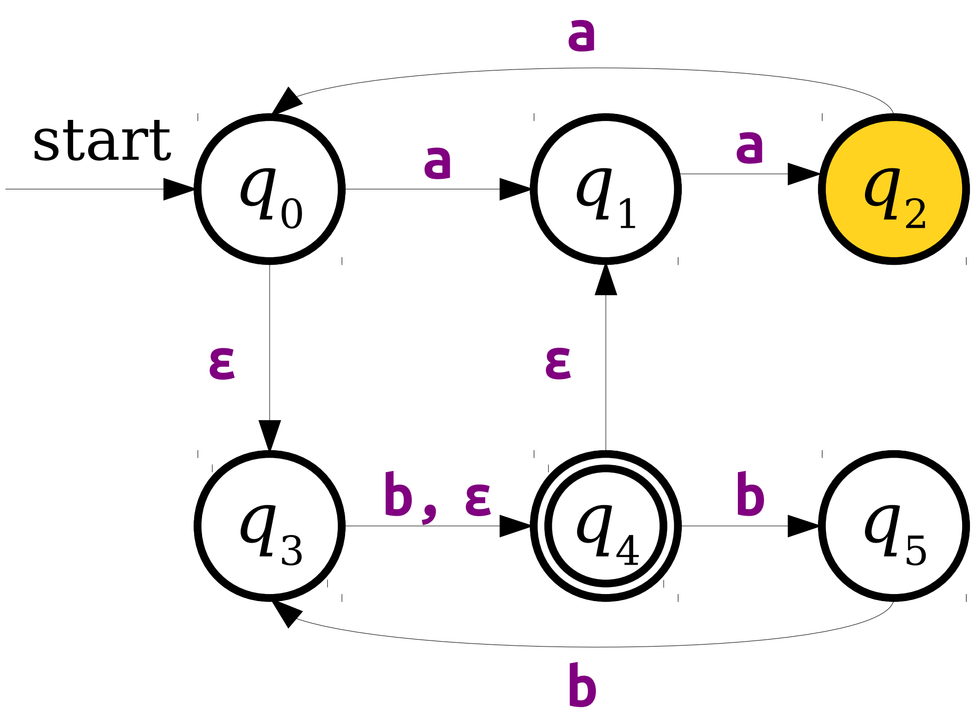

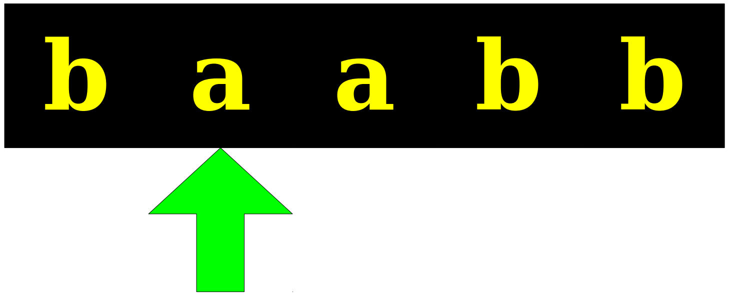

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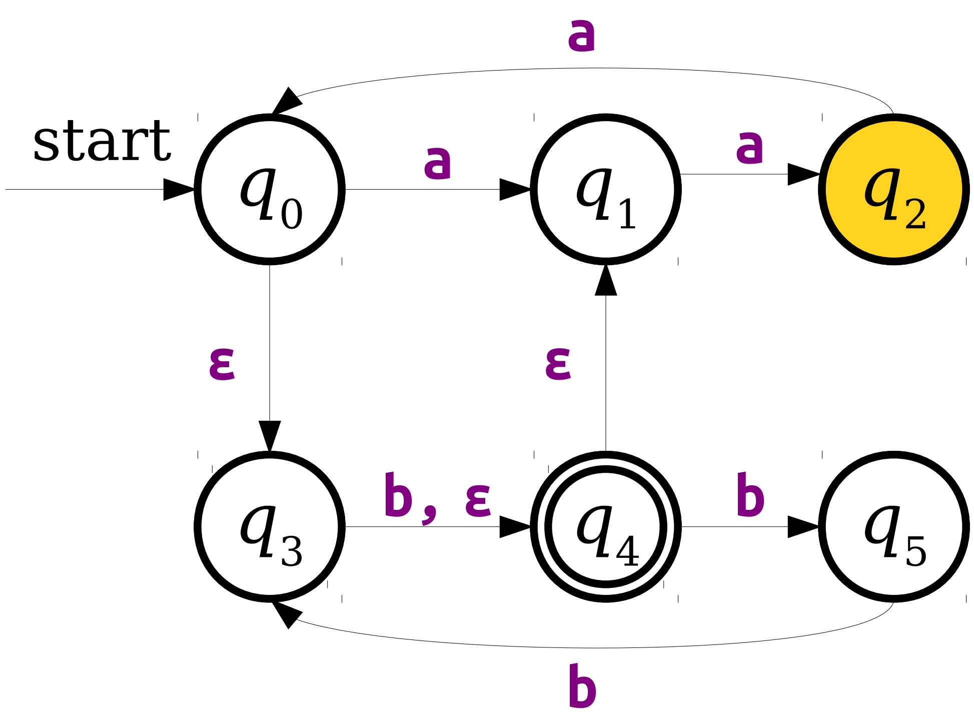

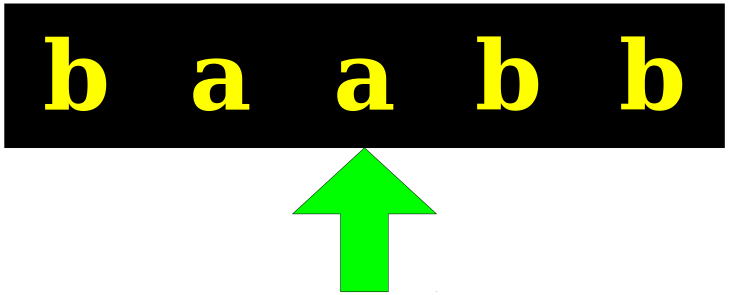

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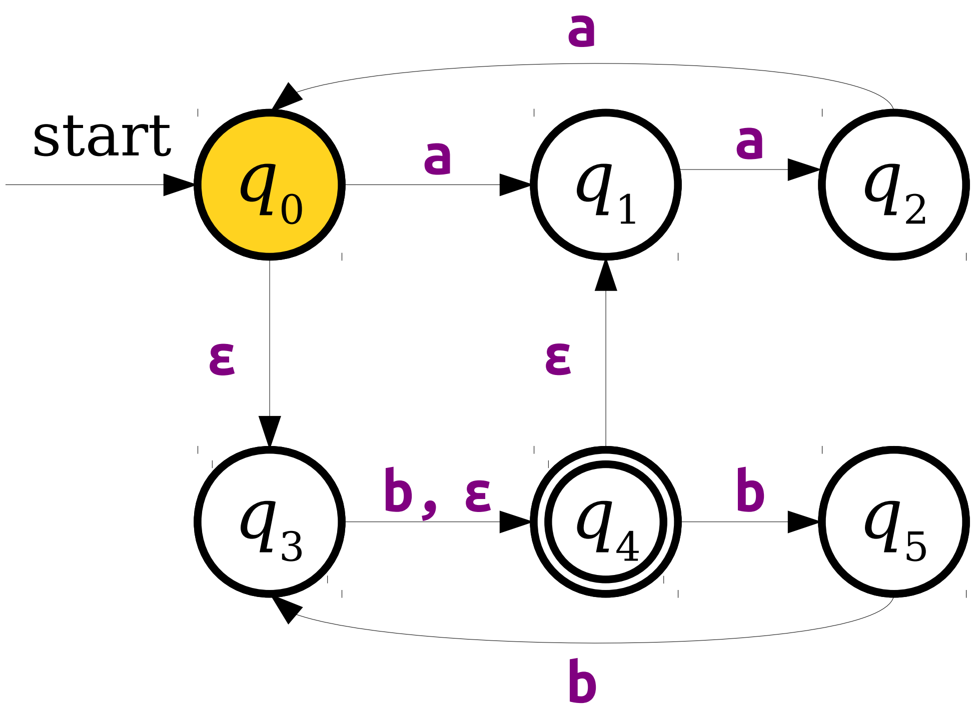

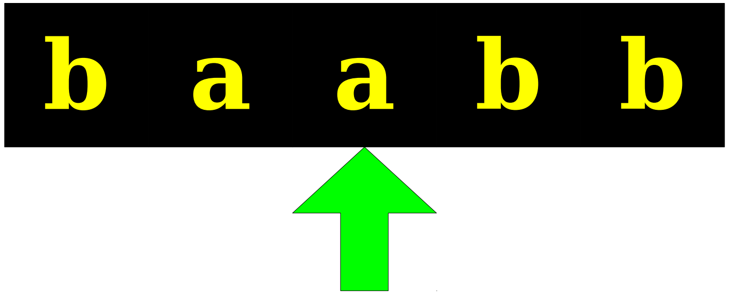

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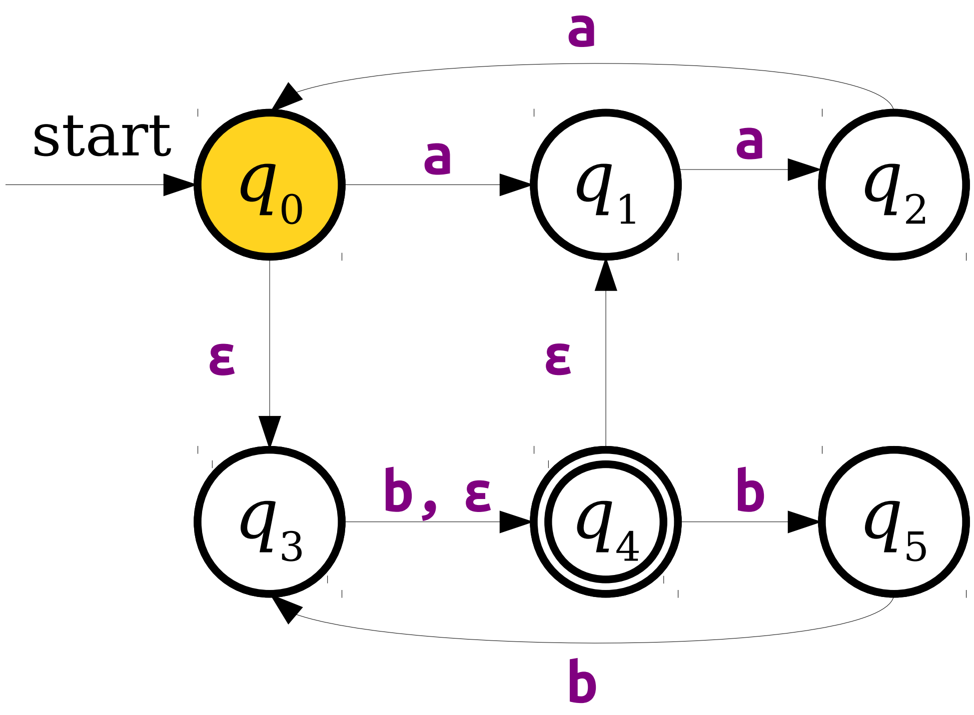

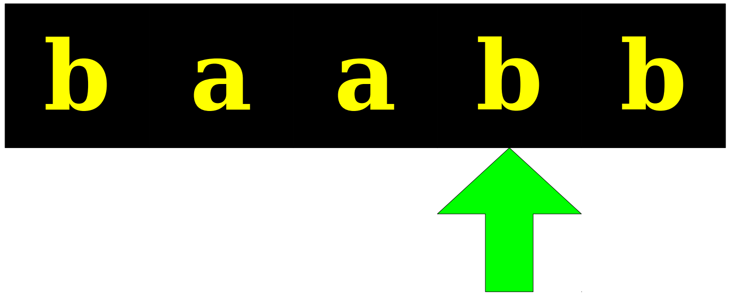

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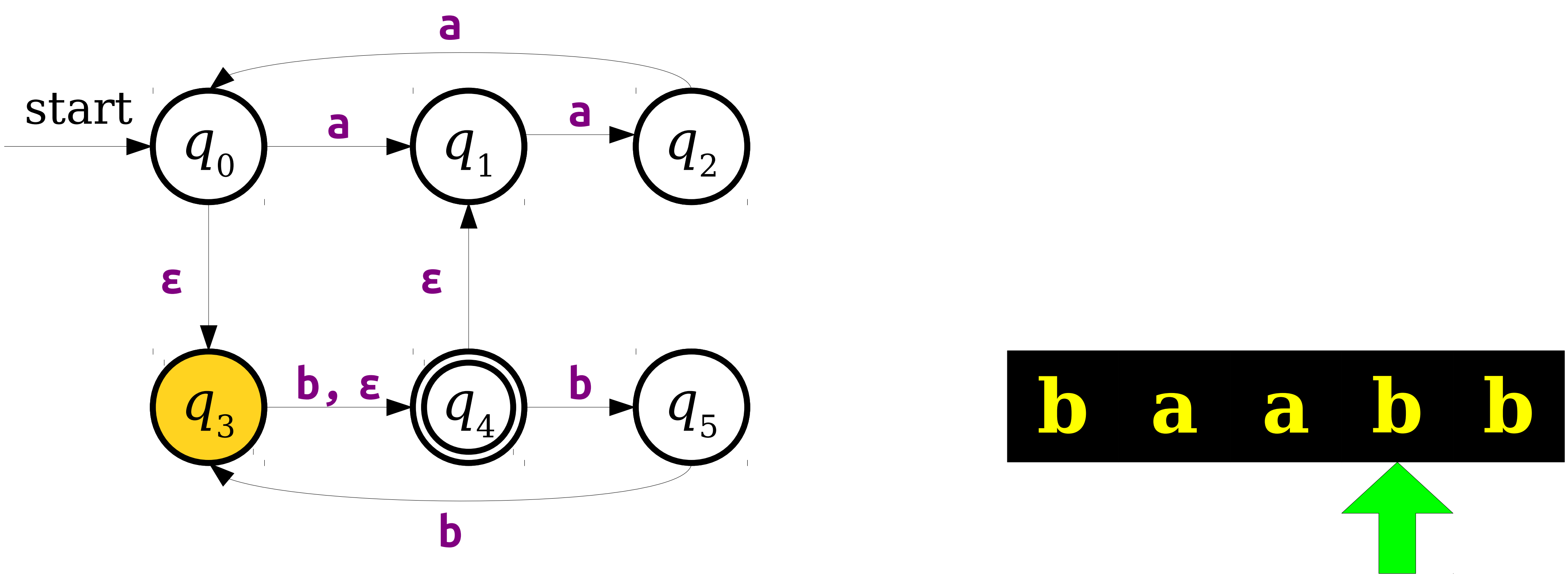

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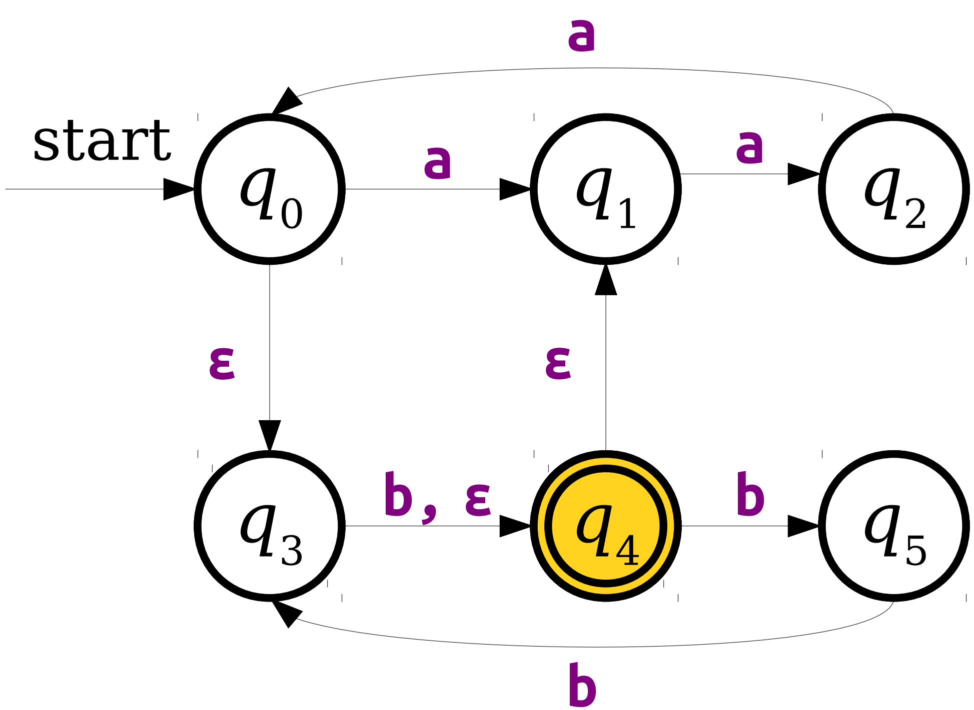

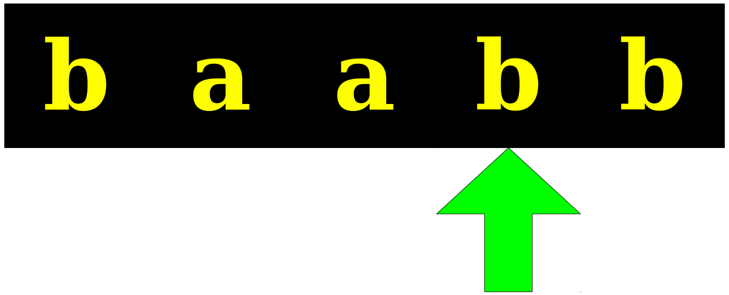

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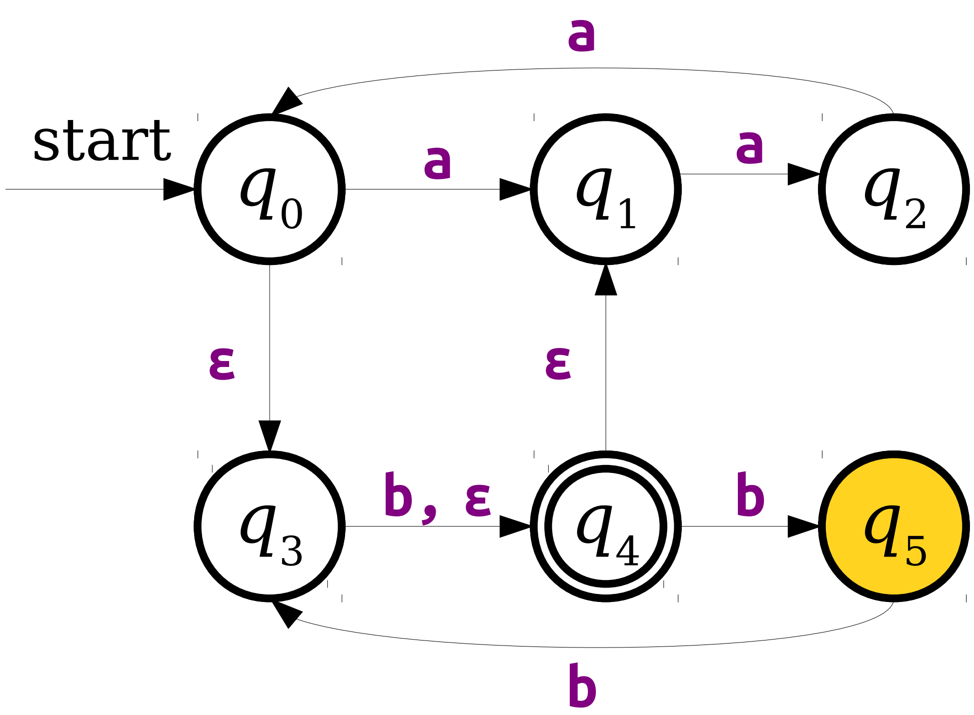

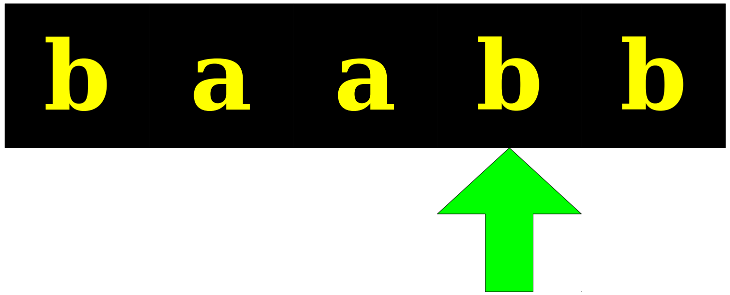

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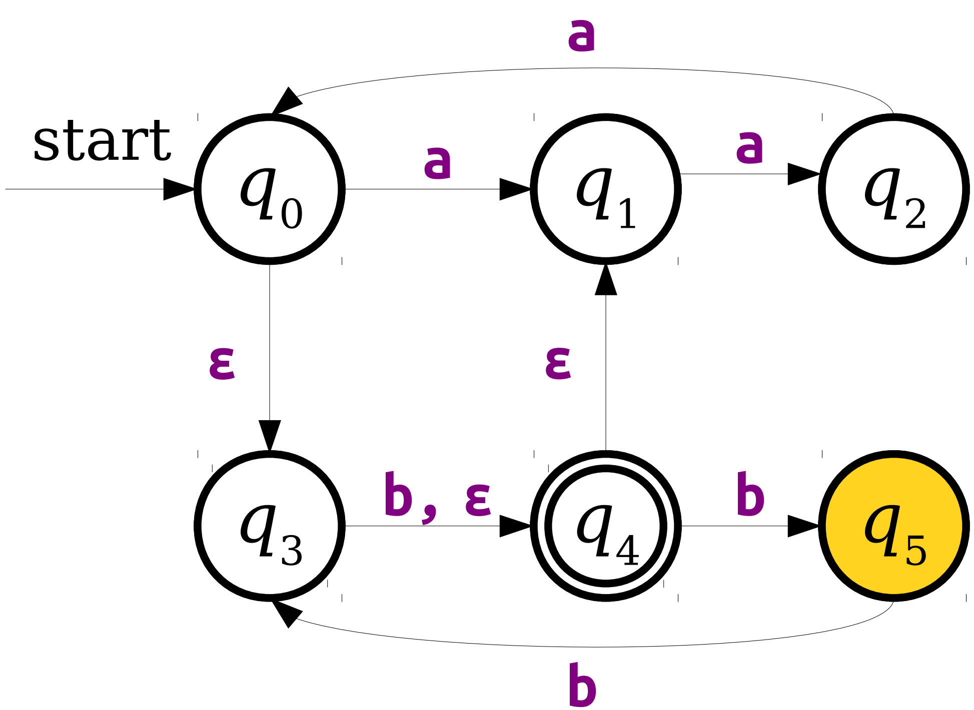

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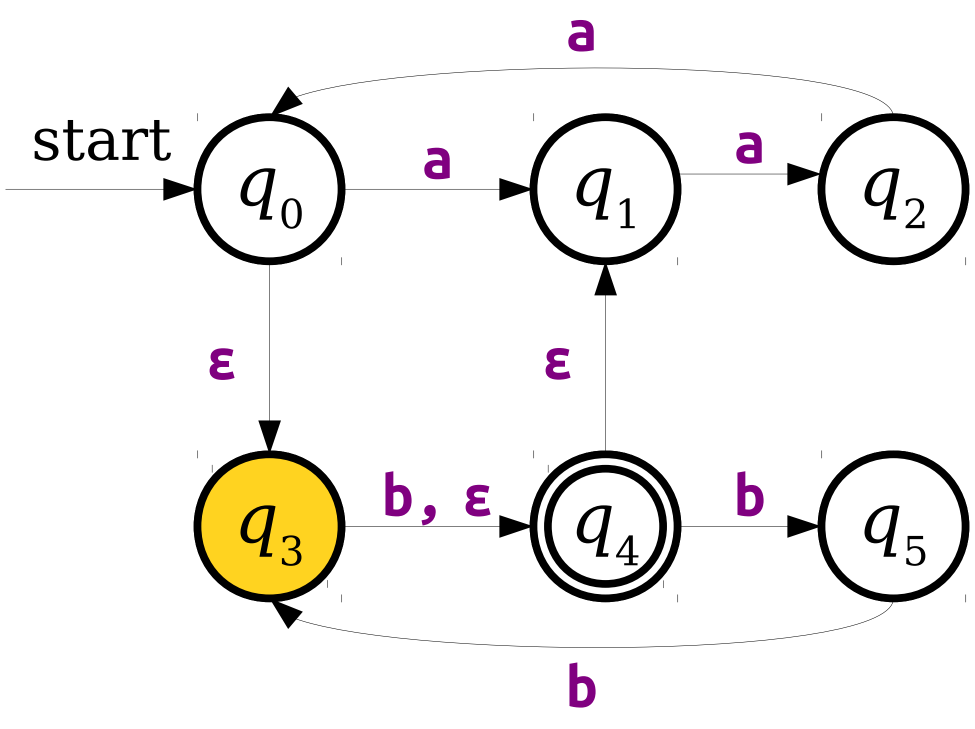

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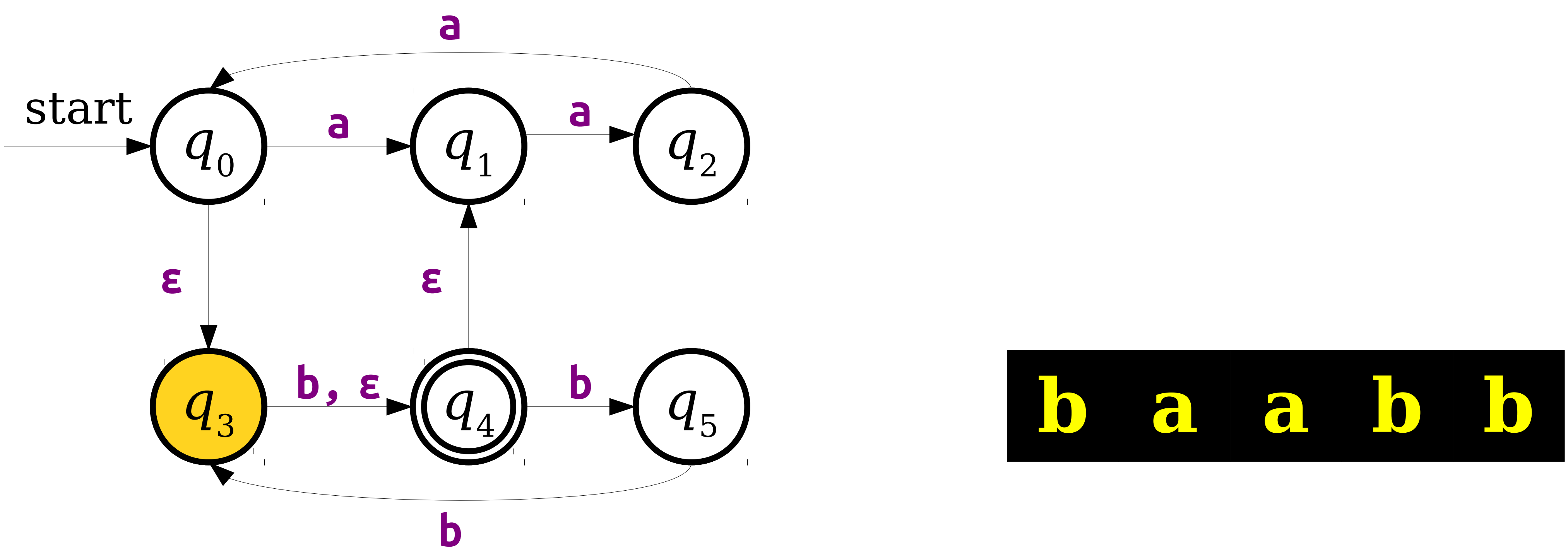

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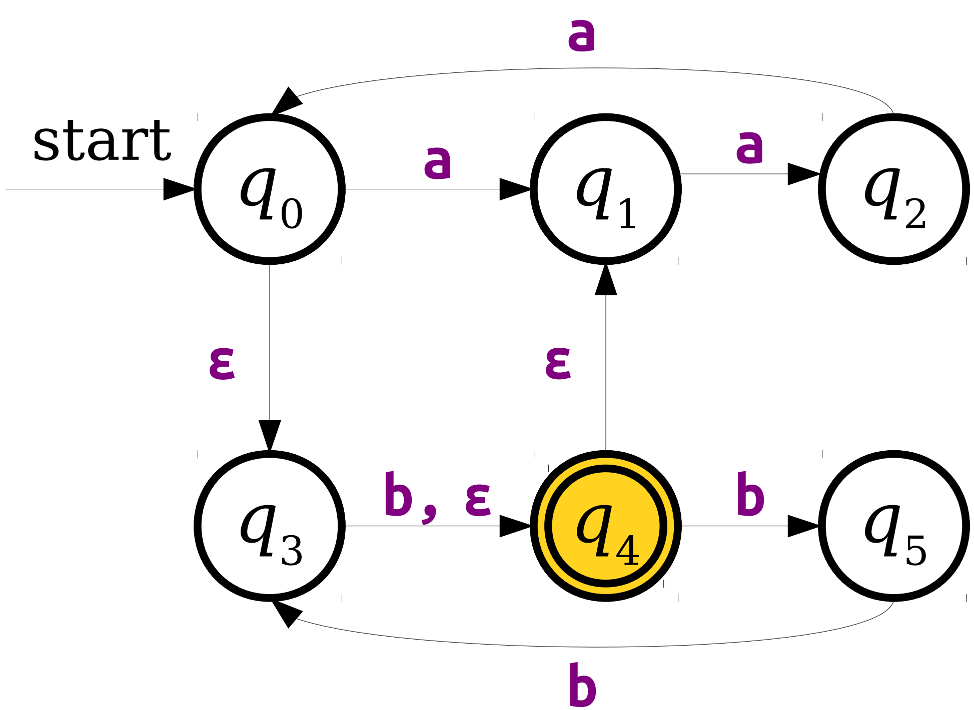
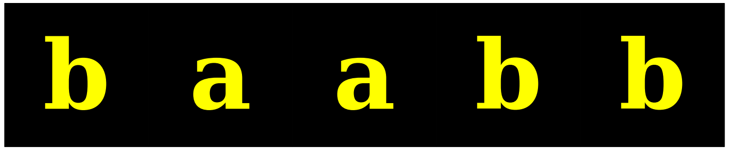




Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- The NFA accepts if *any* of the states that are active at the end are accepting states. It rejects otherwise.

Just how powerful *are* NFAs?

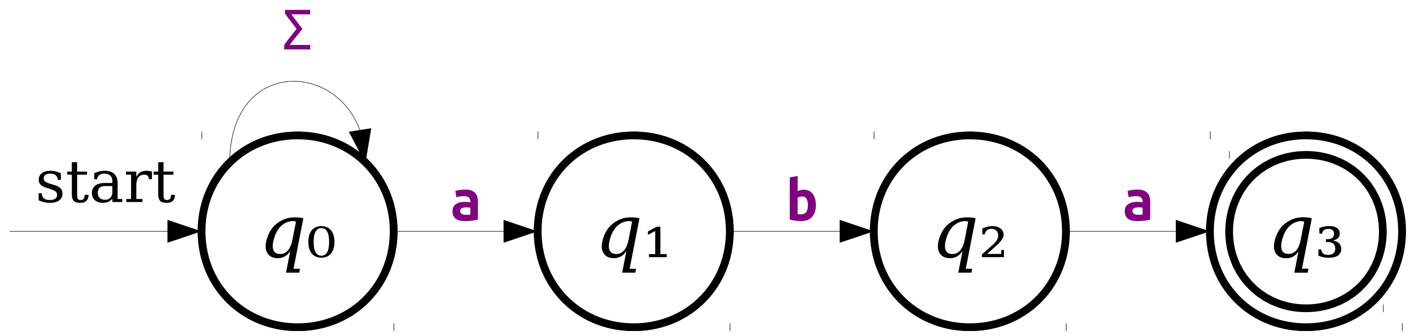
New Stuff!

NFAs and DFAs

- Any language that can be accepted by a DFA can be accepted by an NFA.
- Why?
  - Every DFA essentially already *is* an NFA!
- ***Question:*** Can any language accepted by an NFA also be accepted by a DFA?
- Surprisingly, the answer is ***yes***!

Thought Experiment:

How would you simulate an NFA in software?



Σ
start
q0
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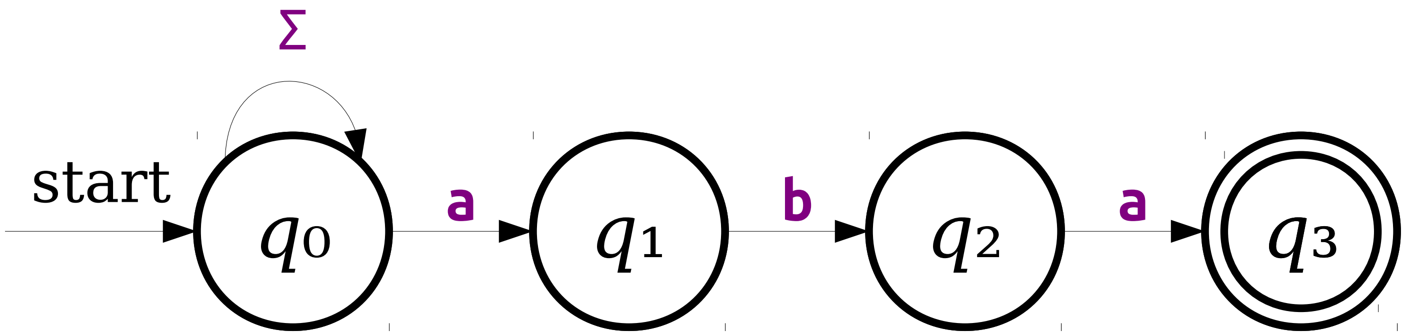
q3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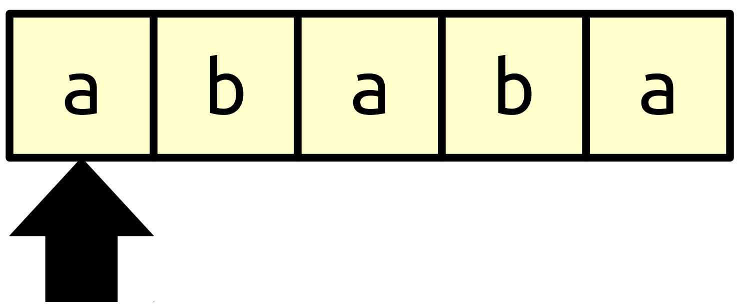
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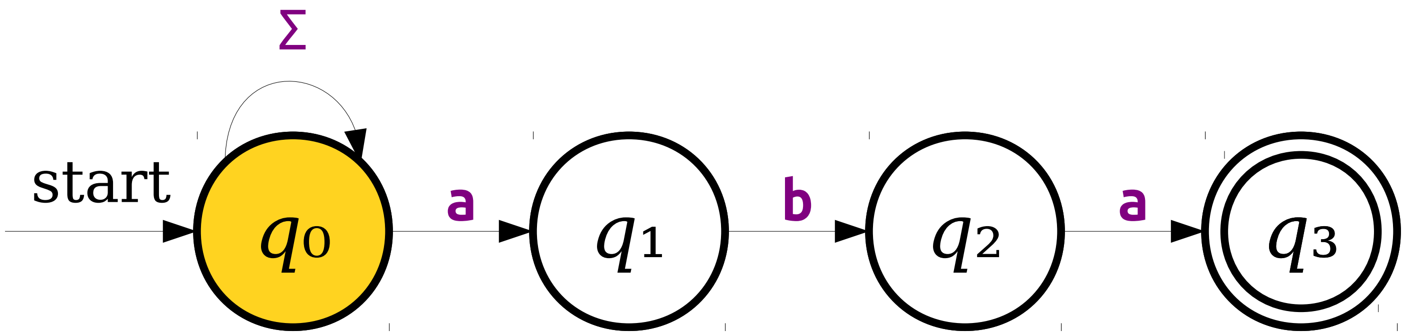
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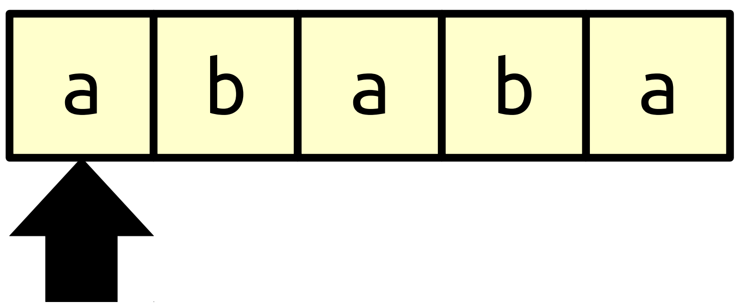
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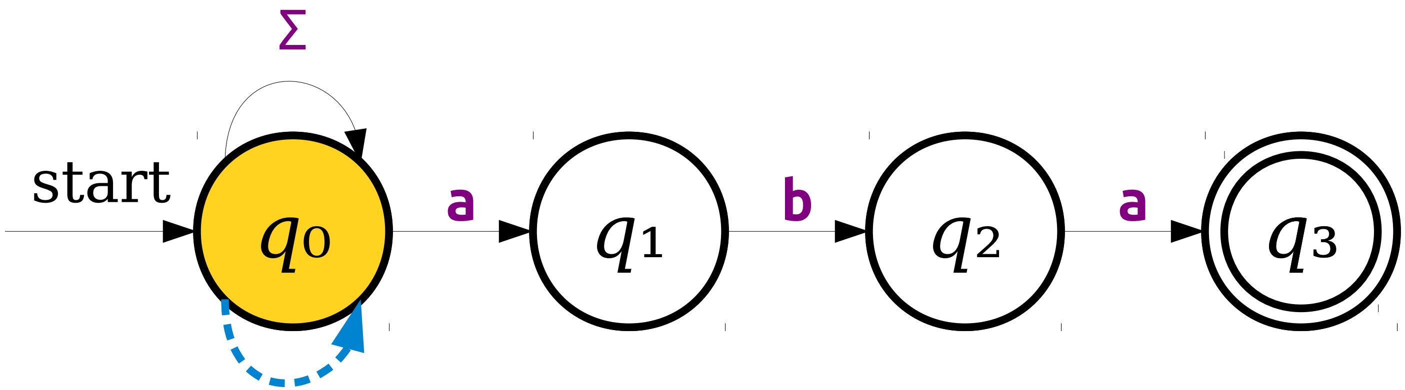
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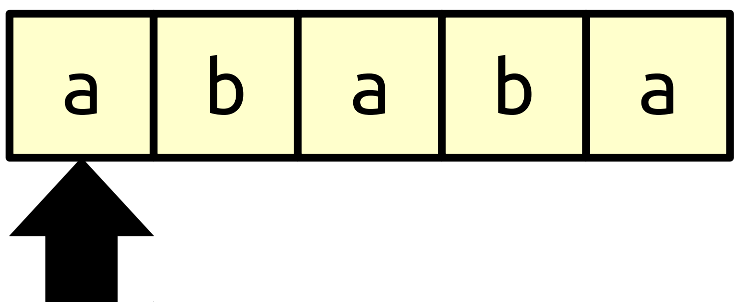
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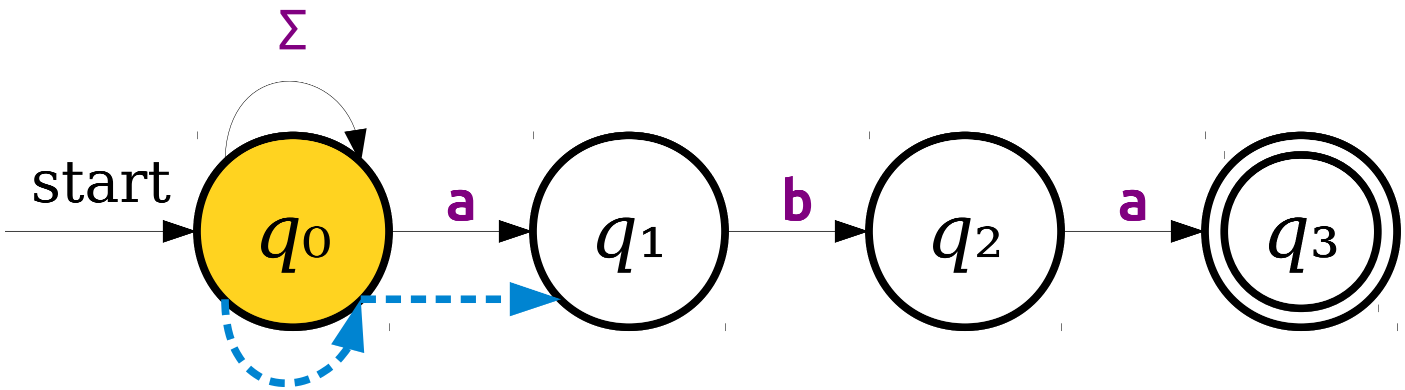
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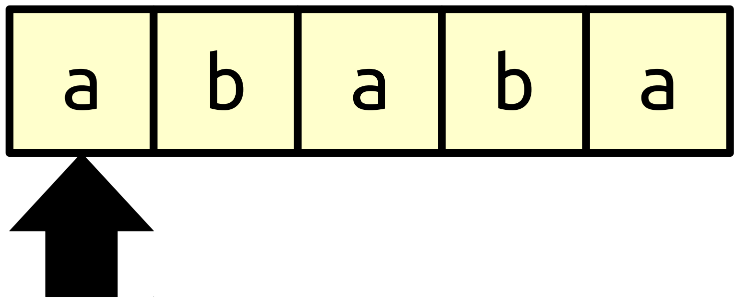
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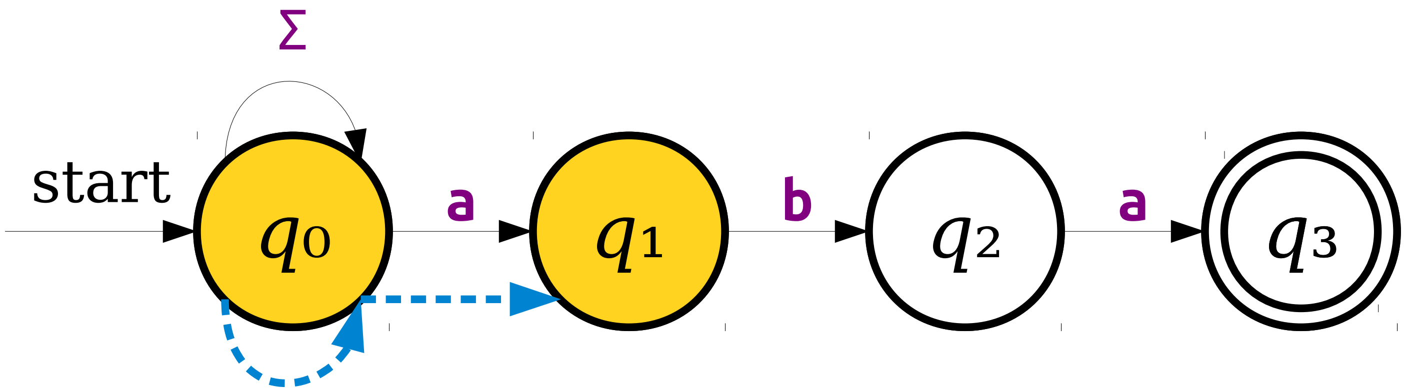
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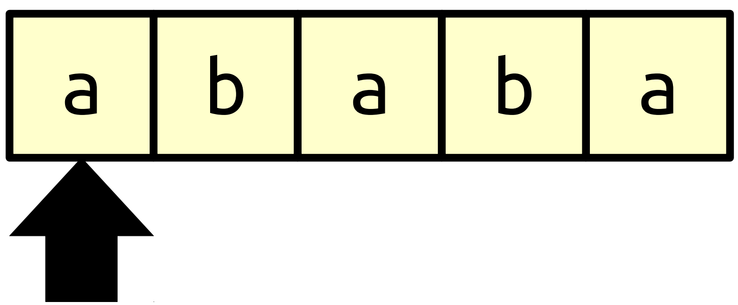
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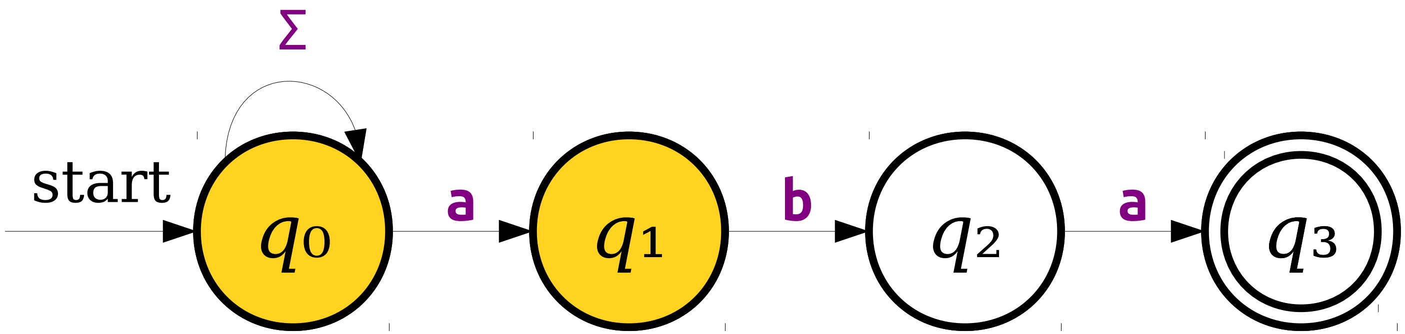
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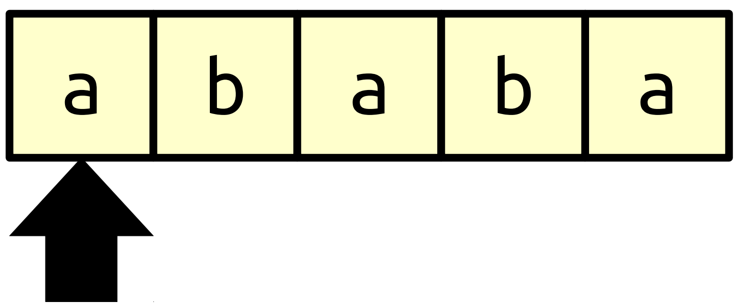
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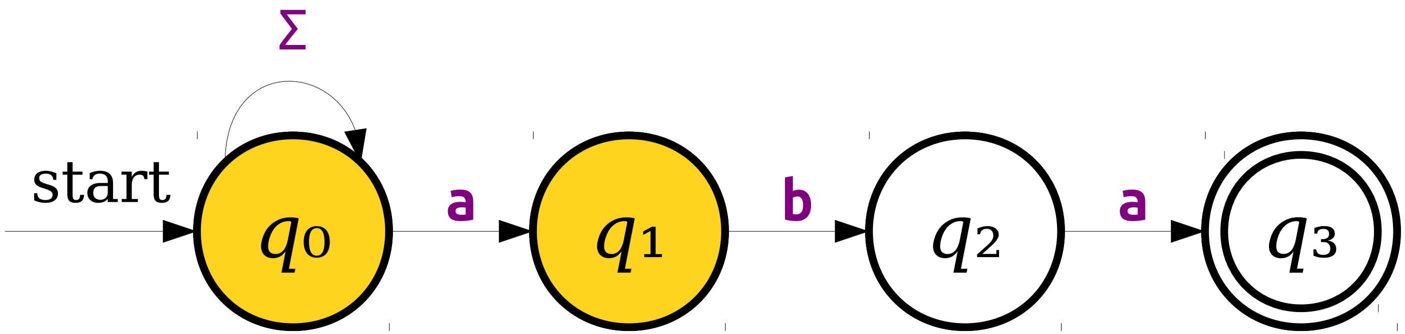
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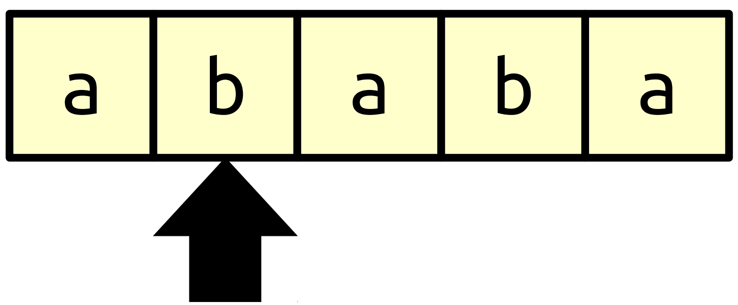
a
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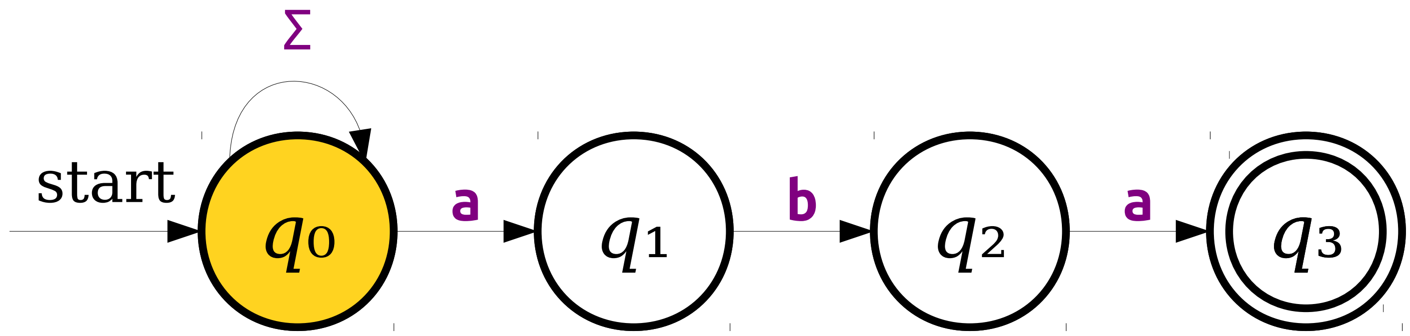
b
a



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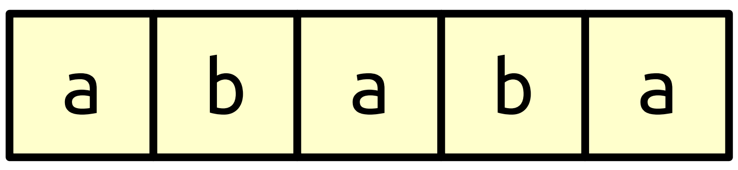

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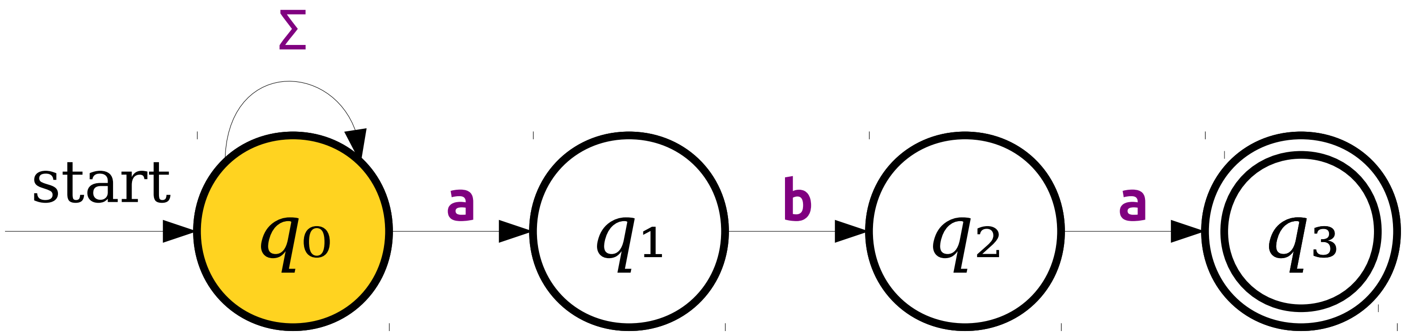
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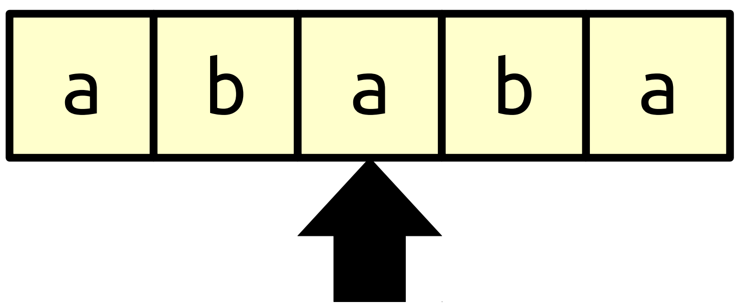
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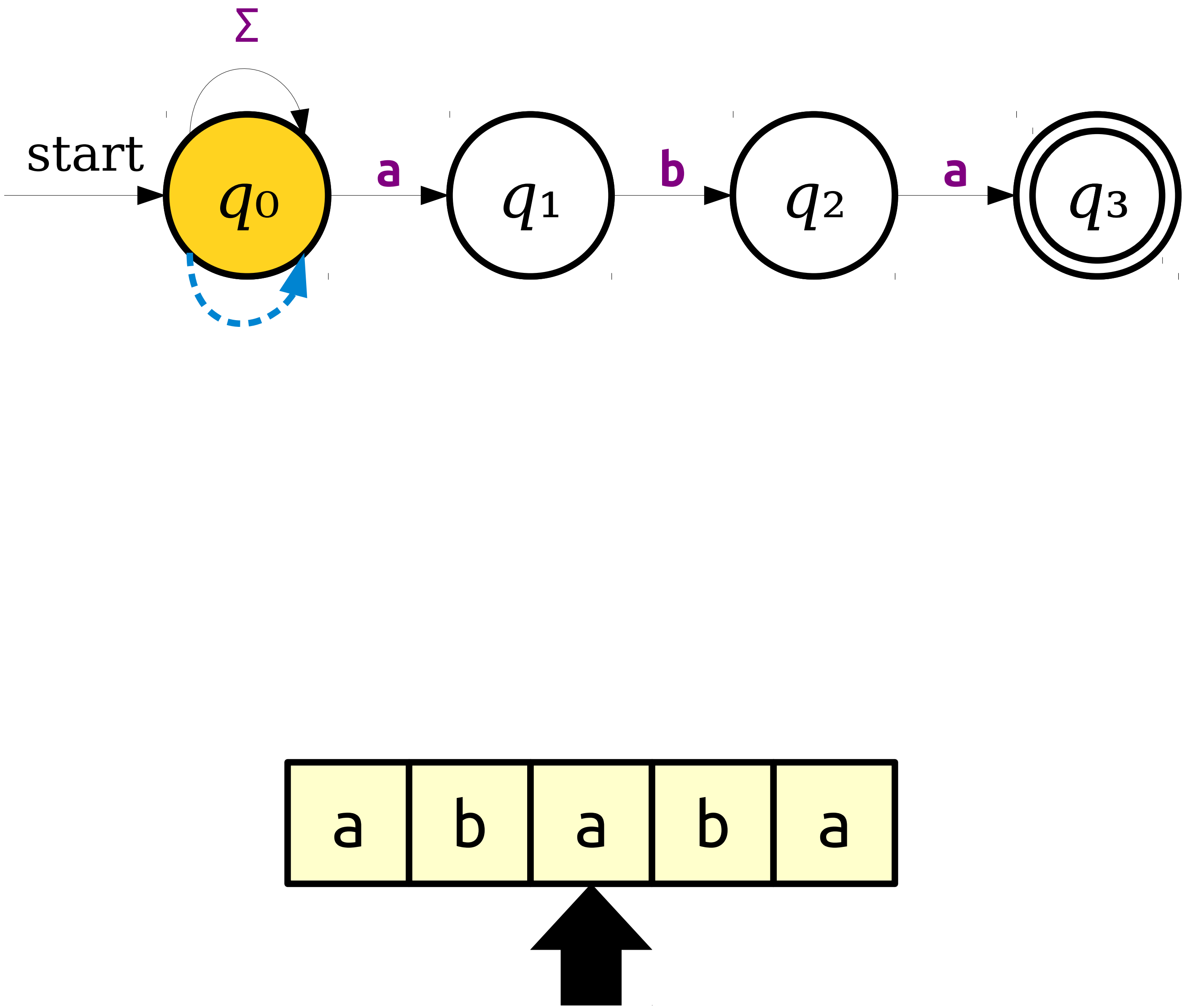

Σ
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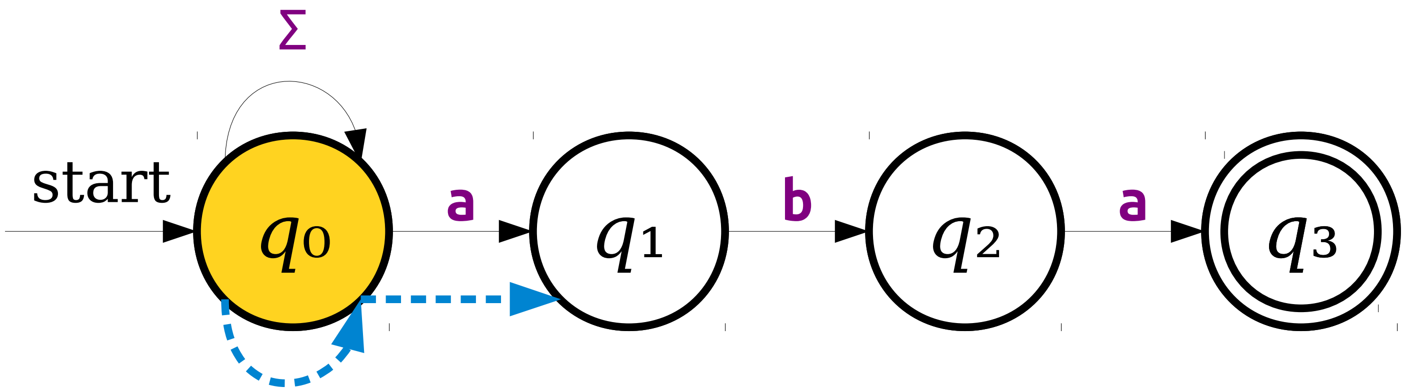
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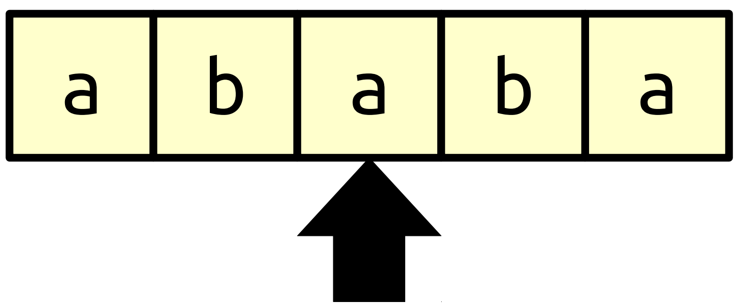
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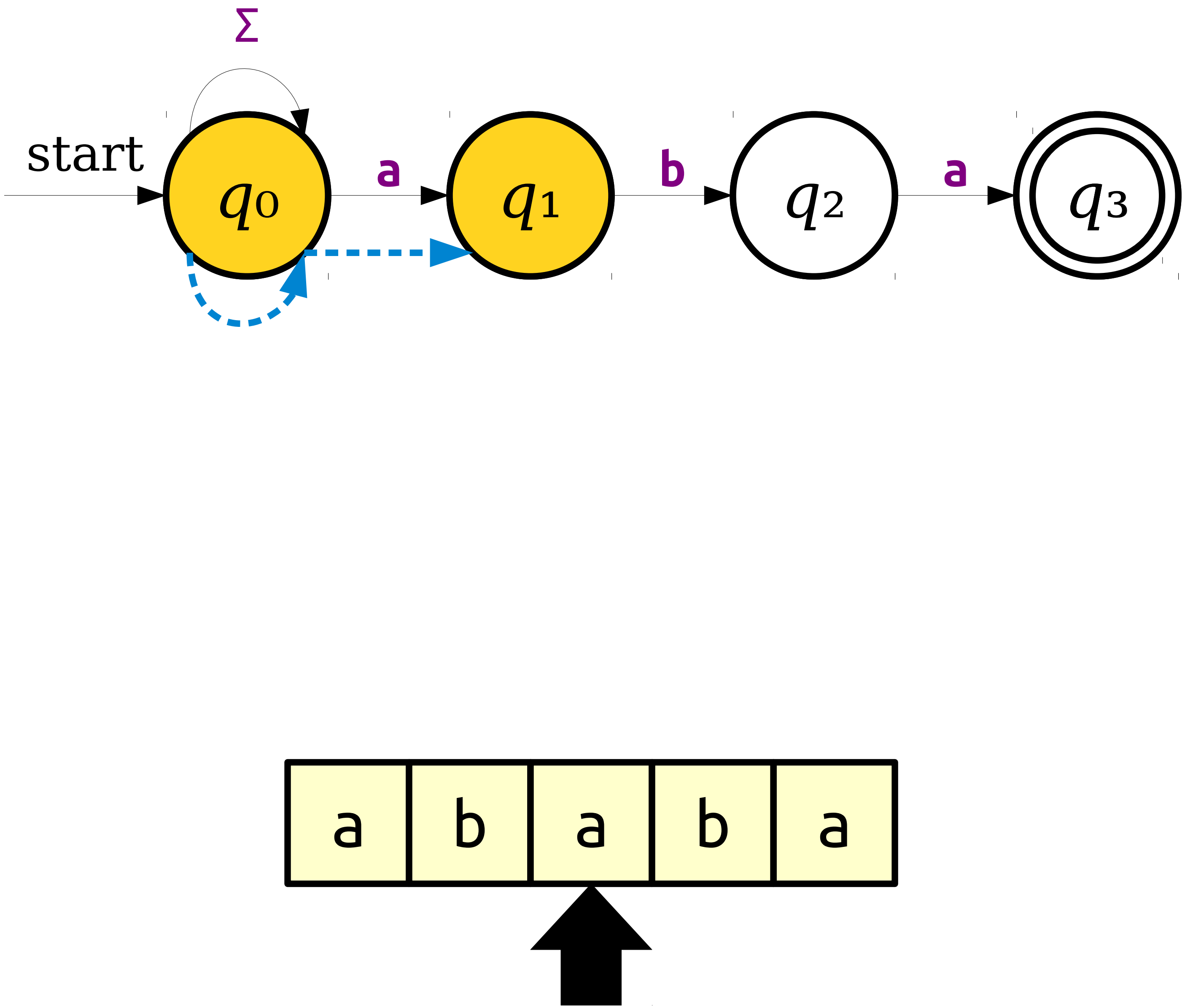

Σ
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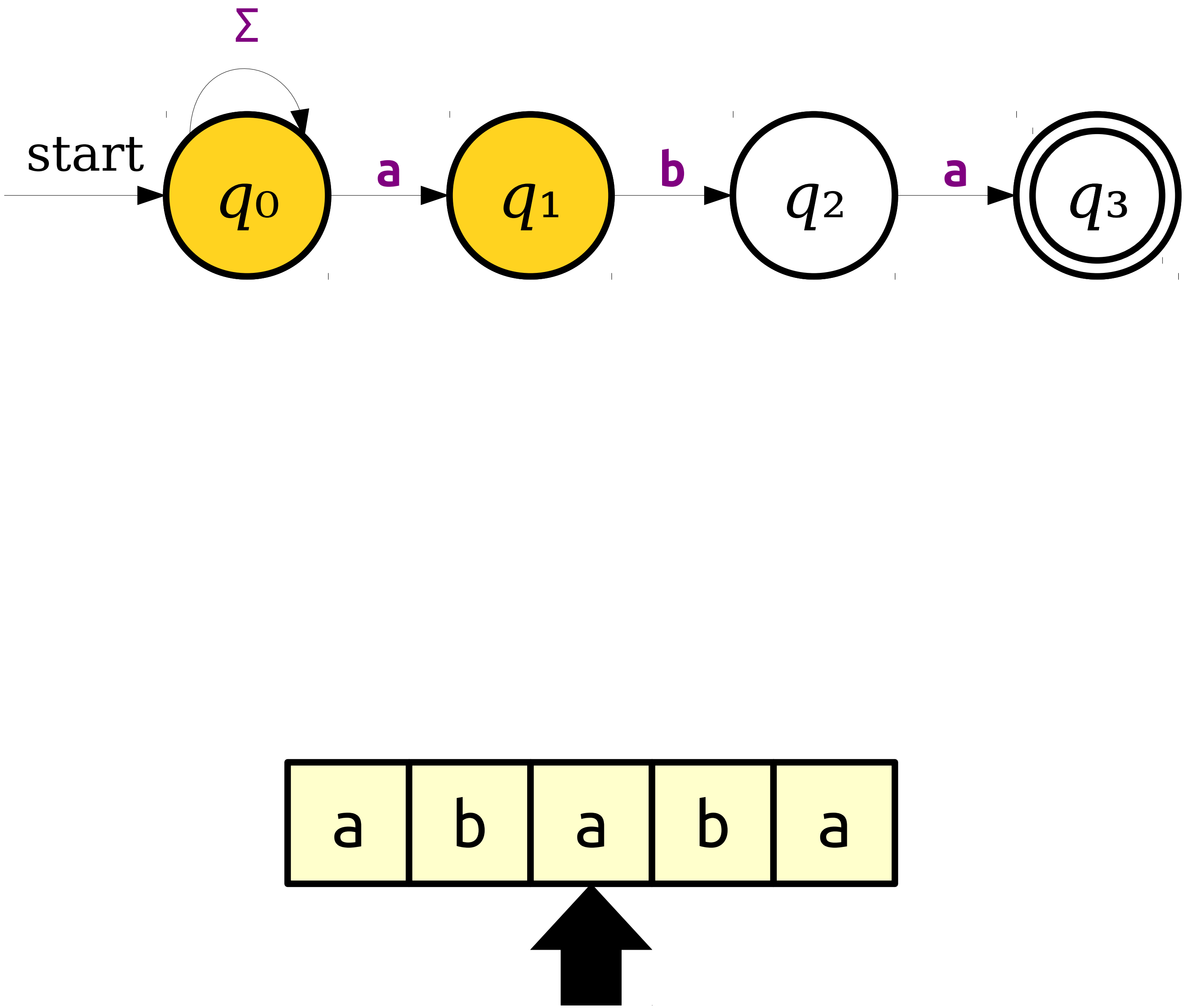
Σ
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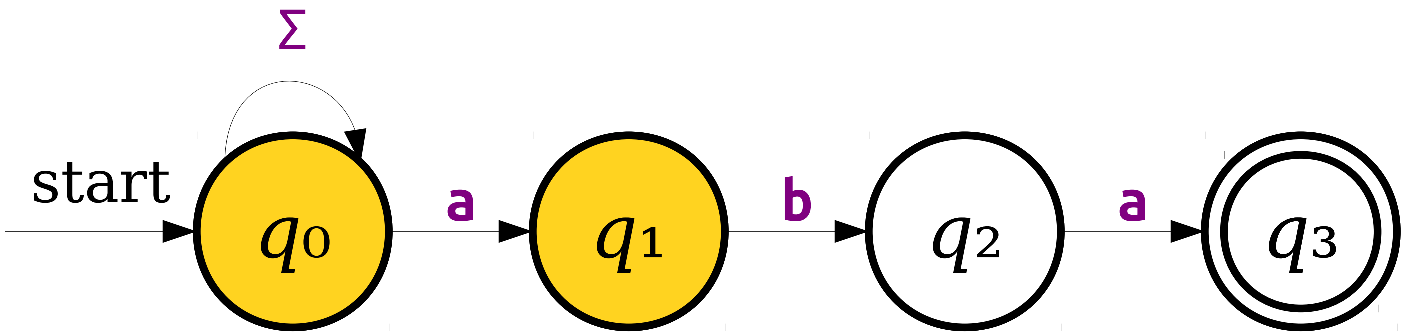
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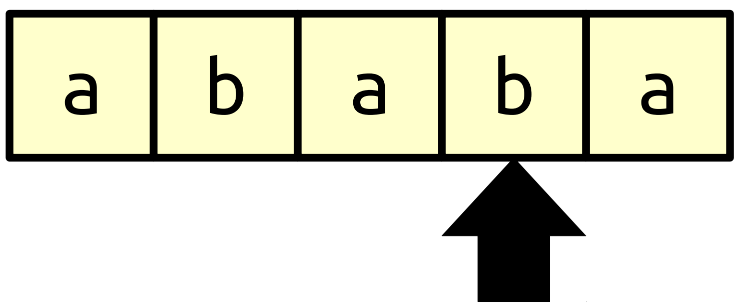
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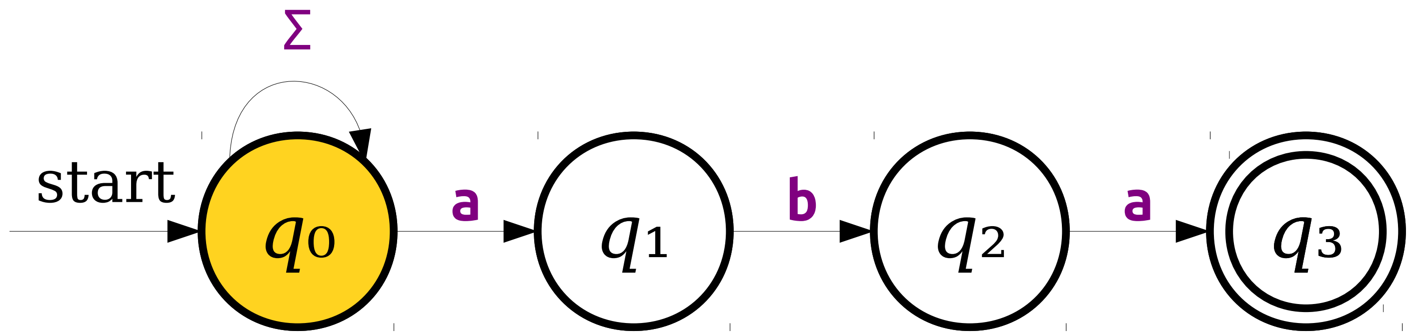
Σ
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q_1
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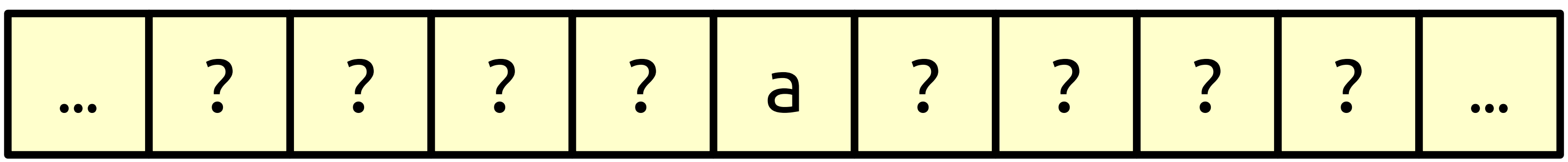
q_2
a
q_3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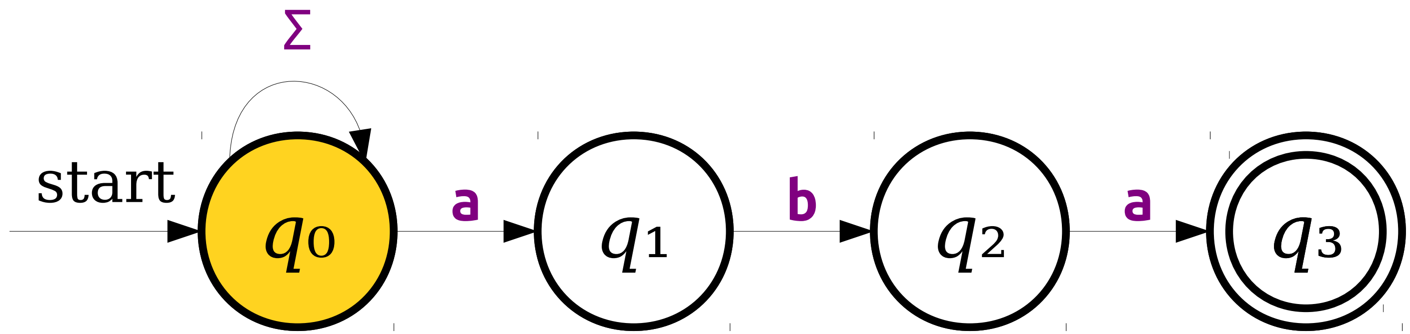
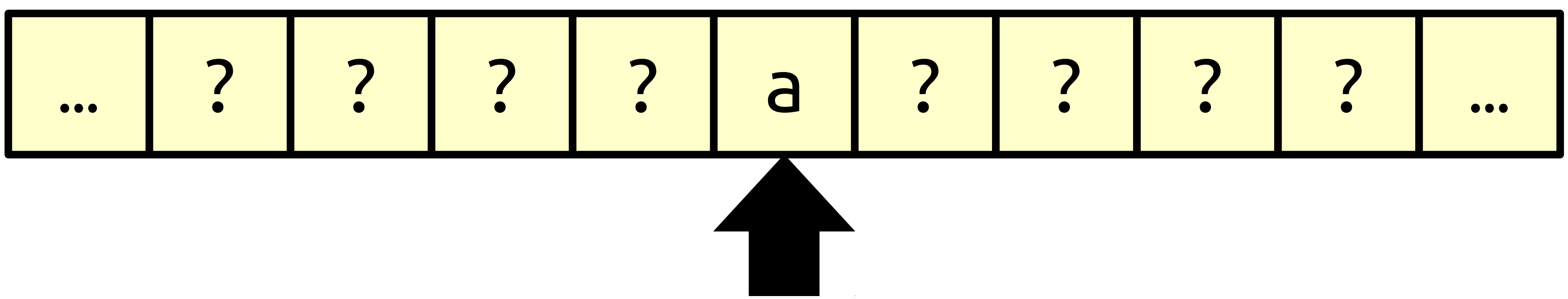

Σ
start
q_0
a
q_1
b
q_2
a
q_3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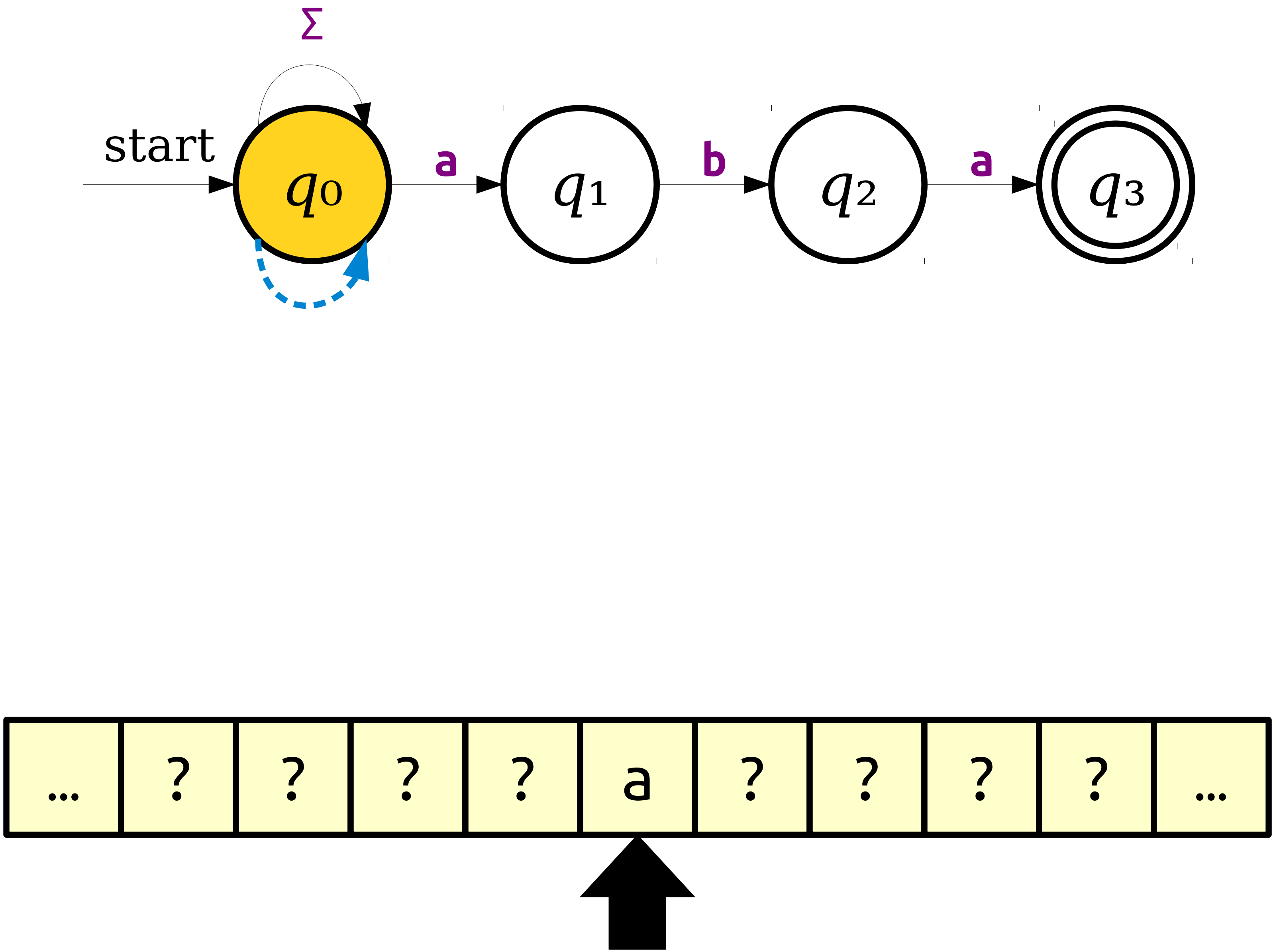
Σ
start
q_0
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q_1
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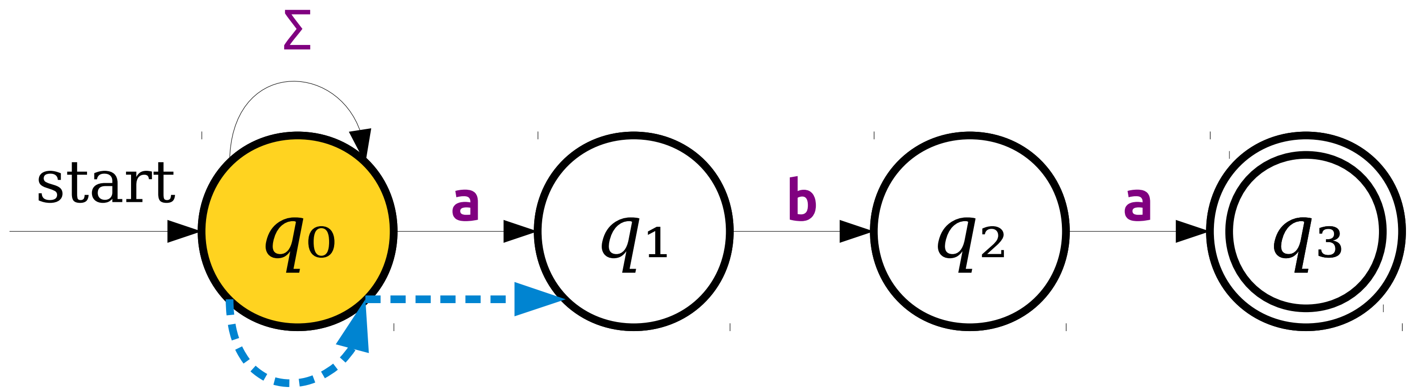
Σ
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q0
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q2
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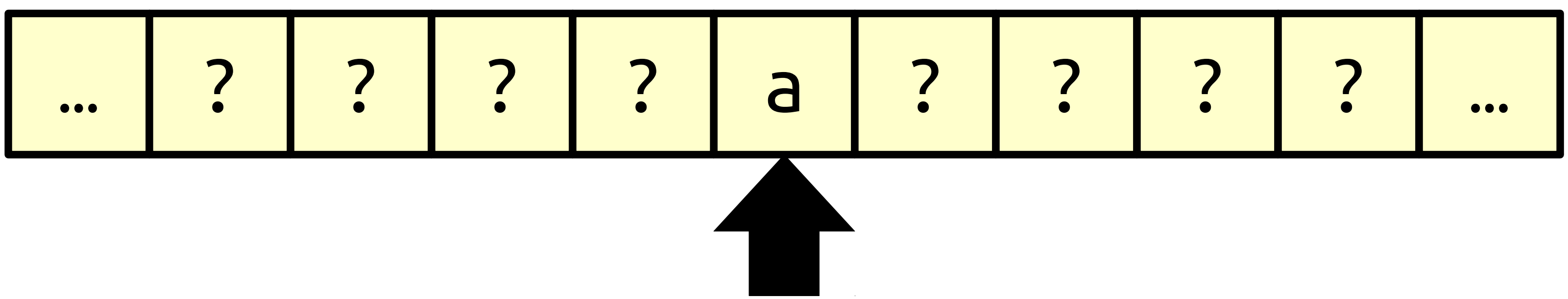
q3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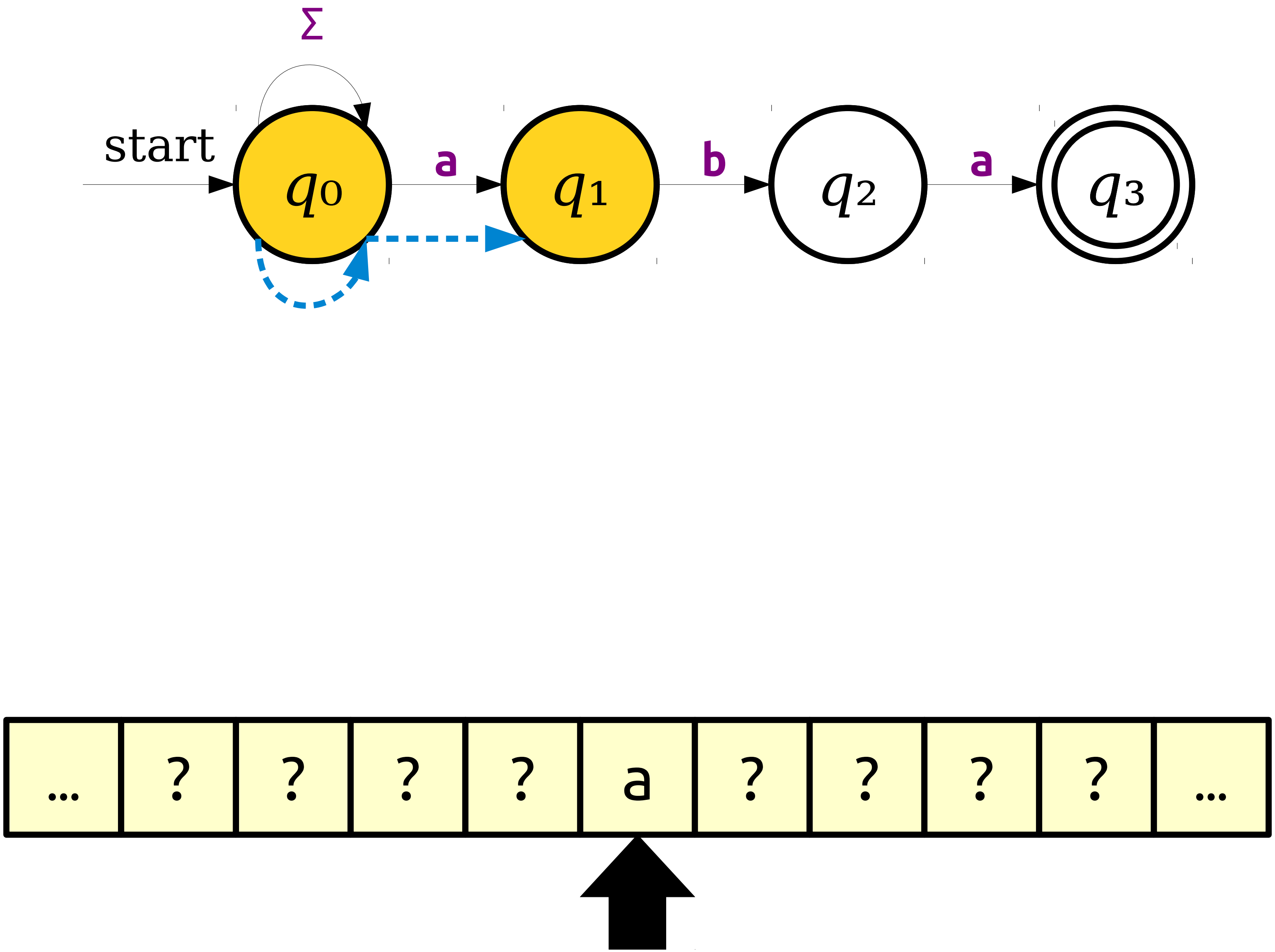

Σ
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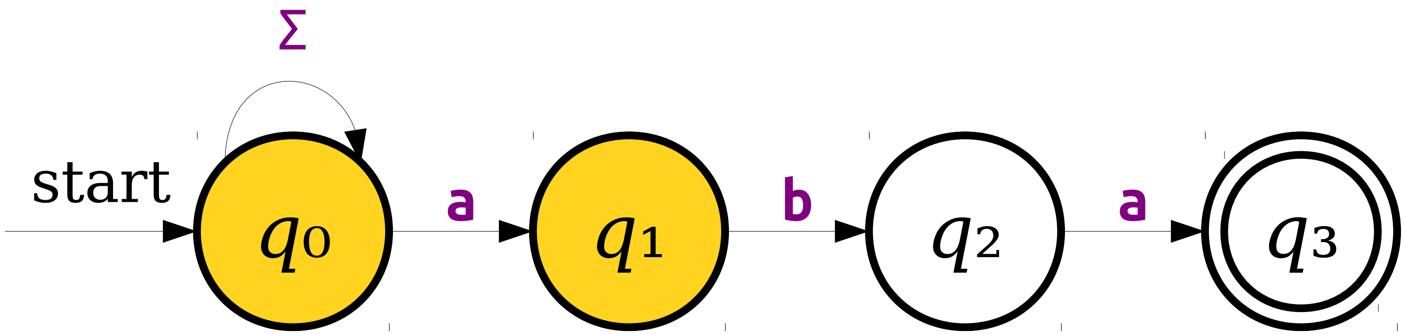
Σ
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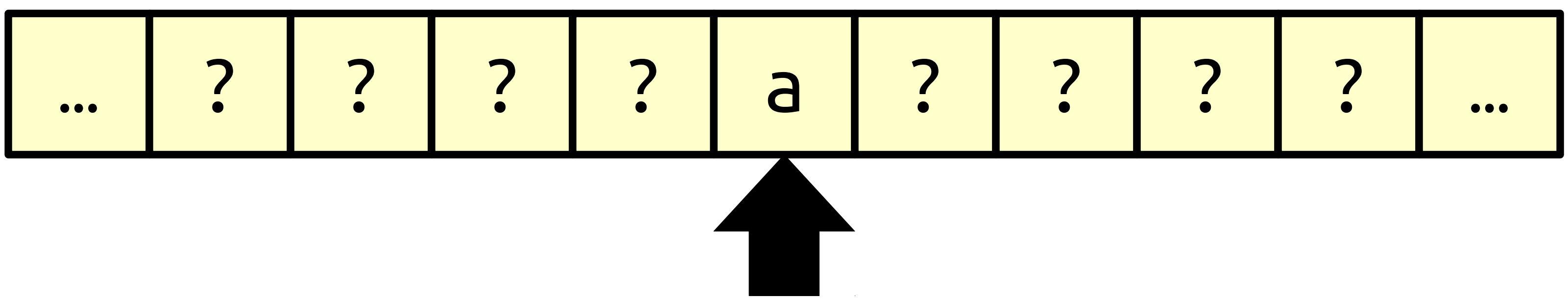
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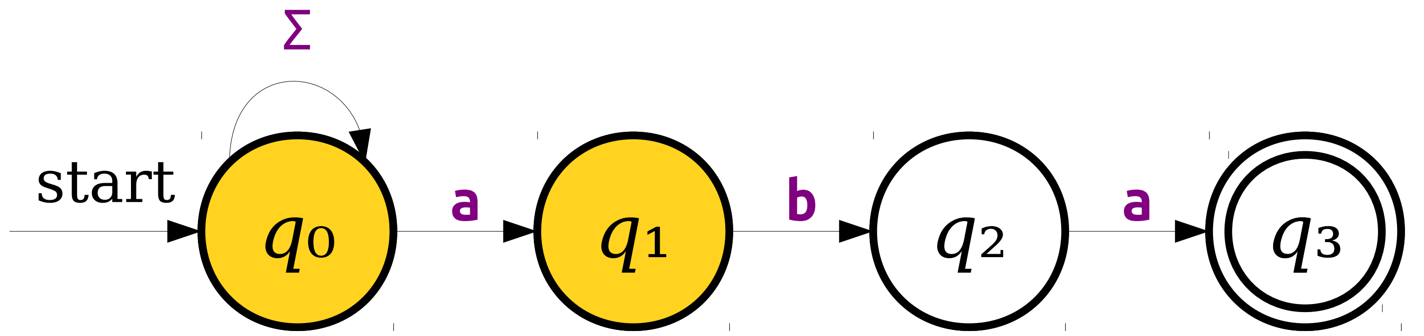
Σ
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q1
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q2
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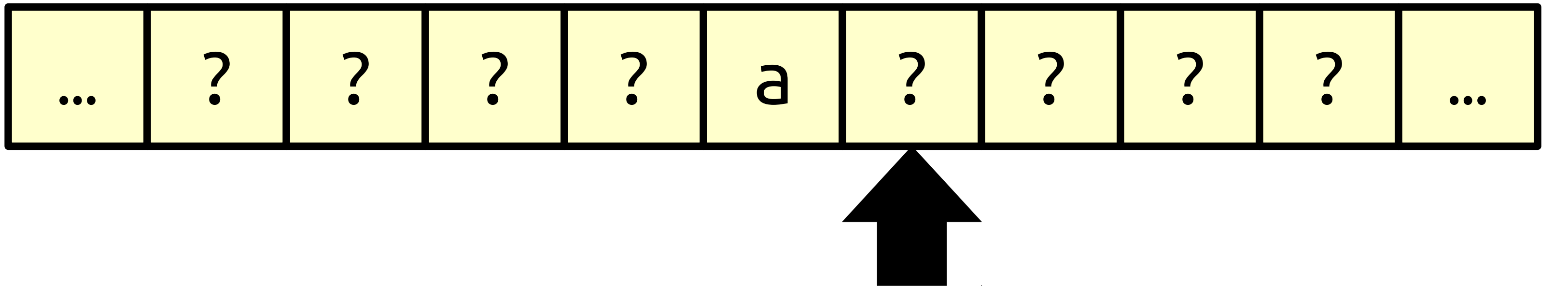
q3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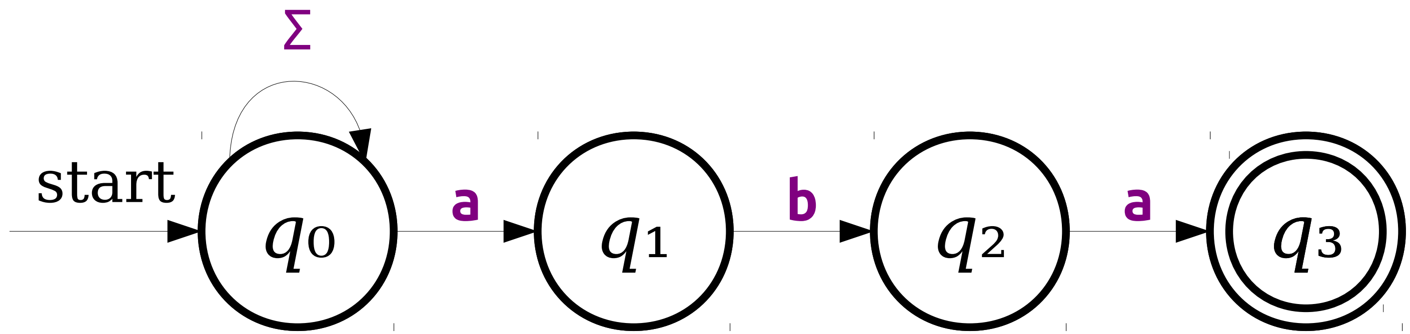
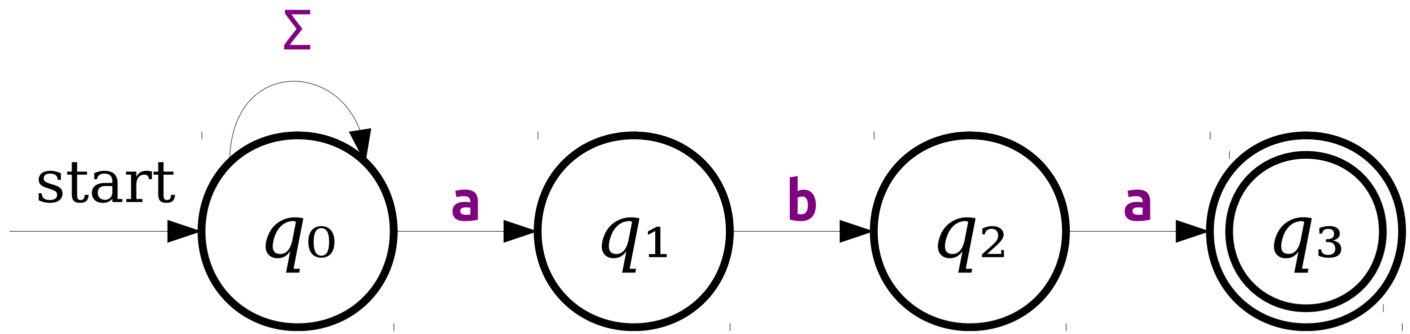
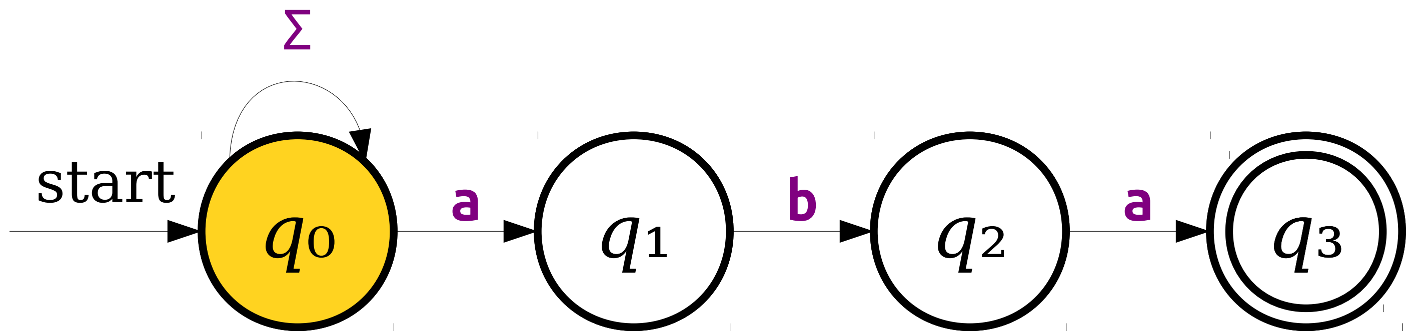


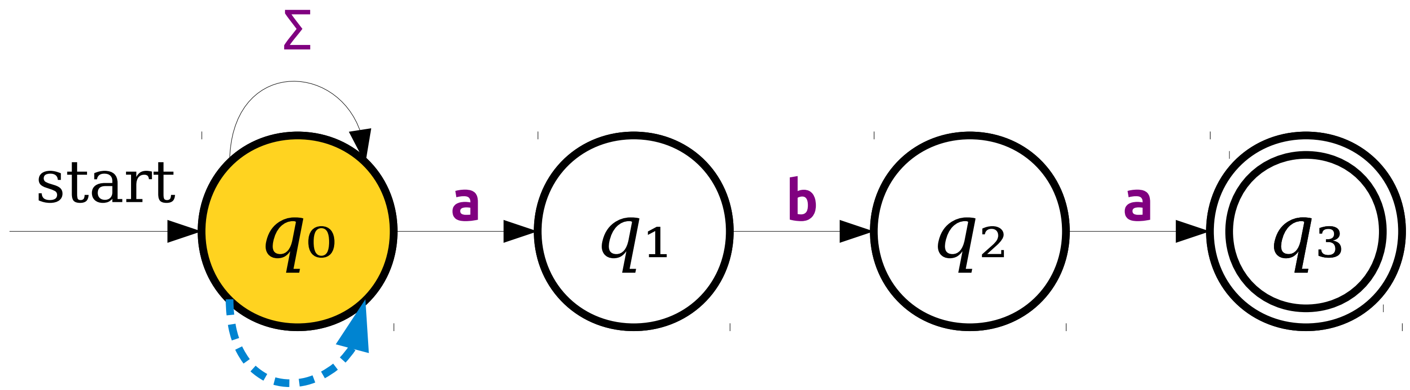
	a
$\{q_0\}$	$\{q_0, q_1\}$



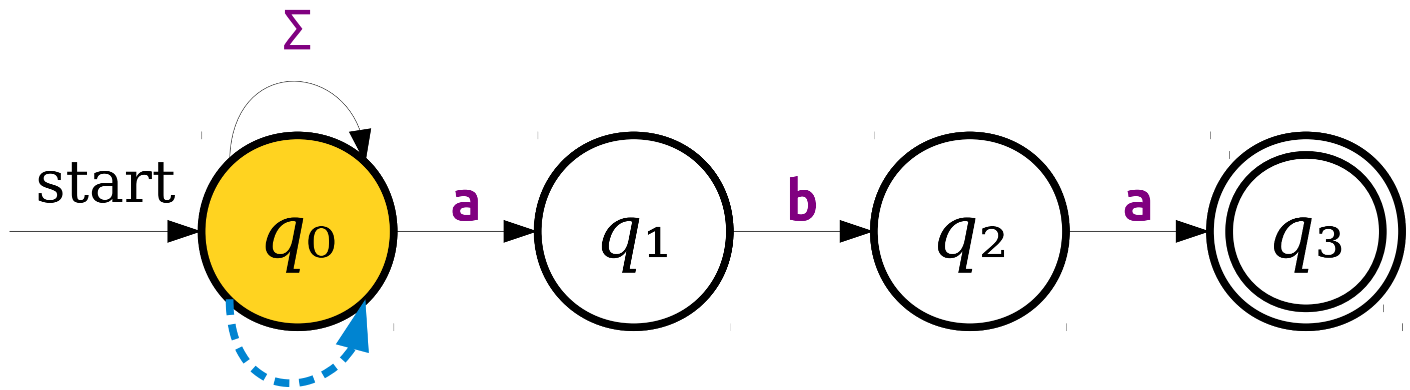
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



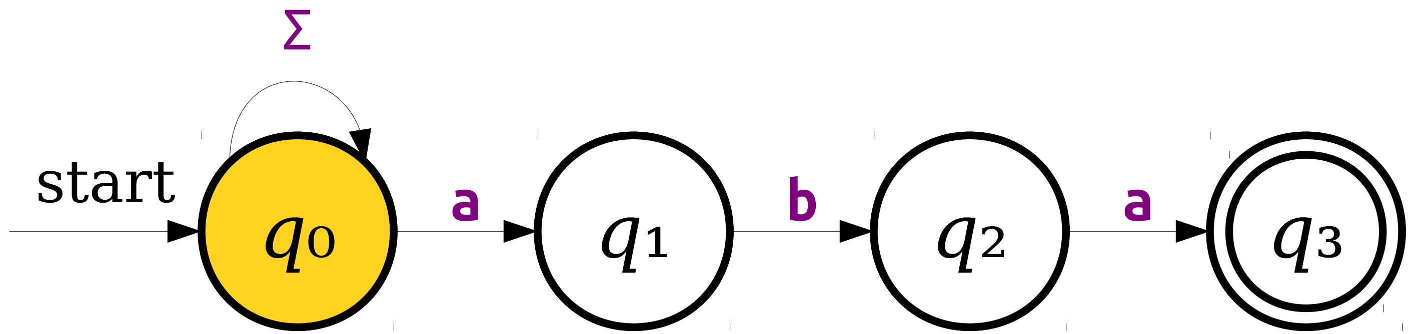
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



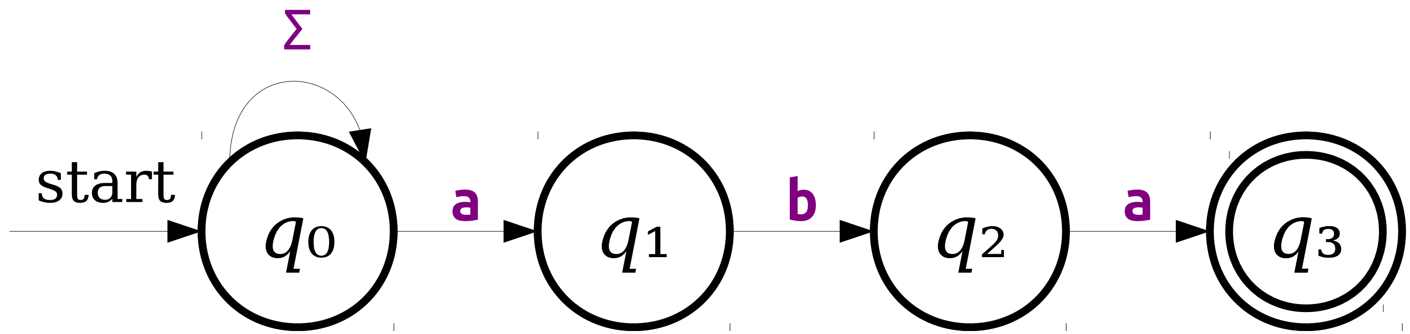
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



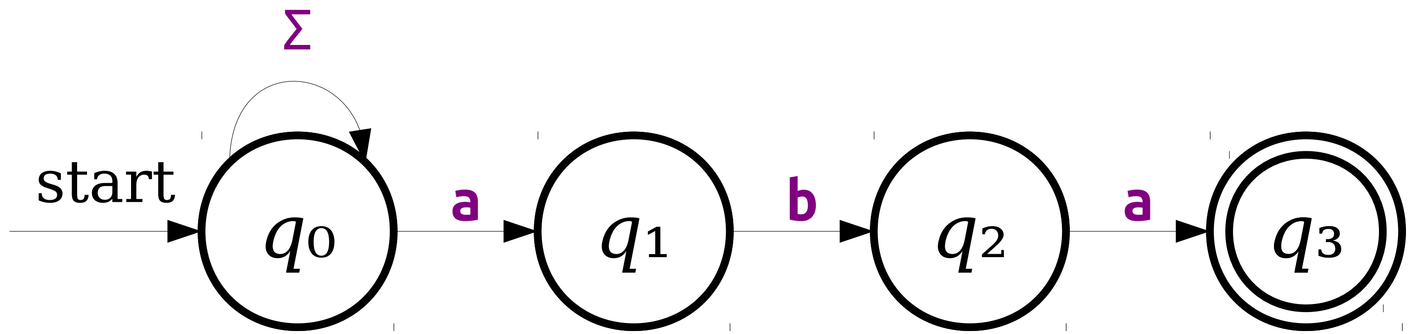
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$



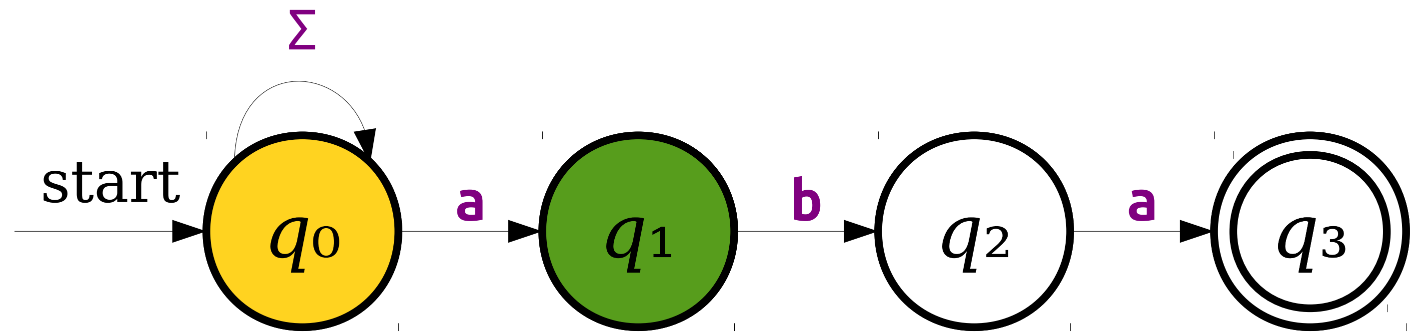
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$



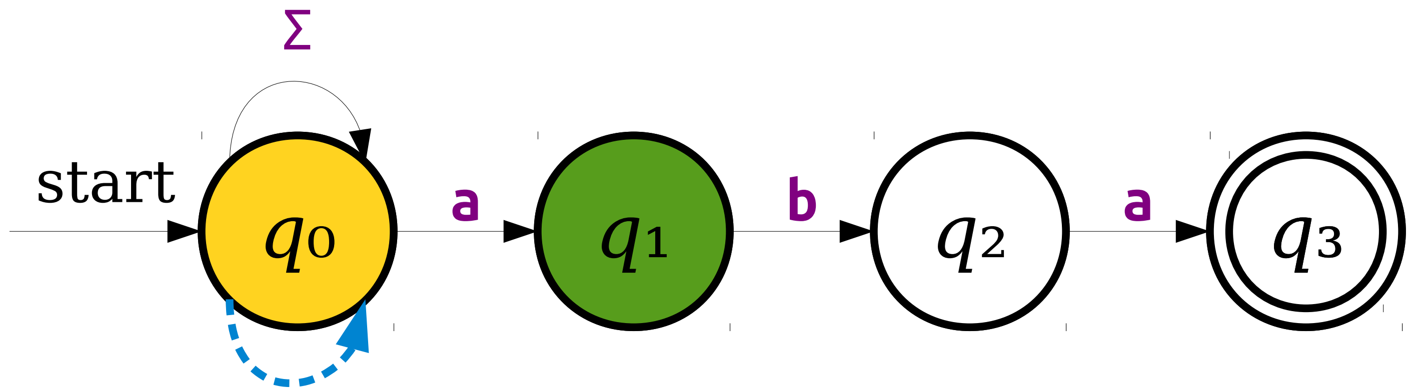
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$



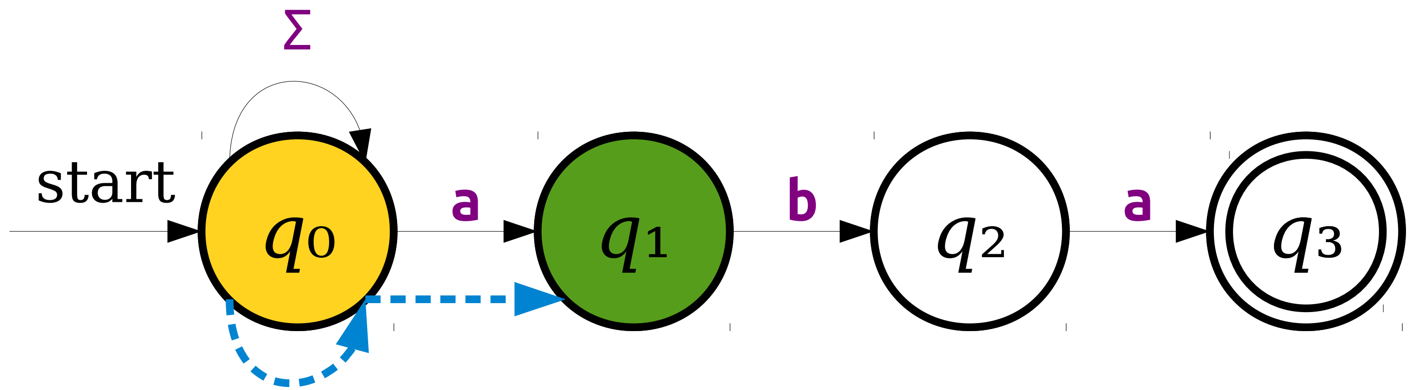
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



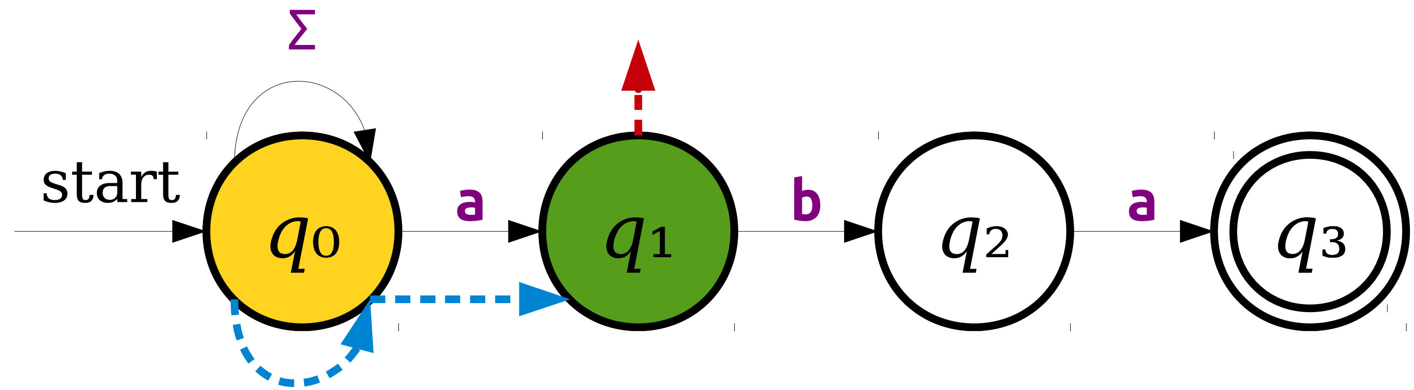
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



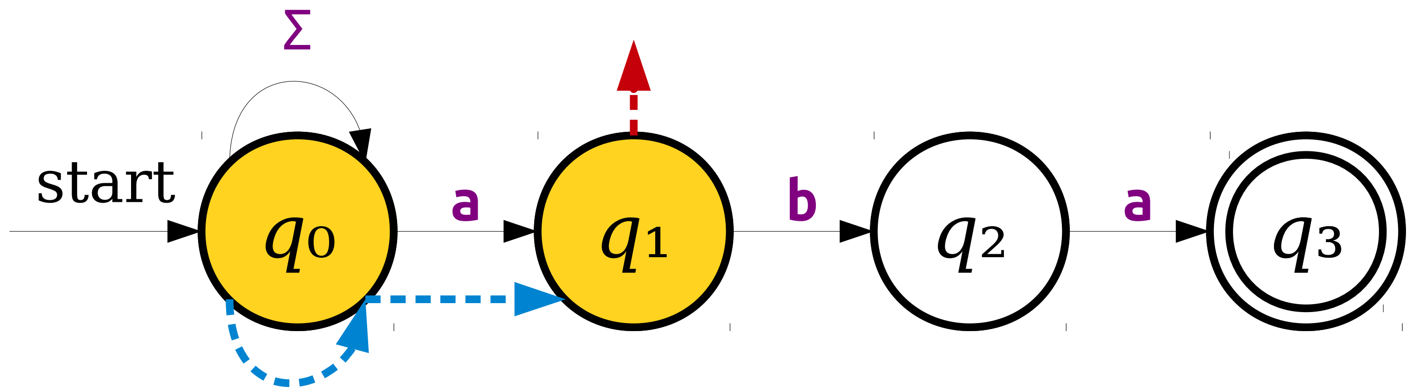
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



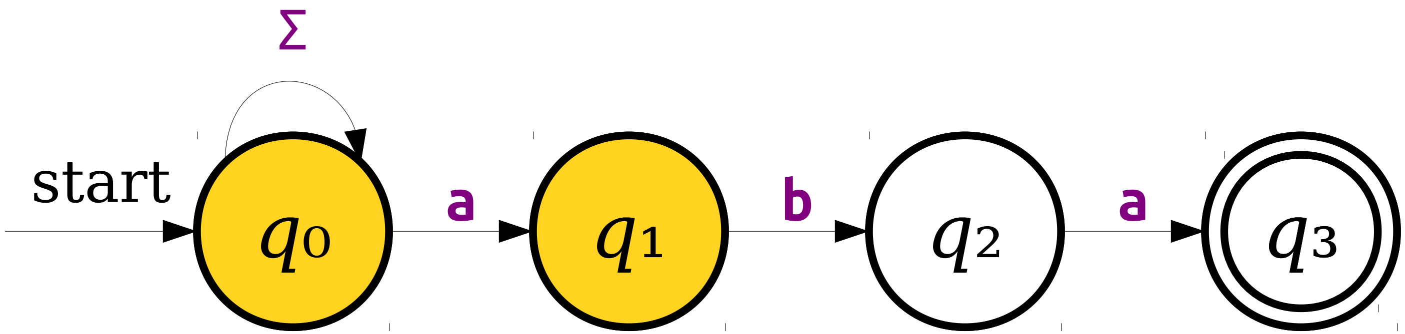
	a	b
$\{q_0\}$	$\{q_0,\ q_1\}$	$\{q_0\}$
$\{q_0,\ q_1\}$		



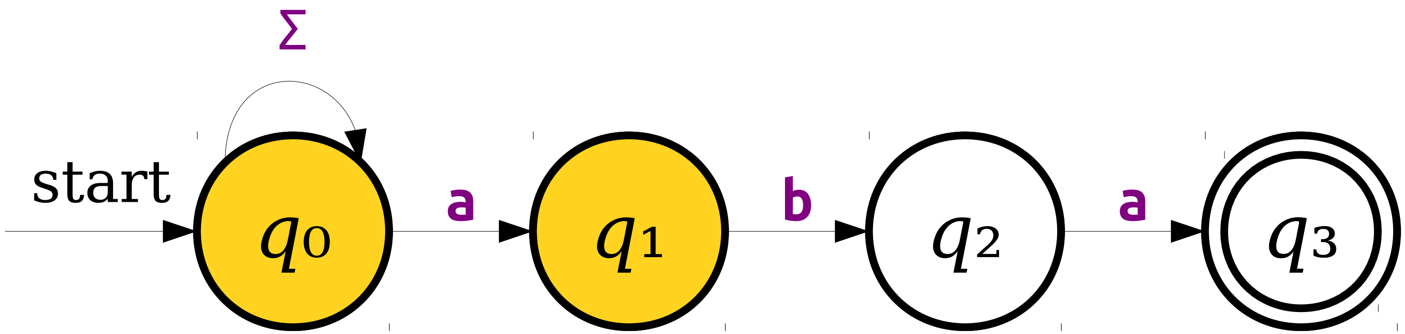
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



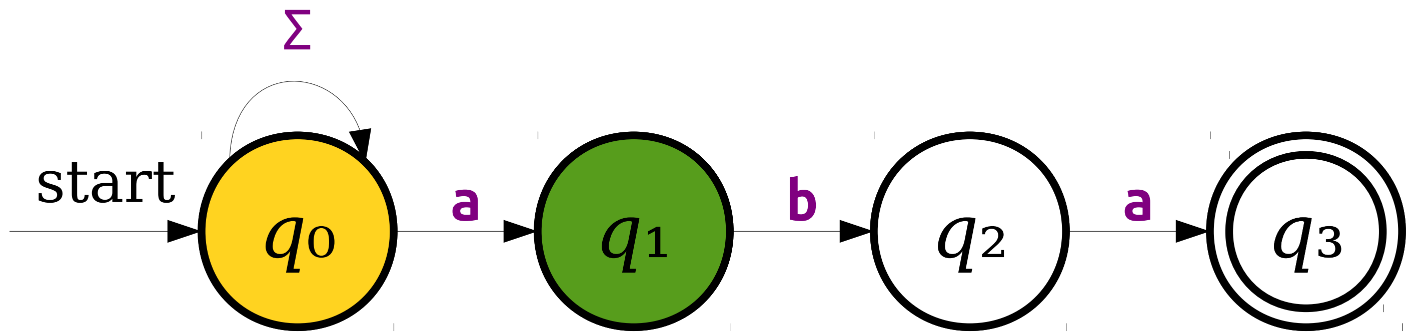
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



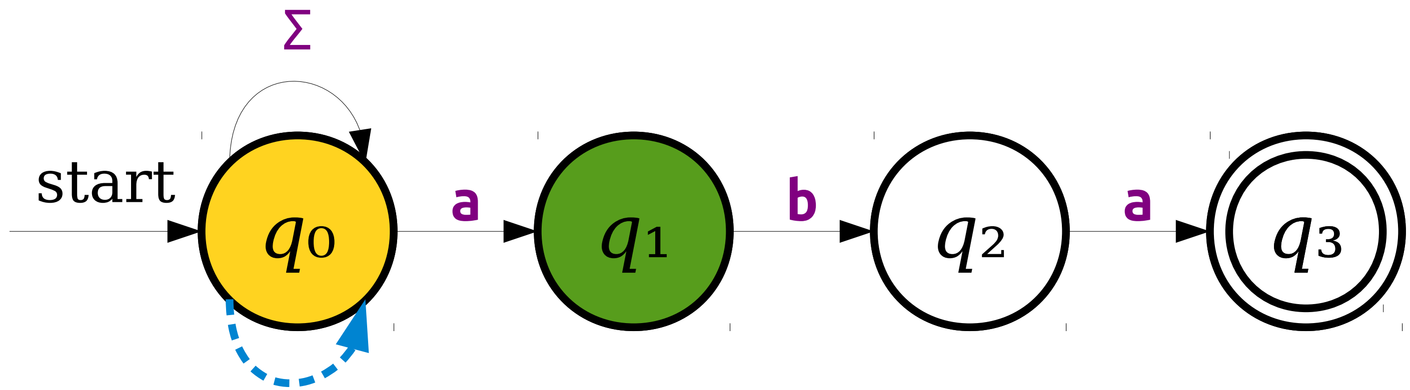
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



	a	b
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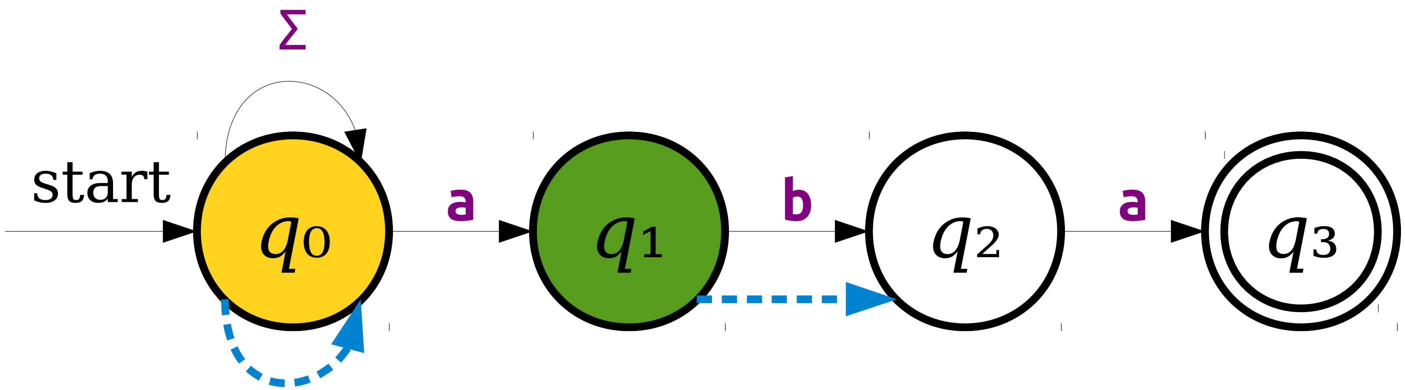
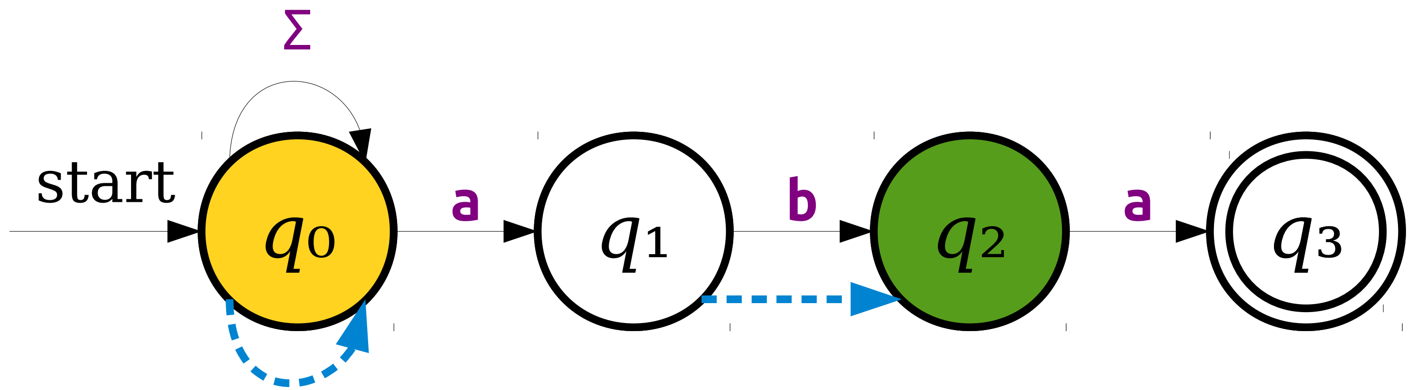
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



	a	b
$\{q_0\}$	$\{q_0,\ q_1\}$	$\{q_0\}$
$\{q_0,\ q_1\}$	$\{q_0,\ q_1\}$	



	a	b
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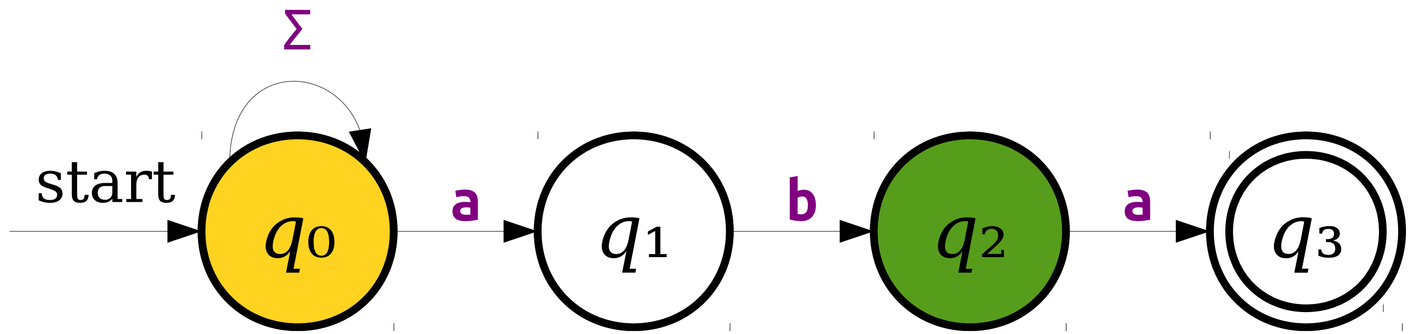

	a	b
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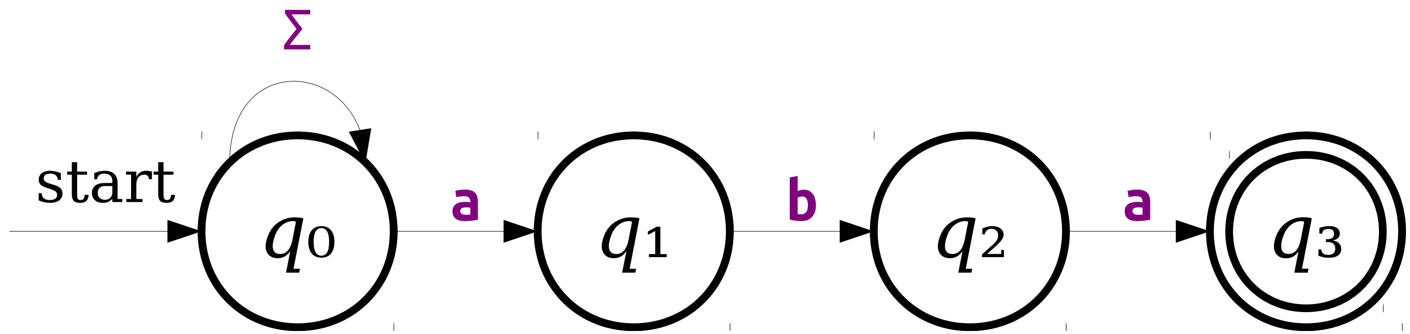
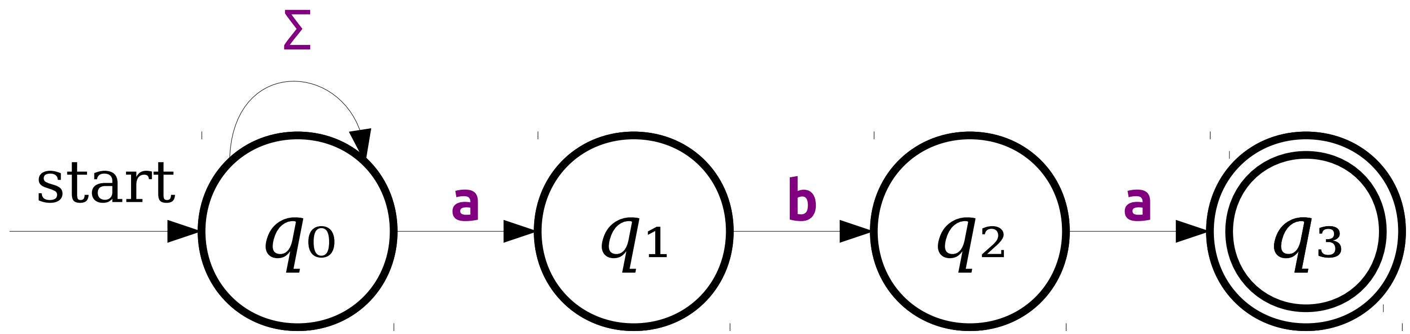
$\{q_0,\ q_1\}$	$\{q_0,\ q_1\}$	$\{q_0,\ q_2\}$



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



	a	b
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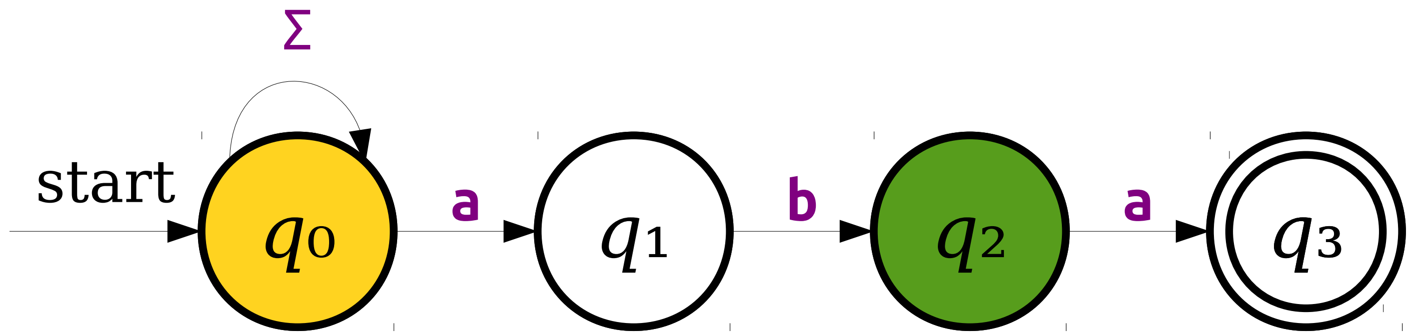
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



	a	b
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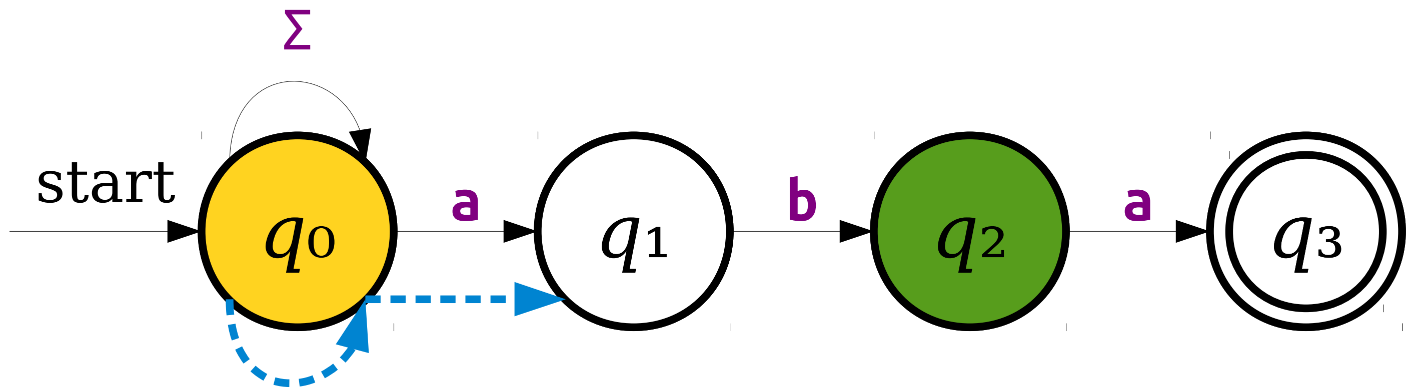
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



	a	b
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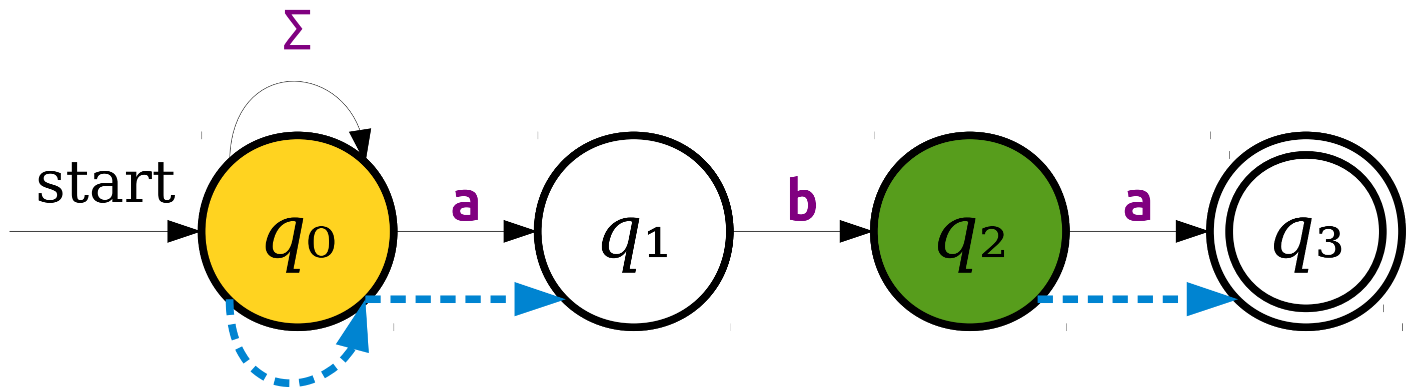

	a	b
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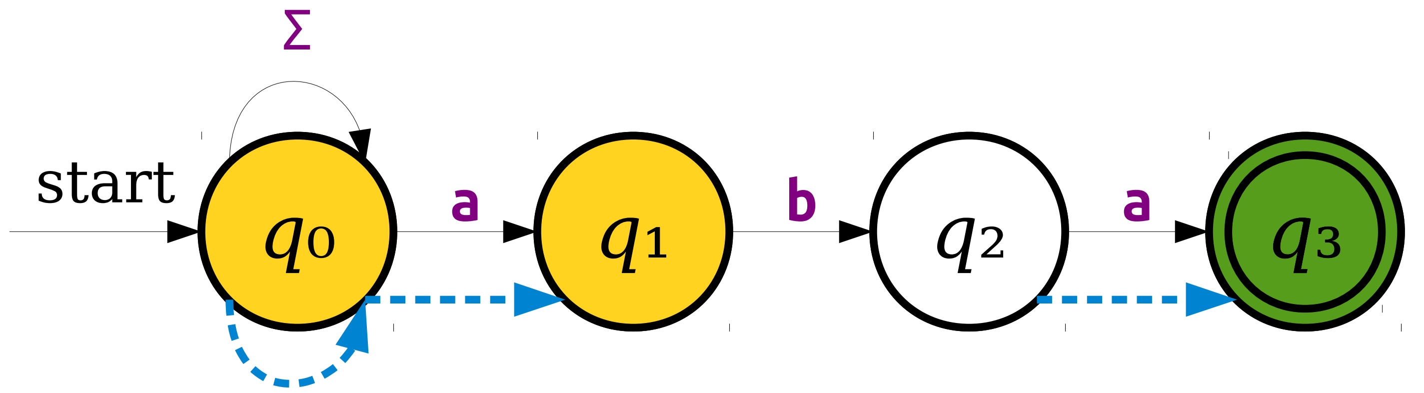
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



	a	b
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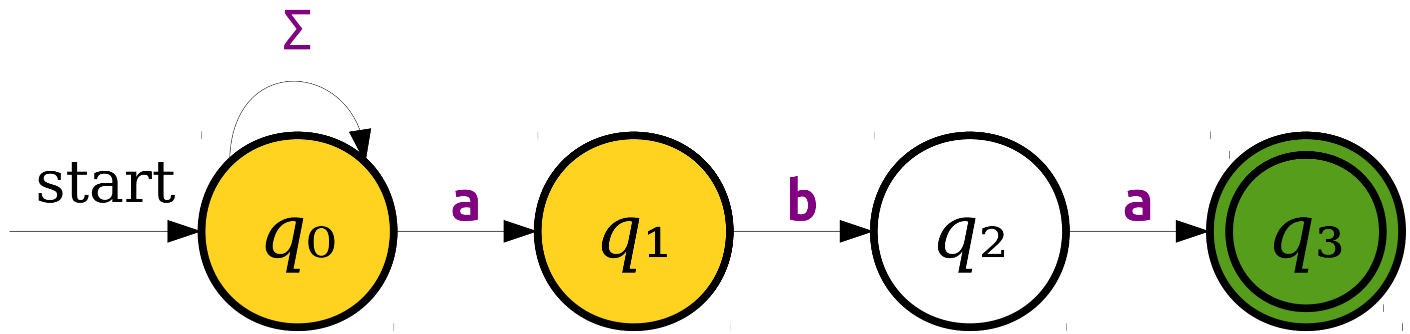

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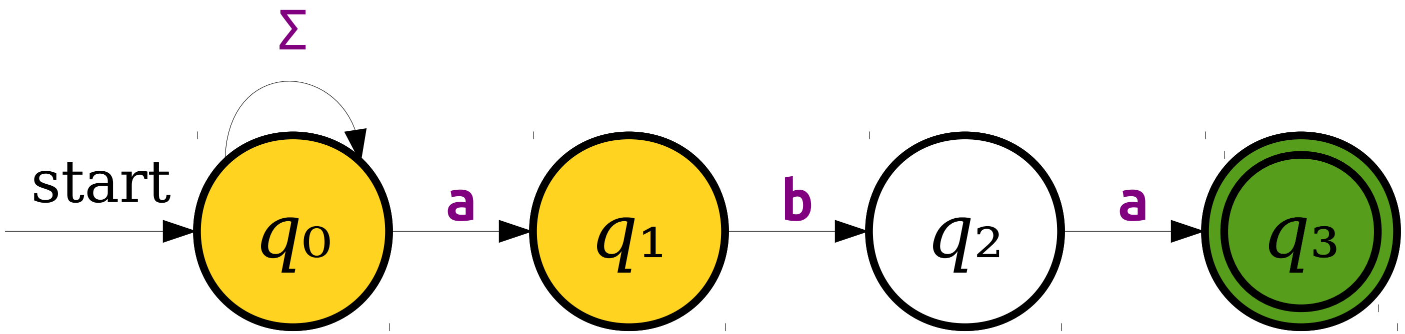

	a	b
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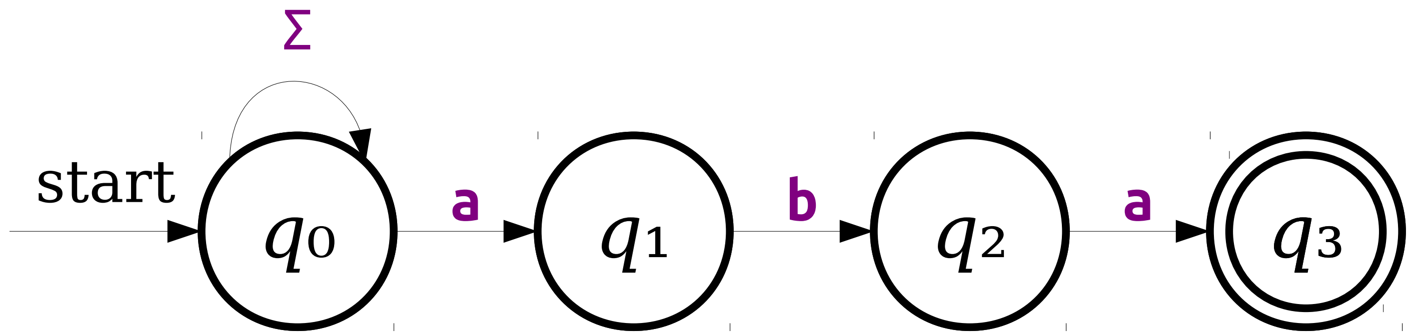

	a	b
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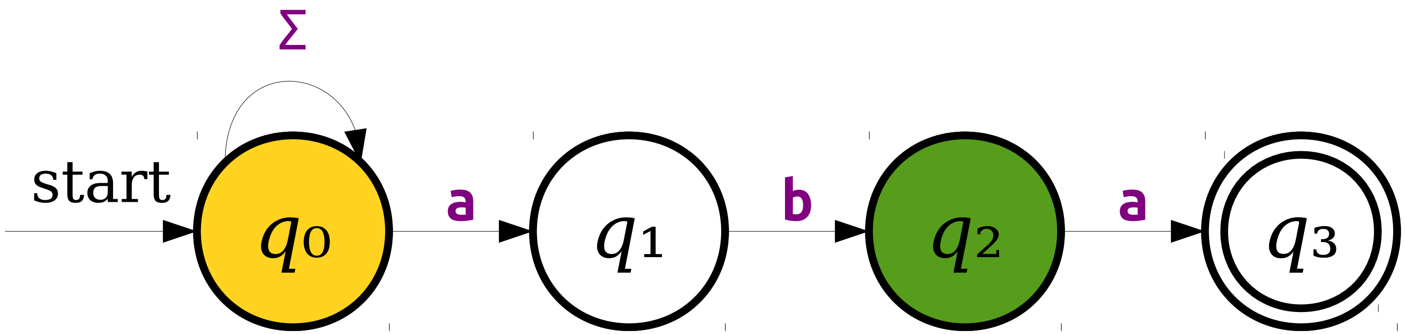

	a	b
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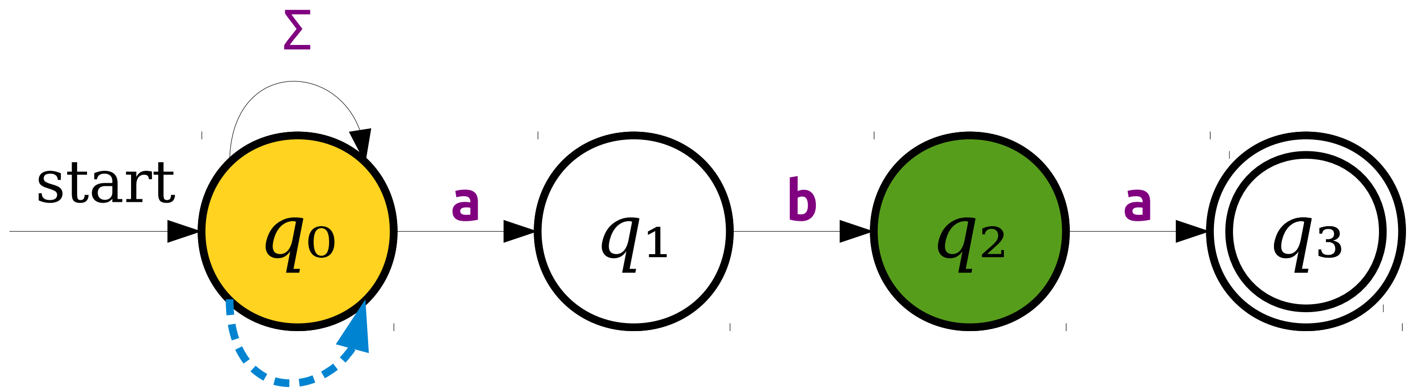

	a	b
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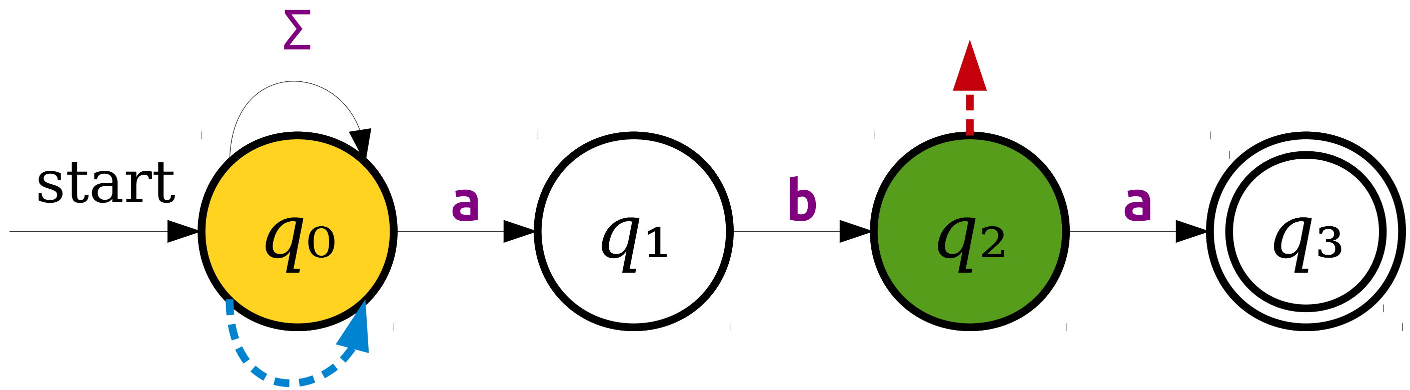

	a	b
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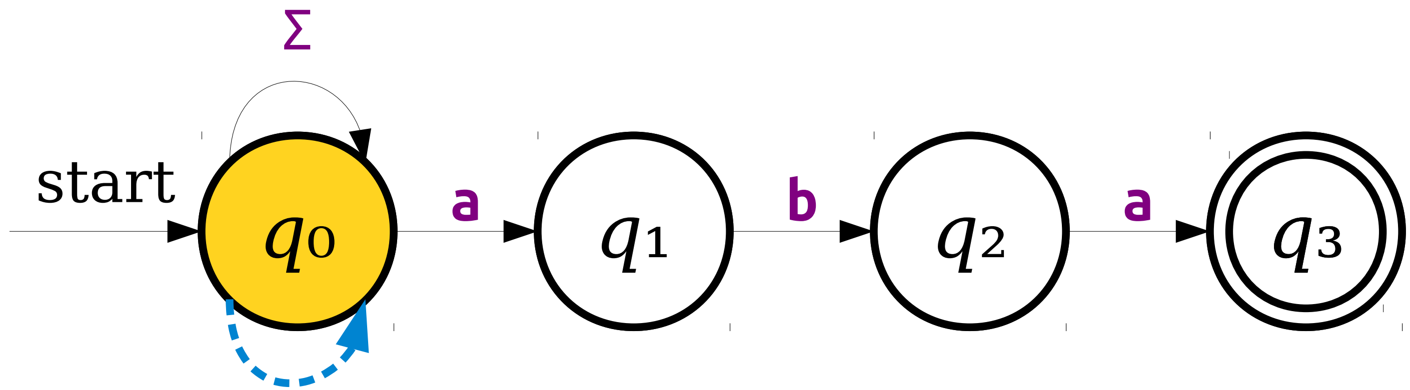

	a	b
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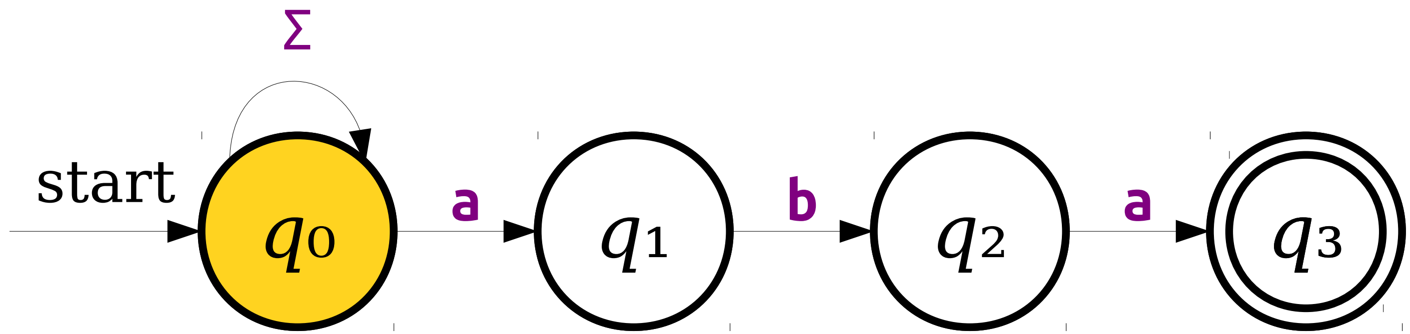

	a	b
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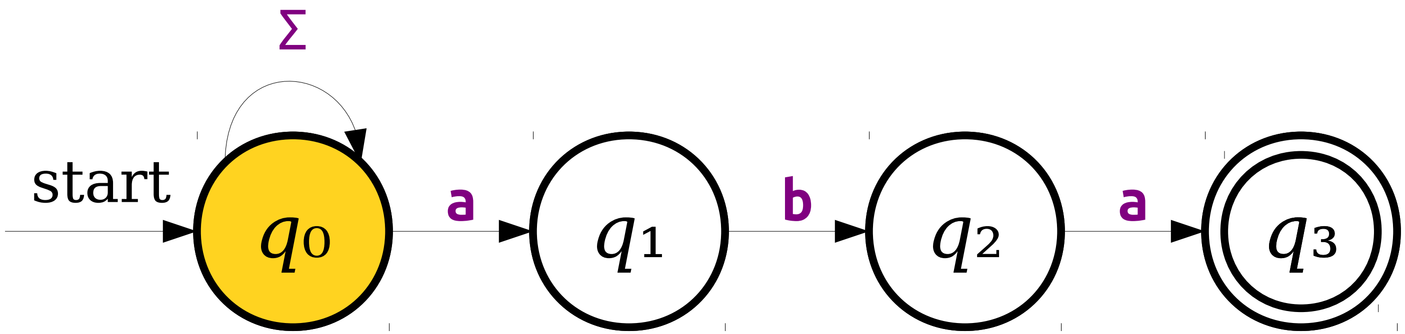

	a	b
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$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
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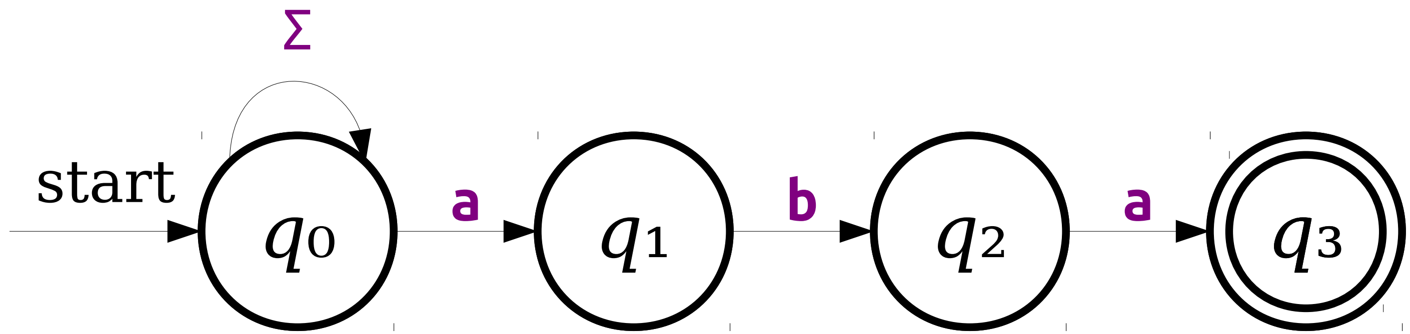

	a	b
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$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
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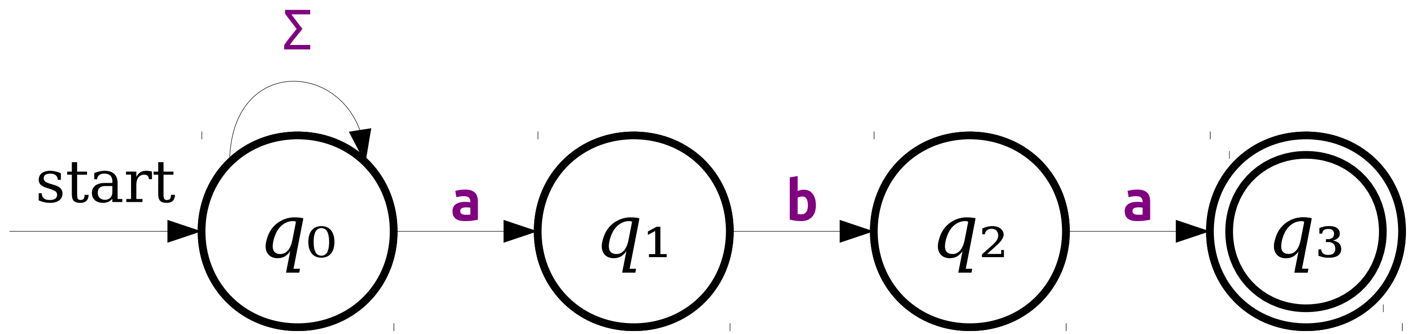

	a	b
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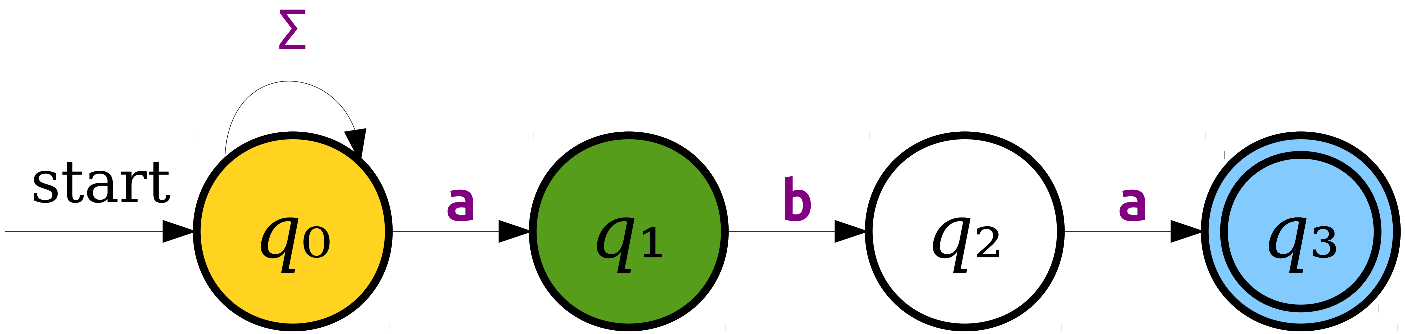
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



	a	b
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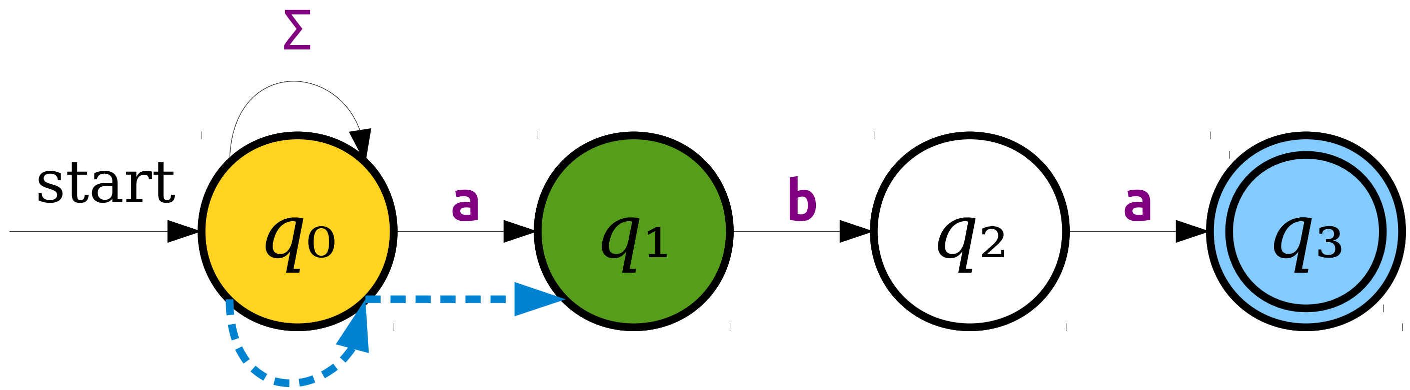
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



	a	b
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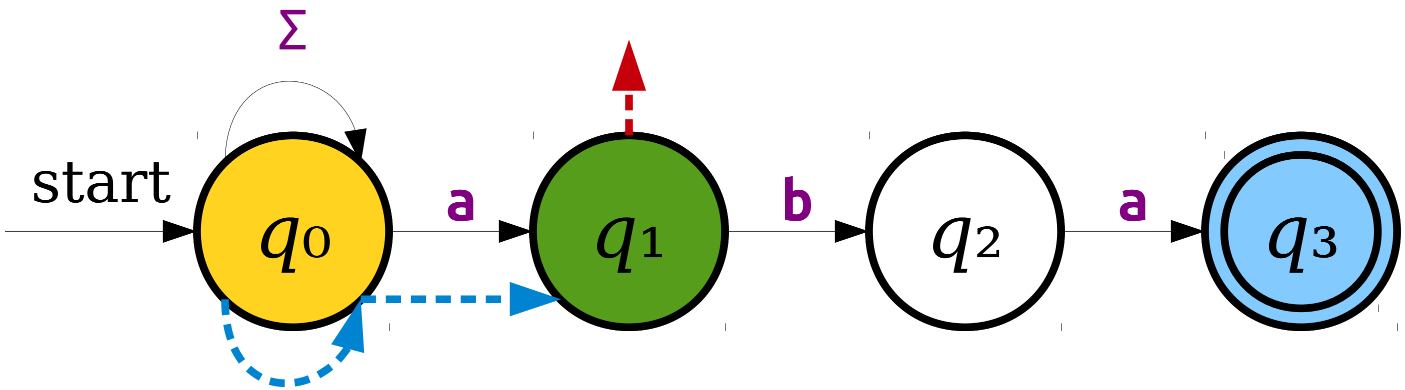
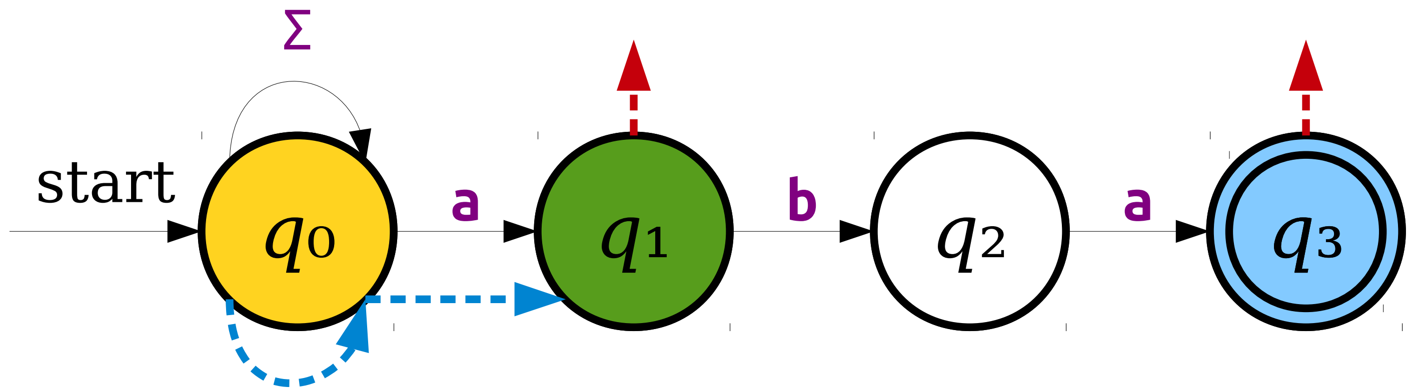
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



	a	b
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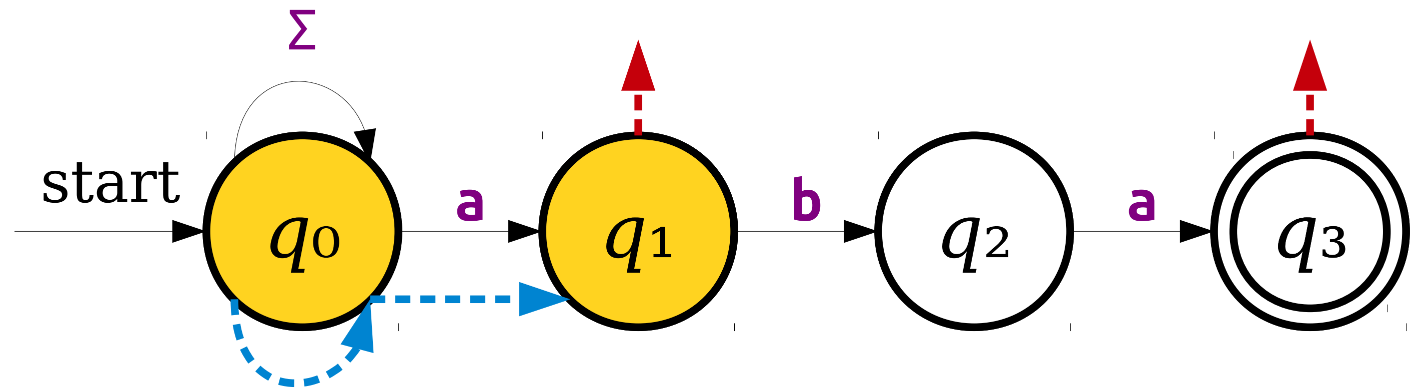
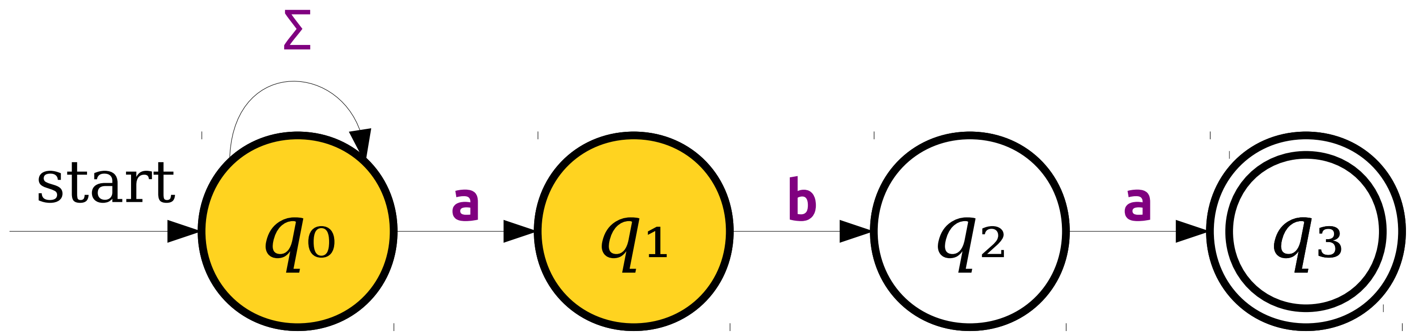
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



	a	b
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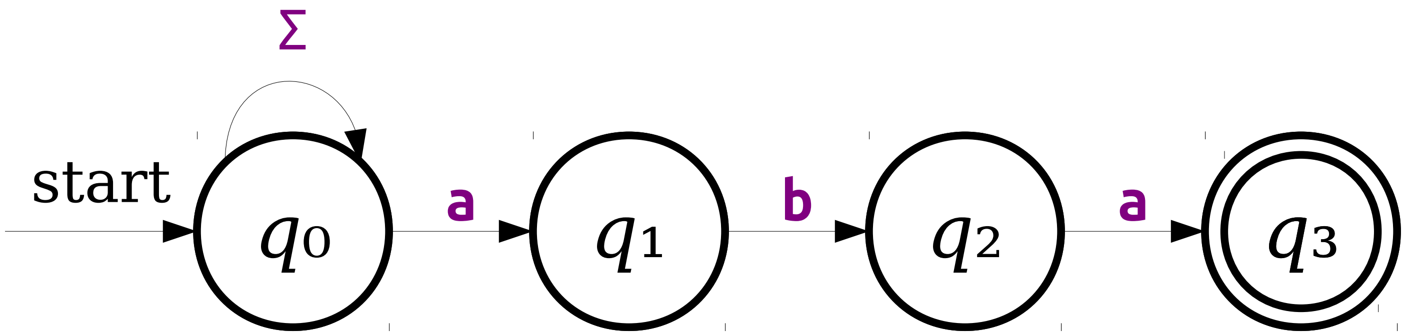
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



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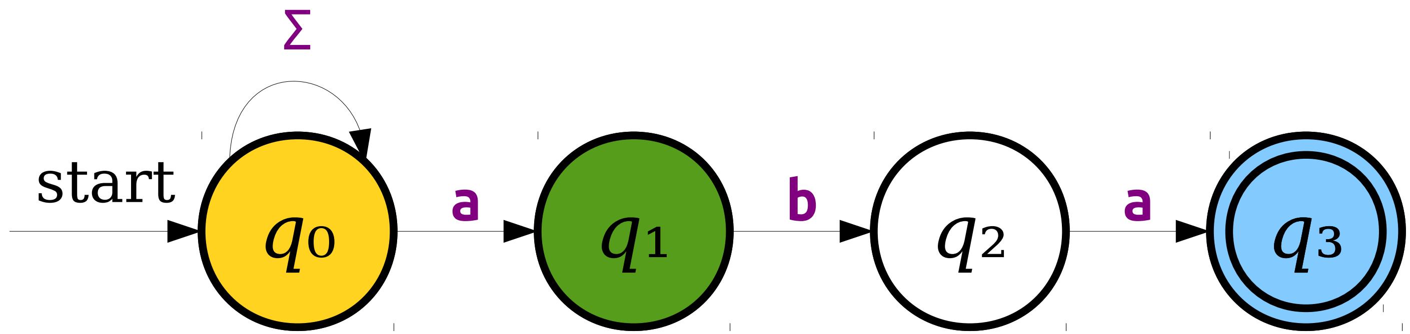
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



	a	b
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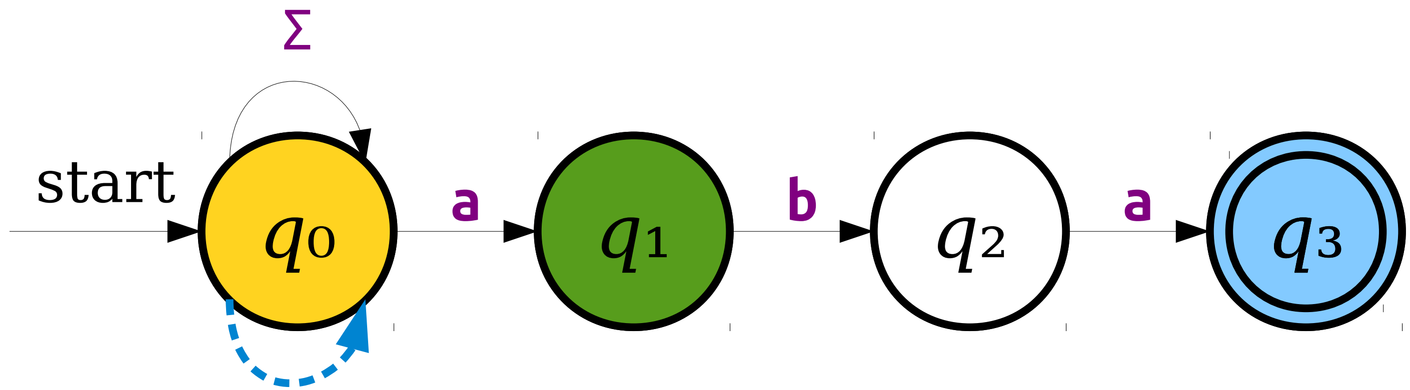
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



	a	b
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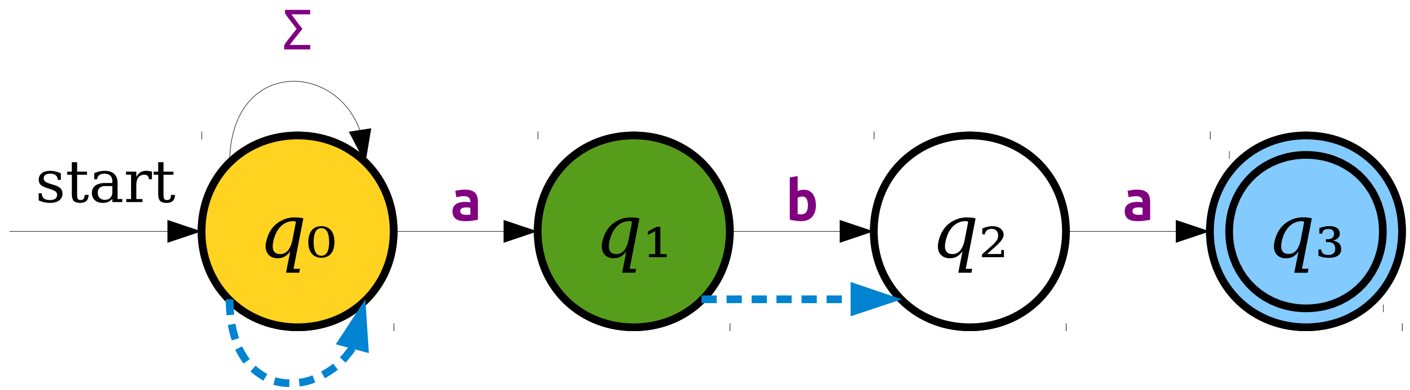
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



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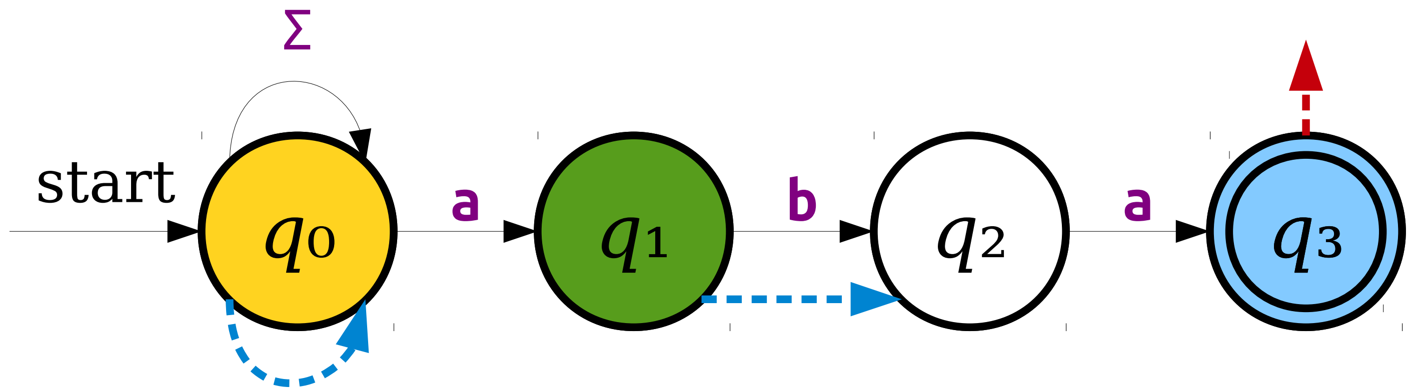

	a	b
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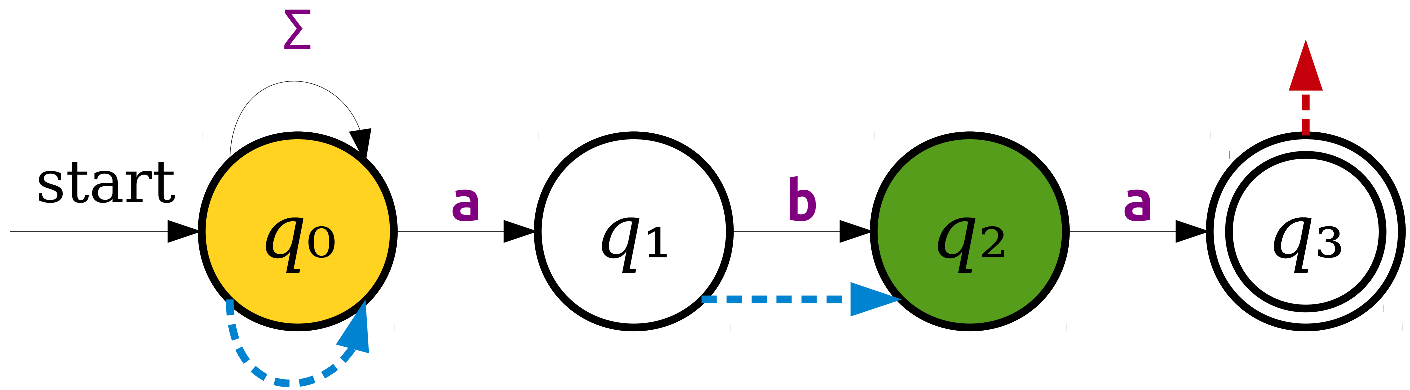
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



	a	b
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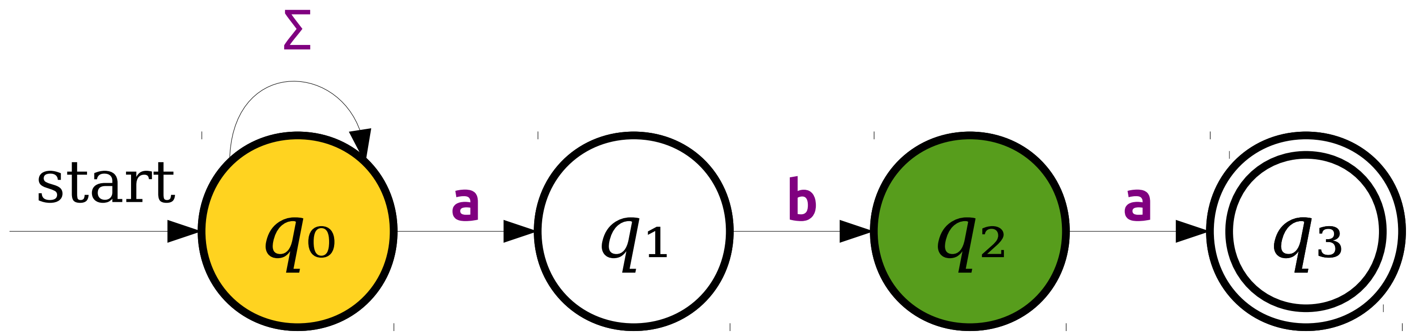
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



	a	b
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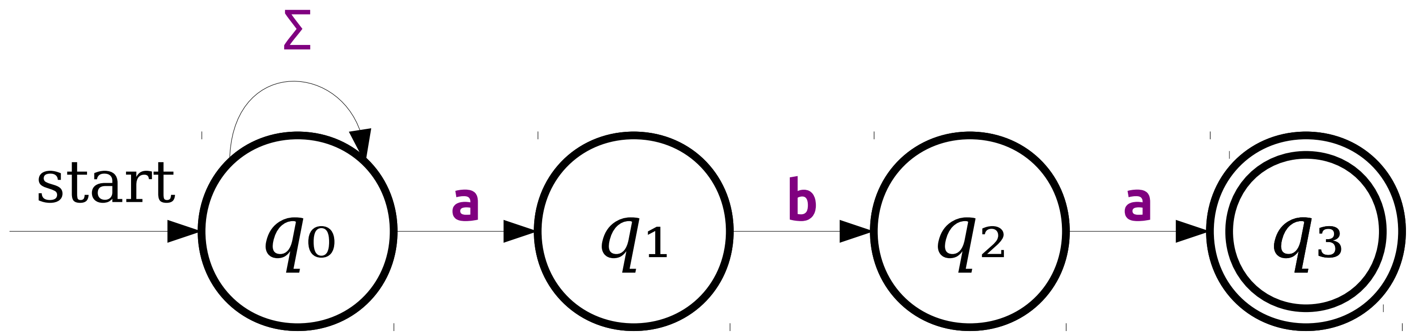
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



	a	b
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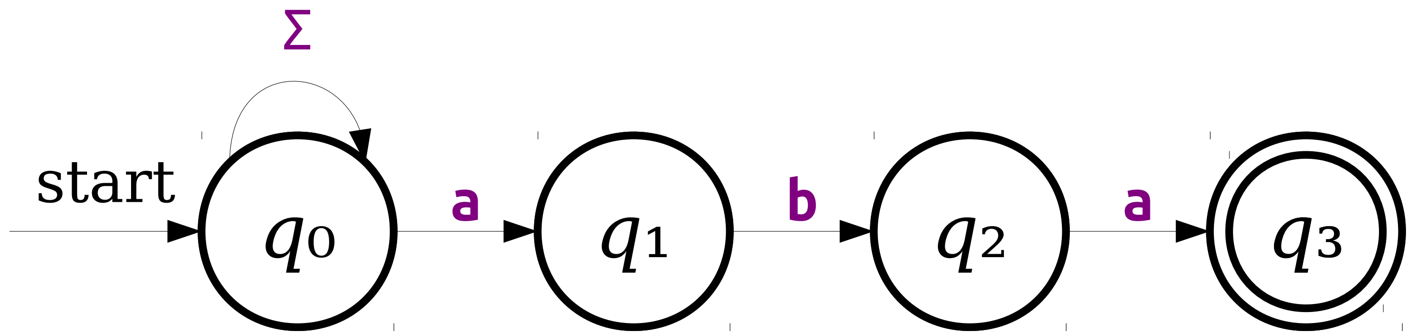
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



	a	b
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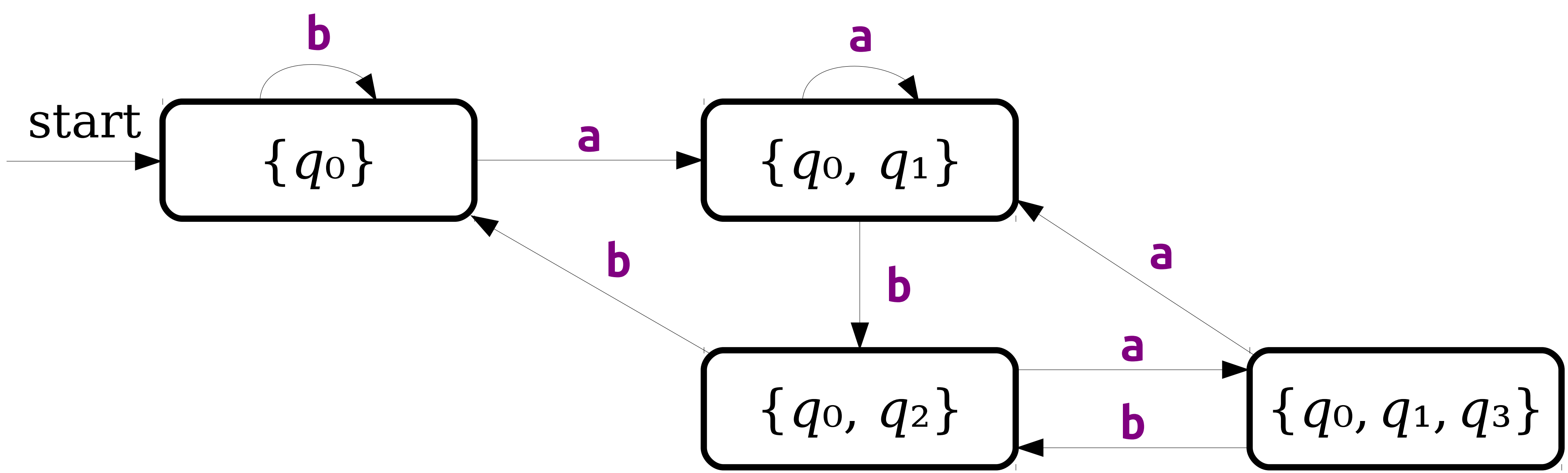
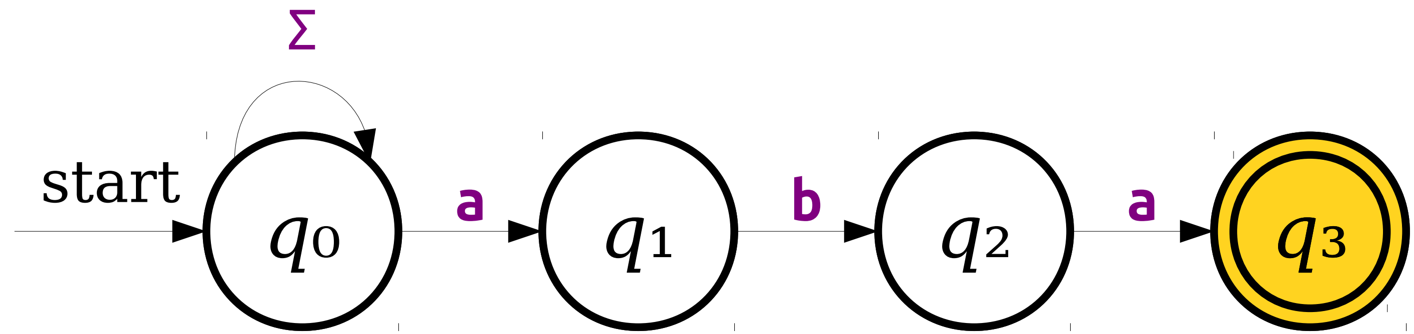
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$





	a	b
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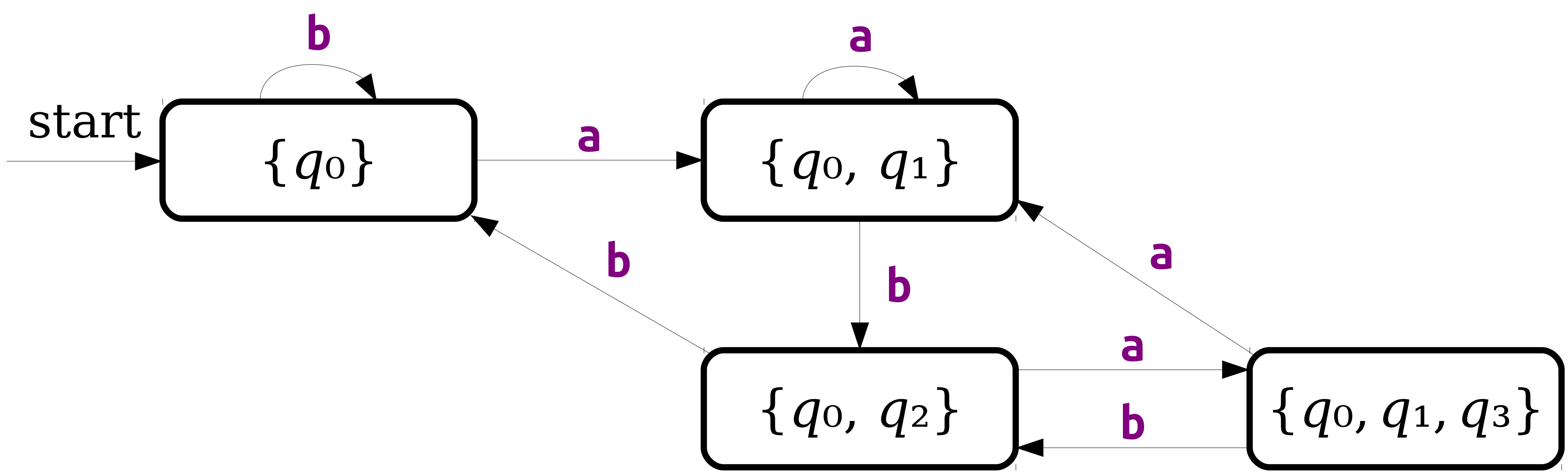
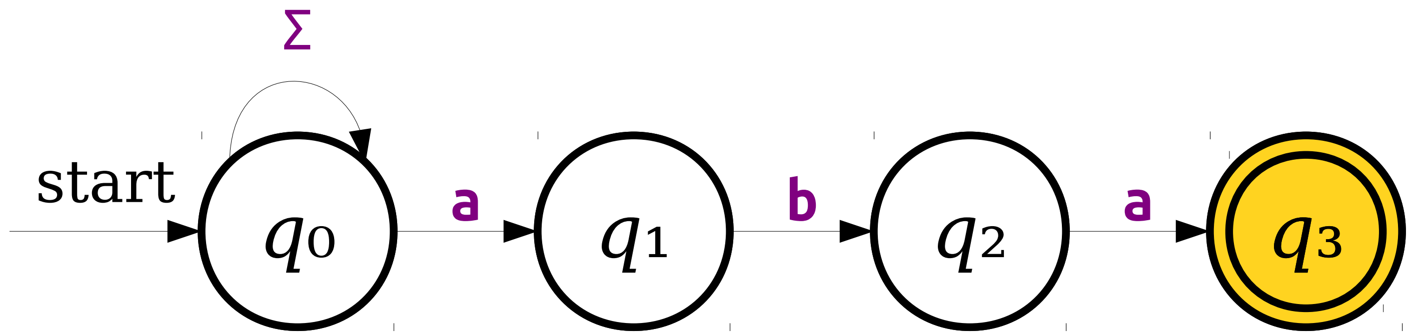
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$





	a	b
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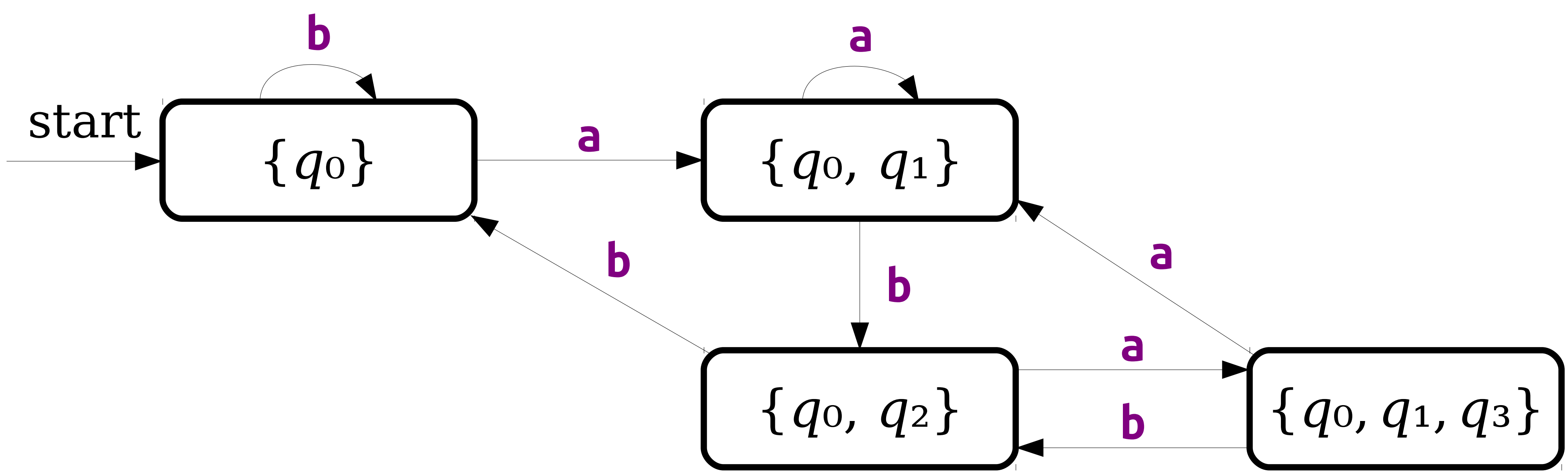
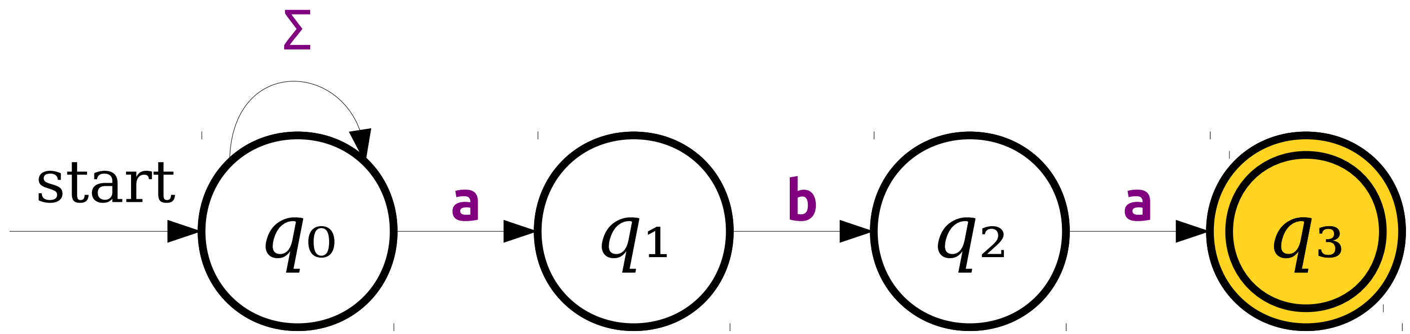
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$





	a	b
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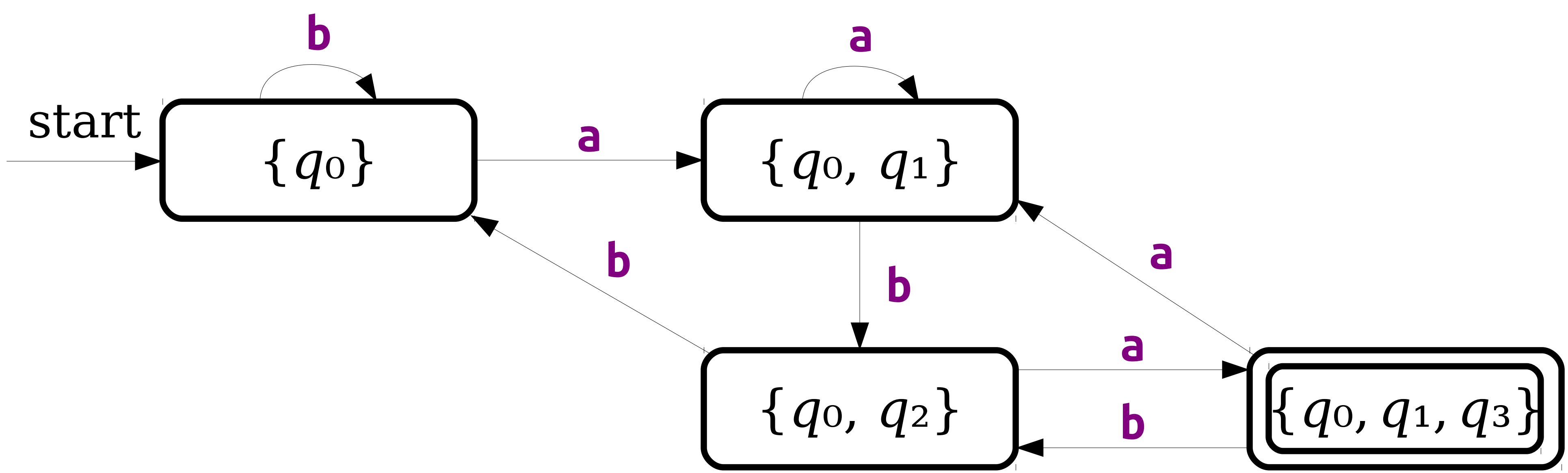
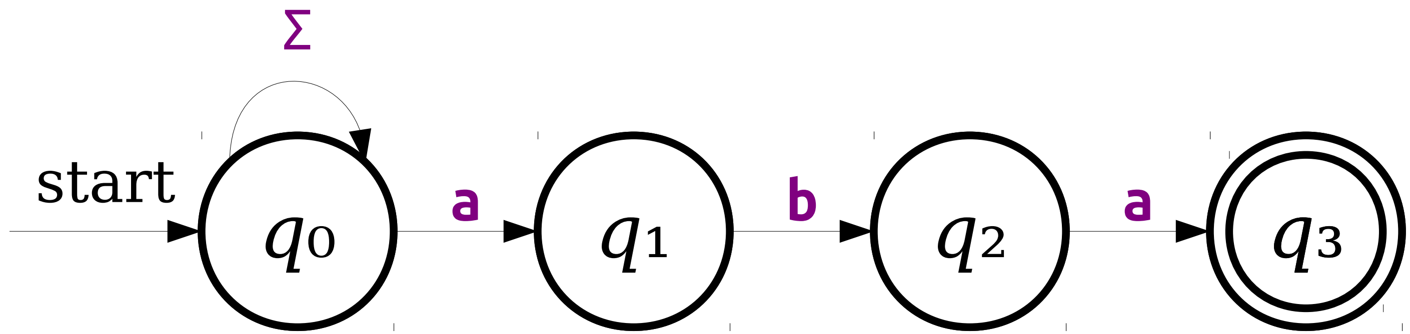
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$





	a	b
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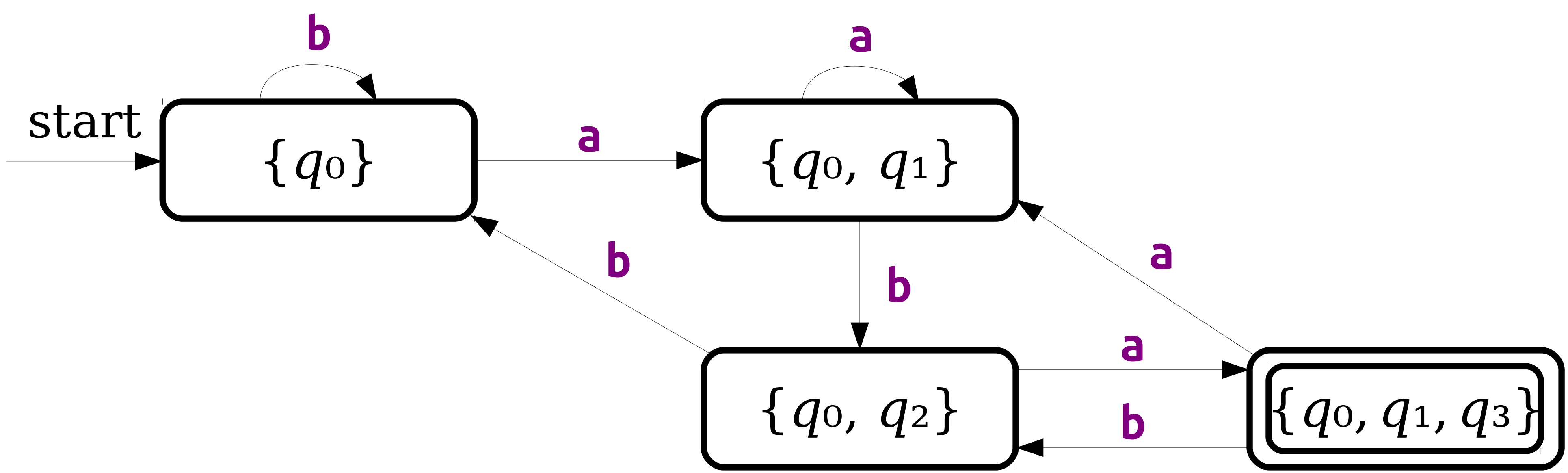
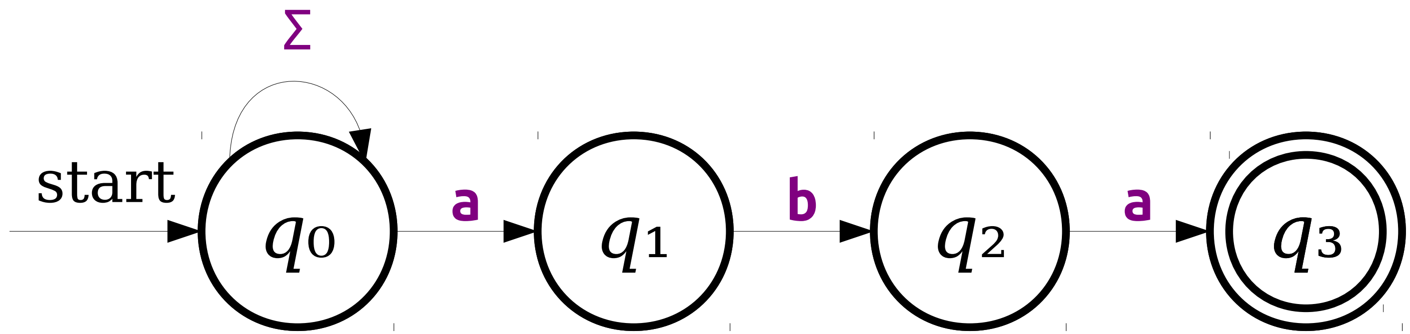
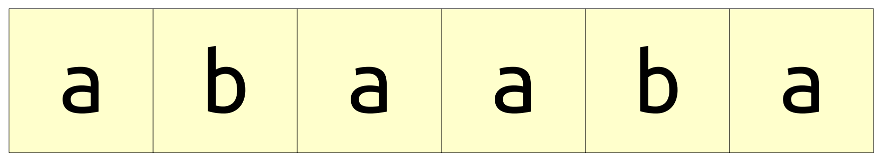
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

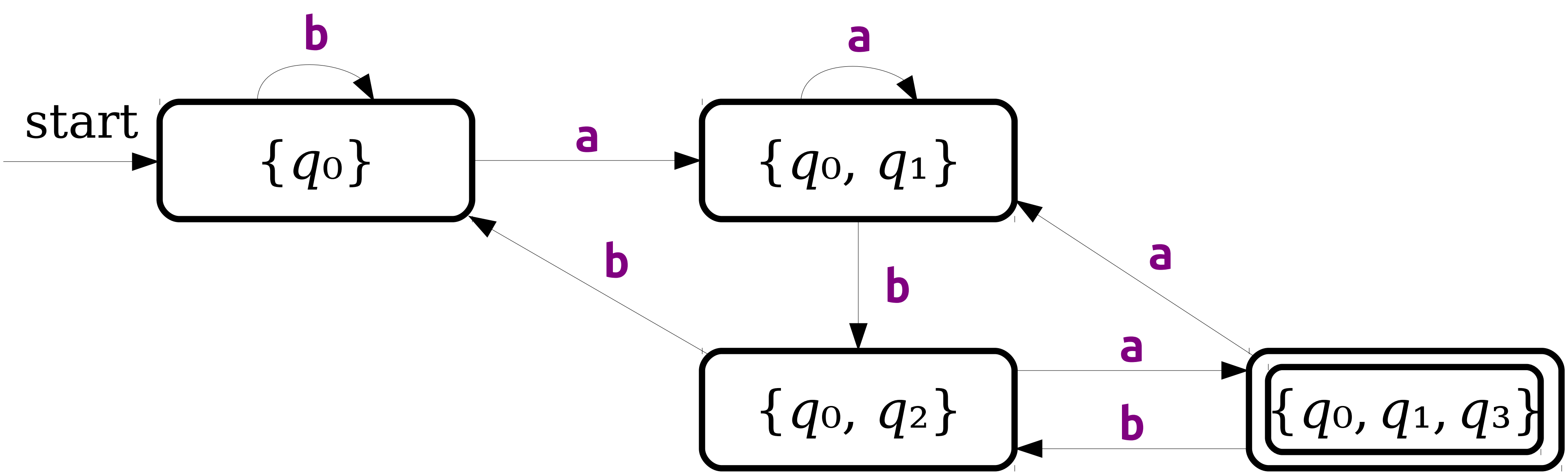




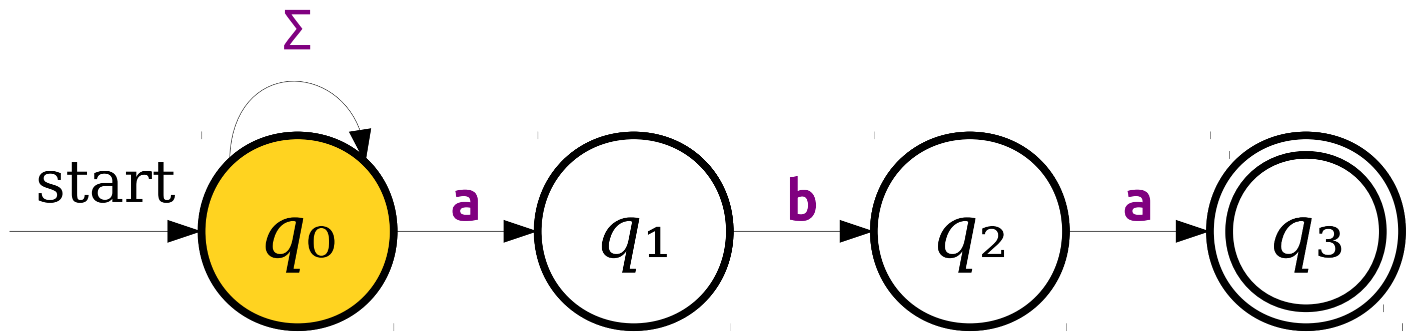
Σ
start
q0
a
q1
b
q2
a
q3

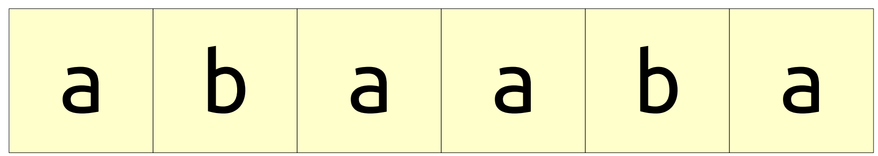


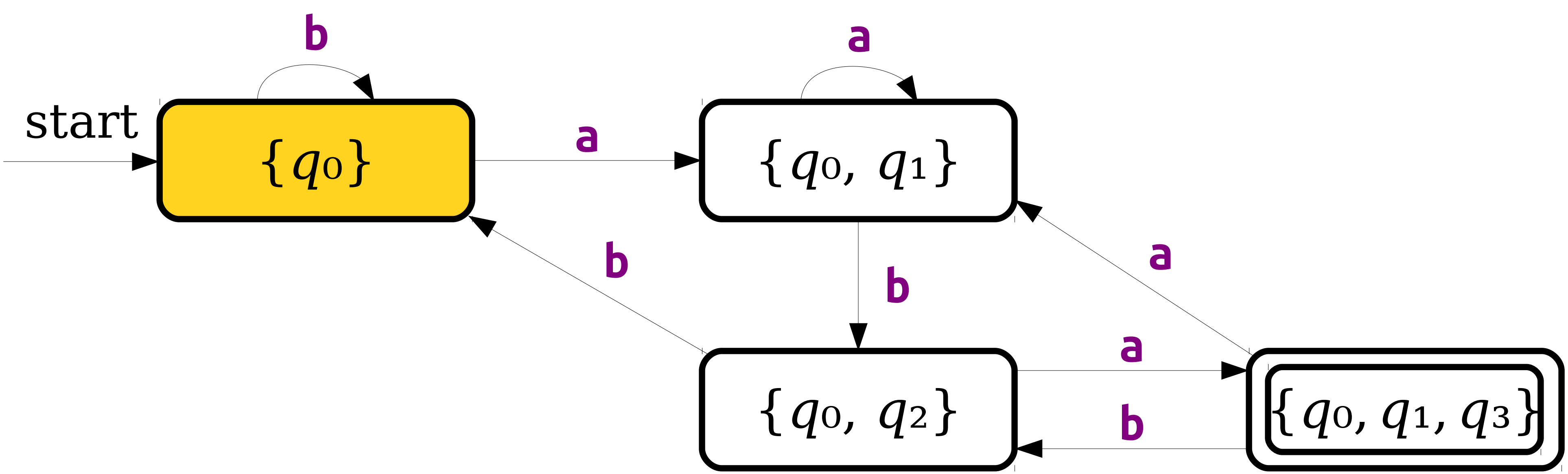
a b a a b a

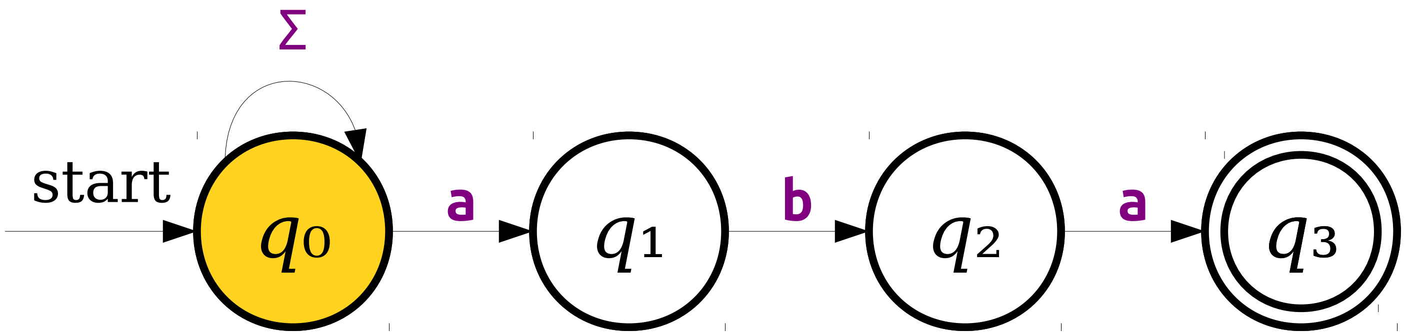


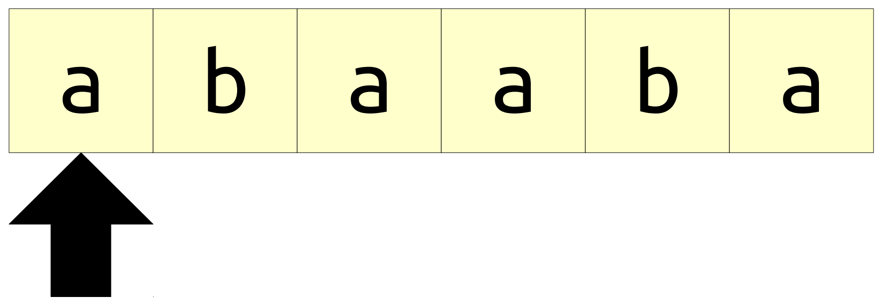
b
a
start
{q0}
a
{q0, q1}
b
b
a
a
{q0, q2}
b
{q0, q1, q3}

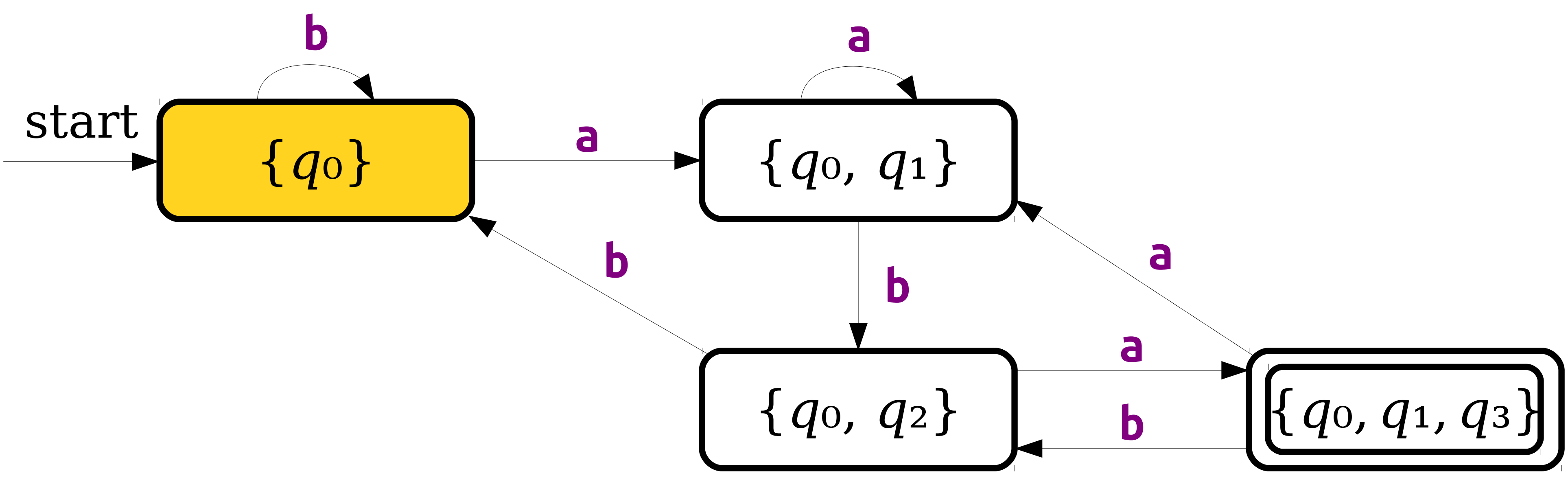

start
Σ
q_0
a
q_1
b
q_2
a
q_3

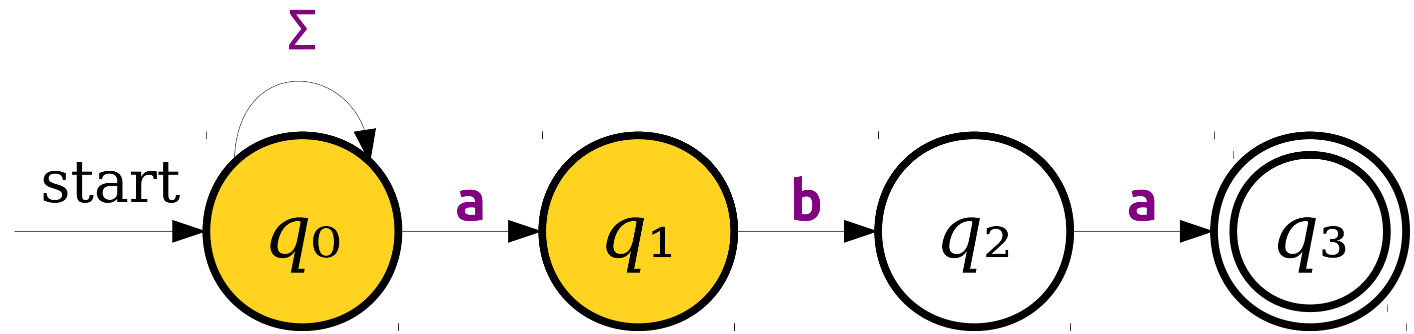

a b a a b a


start
b
{q_0}
a
{q_0, q_1}
a
b
b
a
{q_0, q_2}
a
b
{q_0, q_1, q_3}

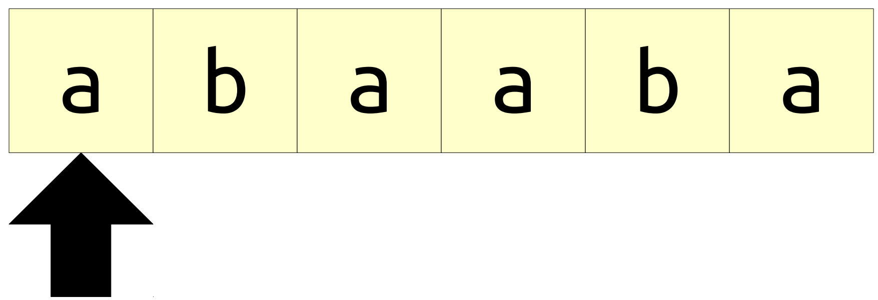

Σ
start
q_0
a
q_1
b
q_2
a
q_3


a b a a b a

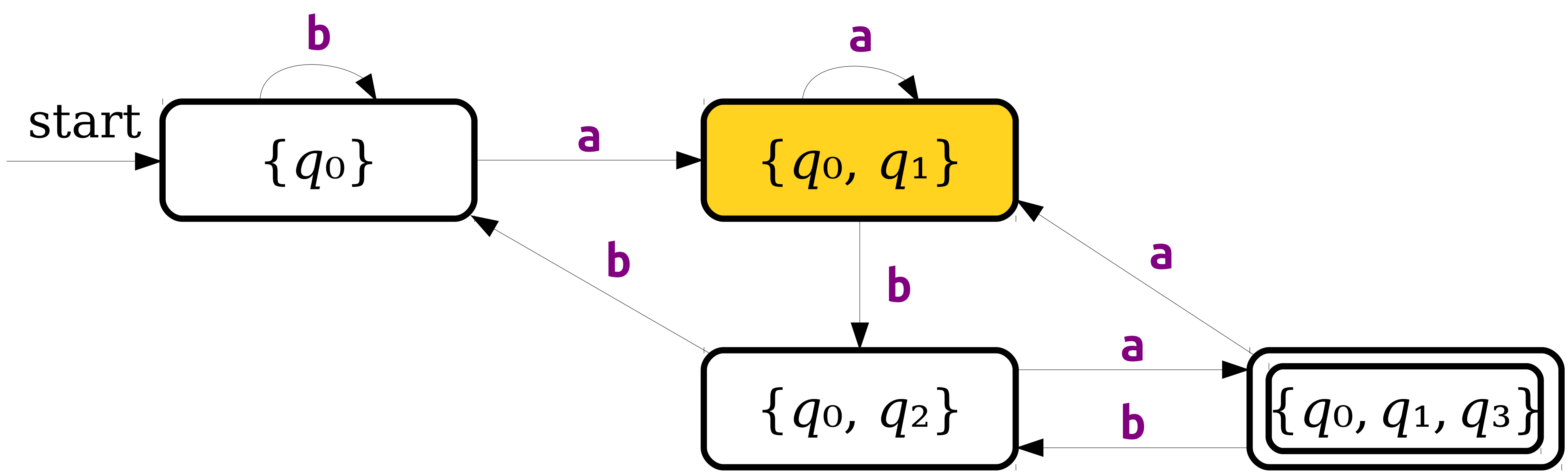

b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
b
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$



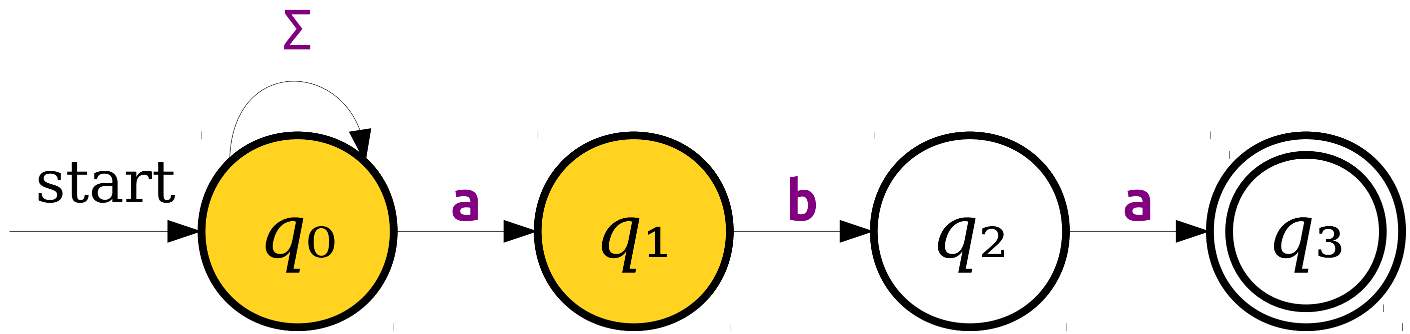
Σ
start
q_0
a
q_1
b
q_2
a
q_3

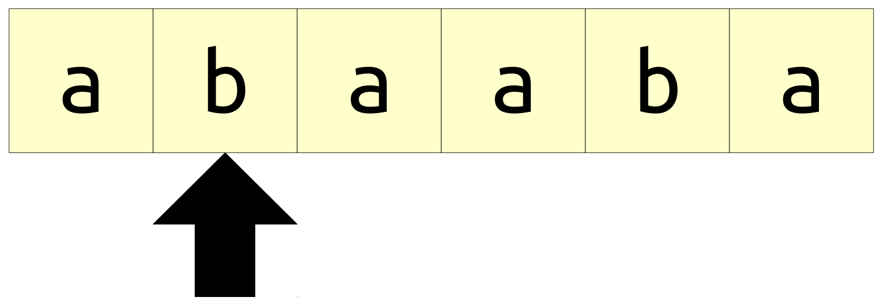


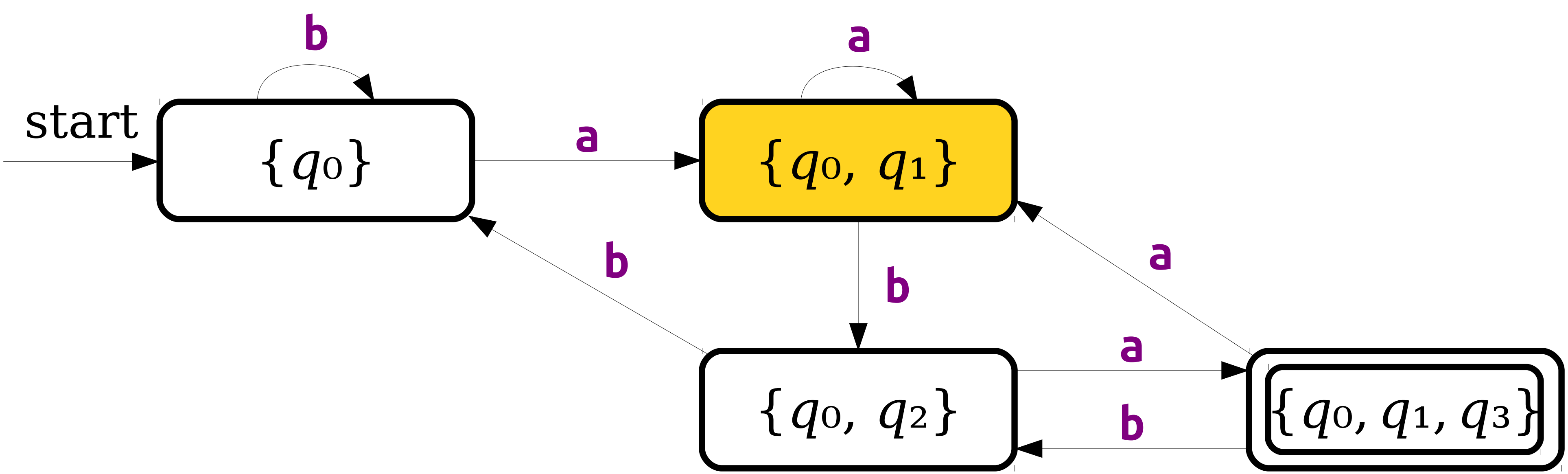
a b a a b a

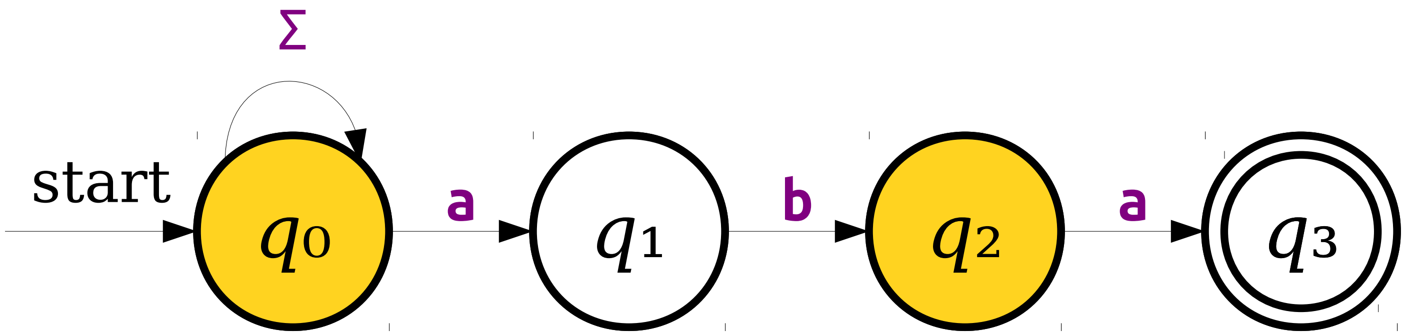


b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
a
b
a
b
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$

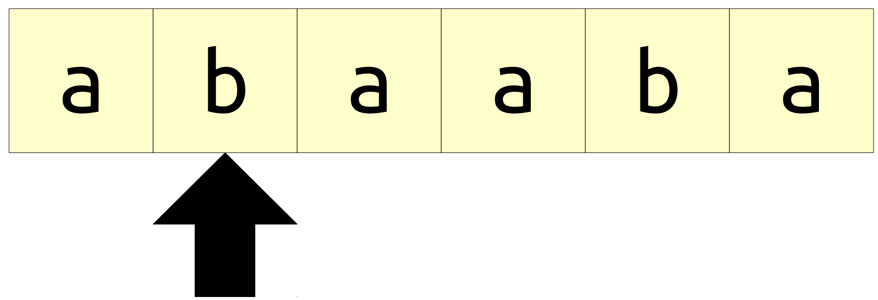

Σ
start
q0
a
q1
b
q2
a
q3


a b a a b a

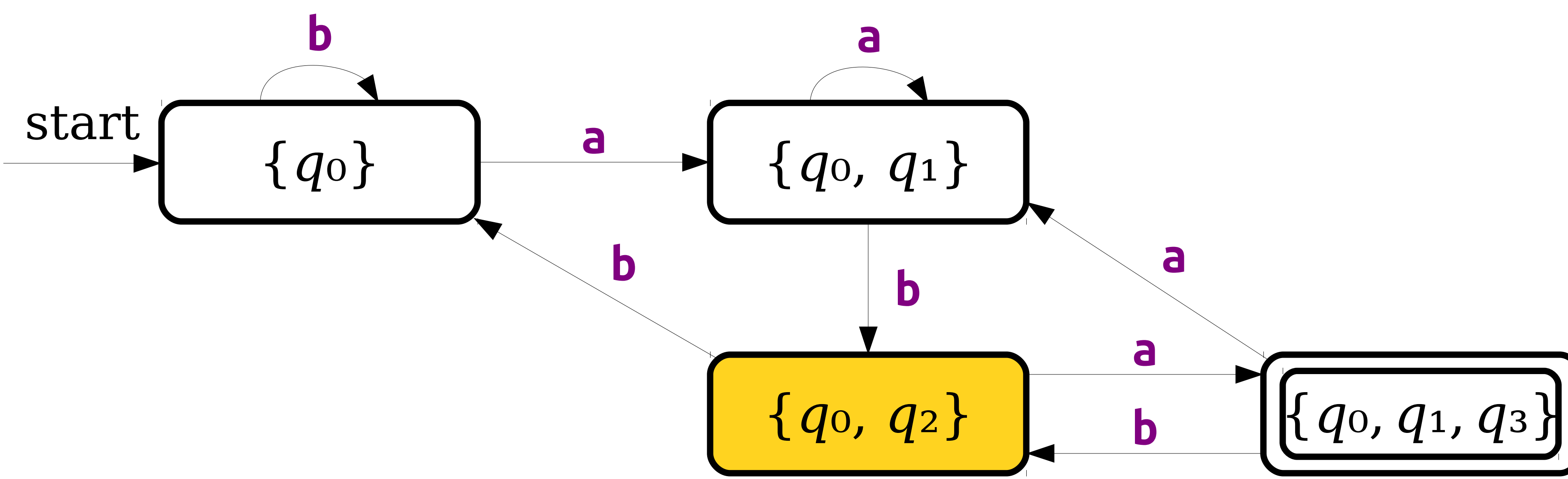

start
{q0}
b
a
{q0, q1}
a
b
b
a
{q0, q2}
a
b
{q0, q1, q3}



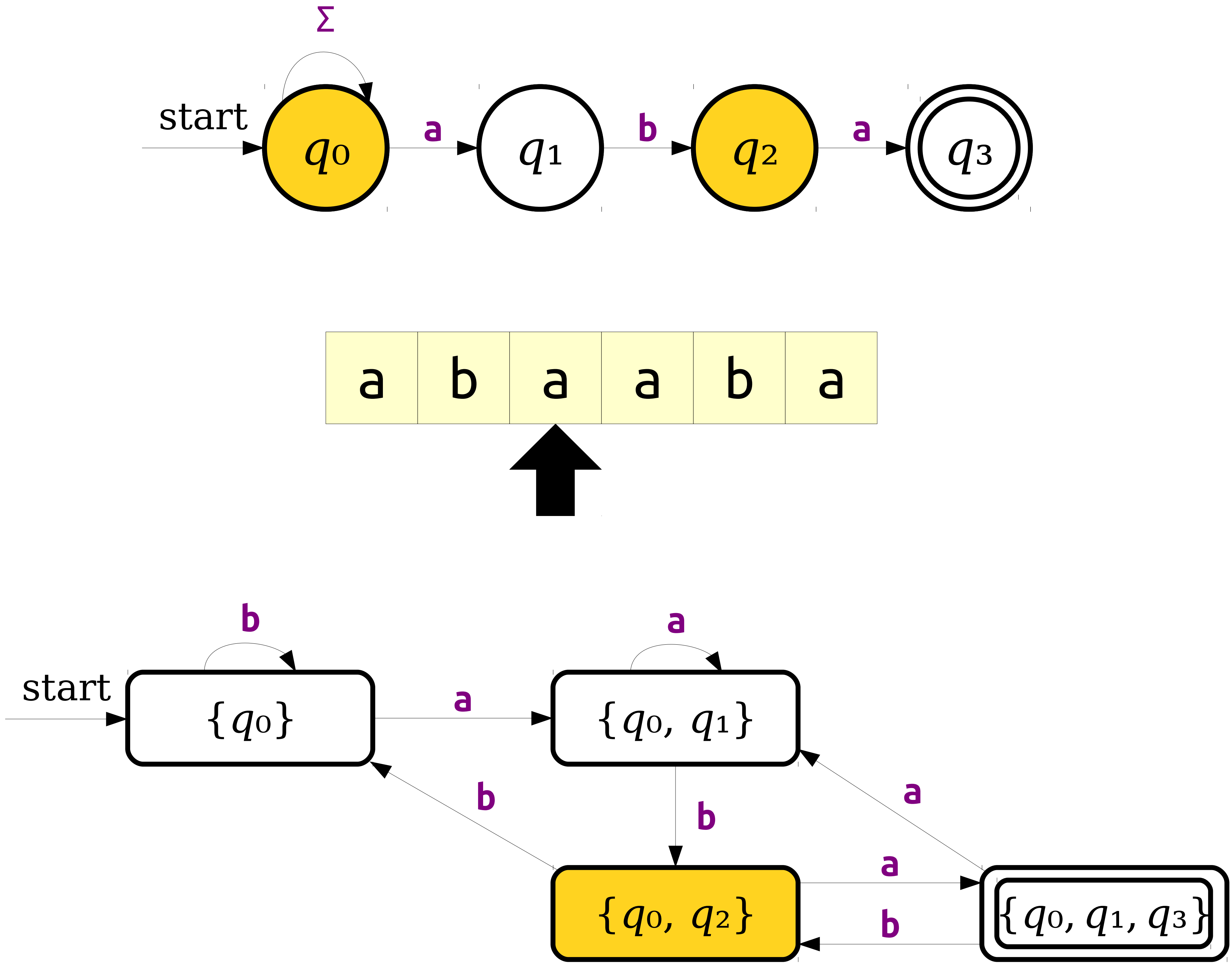
Σ
start
q_0
a
q_1
b
q_2
a
q_3



a b a a b a



start
b
$\{q_0\}$
a
a
$\{q_0, q_1\}$
b
b
a
$\{q_0, q_2\}$
a
b
$\{q_0, q_1, q_3\}$



Σ
start
q0
a
q1
b
q2
a
q3
a b a a b a
b
a
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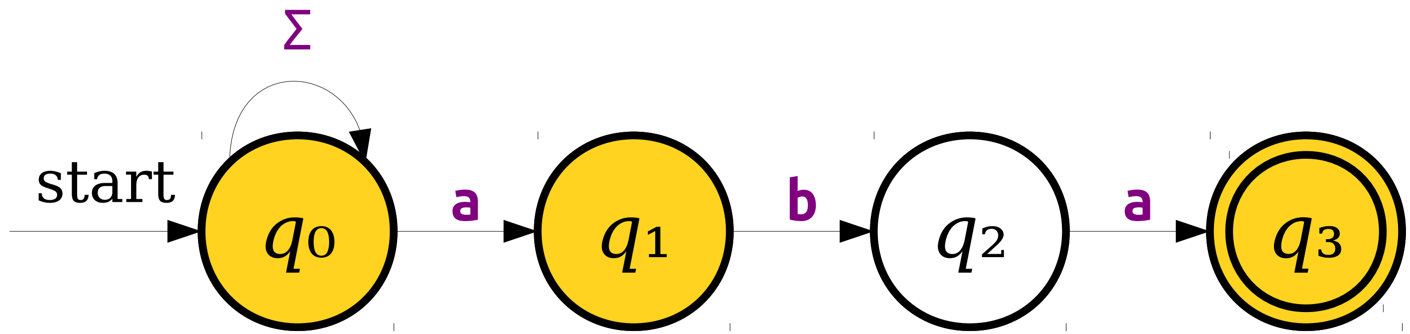
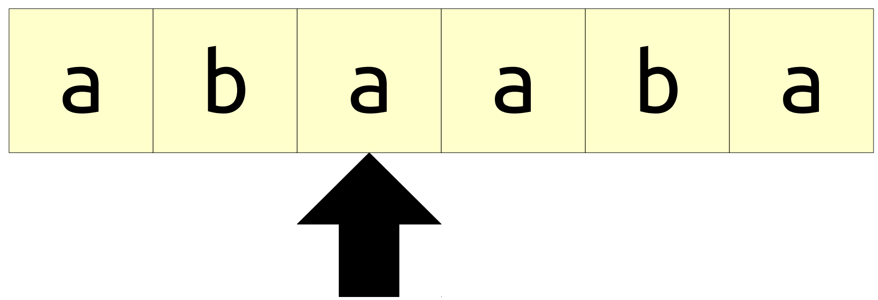
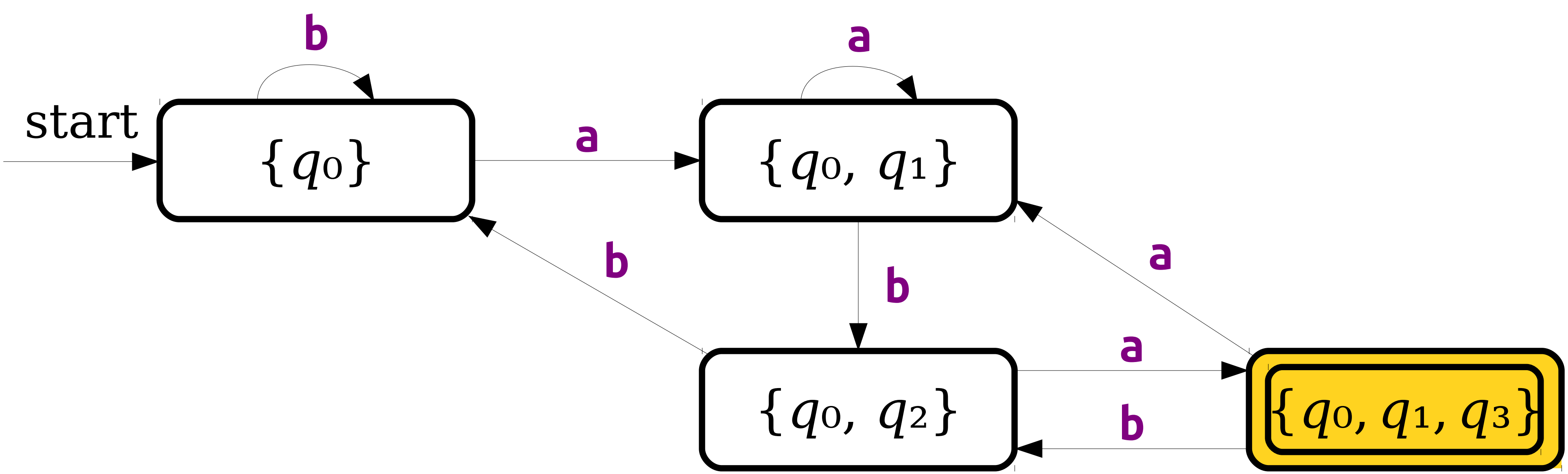
a
{q0, q1}
b
b
a
a
{q0, q2}
b
{q0, q1, q3}



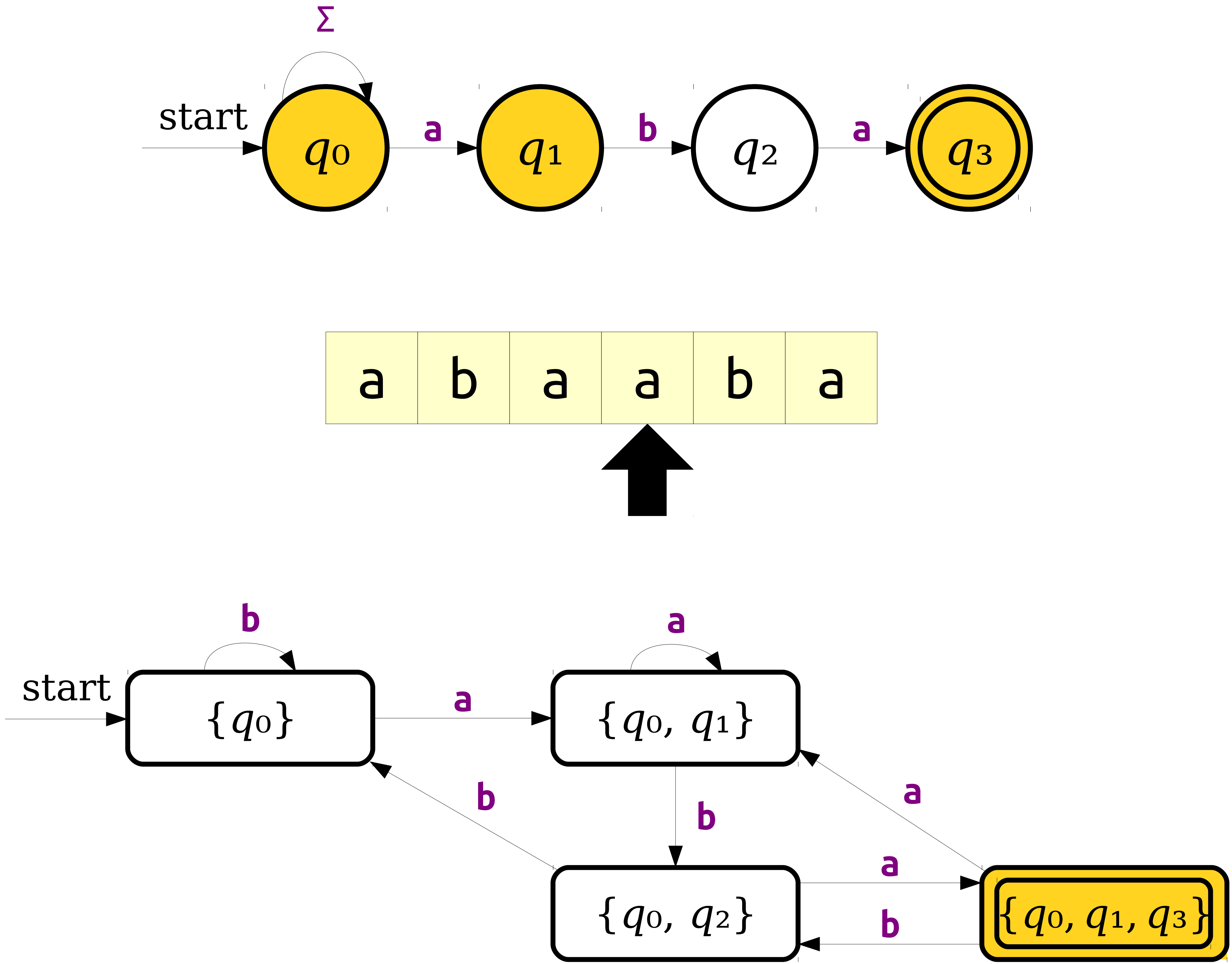
Σ
start
q_0
a
q_1
b
q_2
a
q_3



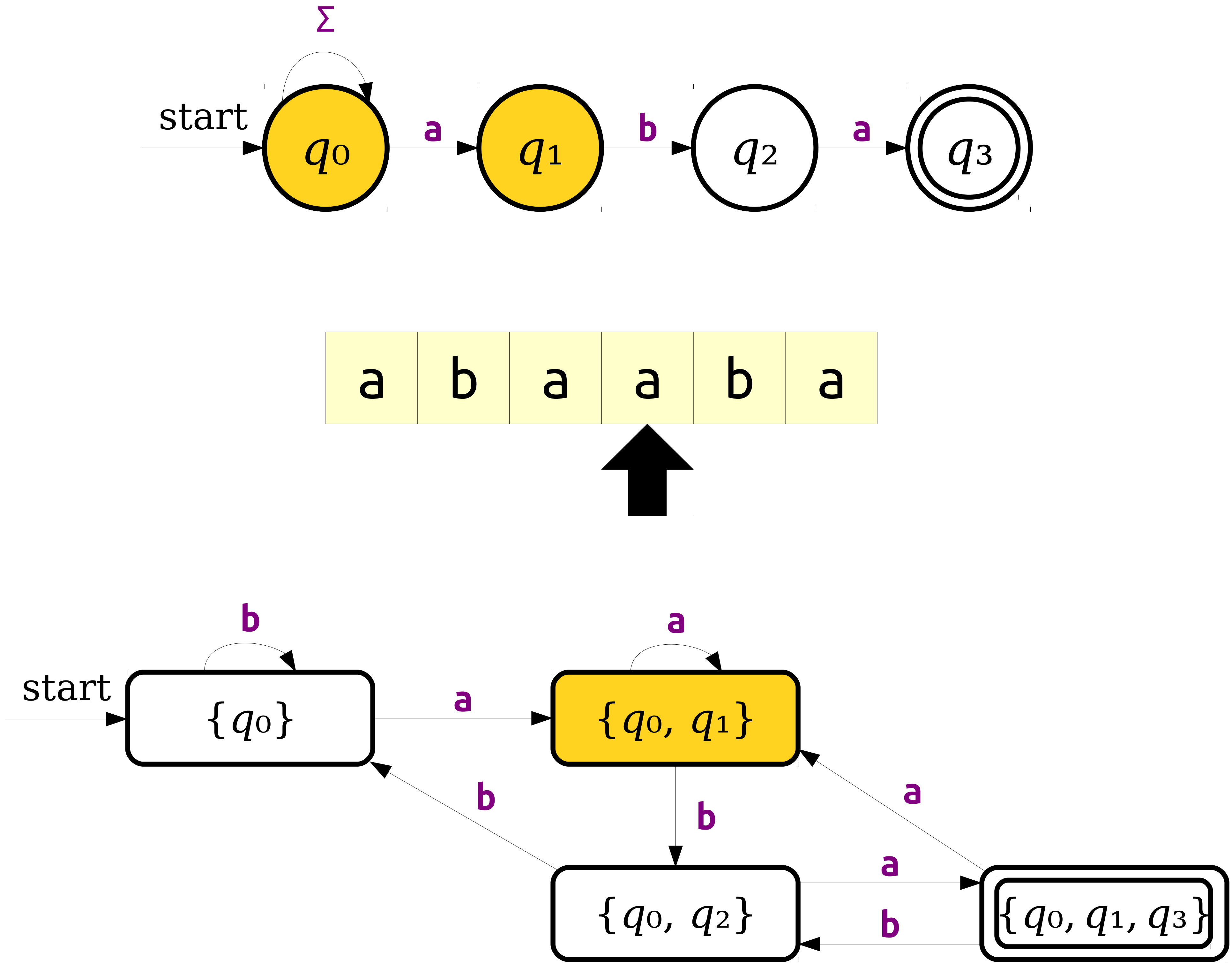
a b a a b a

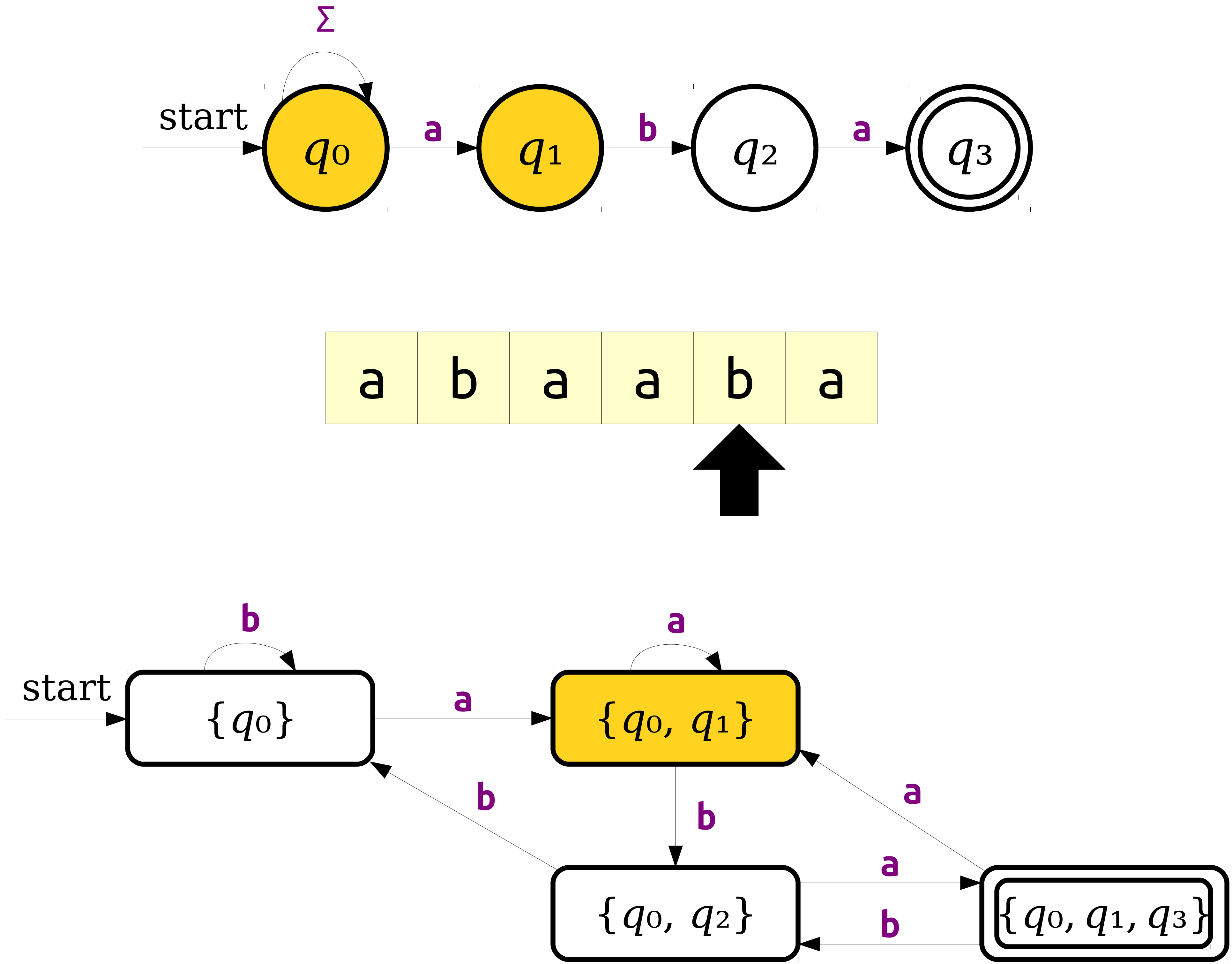


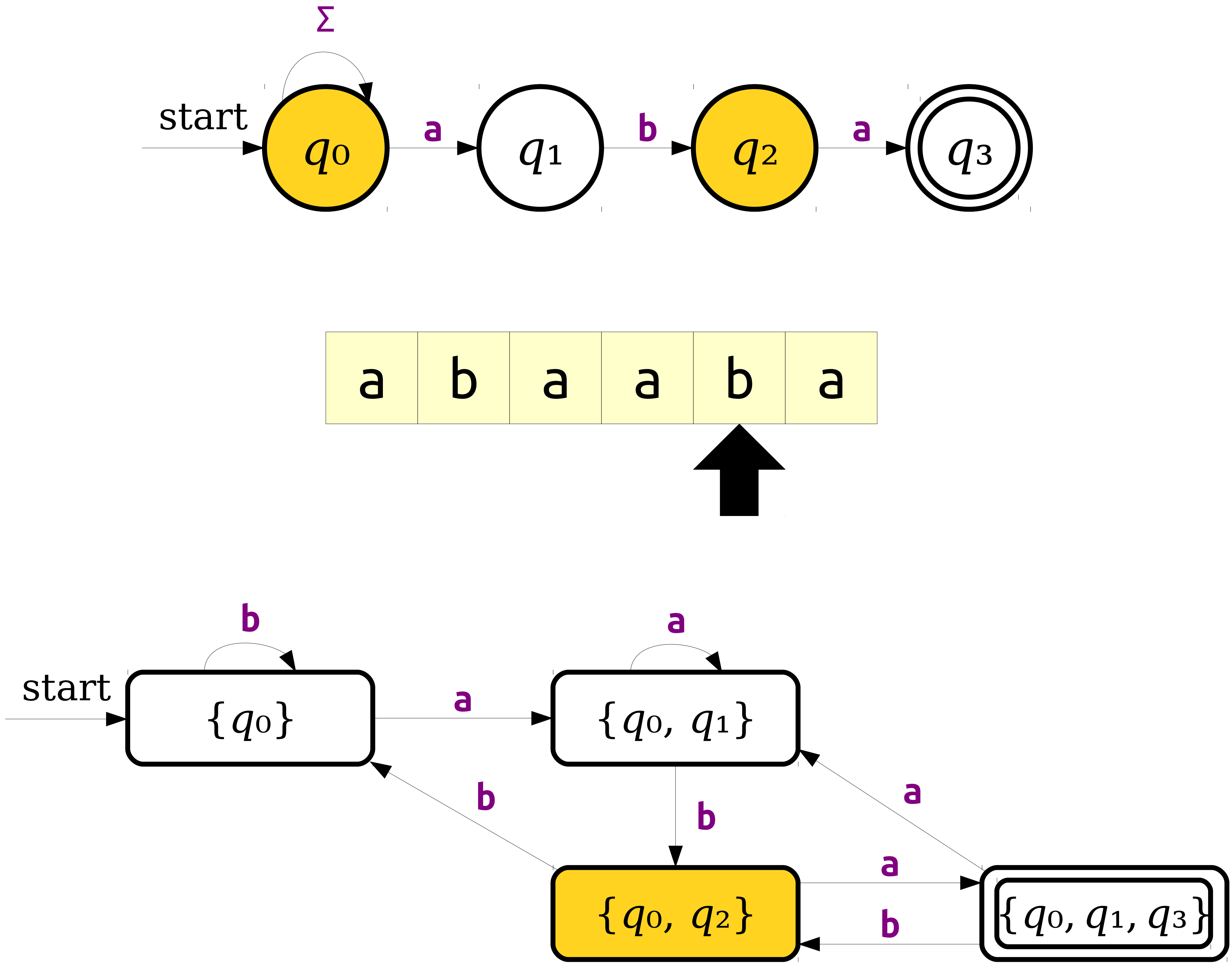
start
b
$\{q_0\}$
a
a
$\{q_0, q_1\}$
b
b
a
$\{q_0, q_2\}$
a
b
$\{q_0, q_1, q_3\}$



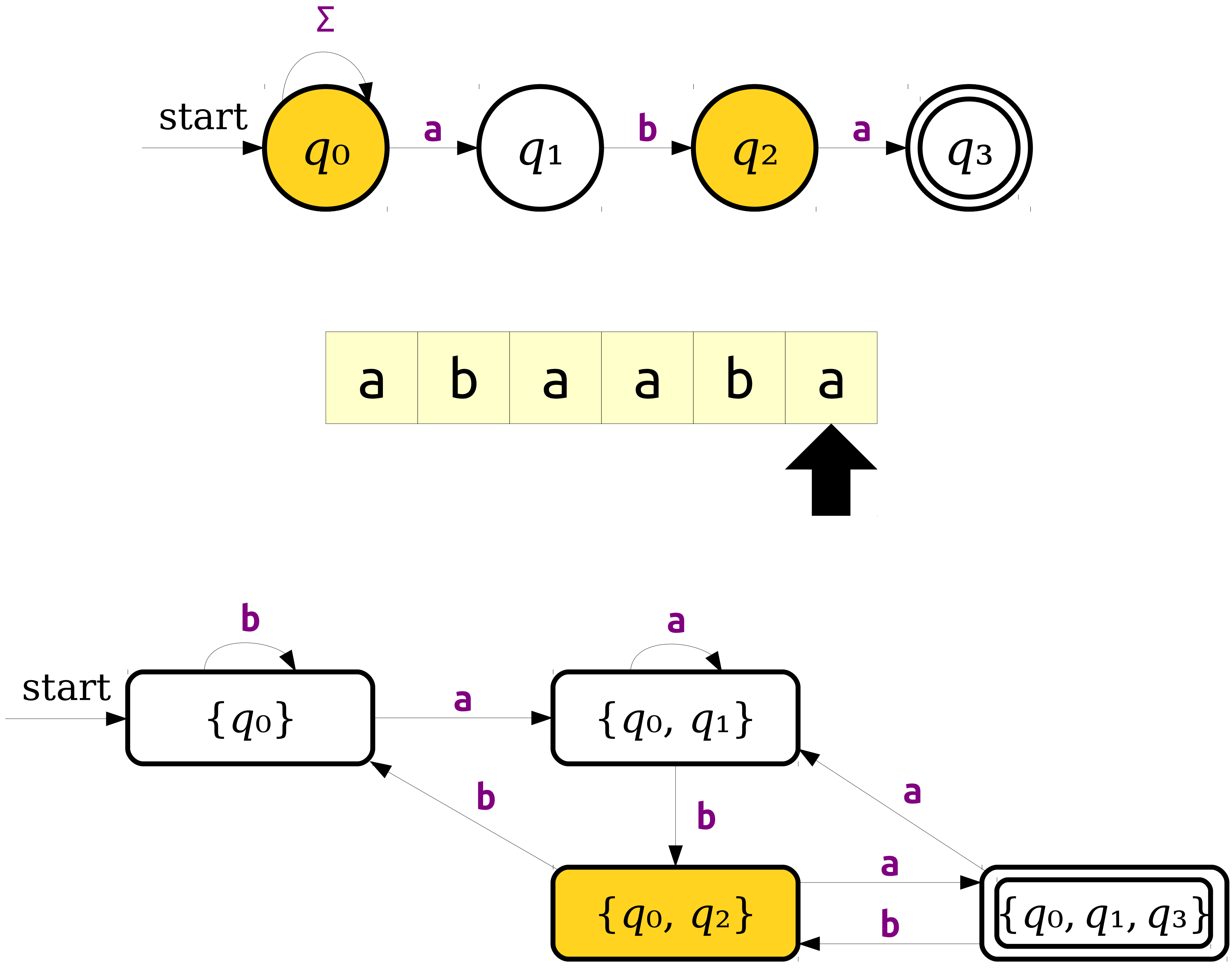
Σ
start
q_0
a
q_1
b
q_2
a
q_3
a b a a b a
b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$


Σ
start
q_0
a
q_1
b
q_2
a
q_3
a b a a b a
b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$

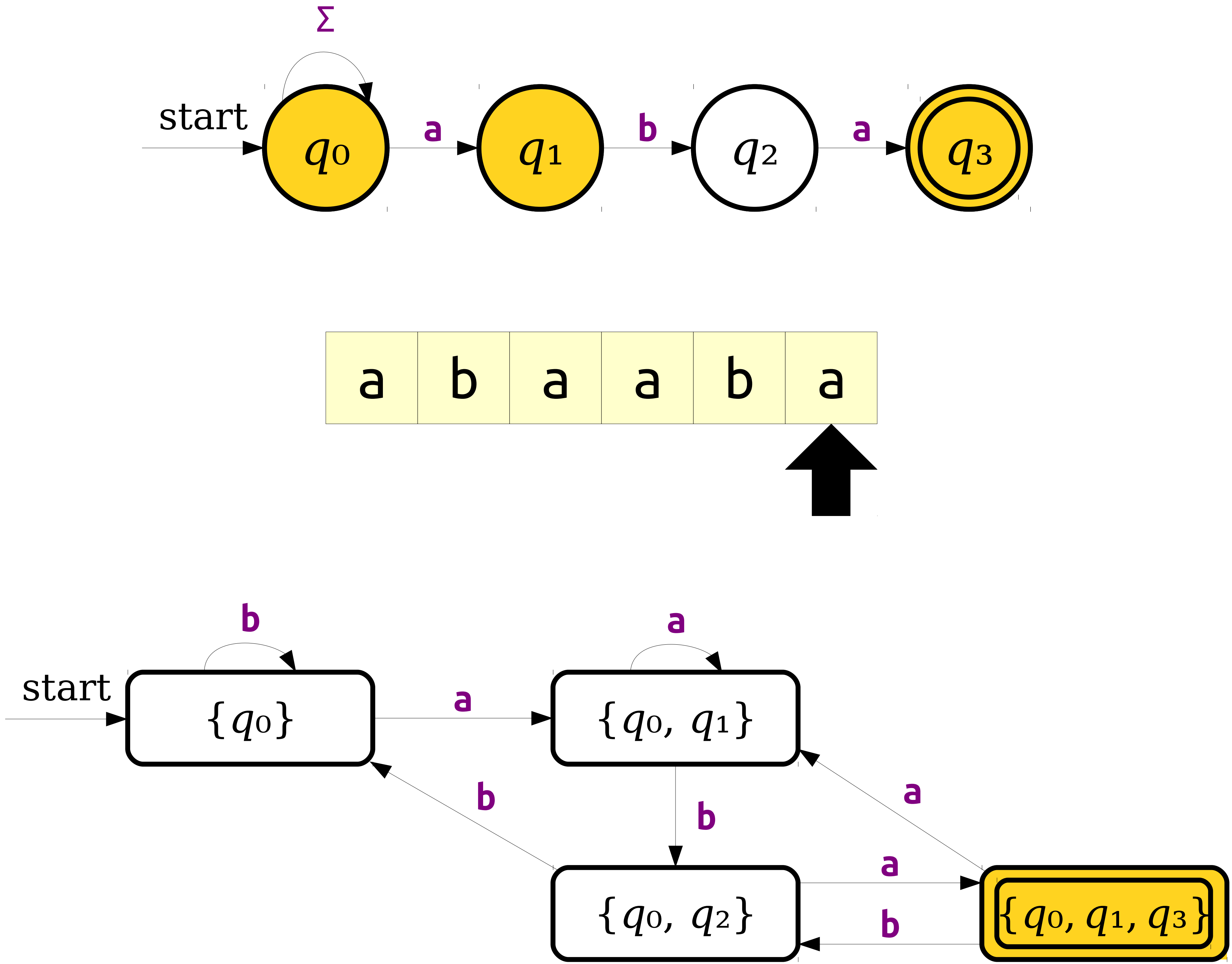

Σ
start
q_0
a
q_1
b
q_2
a
q_3
a b a a b a
b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
b
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$



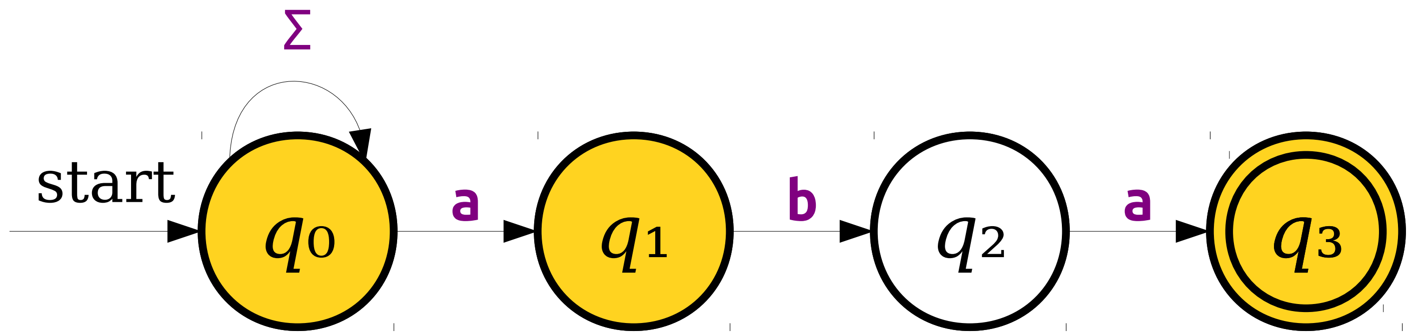
Σ
start
q_0
a
q_1
b
q_2
a
q_3
a b a a b a
b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
b
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$



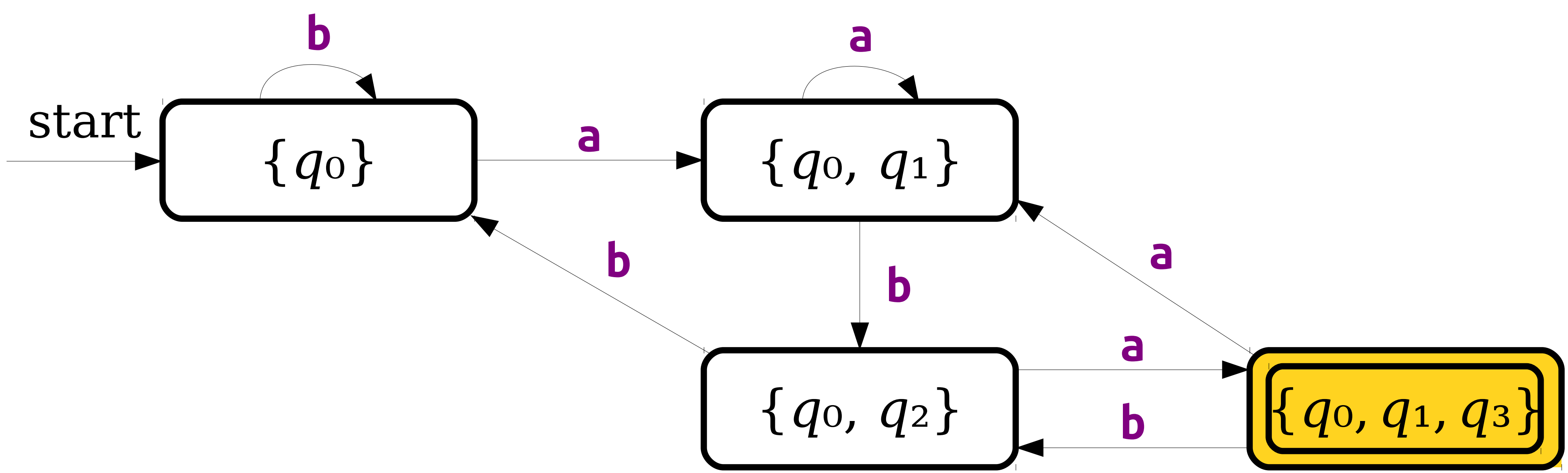
Σ
start
q_0
a
q_1
b
q_2
a
q_3
a b a a b a
b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$



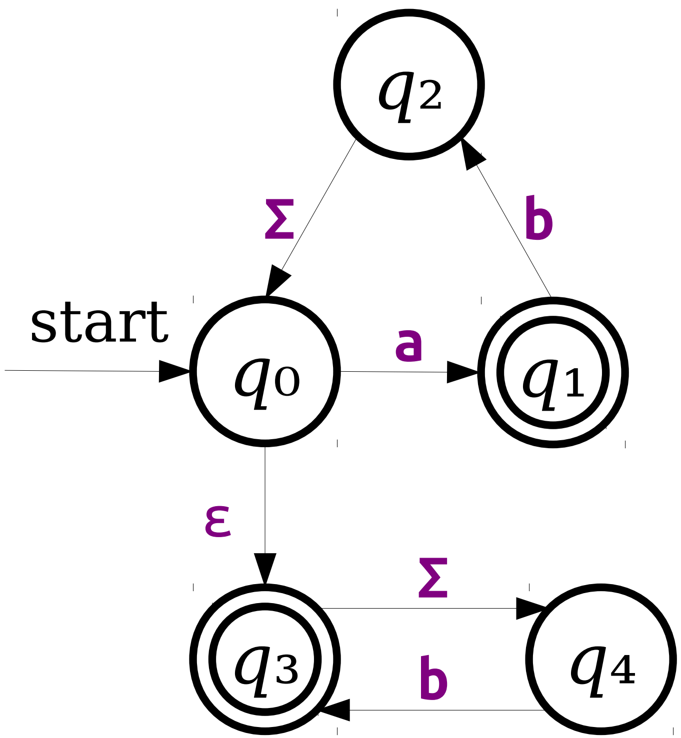
Σ
start
q_0
a
q_1
b
q_2
a
q_3
a b a a b a
b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
$\{q_0, q_2\}$
a
b
$\{q_0, q_1, q_3\}$



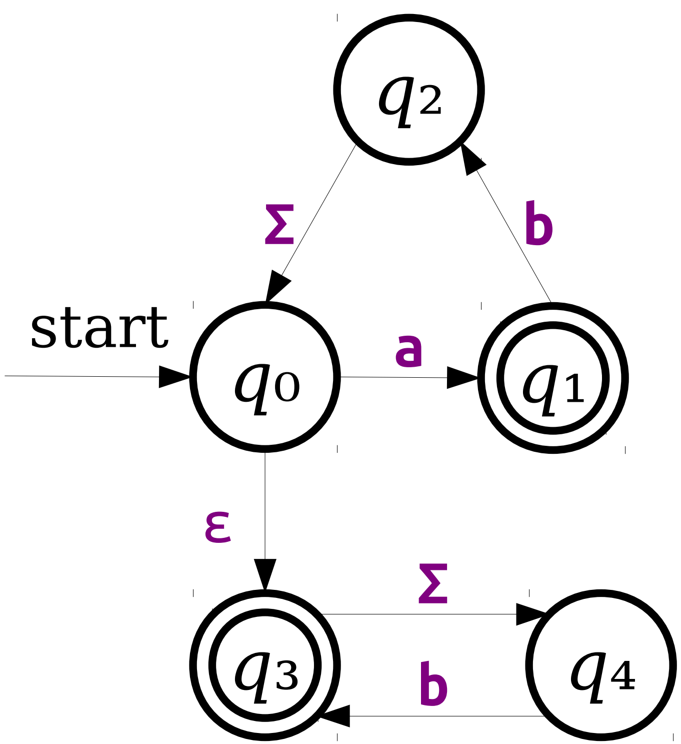
a	b	a	a	b	a



Once More, With Epsilons!

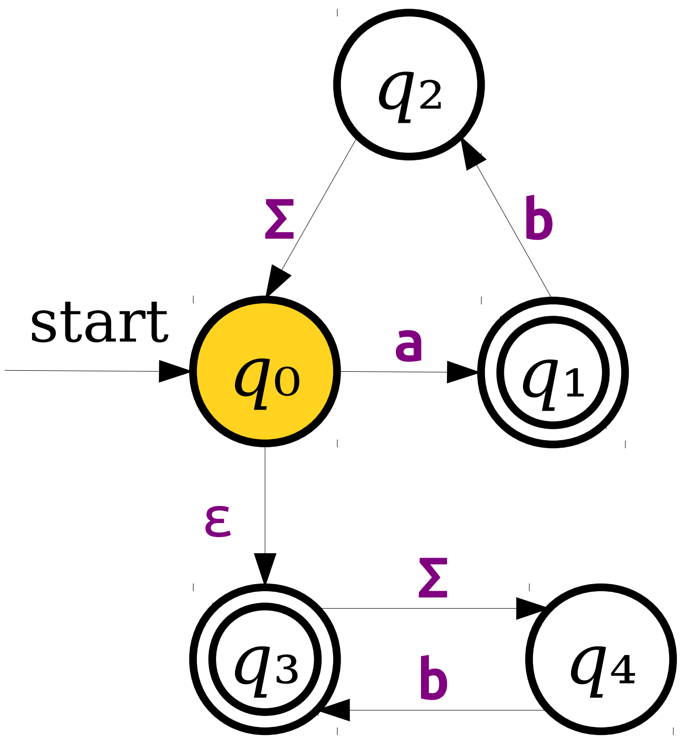


Once More, With Epsilons!



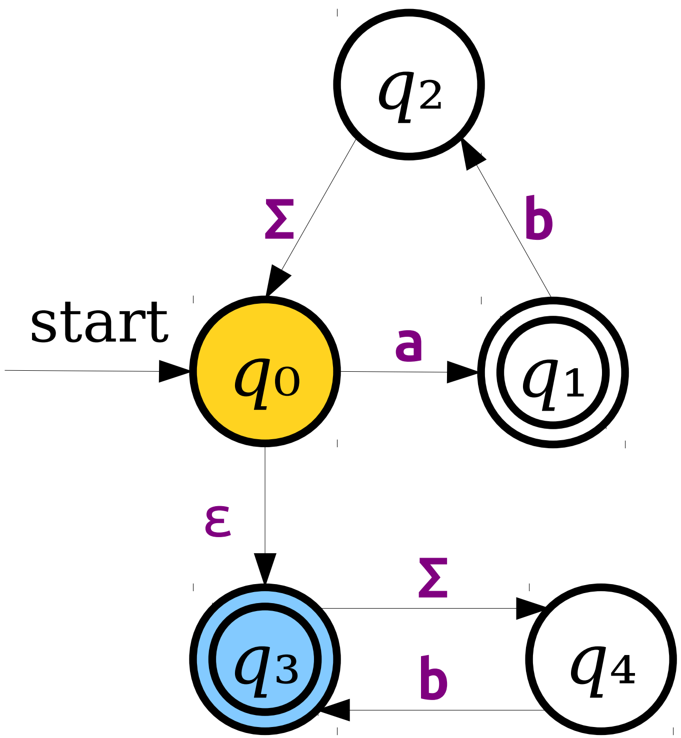
	a	b

Once More, With Epsilons!



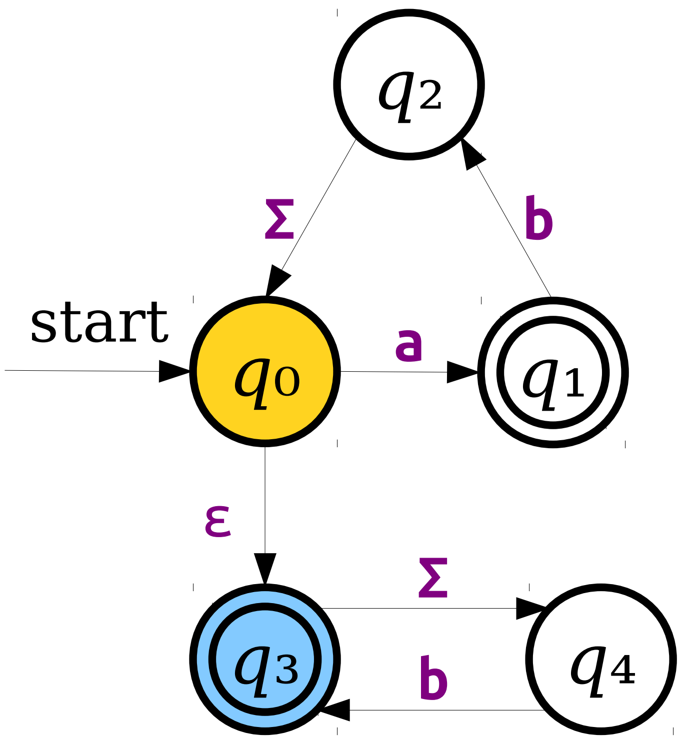
	a	b

Once More, With Epsilons!



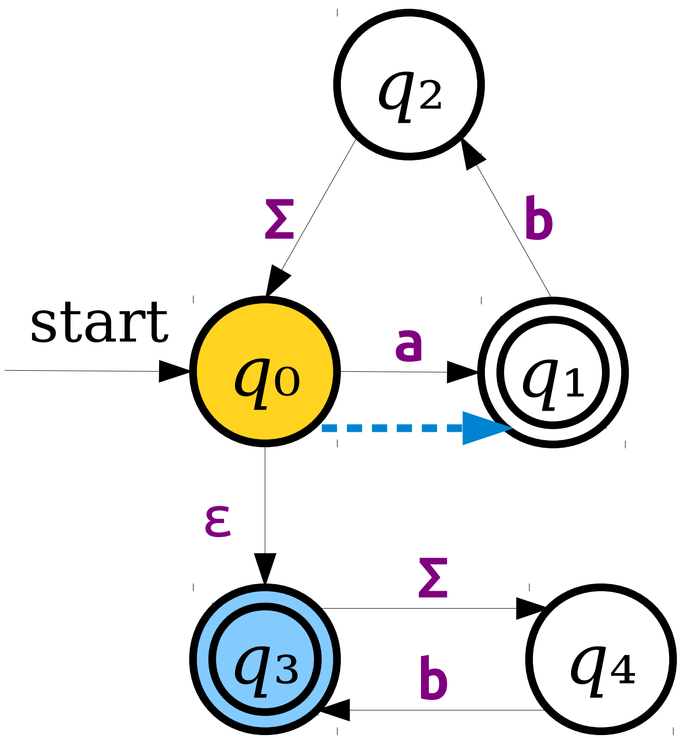
	a	b

Once More, With Epsilons!



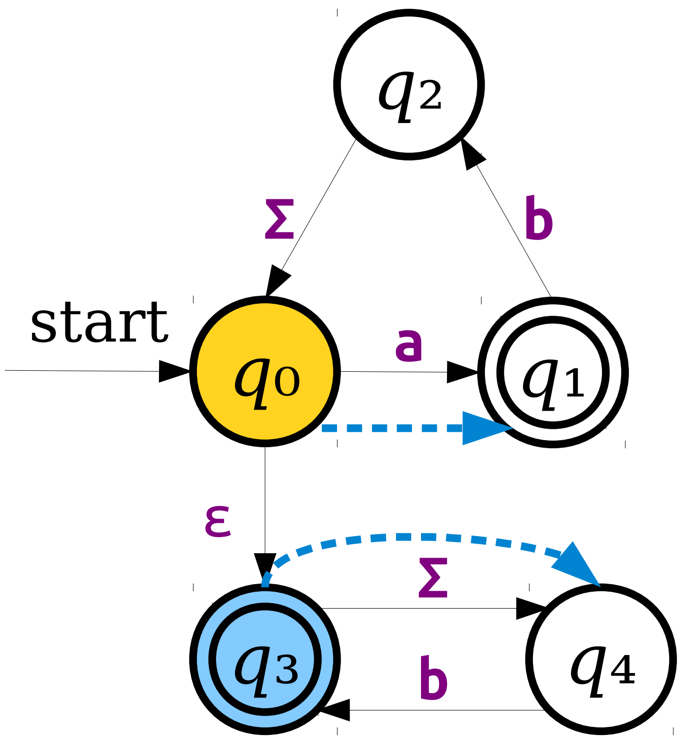
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



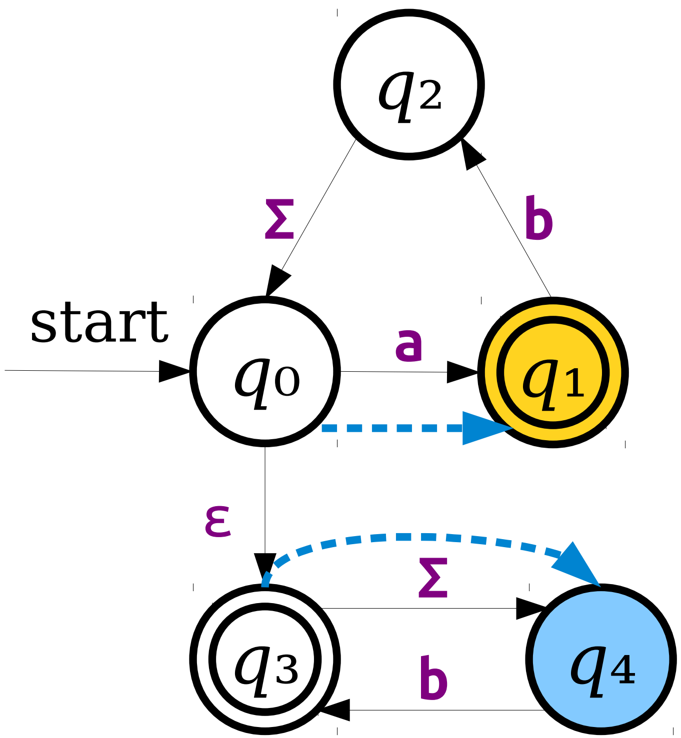
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



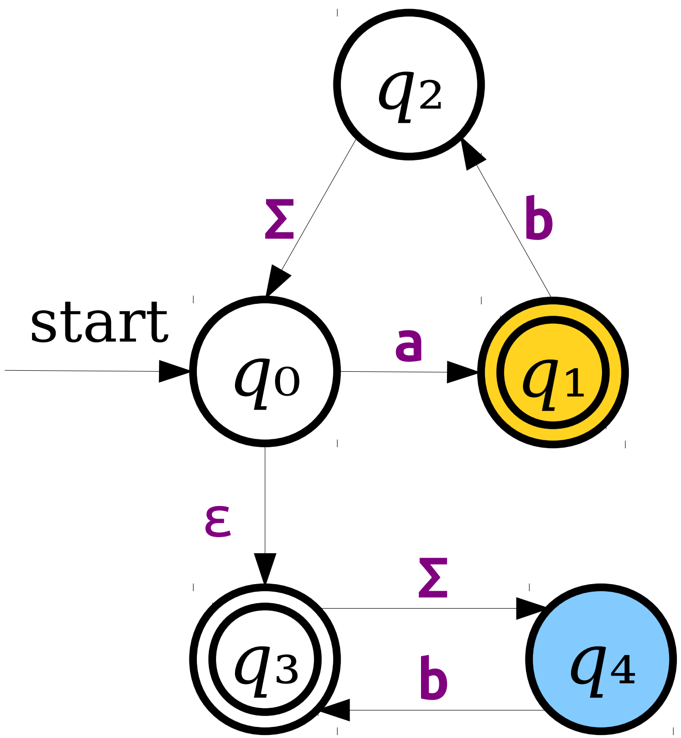
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



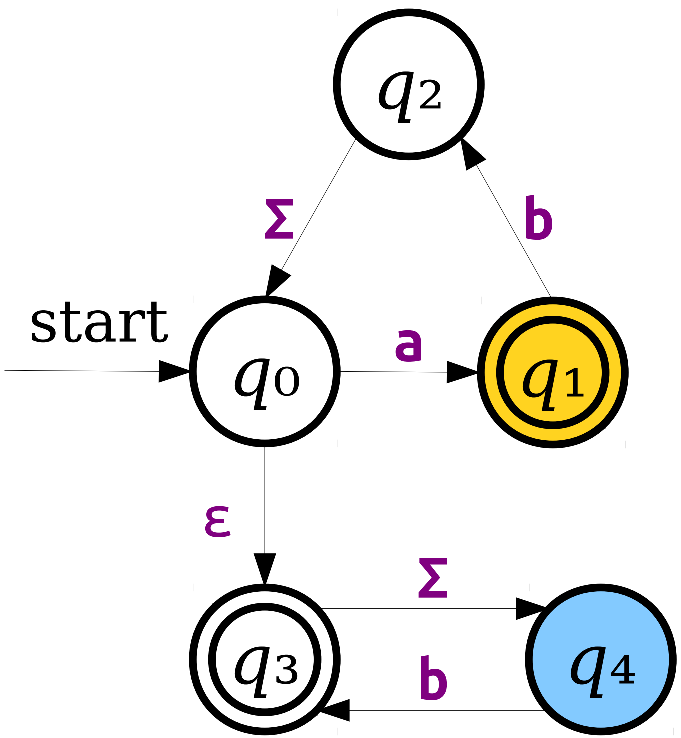
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



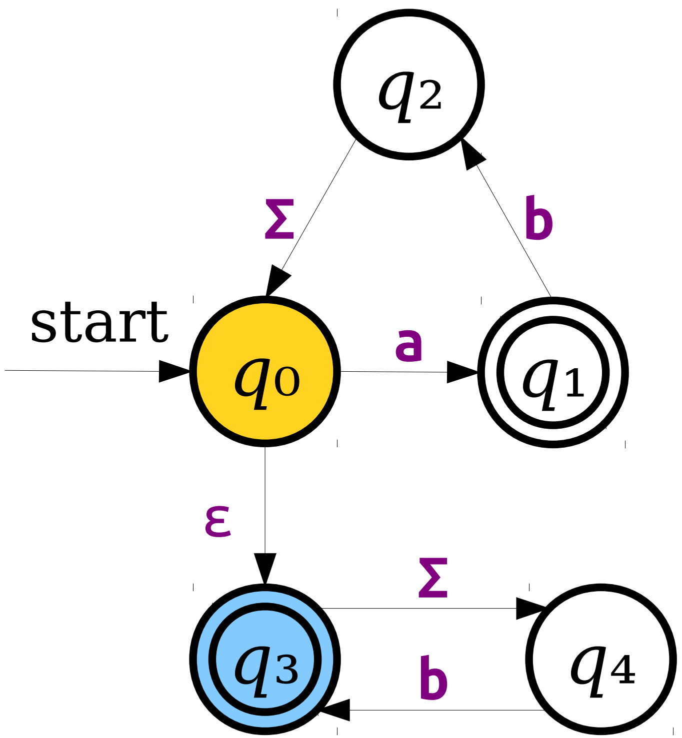
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



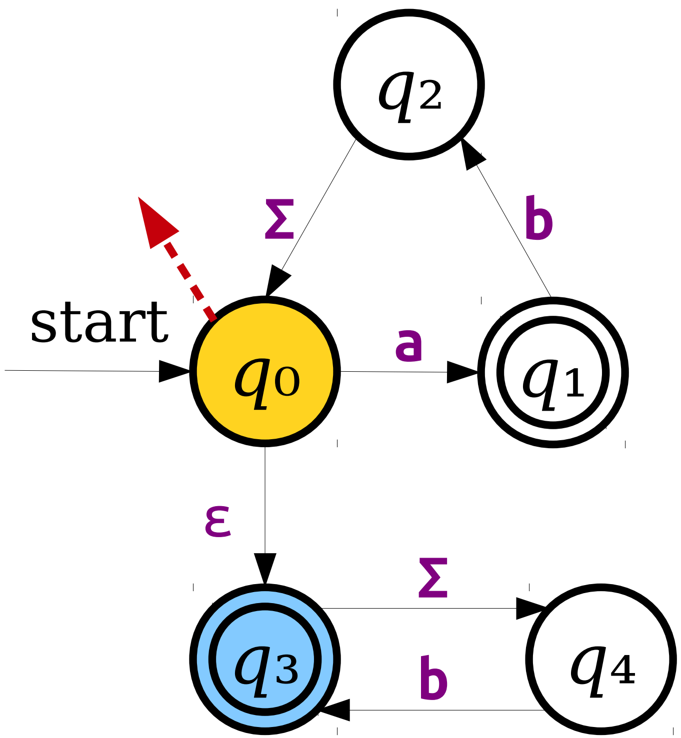
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



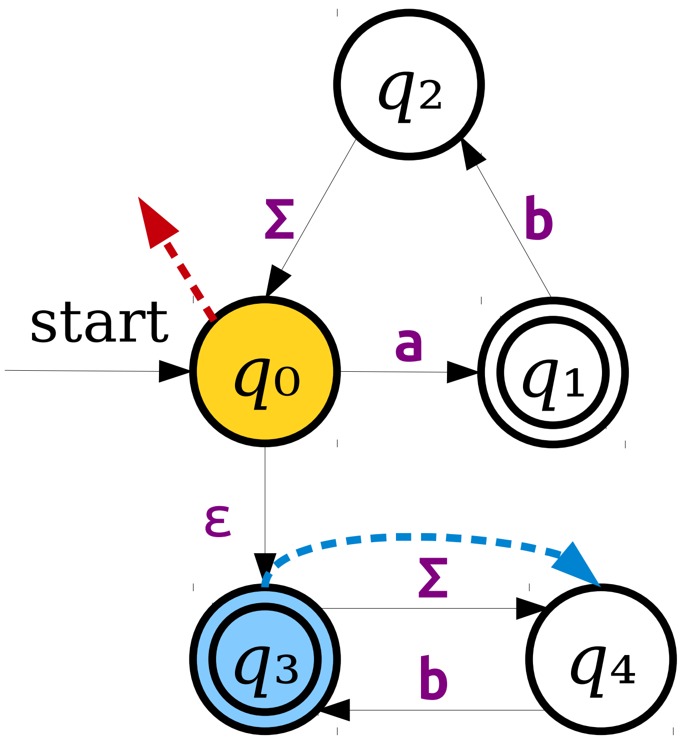
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



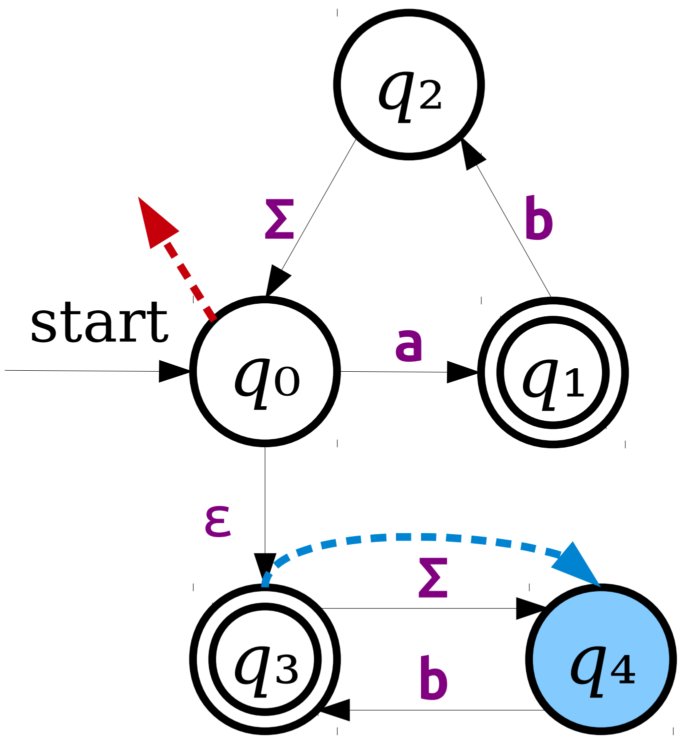
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



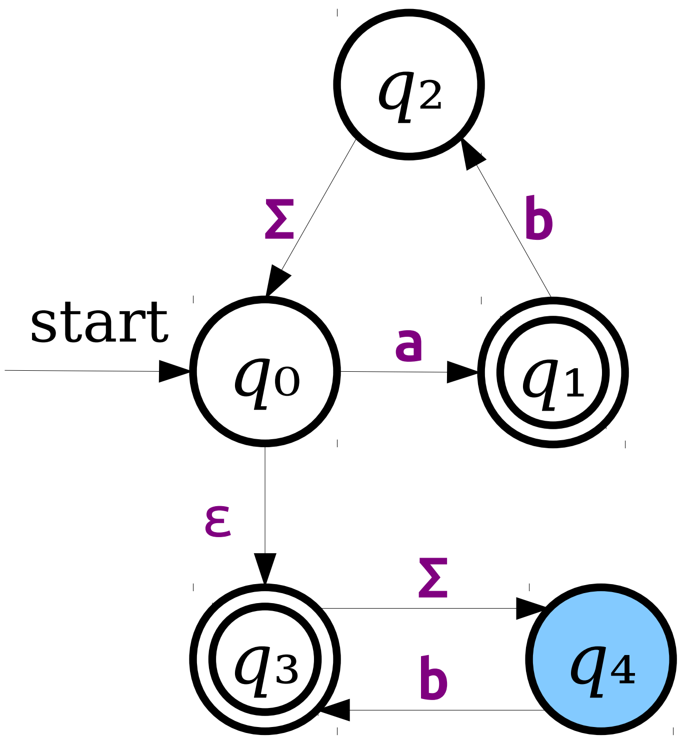
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



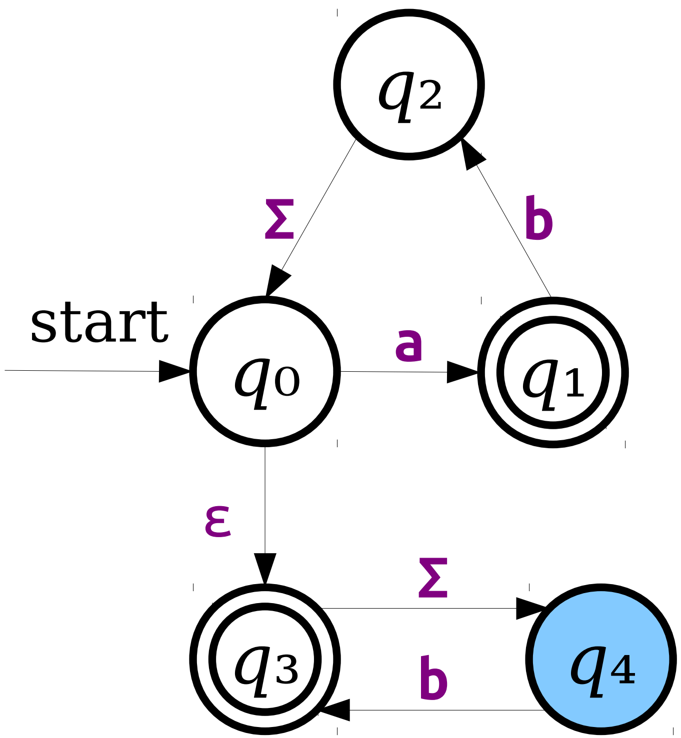
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



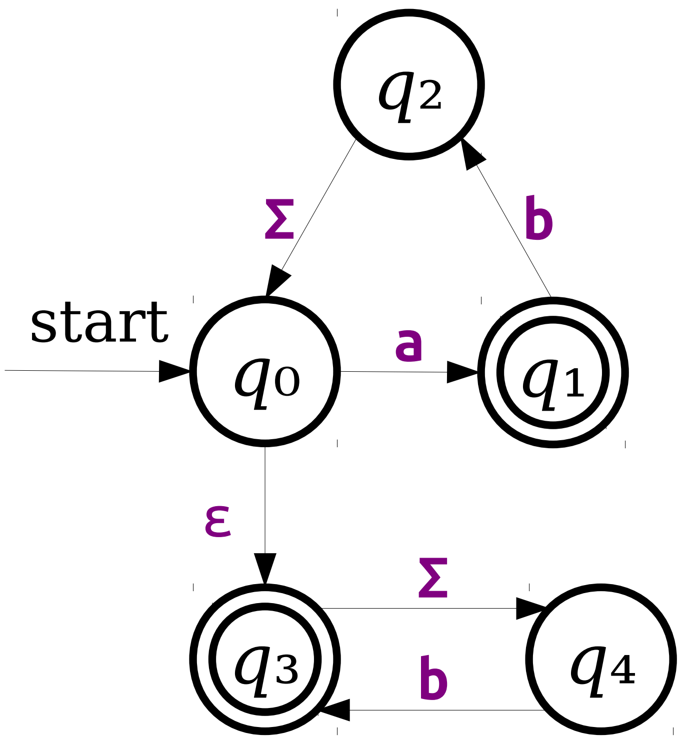
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



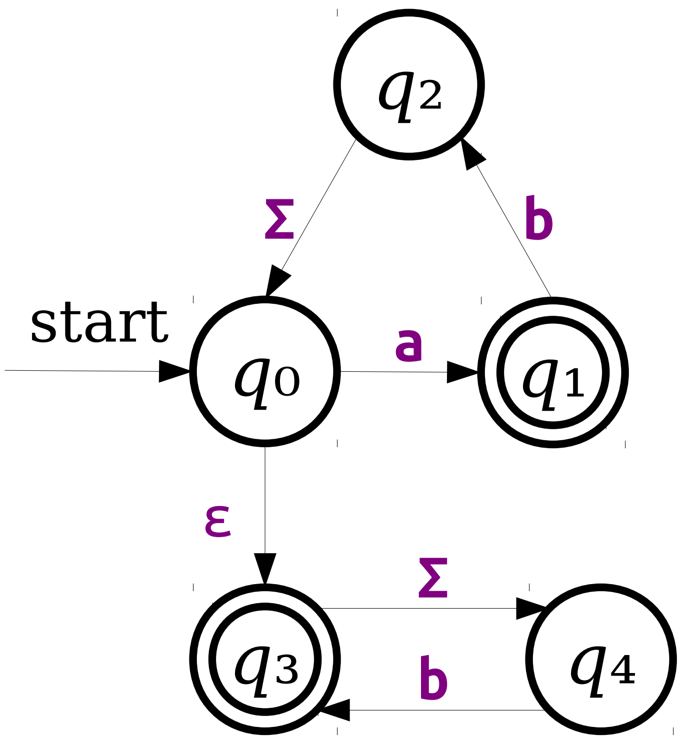
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$

Once More, With Epsilons!



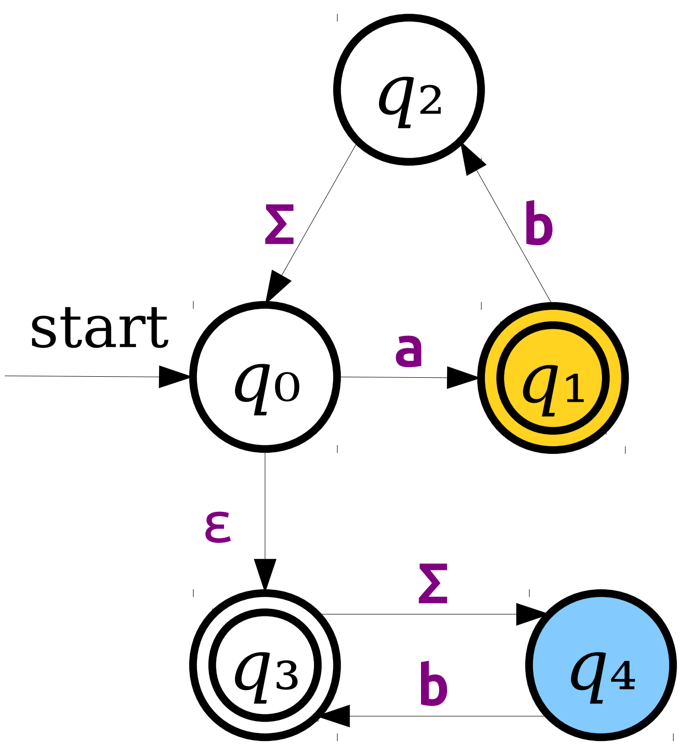
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$

Once More, With Epsilons!



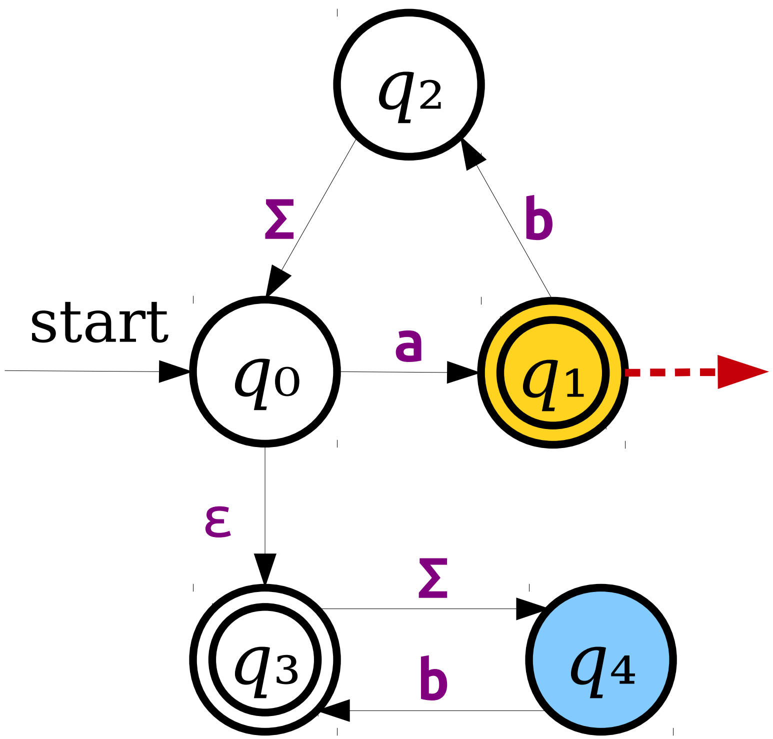
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



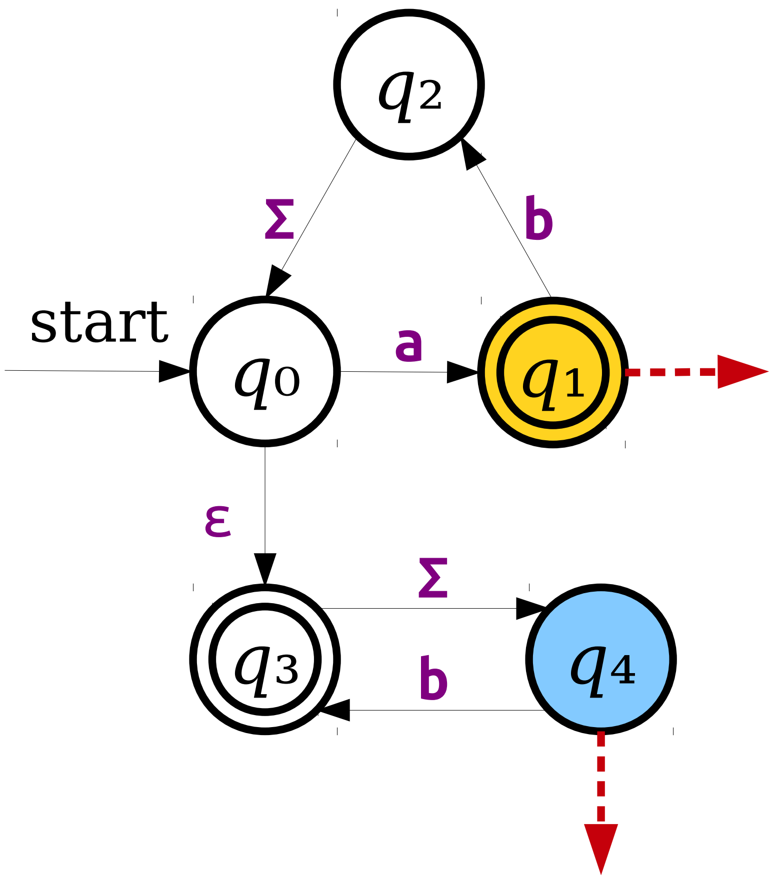
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



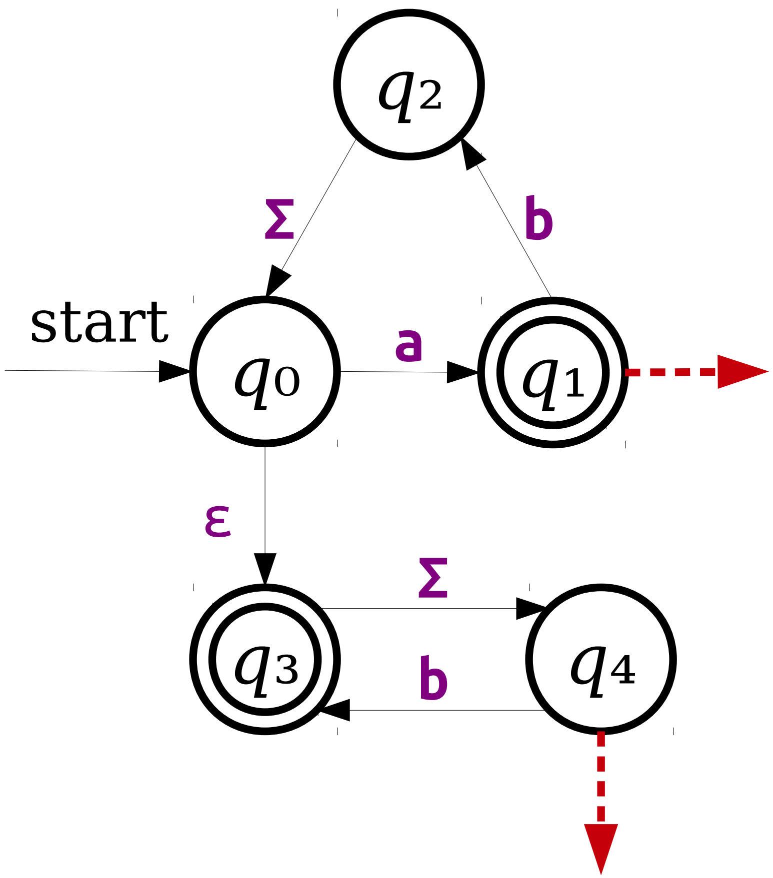
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



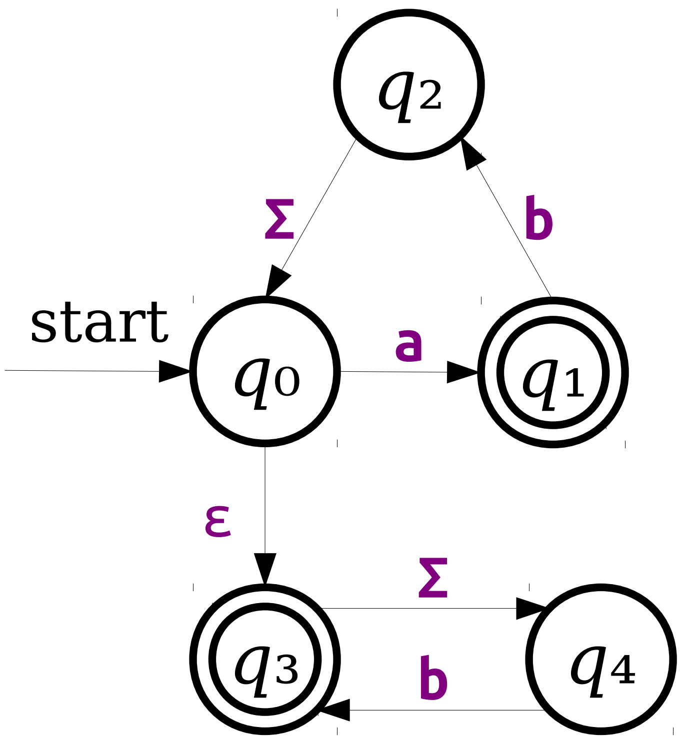
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



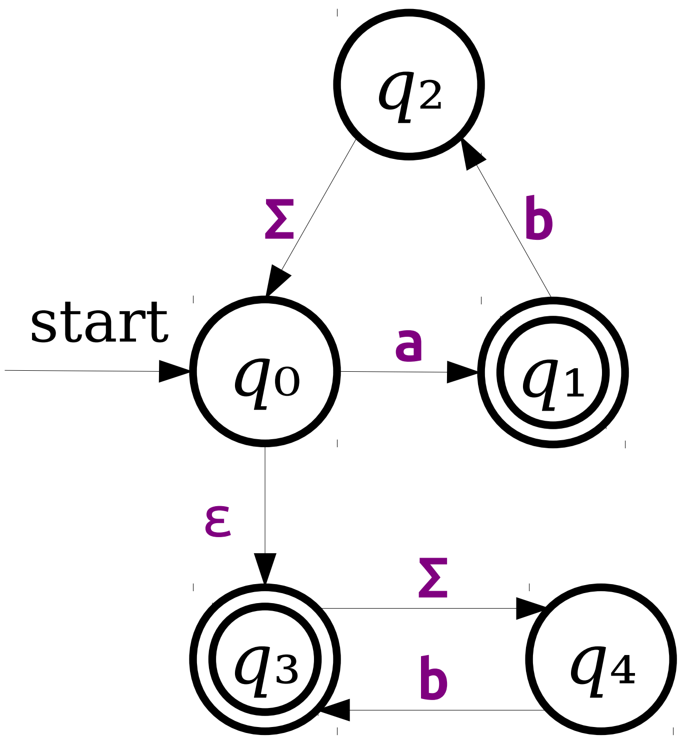
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



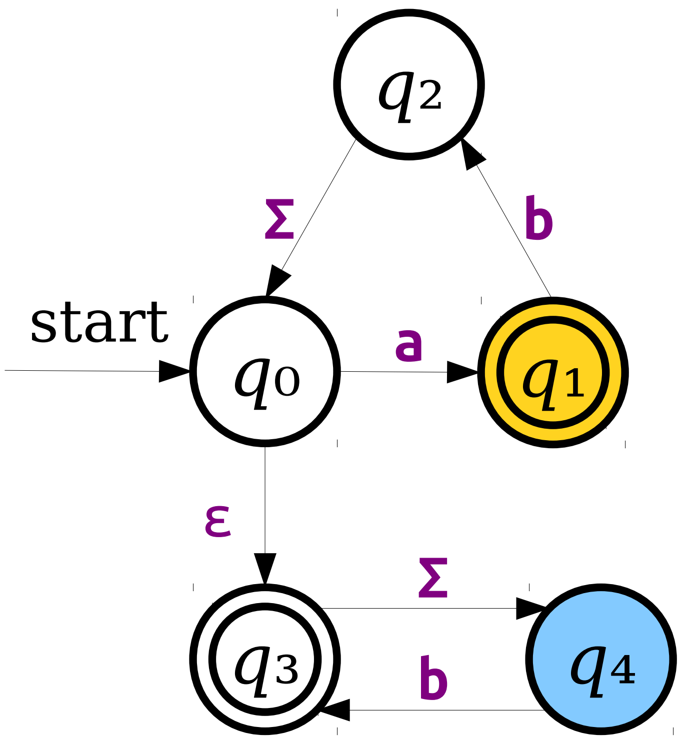
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



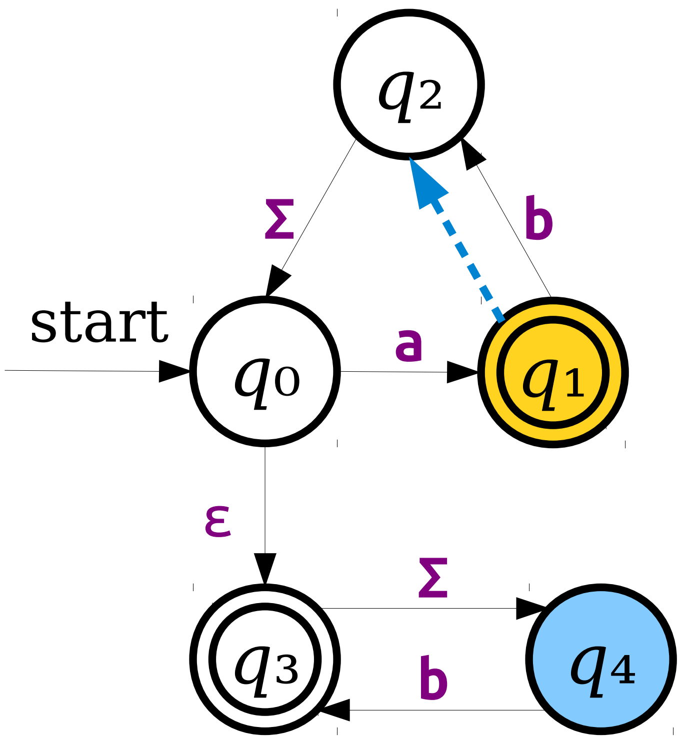
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



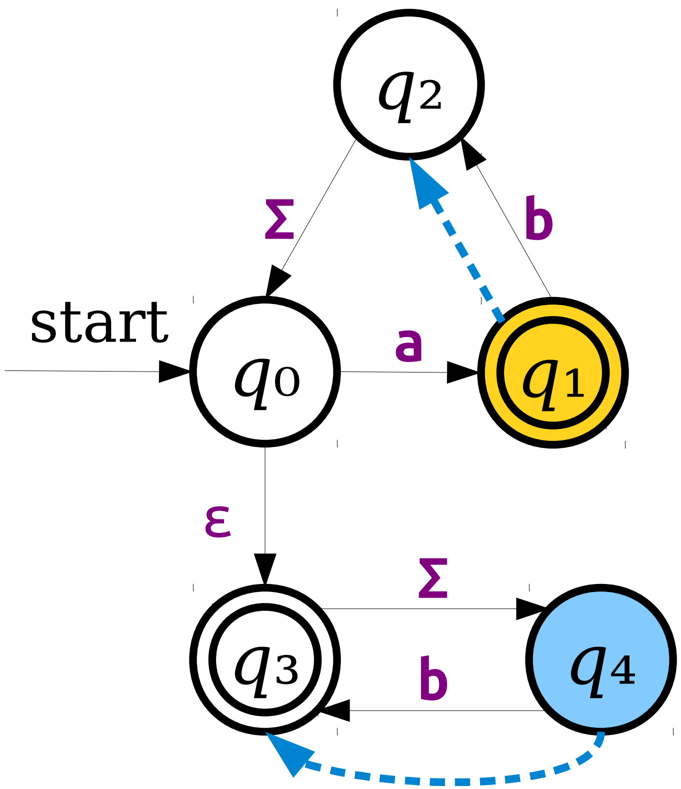
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	

Once More, With Epsilons!



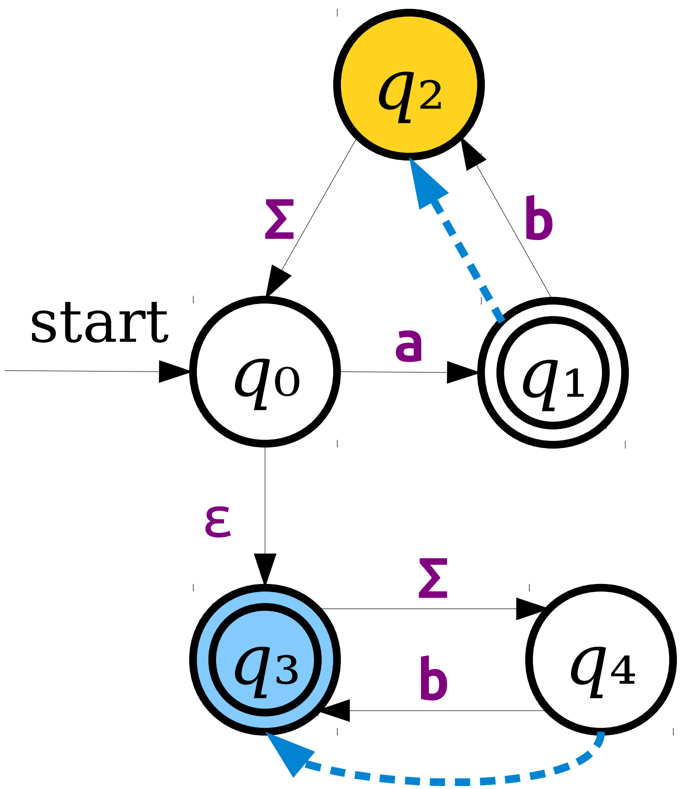
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



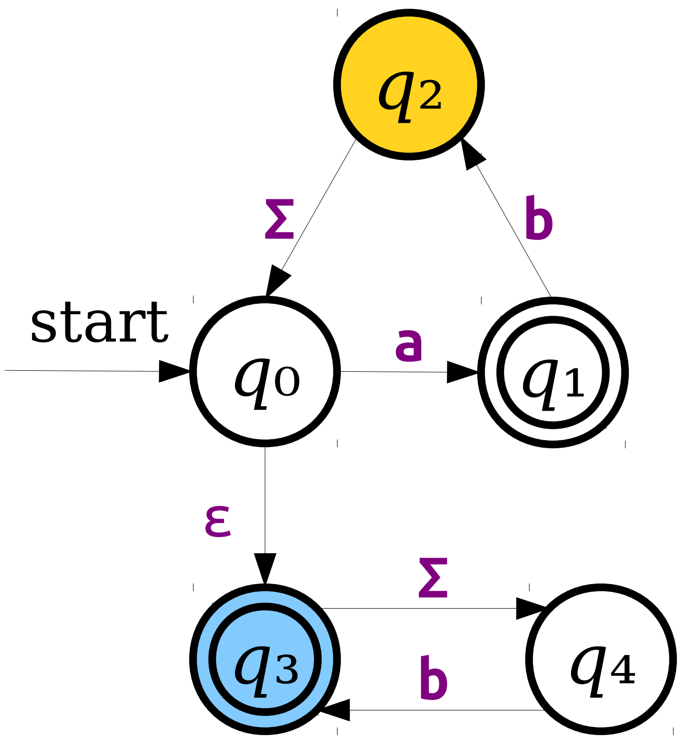
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	

Once More, With Epsilons!



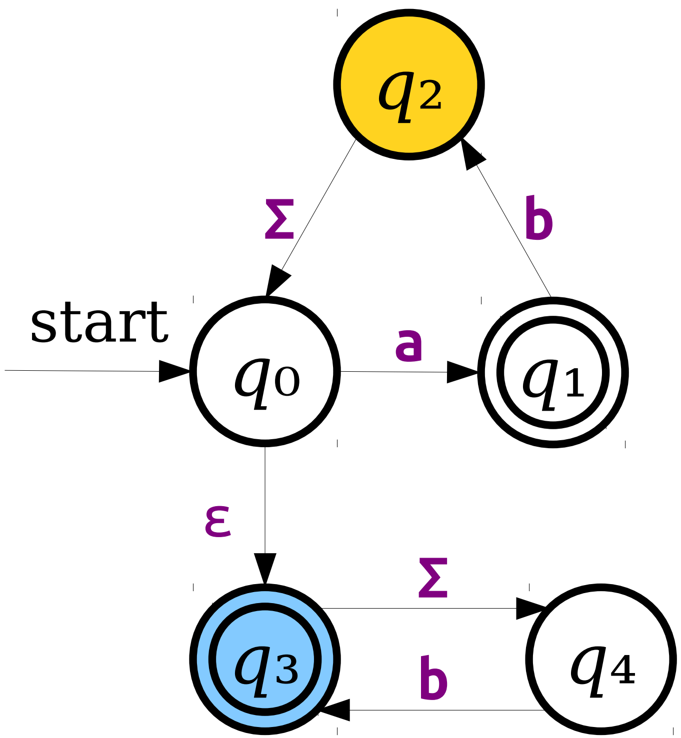
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	

Once More, With Epsilons!



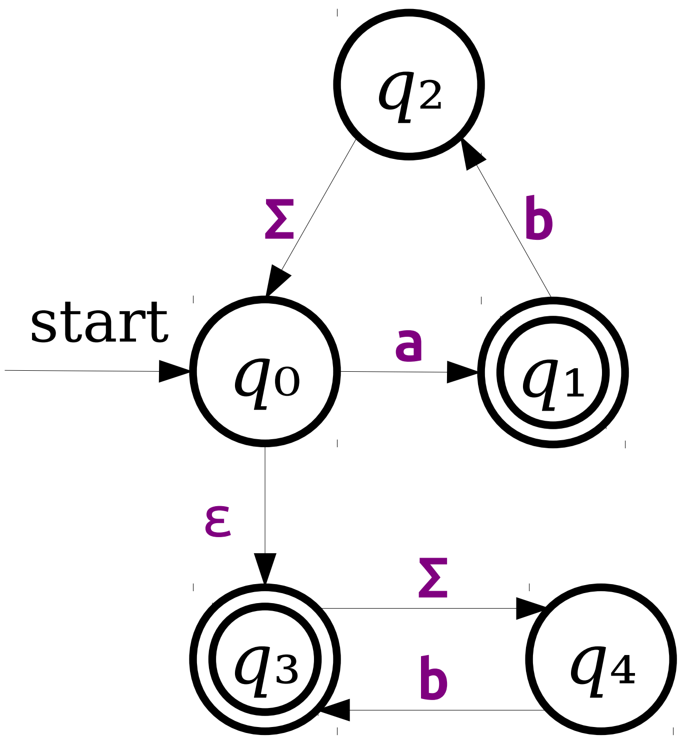
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	

Once More, With Epsilons!



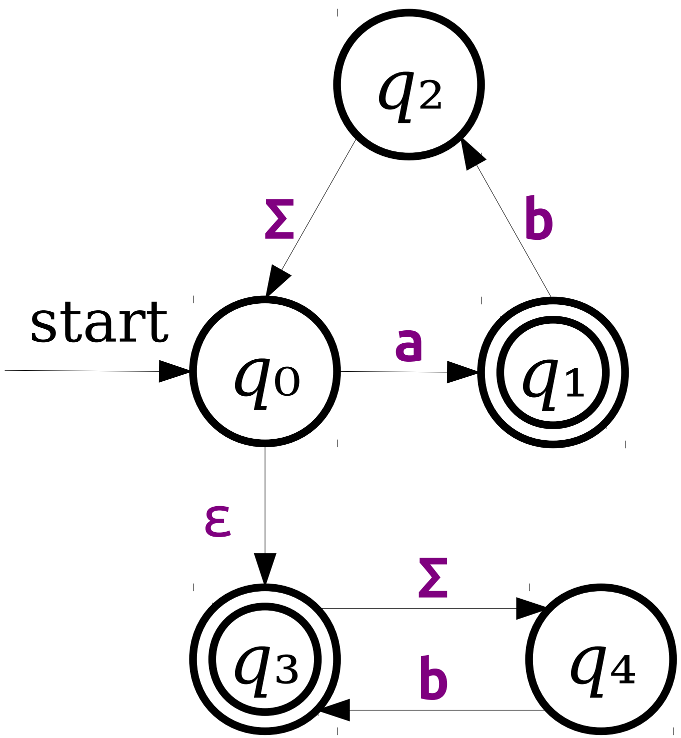
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$

Once More, With Epsilons!



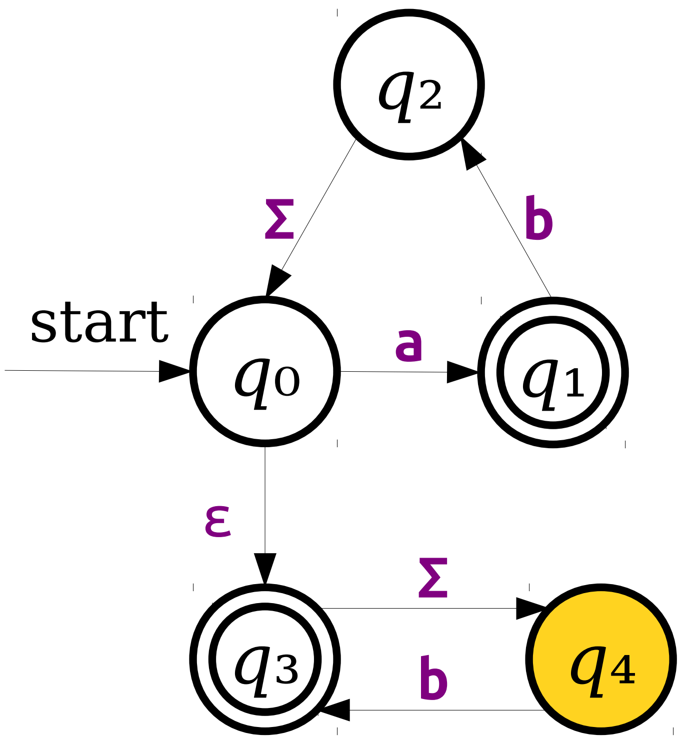
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$

Once More, With Epsilons!



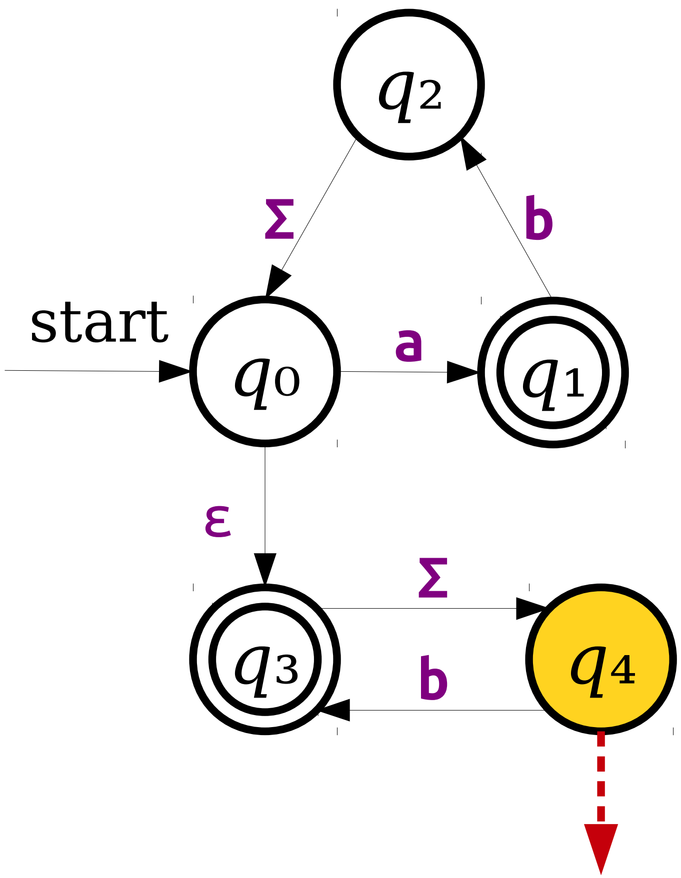
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



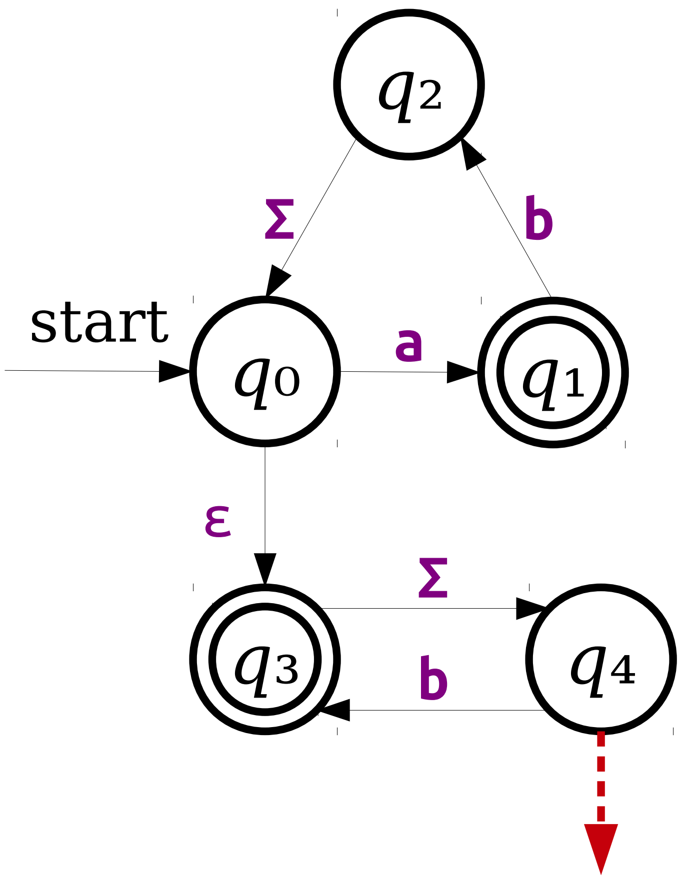
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



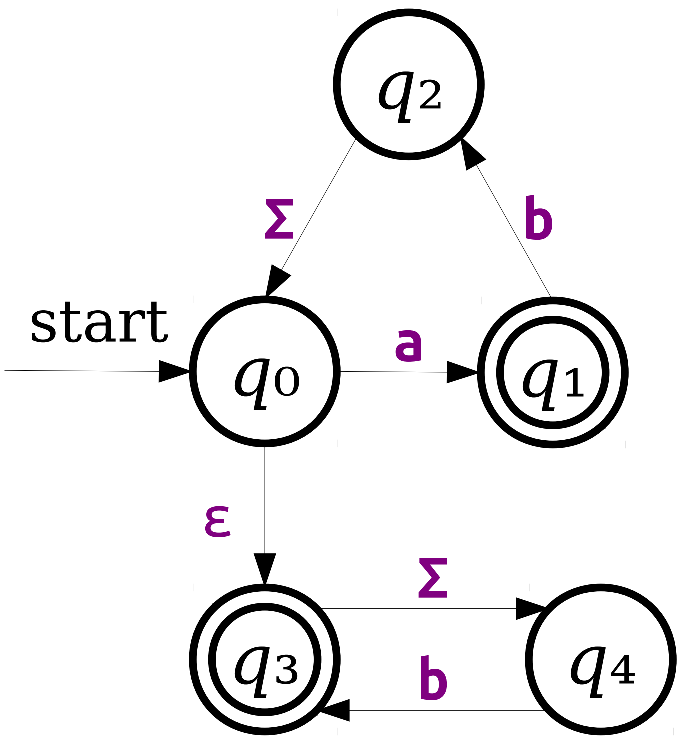
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



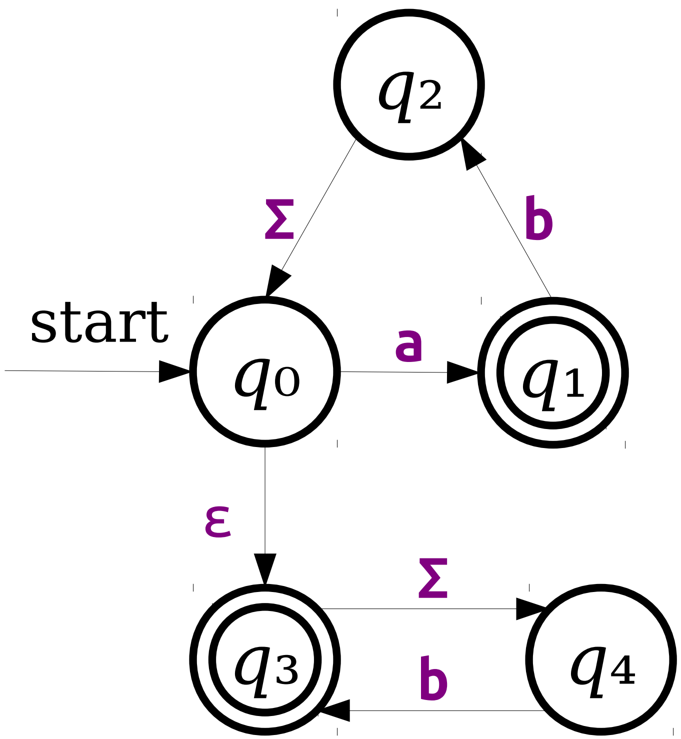
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



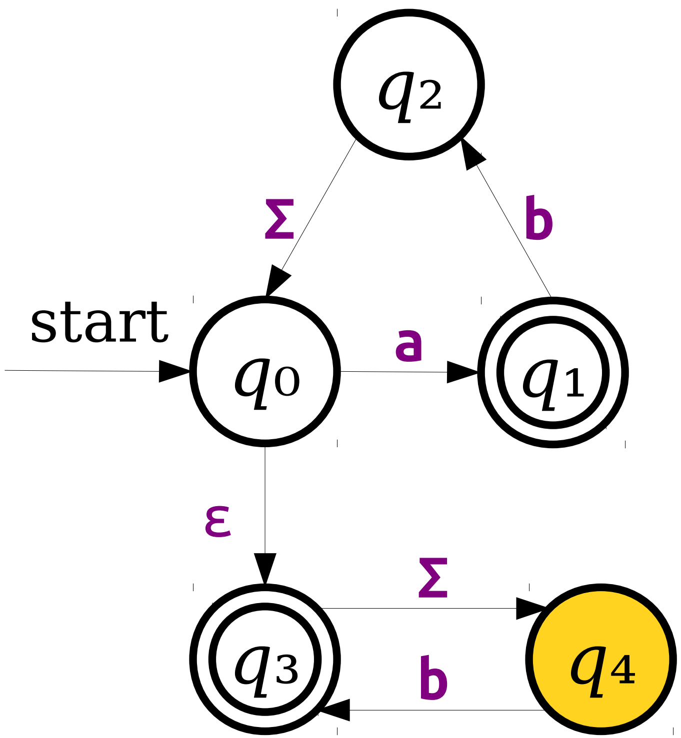
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



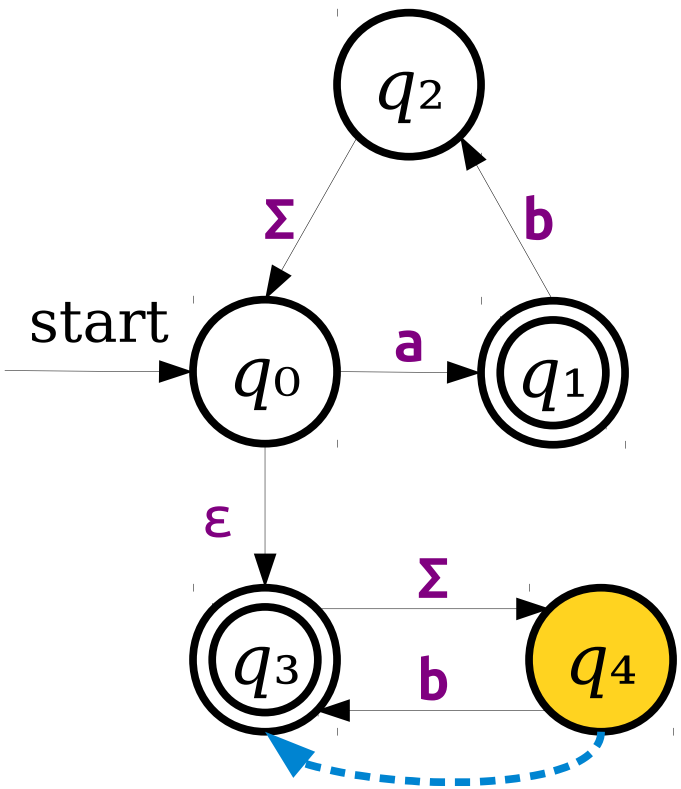
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	

Once More, With Epsilons!



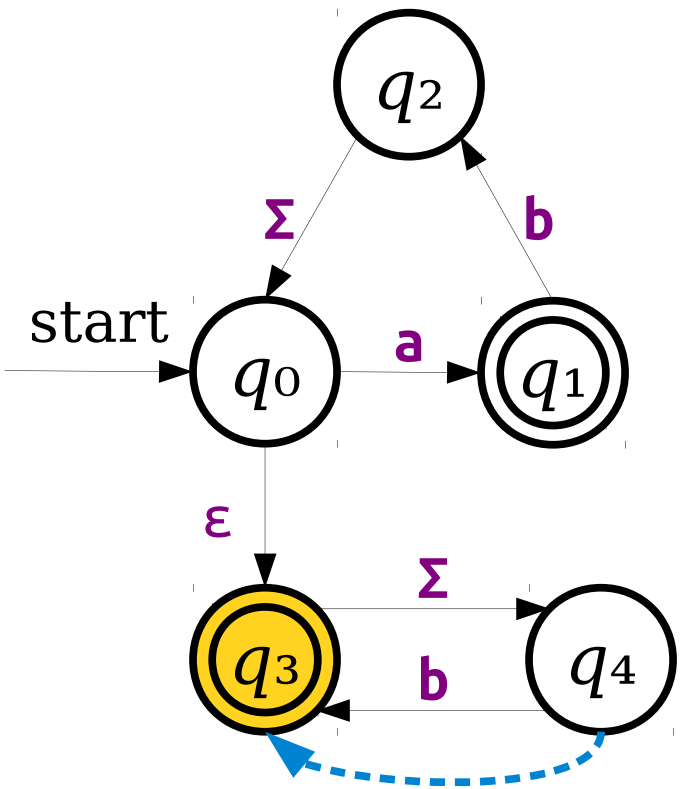
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	

Once More, With Epsilons!



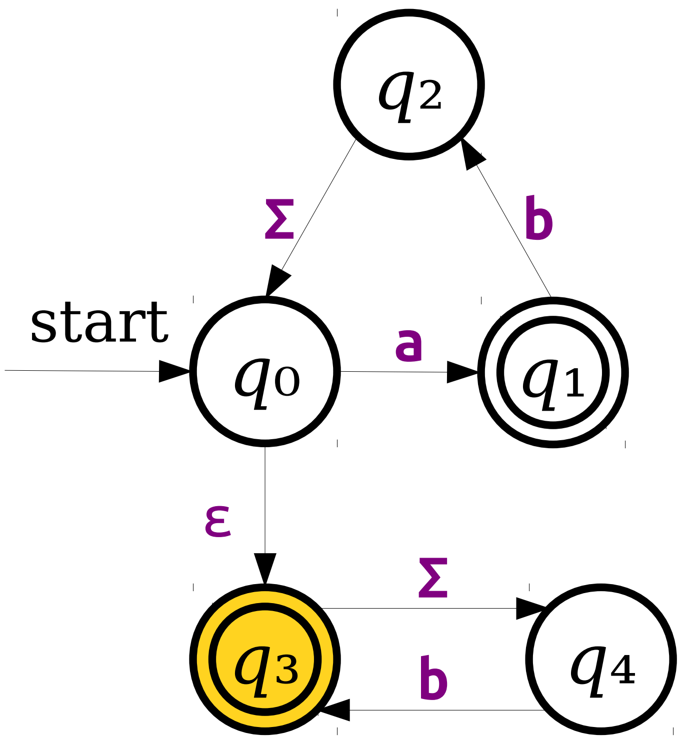
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	

Once More, With Epsilons!



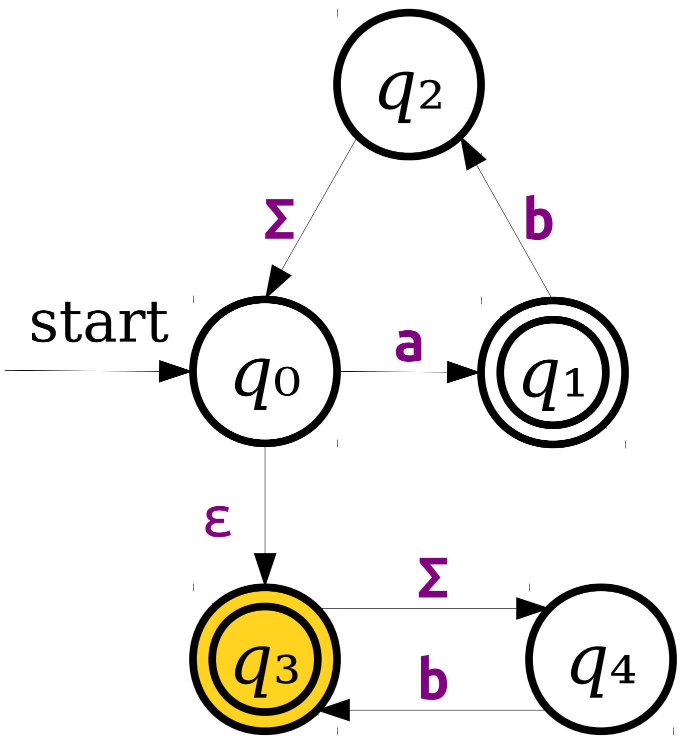
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	

Once More, With Epsilons!



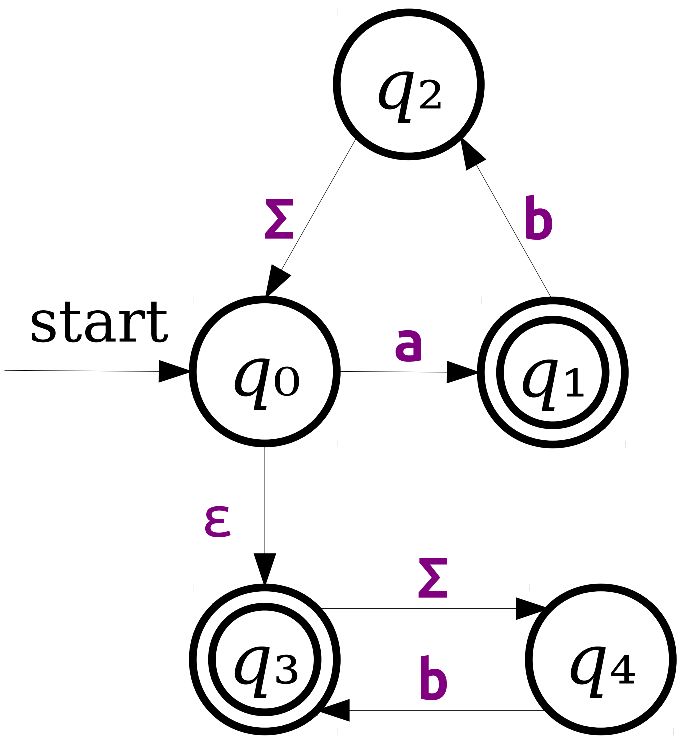
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	

Once More, With Epsilons!



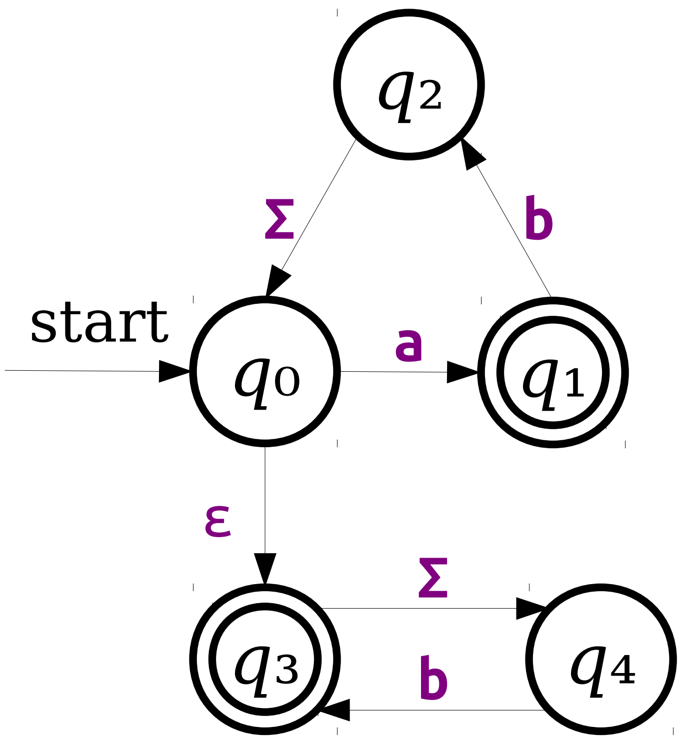
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$

Once More, With Epsilons!



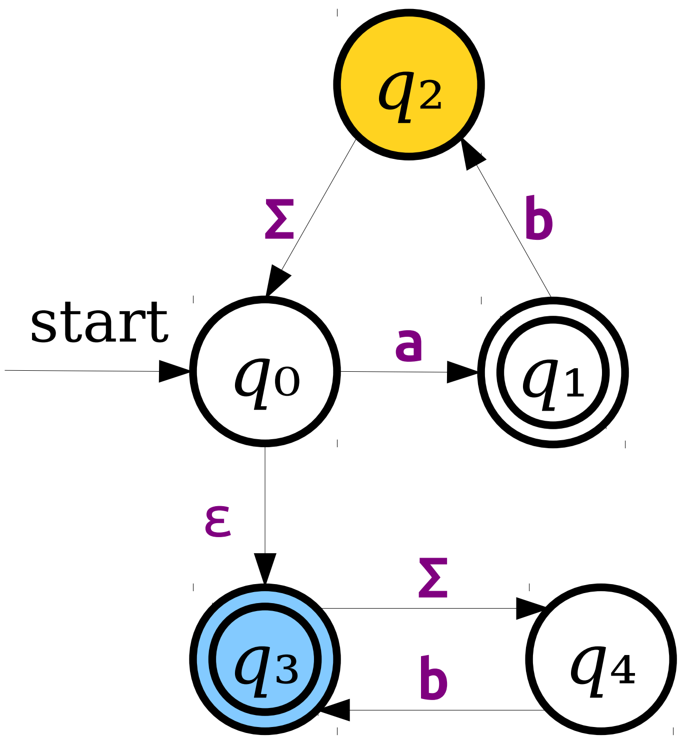
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$

Once More, With Epsilons!



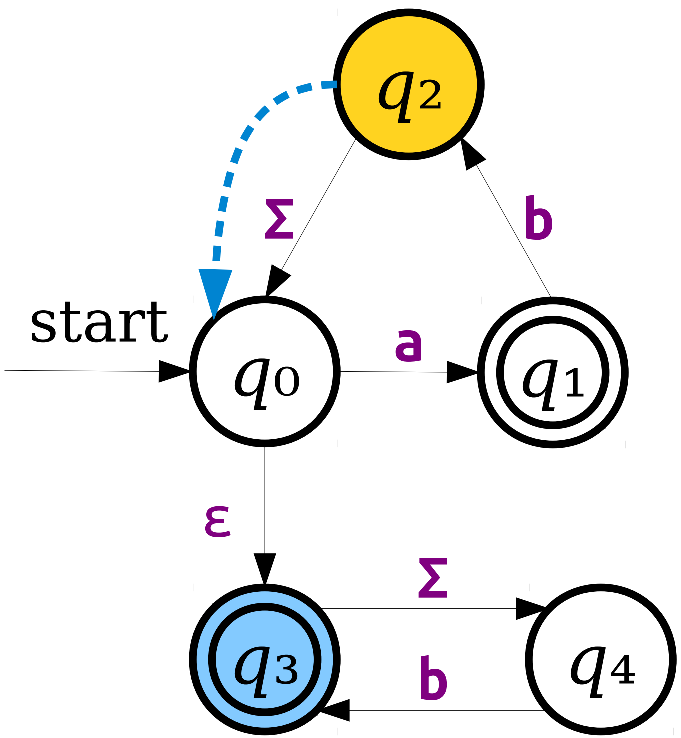
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



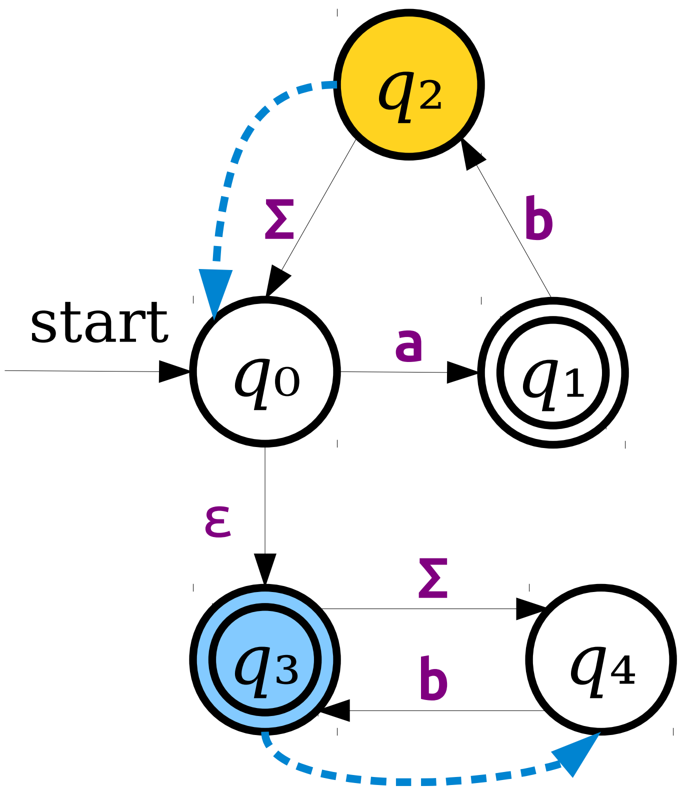
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



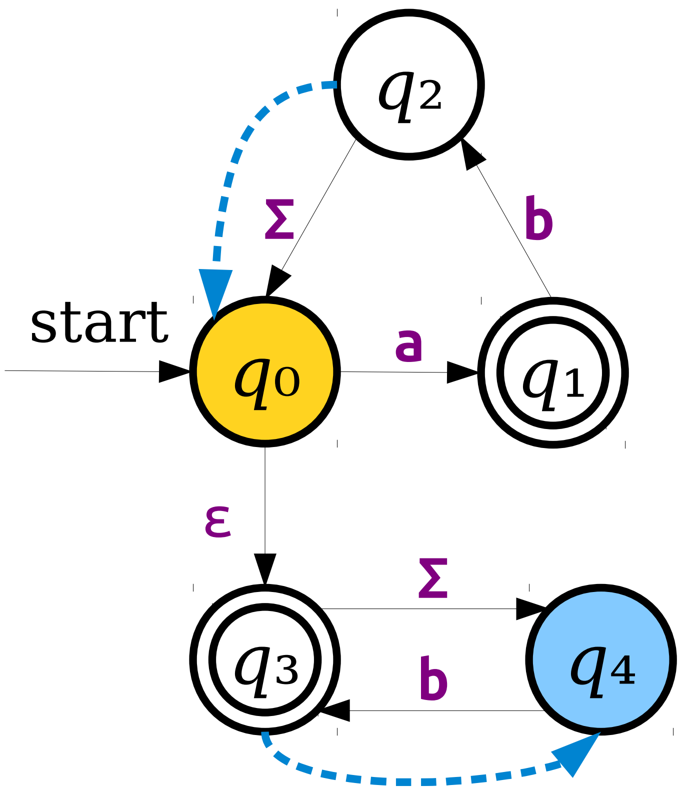
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



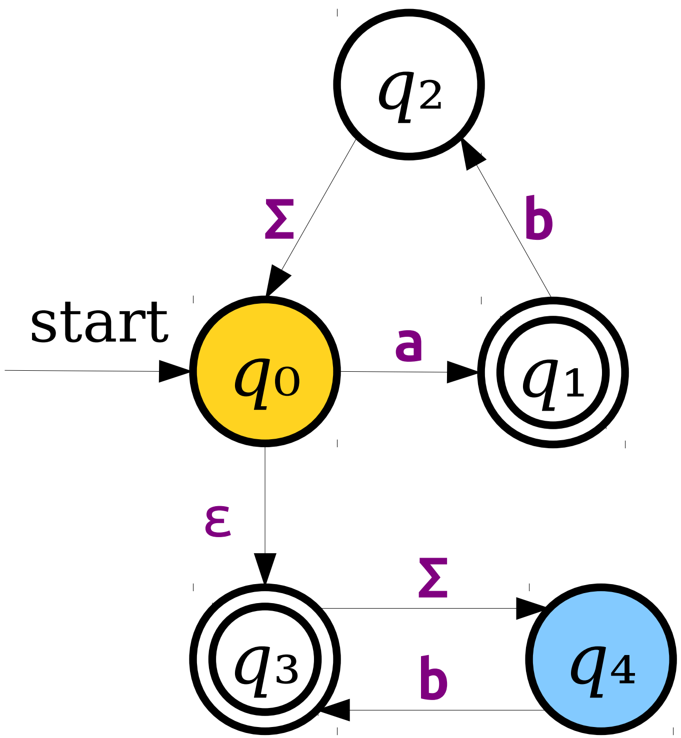
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



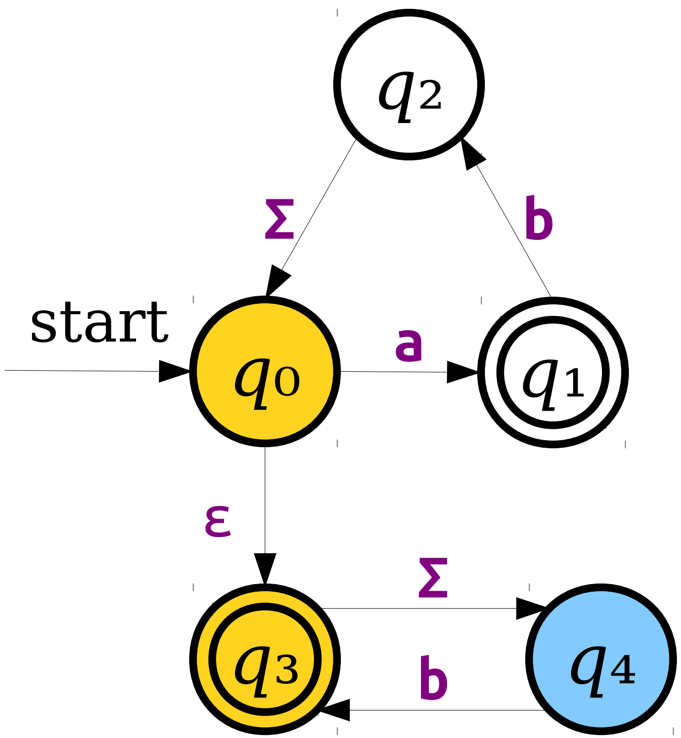
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



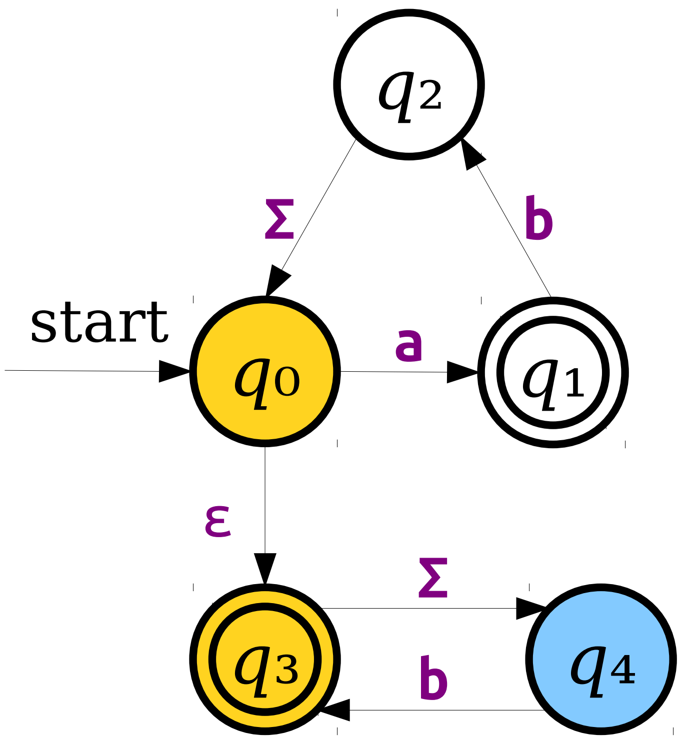
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



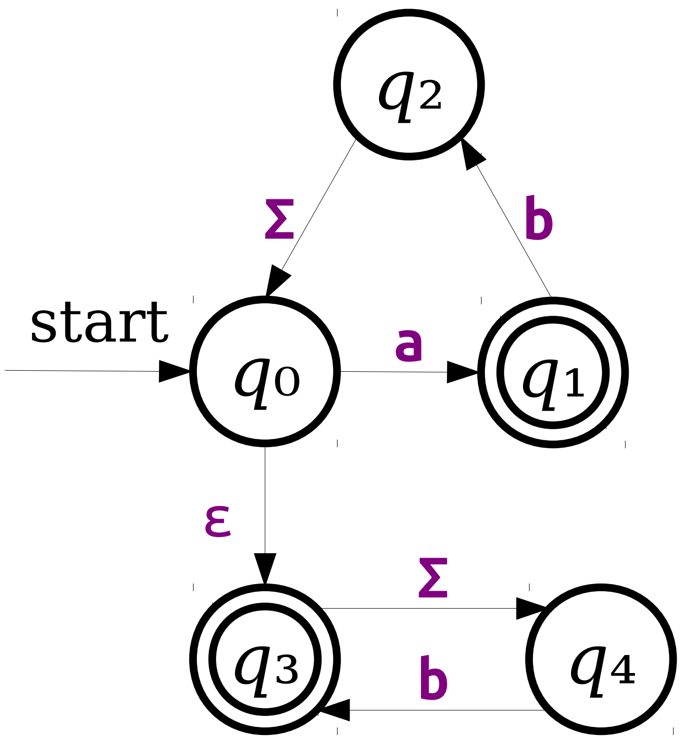
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



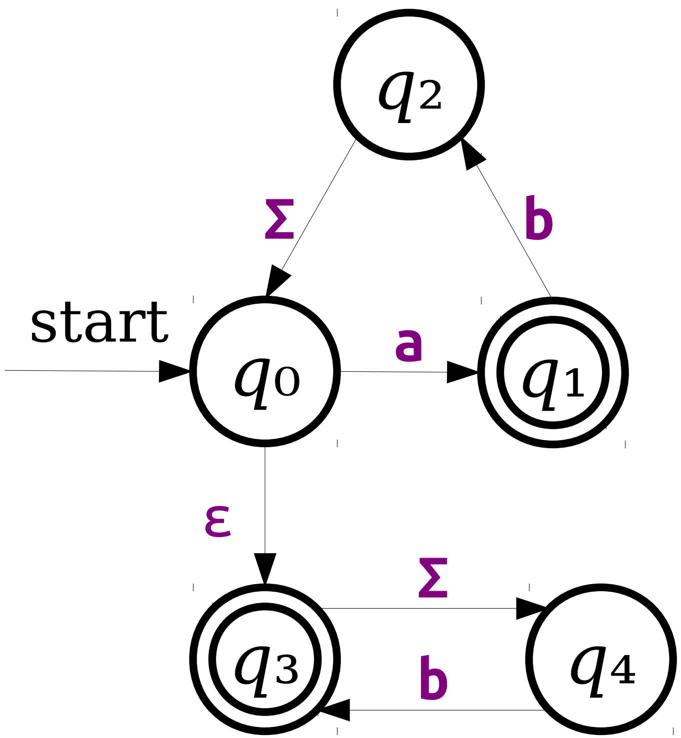
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$

Once More, With Epsilons!



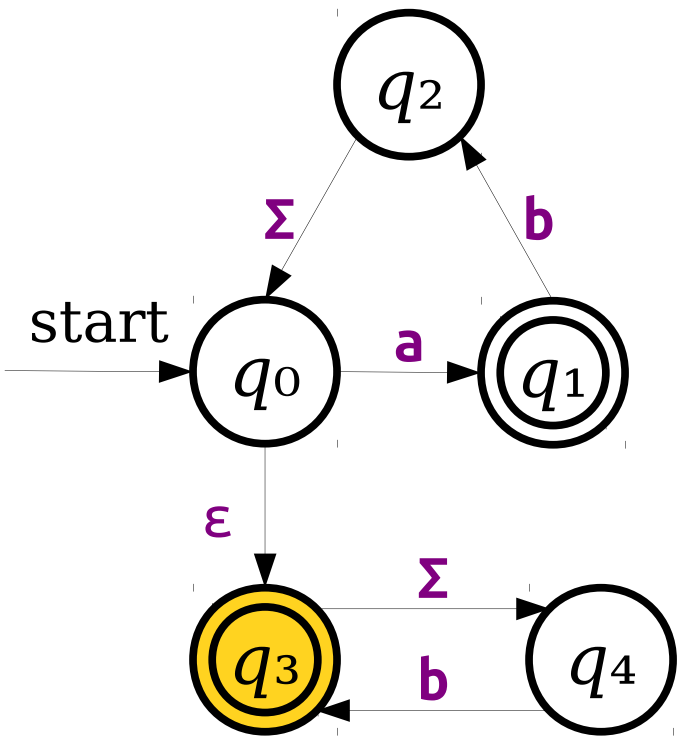
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$

Once More, With Epsilons!



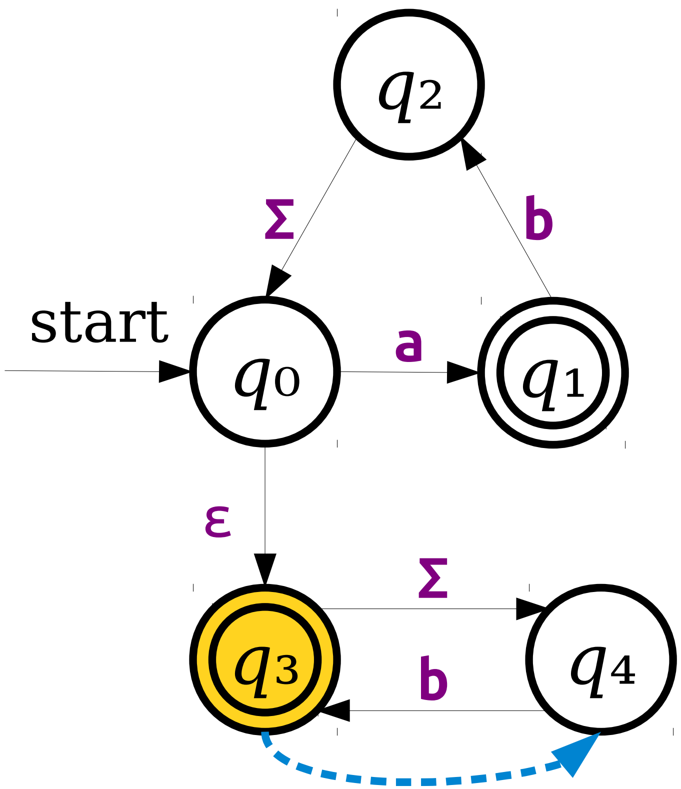
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



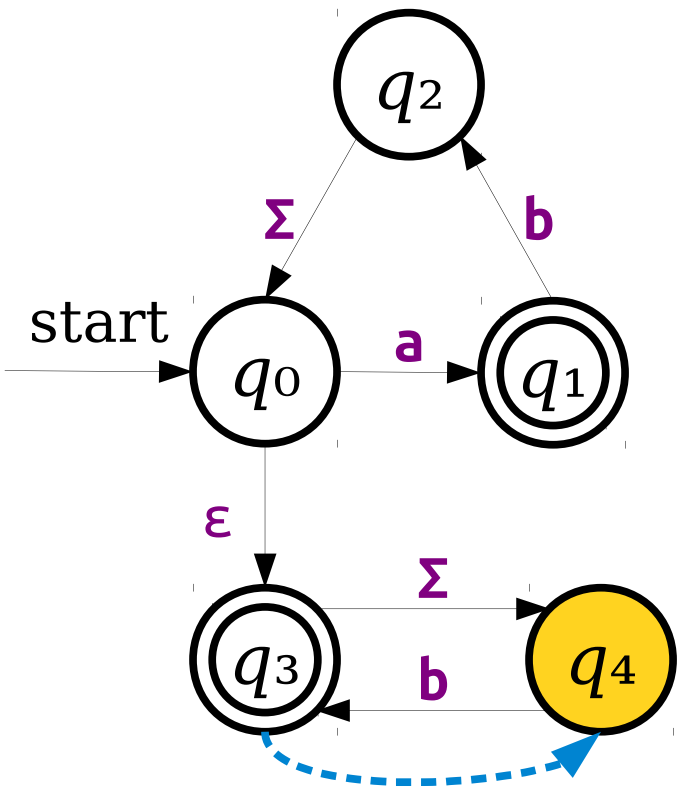
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



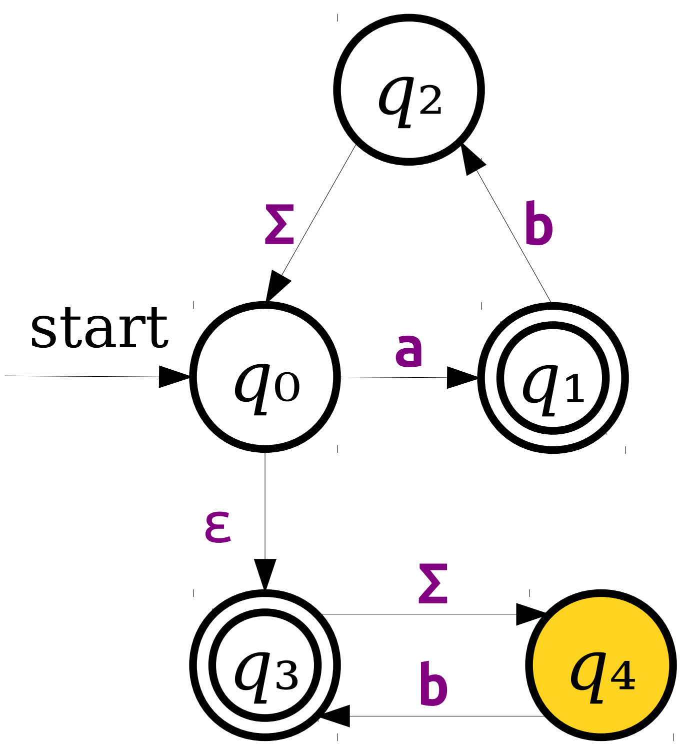
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



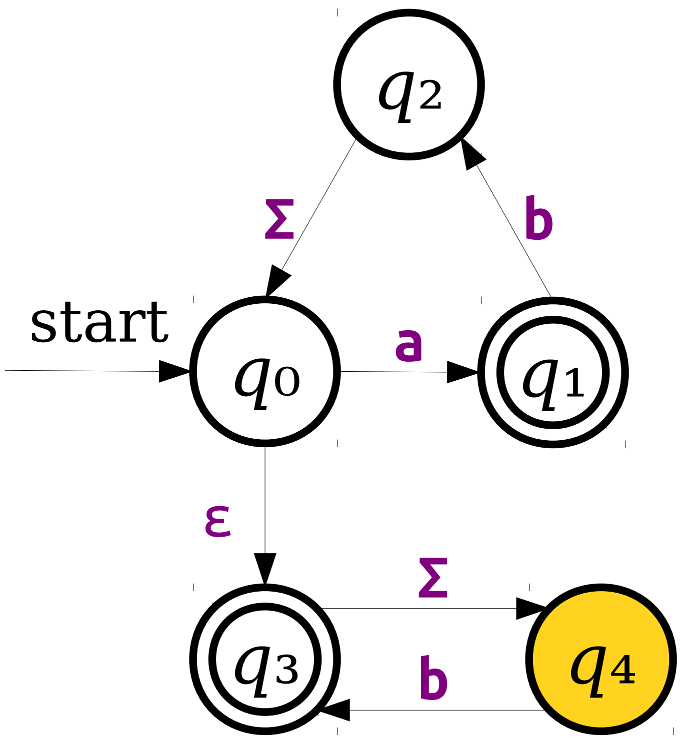
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



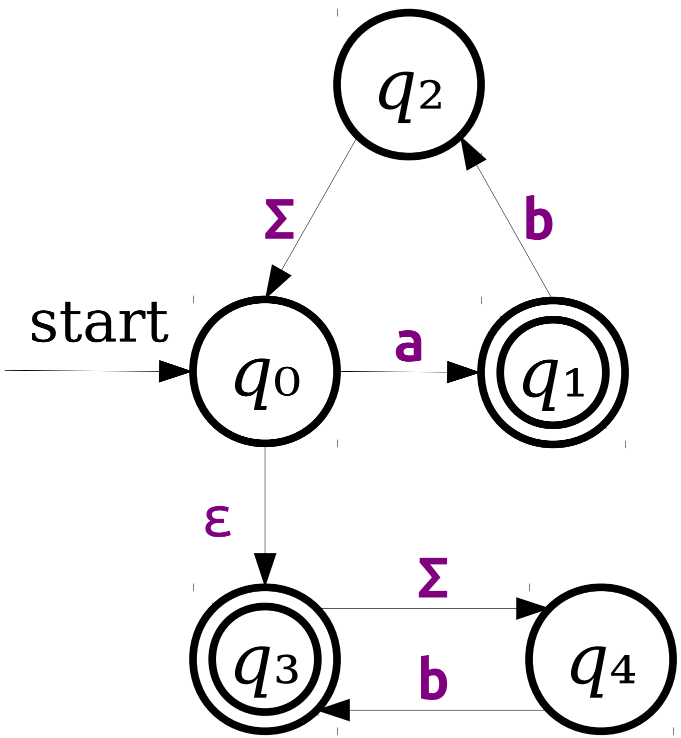
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



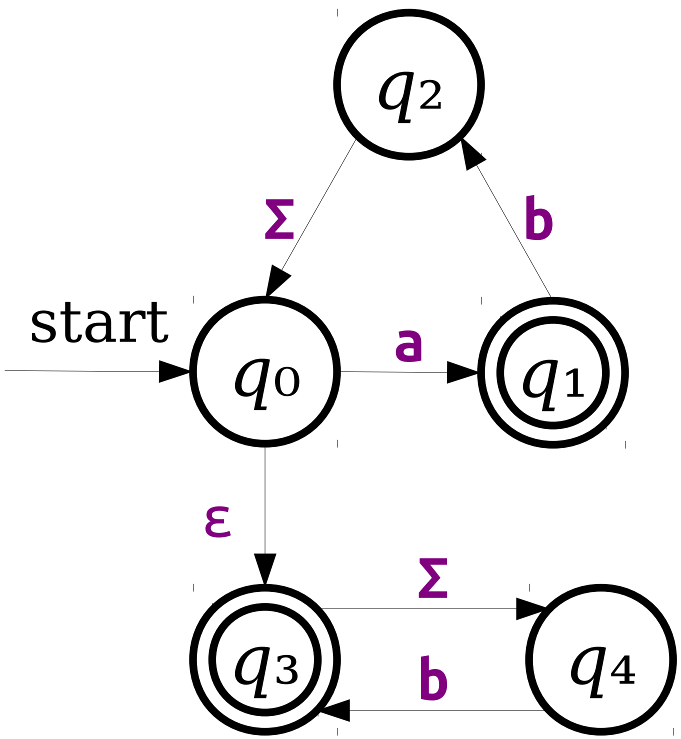
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$

Once More, With Epsilons!



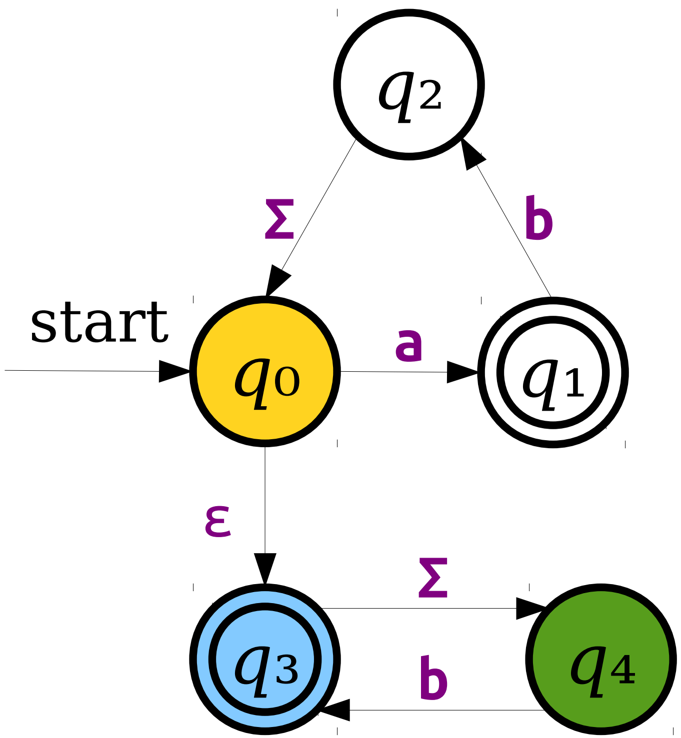
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$

Once More, With Epsilons!



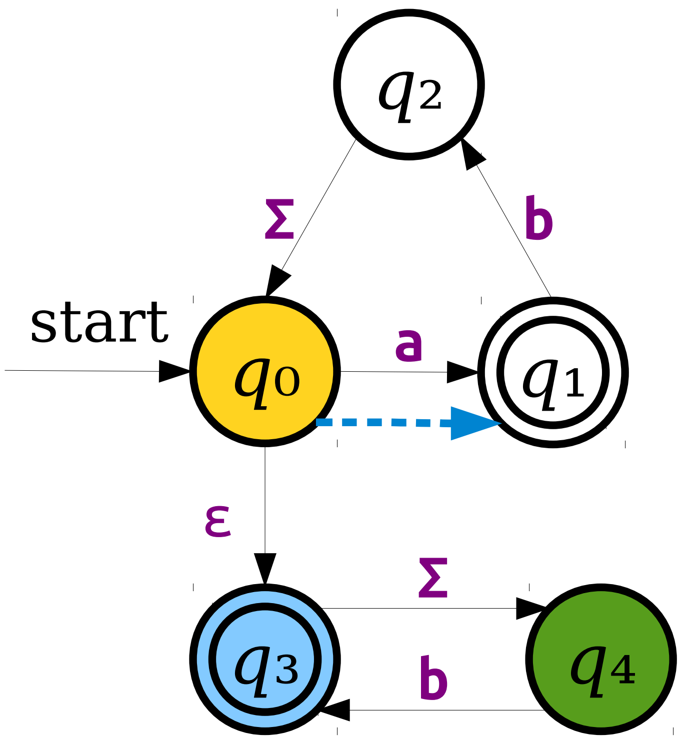
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



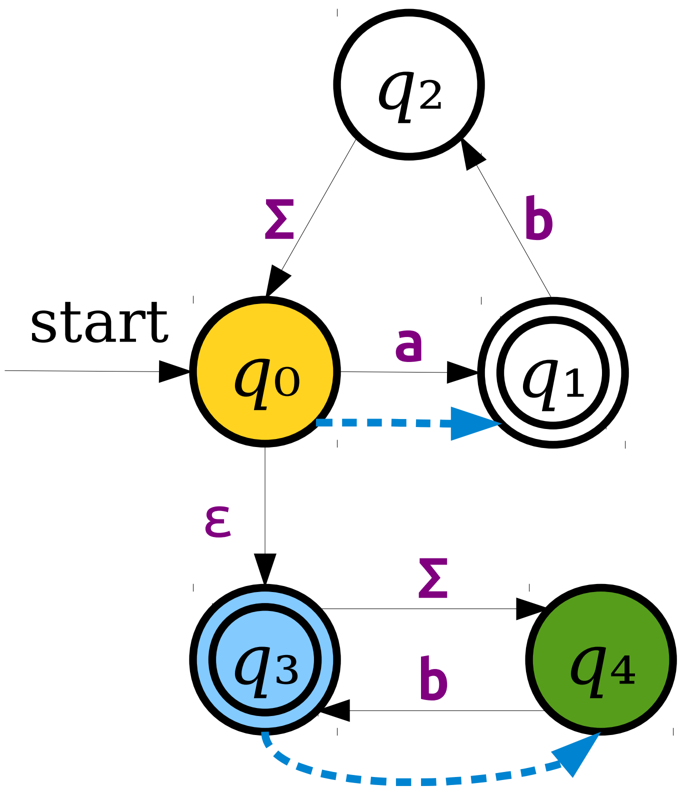
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



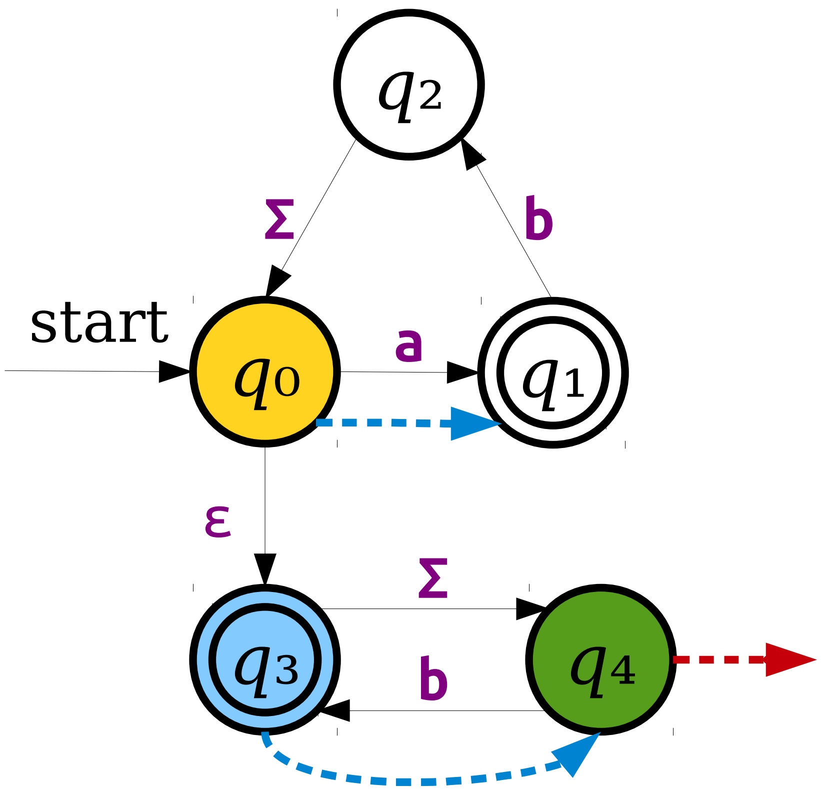
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



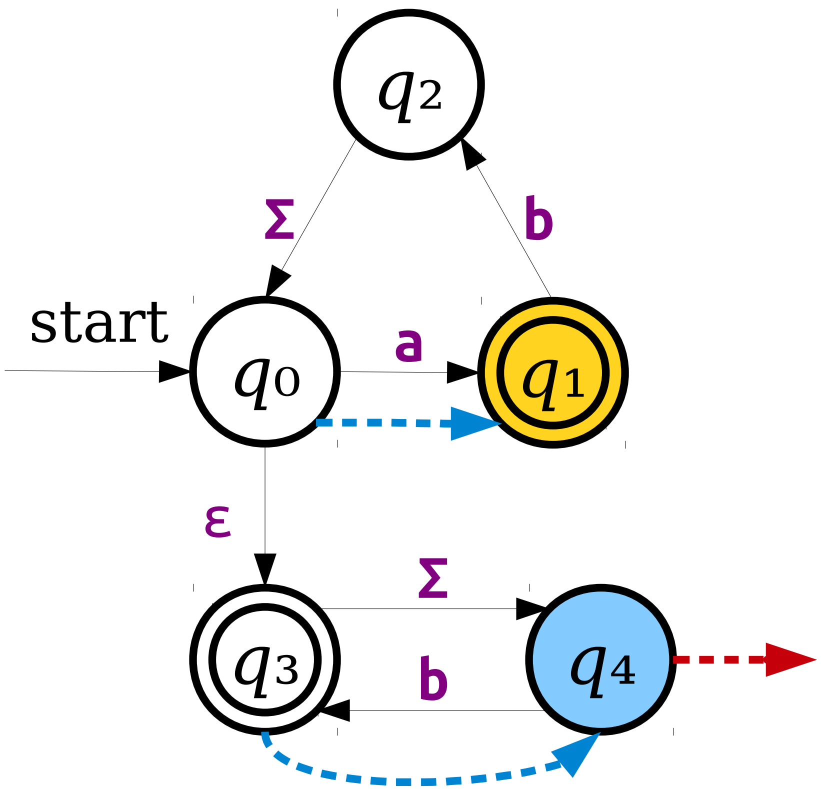
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



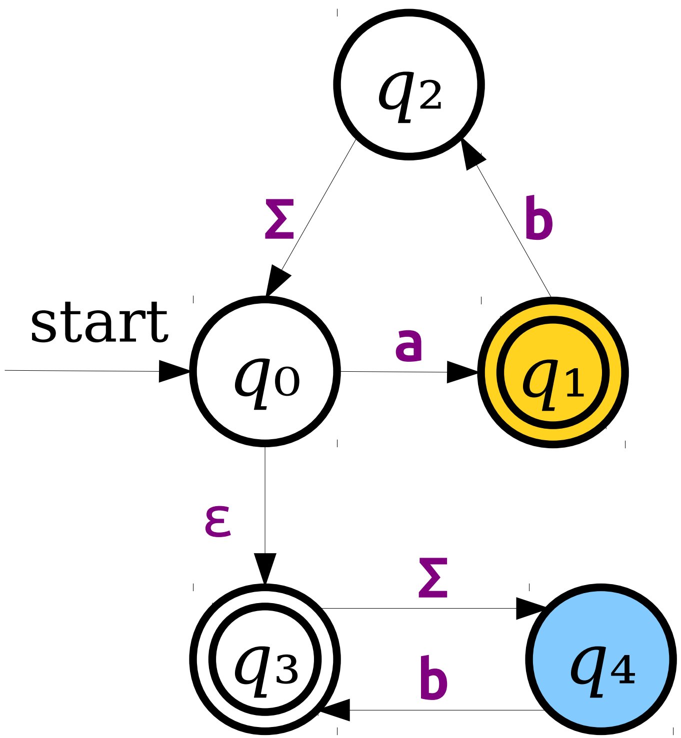
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
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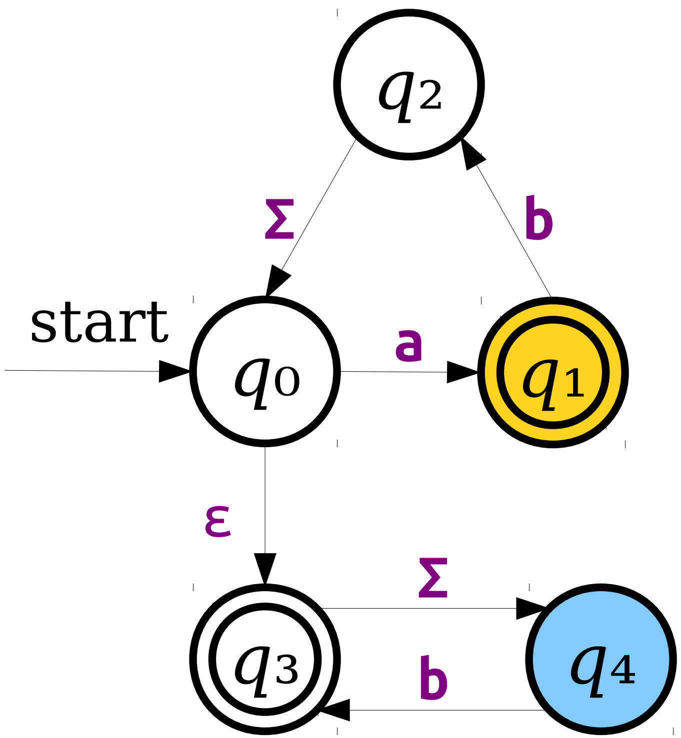
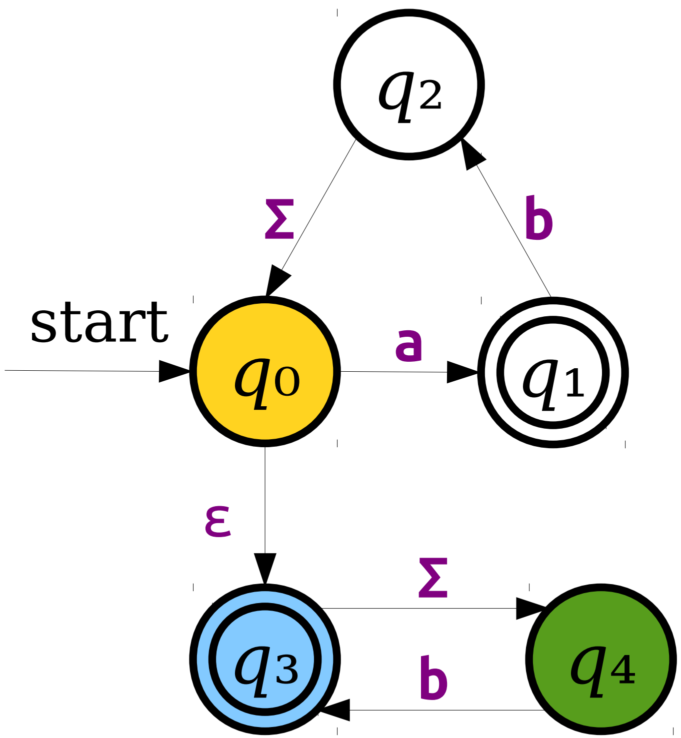
Once More, With Epsilons!



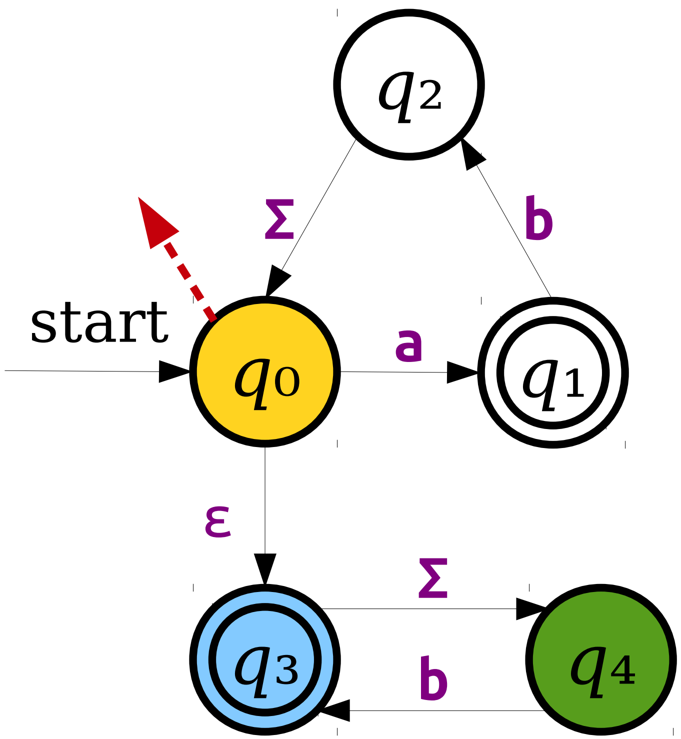
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



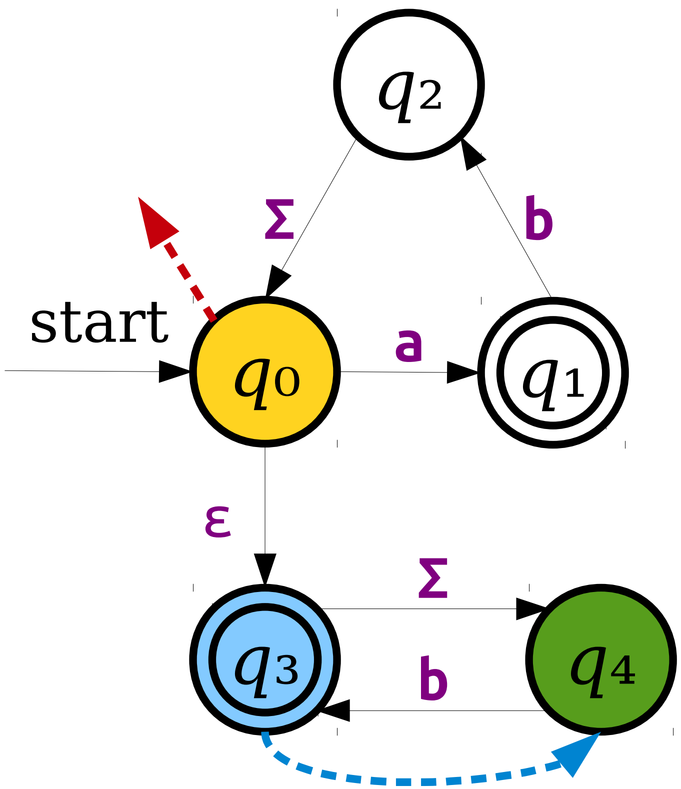
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



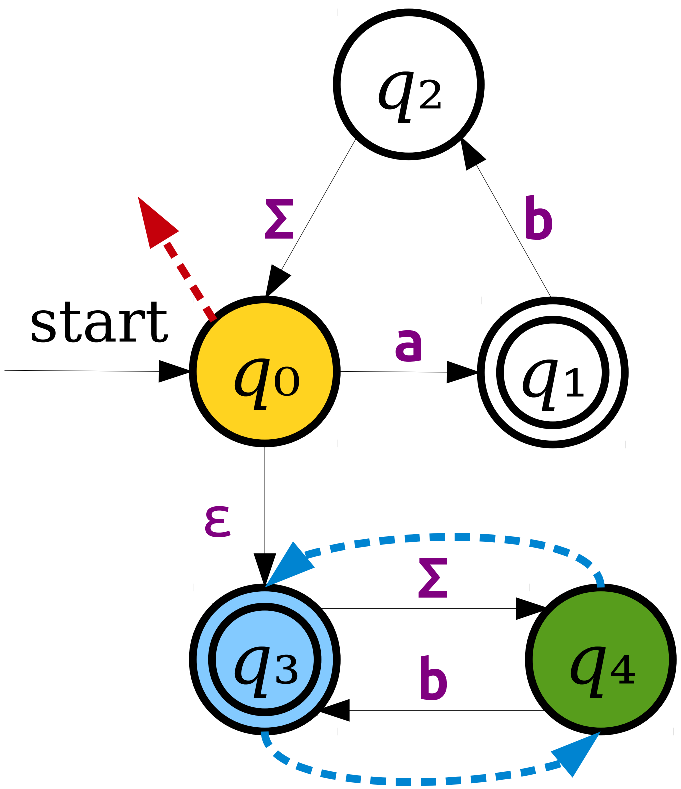
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



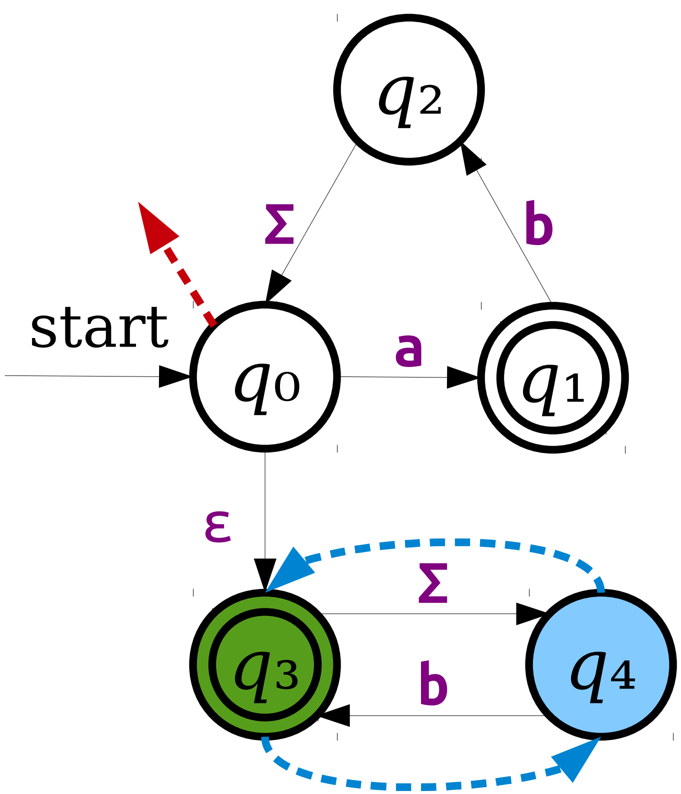
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



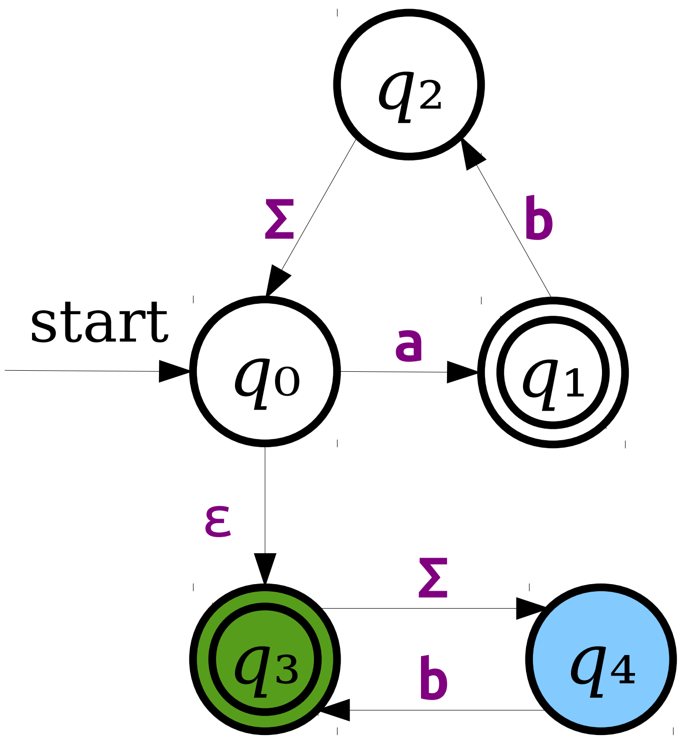
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



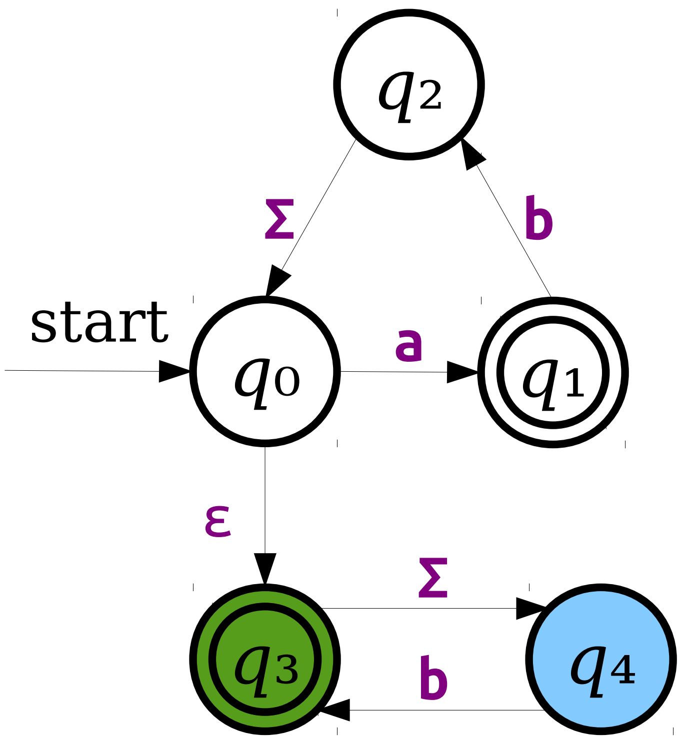
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



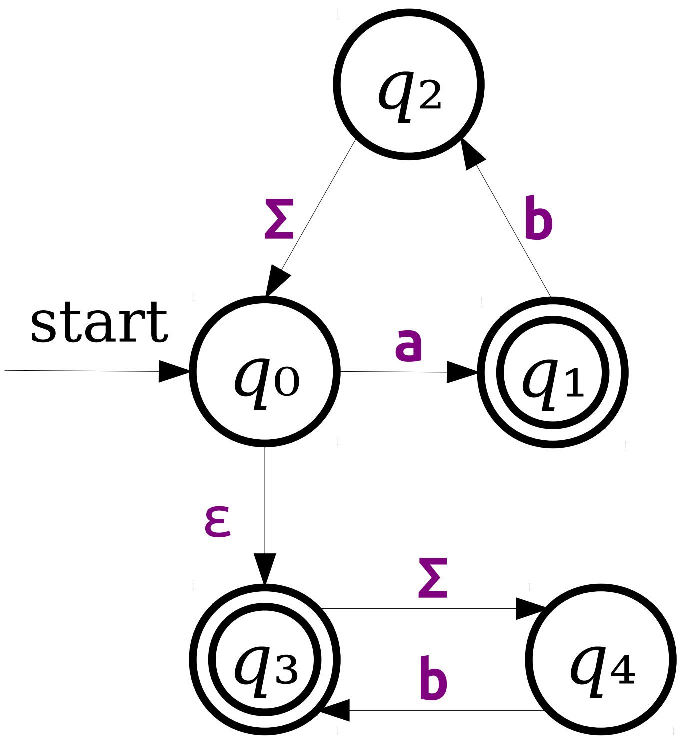
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



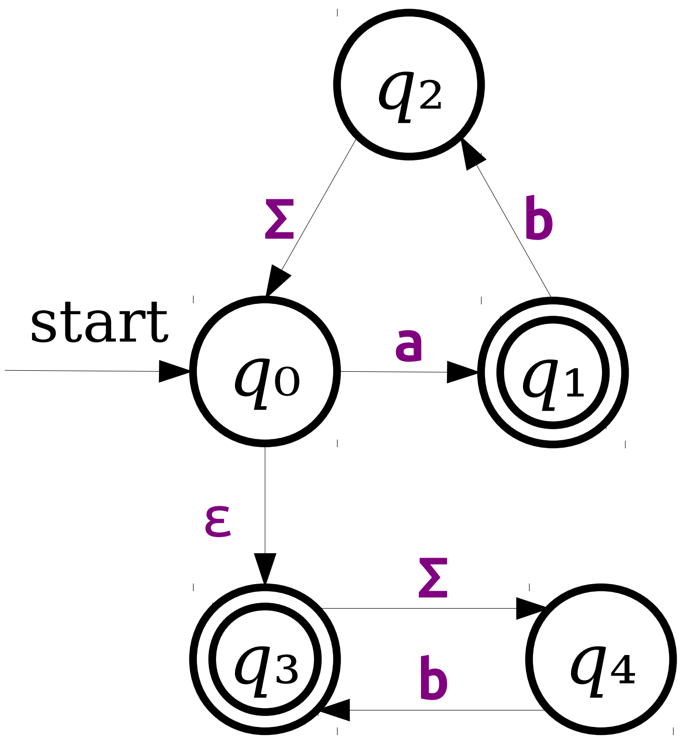
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
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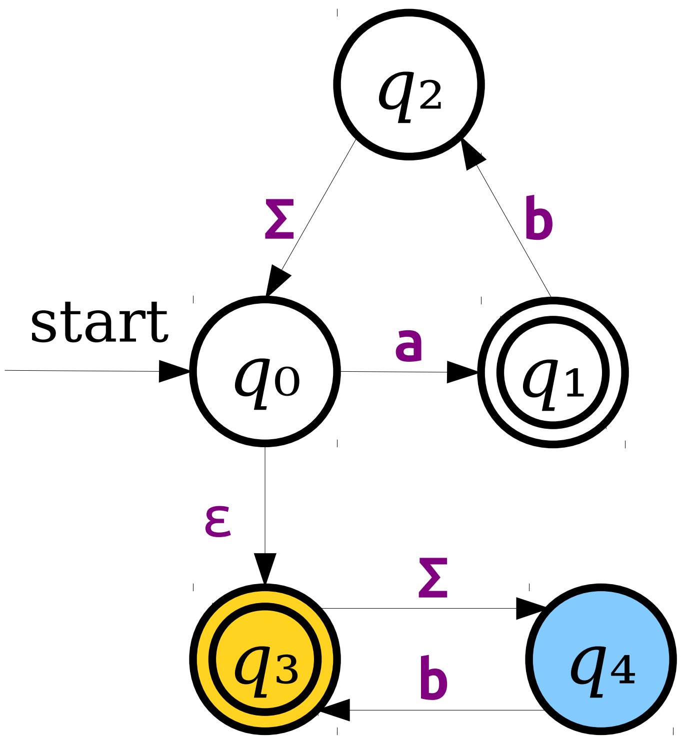

	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
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$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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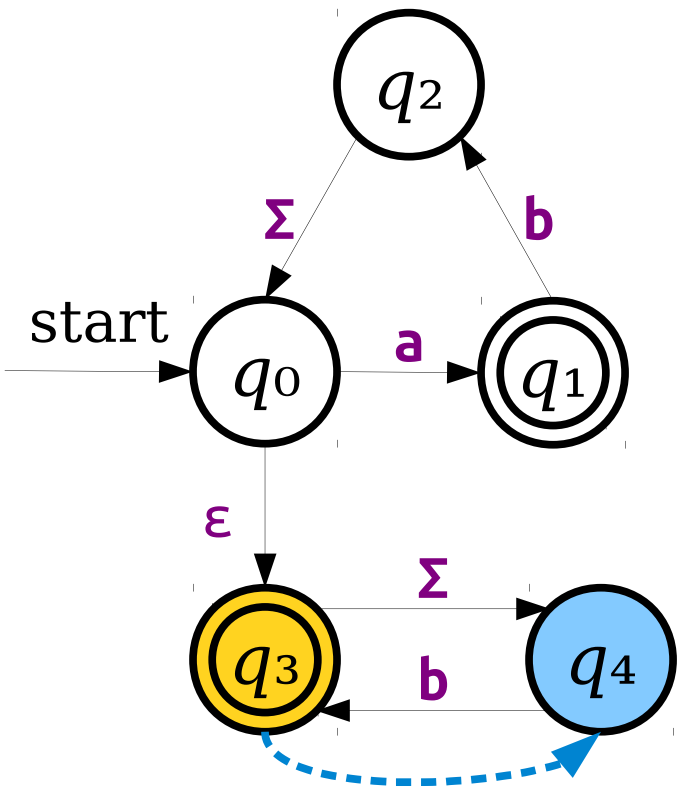

	a	b
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$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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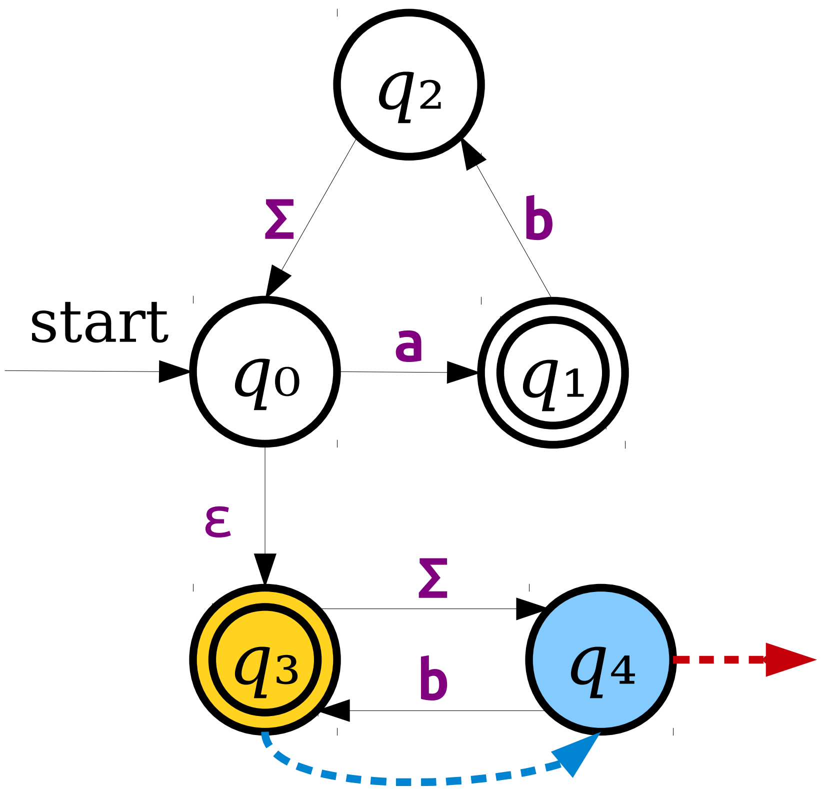

	a	b
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$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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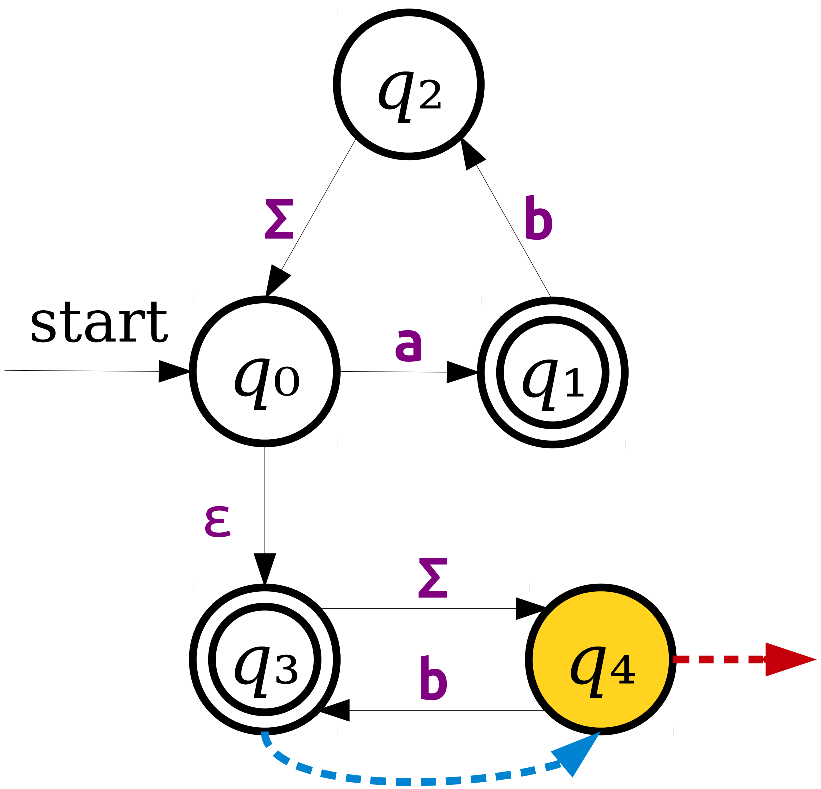

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$\{q_4\}$	$\varnothing$	$\{q_3\}$
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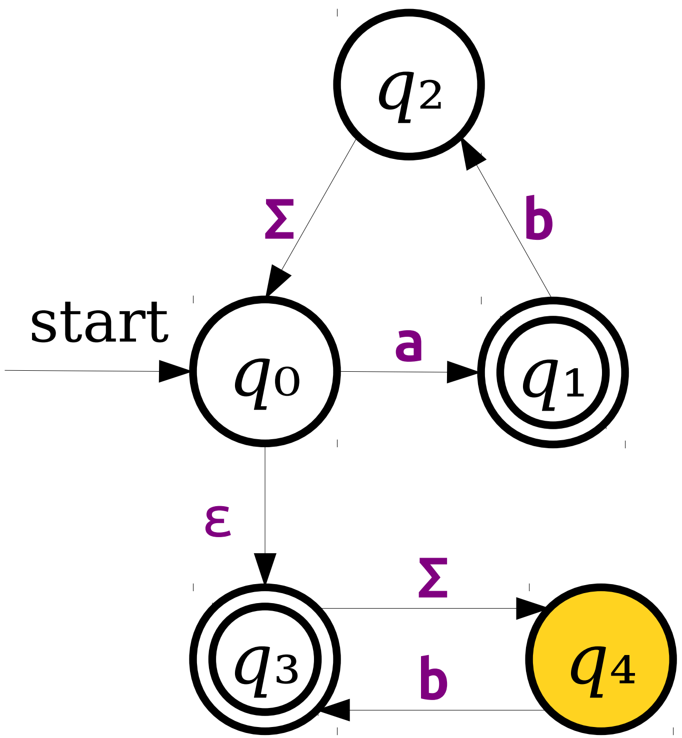

	a	b
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$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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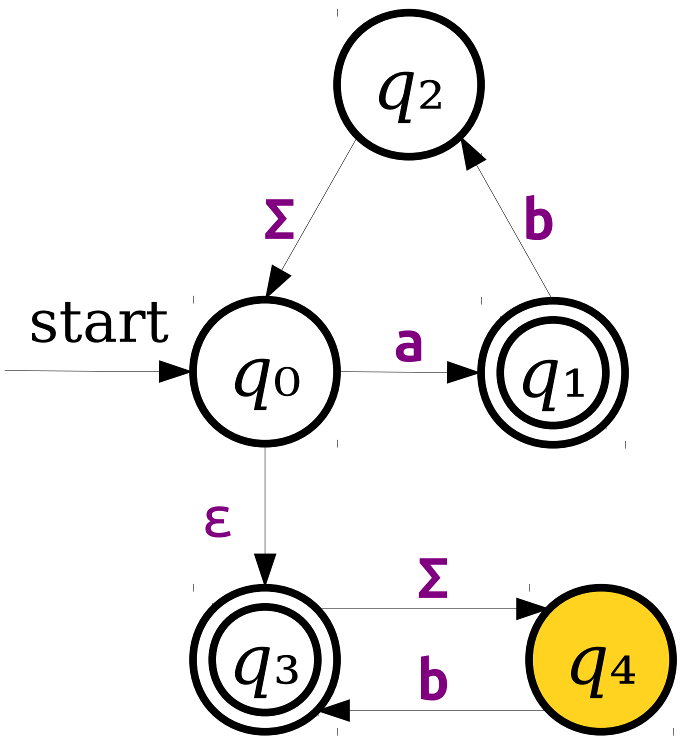

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$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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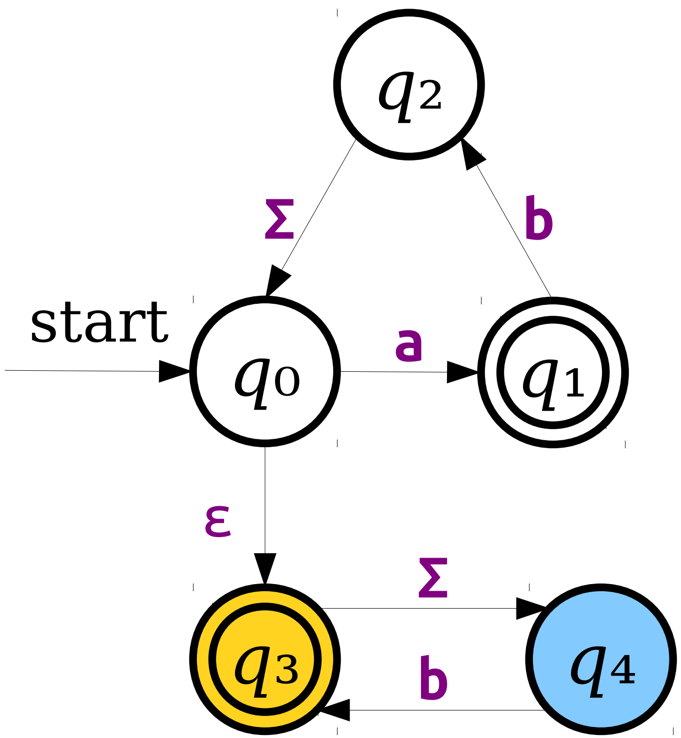

	a	b
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$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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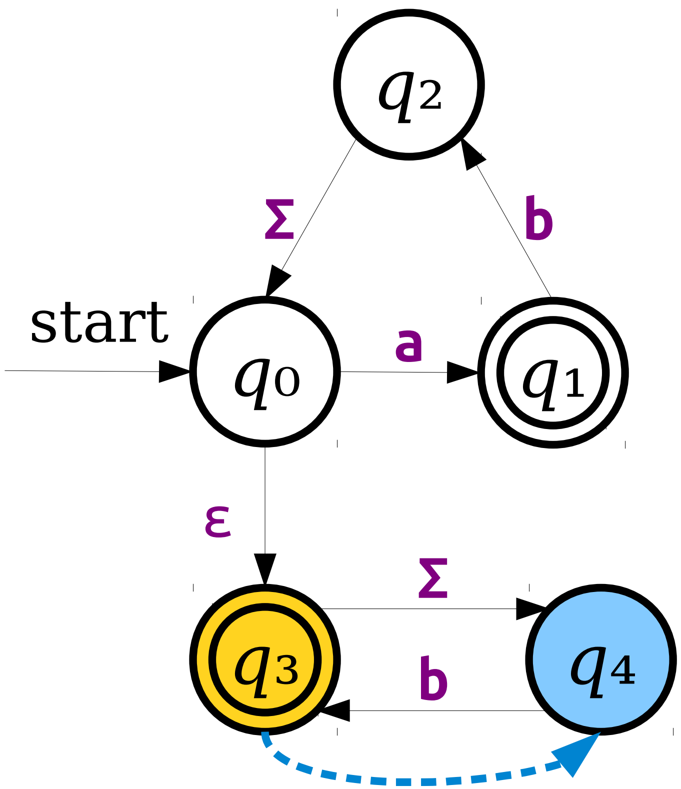

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$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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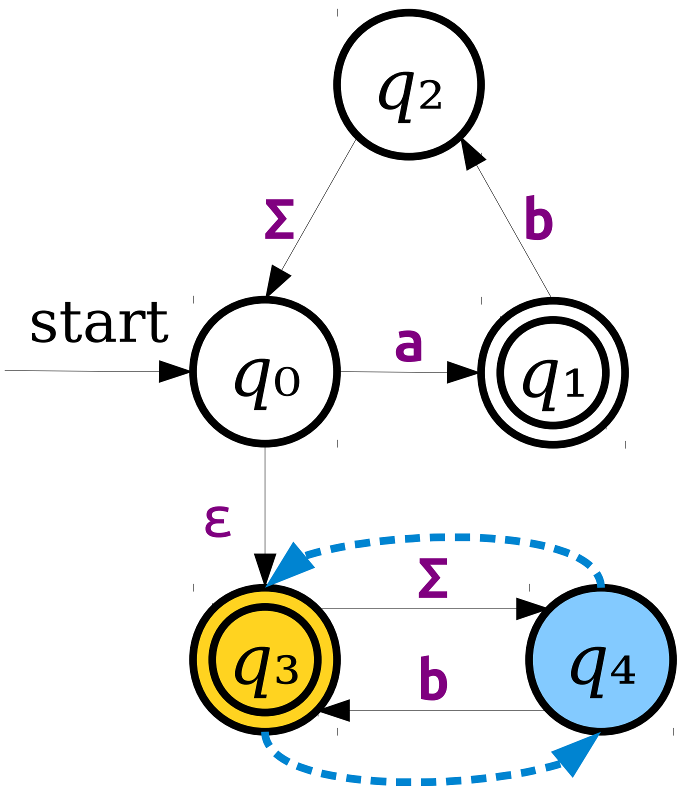

	a	b
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$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
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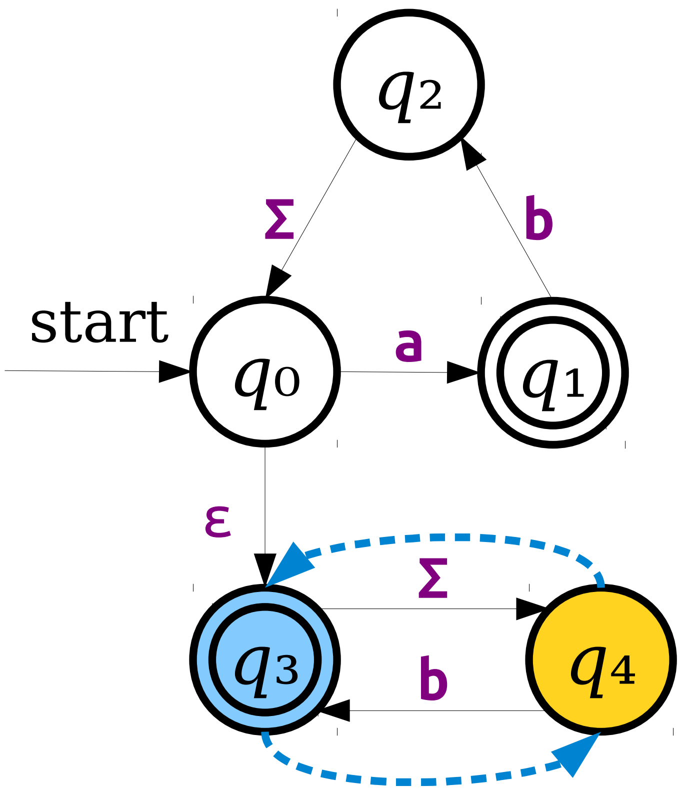

	a	b
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$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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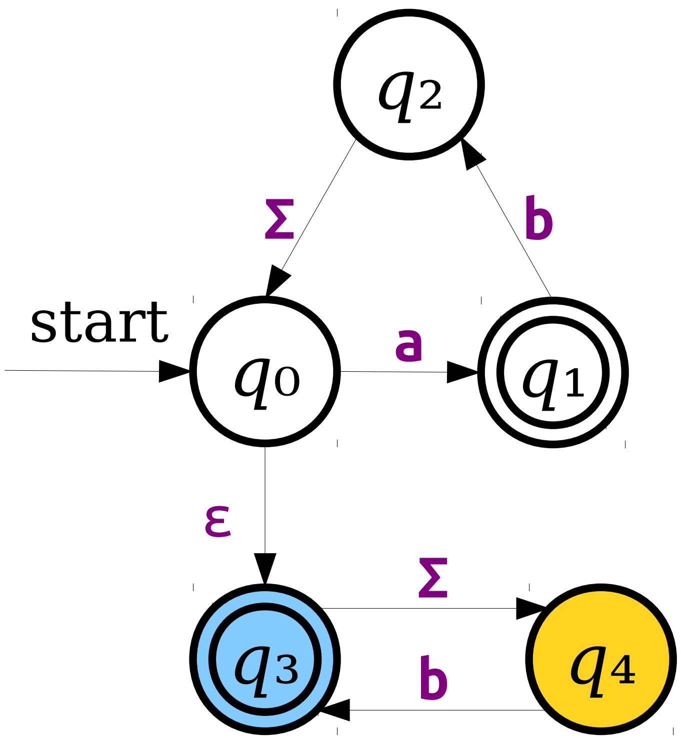

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$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
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$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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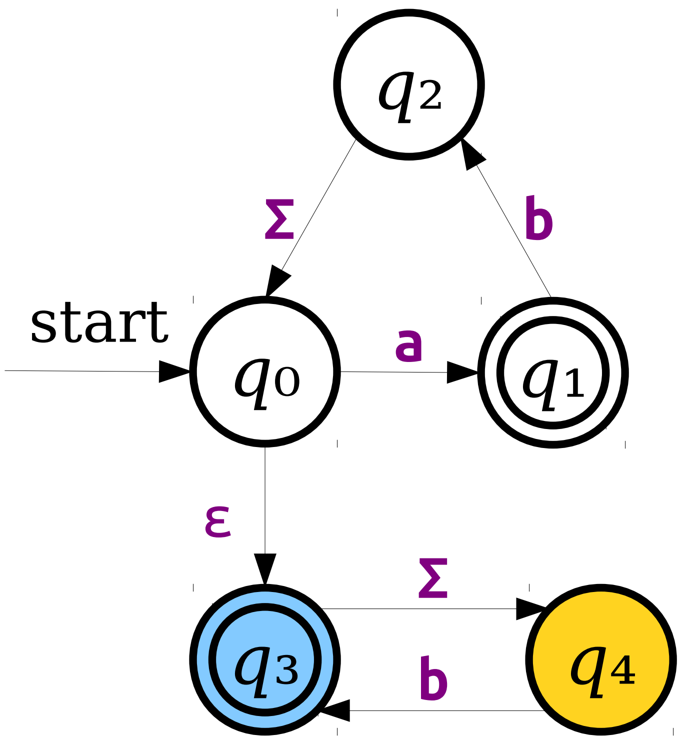

	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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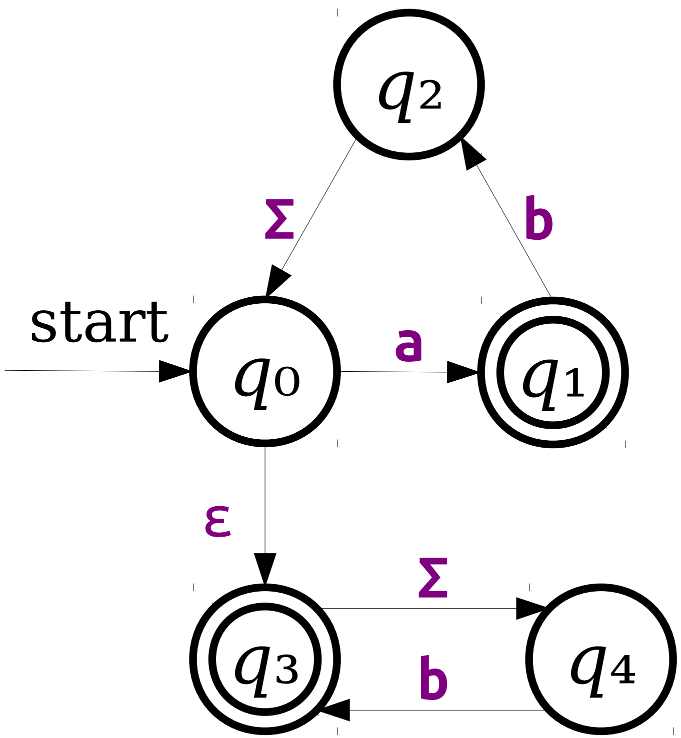

	a	b
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$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
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$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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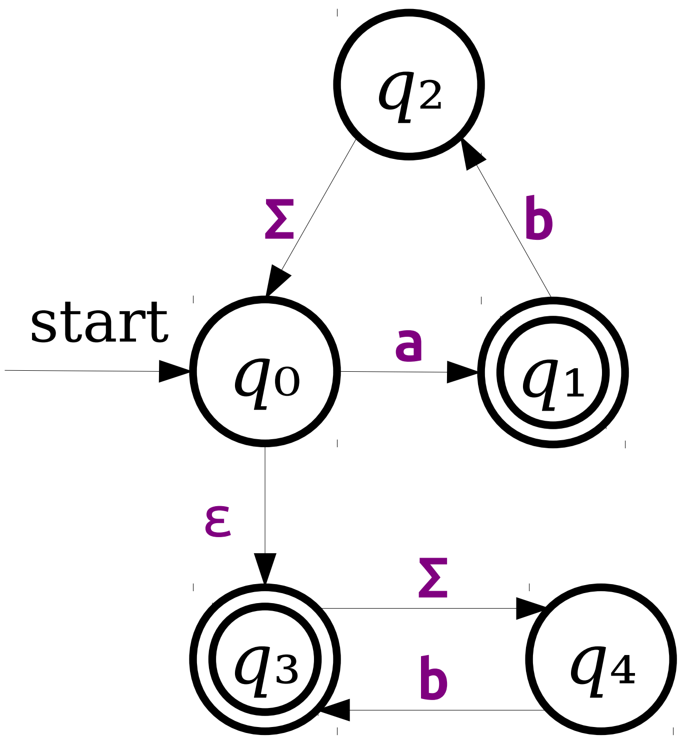

	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
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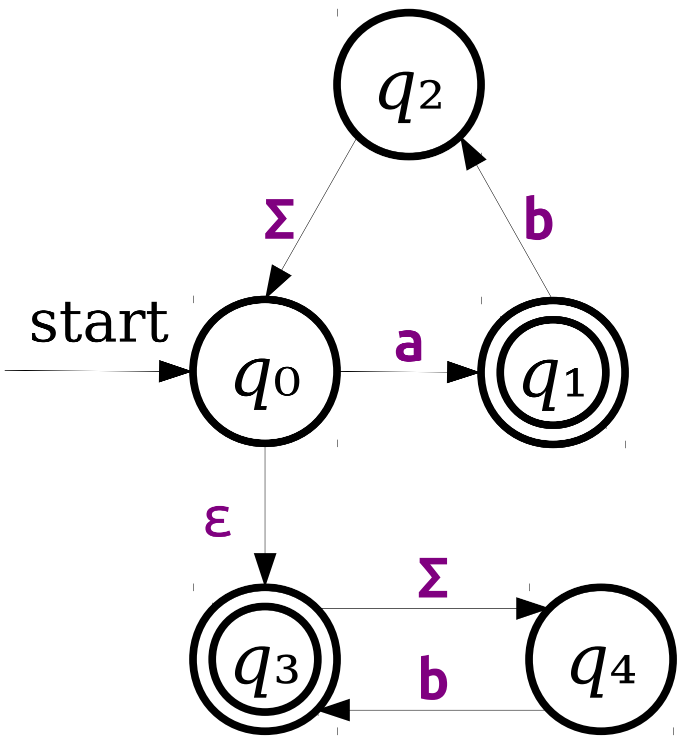

	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
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$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
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$\emptyset$	$\emptyset$	$\emptyset$

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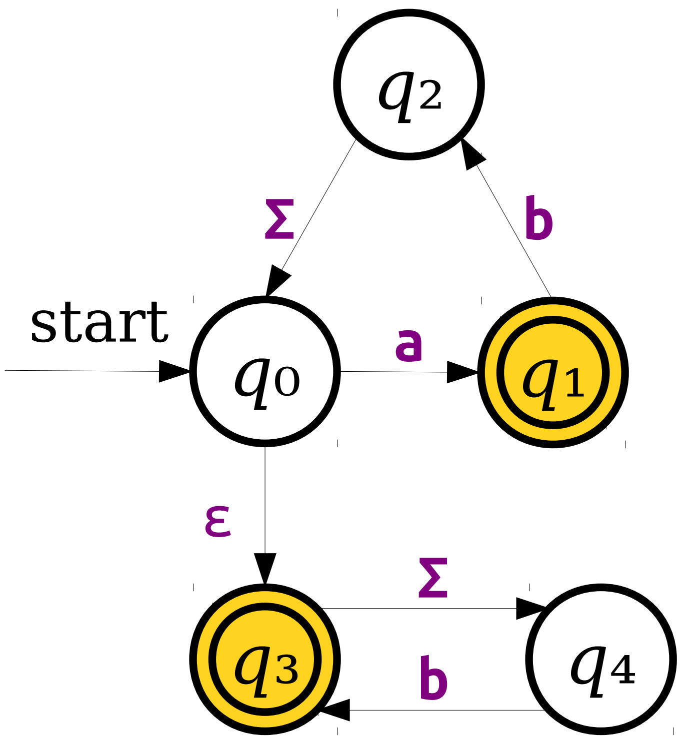

	a	b
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$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
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$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
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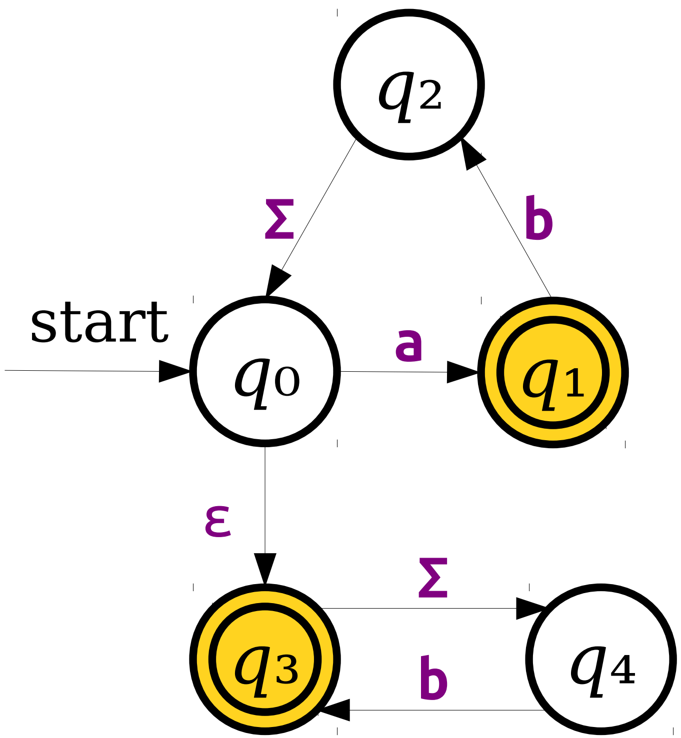

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$*\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
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$\emptyset$	$\emptyset$	$\emptyset$

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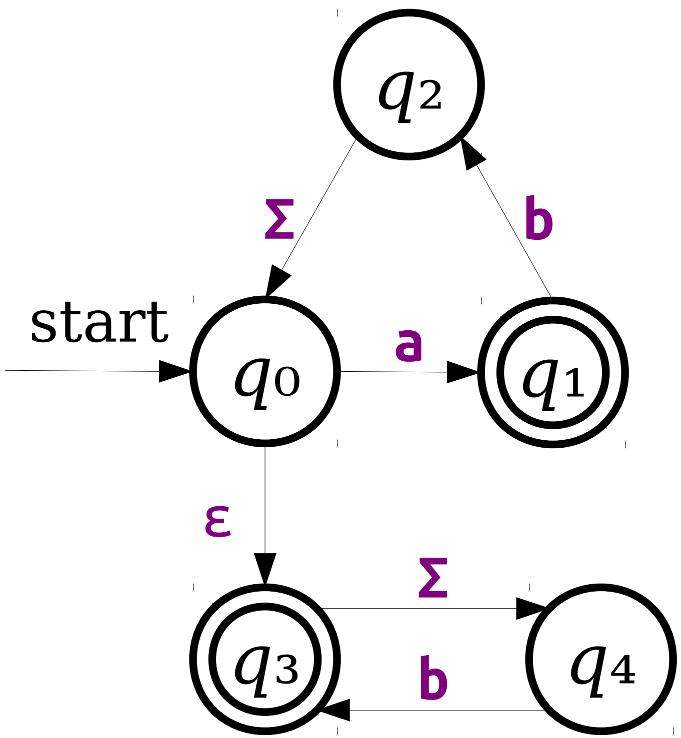

	a	b
$*\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$*\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$*\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$*\{q_3\}$	$\{q_4\}$	$\{q_4\}$
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$*\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\varnothing$	$\varnothing$	$\varnothing$

The Subset Construction

- This construction for transforming an NFA into a DFA is called the ***subset construction*** (or sometimes the ***powerset construction***).
  - Each state in the DFA is associated with a set of states in the NFA.
  - The start state in the DFA corresponds to the start state of the NFA, plus all states reachable via ε-transitions.
  - If a state q in the DFA corresponds to a set of states S in the NFA, then the transition from state q on a character a is found as follows:
    - Let S' be the set of states in the NFA that can be reached by following a transition labeled a from any of the states in S. *(This set may be empty.)*
    - Let S'' be the set of states in the NFA reachable from some state in S' by following zero or more epsilon transitions.
    - The state q in the DFA transitions on a to a DFA state corresponding to the set of states S''.
- ***Read Sipser for a formal account.***

The Subset Construction

- In converting an NFA to a DFA, the DFA's states correspond to sets of NFA states.
- ***Useful fact:*** $|\wp(S)| = 2^{|S|}$ for any finite set S.
- In the worst-case, the construction can result in a DFA that is *exponentially larger* than the original NFA.
- Interesting challenge: Find a language for which this worst-case behavior occurs (there are infinitely many of them!)

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

An Important Result

Theorem: A language L is regular iff there is some NFA N such that $\mathscr{L}(N) = L$.

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Proof Sketch:

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Proof Sketch: If L is regular, there exists some DFA for it, which we can easily convert into an NFA. If L is accepted by some NFA, we can use the subset construction to convert it into a DFA that accepts the same language, so L is regular. ■

Why This Matters

- We now have two perspectives on regular languages:
  - Regular languages are languages accepted by DFAs.
  - Regular languages are languages accepted by NFAs.
- We can now reason about the regular languages in two different ways.

Time-Out for Announcements!

Problem Set Six

- Problem Set Five was due at 2:30PM today.
- Problem Set Six goes out today. It's due next Friday at 2:30PM.
  - Play around with DFAs, NFAs, language transformations, and their properties!
  - Explore how all the discrete math topics we've talked about so far come into play!

Looking for a Partner?

- I’ve heard from many of you that you’re now looking for a problem set partner.
- Don’t forget that Piazza has a lovely “Search for Teammates” feature that you can use to do this.
- It’s like speed dating for theory!

DFA/NFA Editor

- We have an online DFA/NFA editor you'll use to answer and submit some of the questions for PS6.
- This tool will let you design and test your automata on a number of different inputs.
- You can also use it to explore on your own!

Extra Practice Problems 2

- Solutions are now available to Extra Practice Problems 2.
- We recommend only reading over them if you've actually attempted the problems from EPP2. ☺

Your Questions

“Any tips on finding a good partner for a group project?”

Treat it like a relationship. Make sure you’re clear about your expectations from the beginning. Don’t be afraid to break up with someone if you don’t work well together. And play the long game – if you find someone really on that you work really well with, it can pay off wonderfully over the course of being a student here!

“Why is pset grading left on "discretion of TA"? I asked why I got points off and TA says he didn't grade mine :(”

We have a pretty detailed set of internal criteria that we use and an internal discussion channel to talk about specific cases. There's discretion in the sense of “making a judgment call in a boundary case,” but the TAs don't have full discretionary authority to give whatever grade they feel like.

“what is the MEANING of LIFE”

Haven't a clue. Are we even sure one exists? It probably has something to do with having an enjoyable experience and making things a little bit better for everyone else.

Back to CS103!

Properties of Regular Languages

The Union of Two Languages

- If L_1 and L_2 are languages over the alphabet Σ, the language $L_1 \cup L_2$ is the language of all strings in at least one of the two languages.
- If L_1 and L_2 are regular languages, is $L_1 \cup L_2$?

The Union of Two Languages

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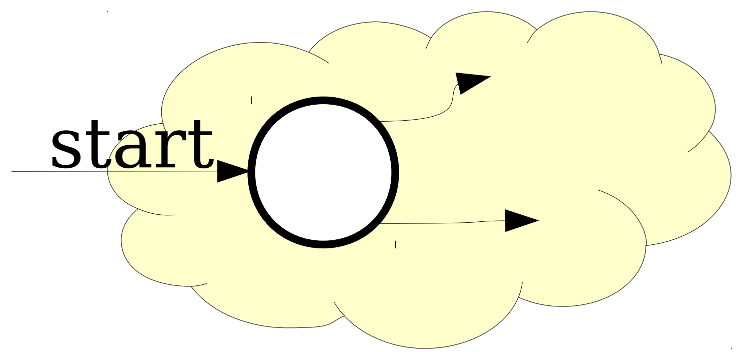
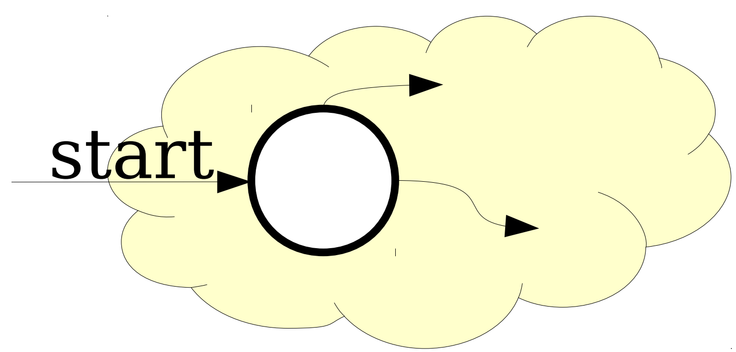


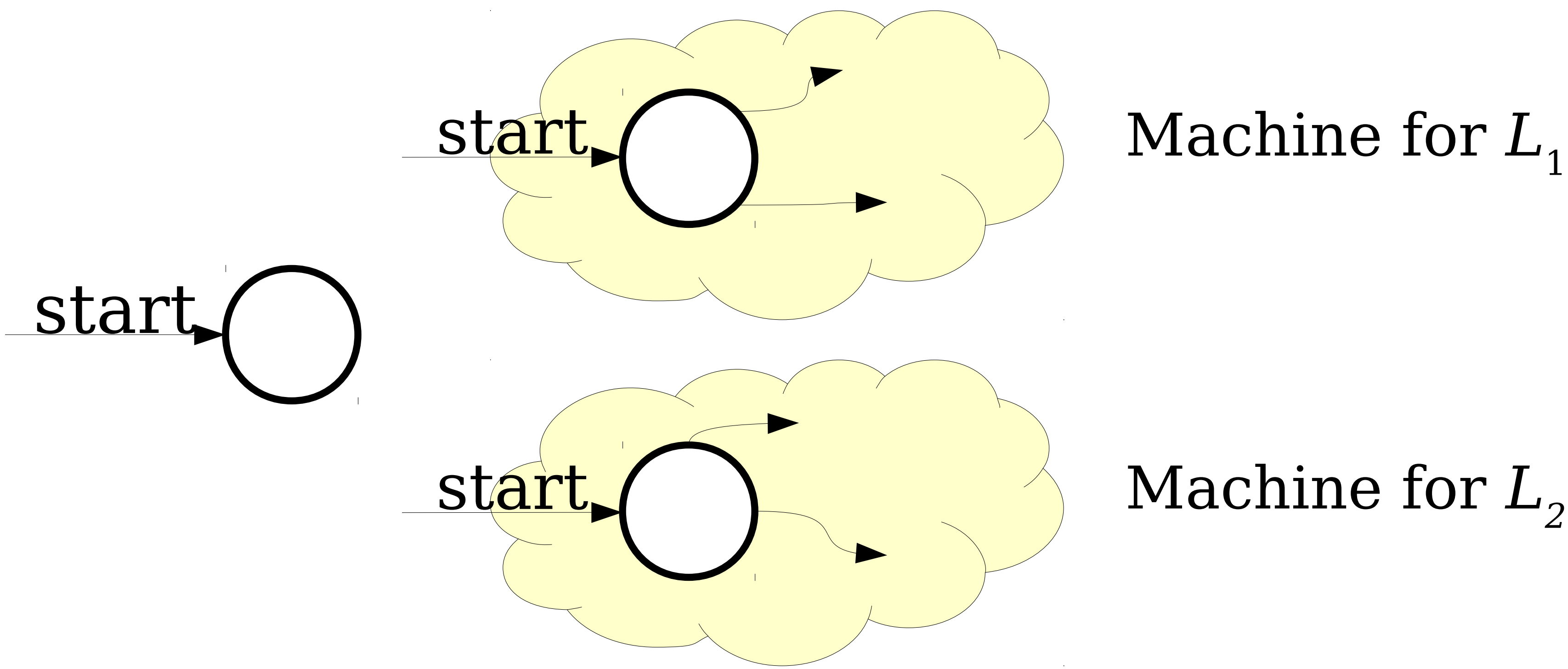
Machine for L_1



Machine for L_2

The Union of Two Languages

- If L_1 and L_2 are languages over the alphabet Σ, the language $L_1 \cup L_2$ is the language of all strings in at least one of the two languages.
- If L_1 and L_2 are regular languages, is $L_1 \cup L_2$?



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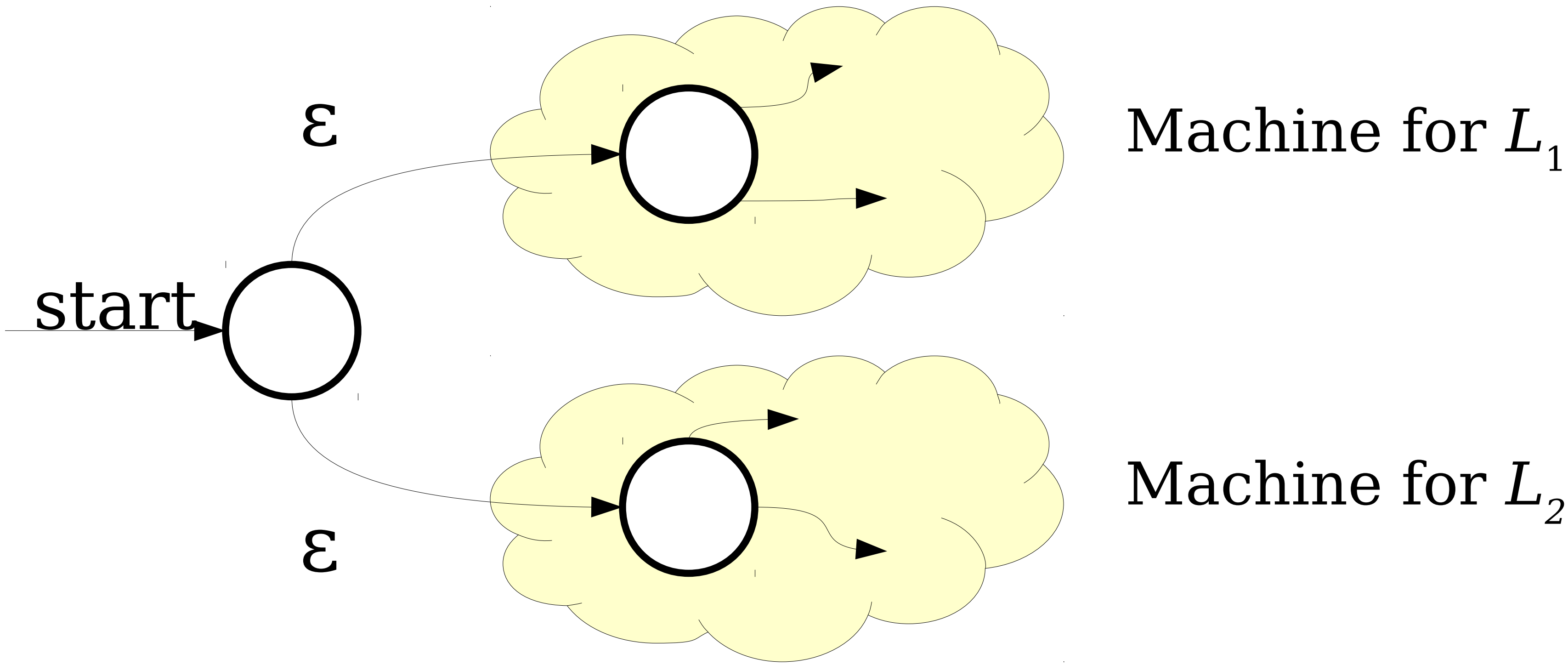

The Union of Two Languages

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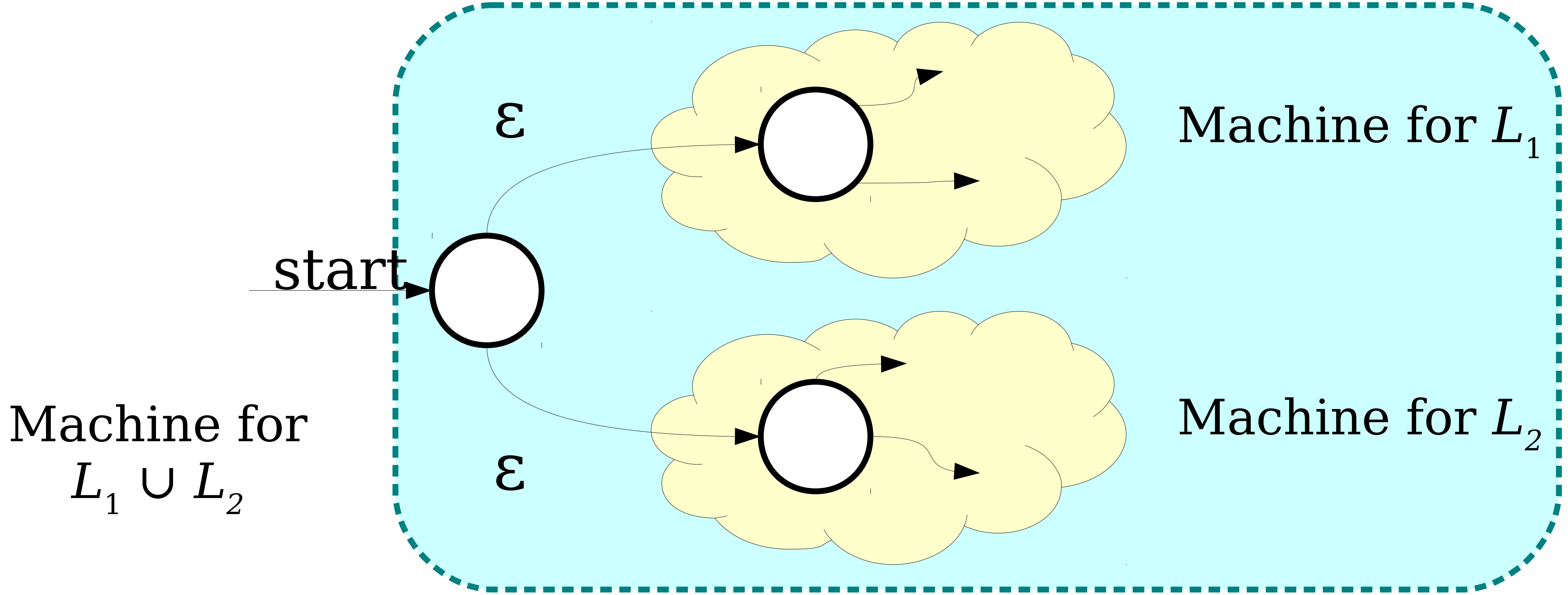

The Intersection of Two Languages

- If L_1 and L_2 are languages over Σ, then $L_1 \cap L_2$ is the language of strings in both L_1 and L_2.
- Question: If L_1 and L_2 are regular, is $L_1 \cap L_2$ regular as well?

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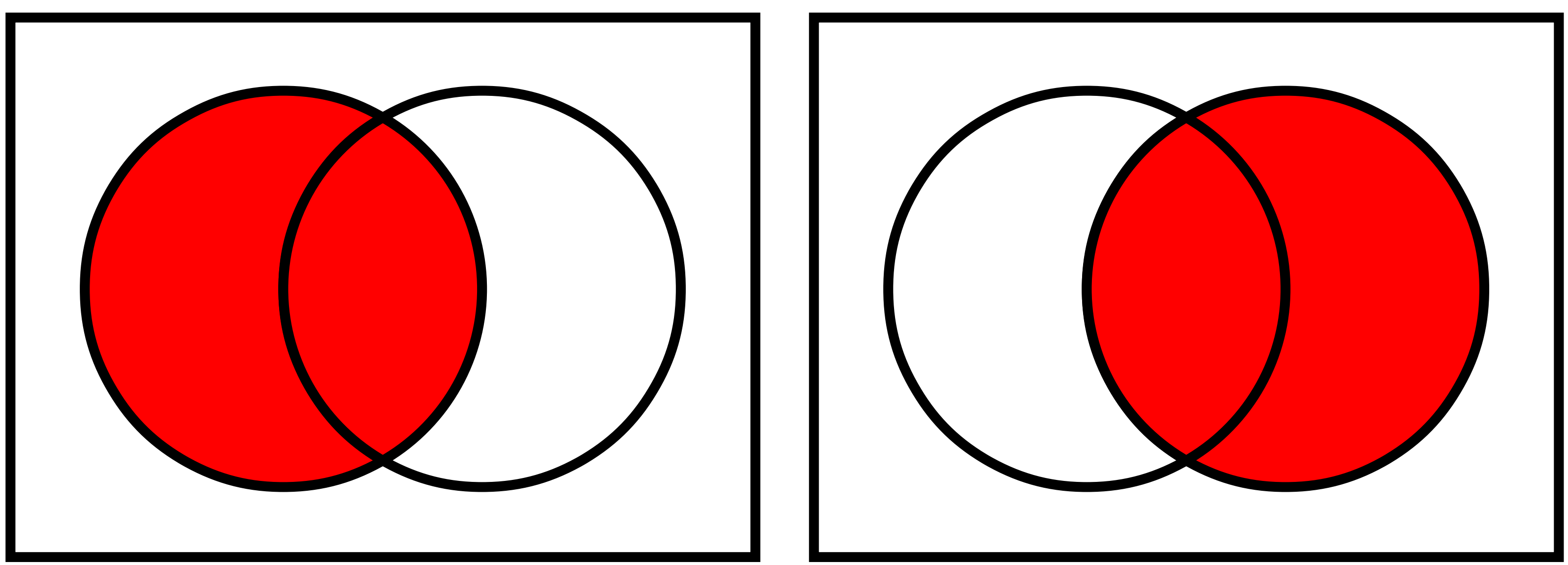

L_1 L_2

The Intersection of Two Languages

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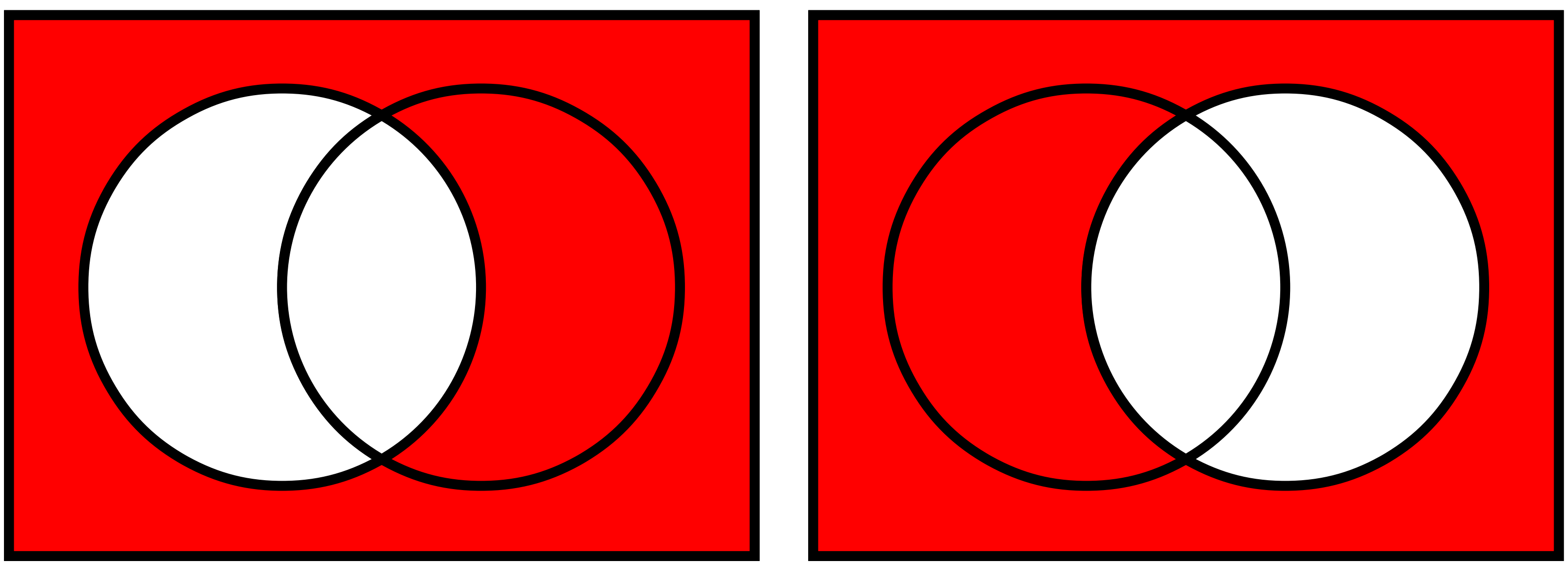

$\overline{L}_1$ $\overline{L}_2$

The Intersection of Two Languages

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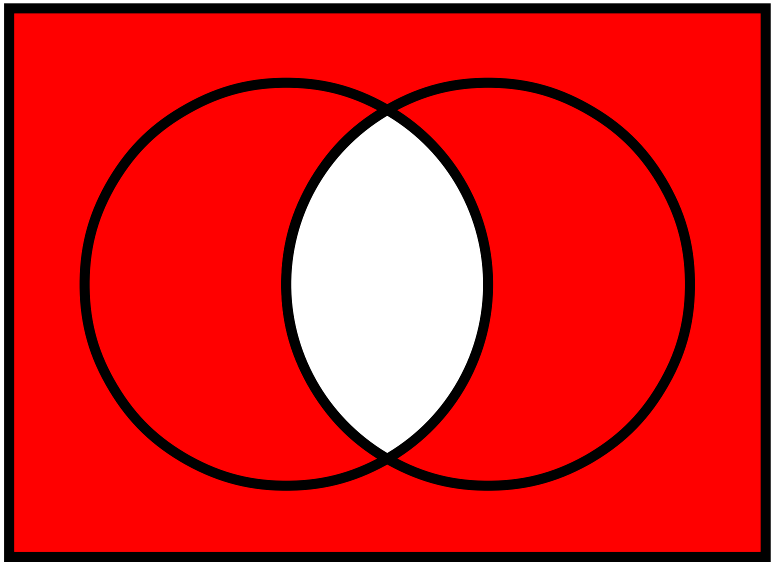

$$\overline{L}_1 \cup \overline{L}_2$$

The Intersection of Two Languages

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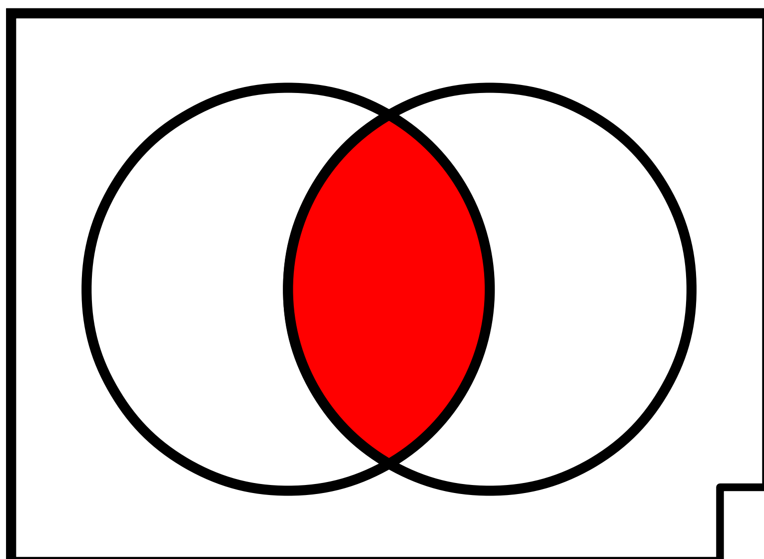

$$\overline{\overline{L_1} \cup \overline{L_2}}$$

Hey, it's De Morgan's laws!

Concatenation

String Concatenation

- If $w \in \Sigma^*$ and $x \in \Sigma^*$, the ***concatenation*** of w and x, denoted wx, is the string formed by tacking all the characters of x onto the end of w.
- Example: if w = **quo** and x = **kka**, the concatenation wx = **quokka**.
- Analogous to the + operator for strings in many programming languages.
- Some facts about concatenation:
  - The empty string ε is the ***identity element*** for concatenation:

$$w\varepsilon = \varepsilon w = w$$

  - Concatenation is ***associative***:

$$wxy = w(xy) = (wx)y$$

Concatenation

- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$\boldsymbol{L_1L_2 = \{ wx \in \Sigma^* \mid w \in L_1 \land x \in L_2 \}}$$

Concatenation Example

- Let Σ = { **a**, **b**, …, **z**, **A**, **B**, …, **Z** } and consider these languages over Σ:
  - ***Noun*** = { **Puppy**, **Rainbow**, **Whale**, … }
  - ***Verb*** = { **Hugs**, **Juggles**, **Loves**, … }
  - ***The*** = { **The** }
- The language ***TheNounVerbTheNoun*** is
  - { **ThePuppyHugsTheWhale**, **TheWhaleLovesTheRainbow**, **TheRainbowJugglesTheRainbow**, … }

Concatenation

- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$L_1L_2 = \{ wx \in \Sigma^* \mid w \in L_1 \land x \in L_2 \}$$

- Two views of L_1L_2:
  - The set of all strings that can be made by concatenating a string in L_1 with a string in L_2.
  - The set of strings that can be split into two pieces: a piece from L_1 and a piece from L_2.
- Conceptually similar to the Cartesian product of two sets, only with strings.

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?

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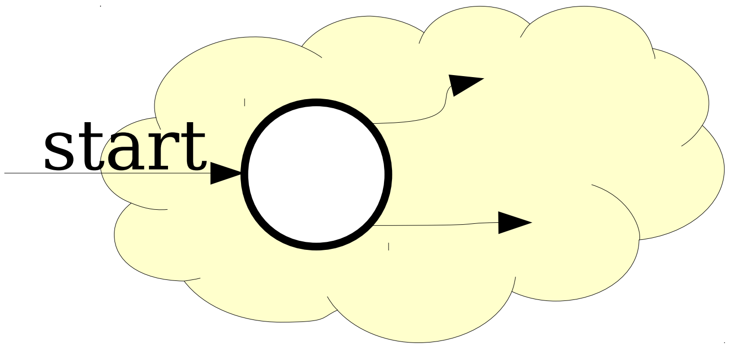
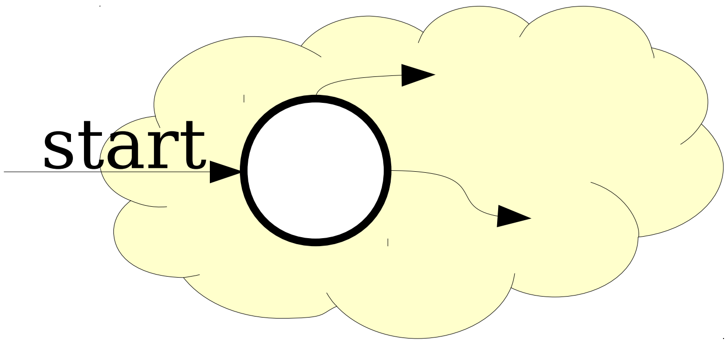


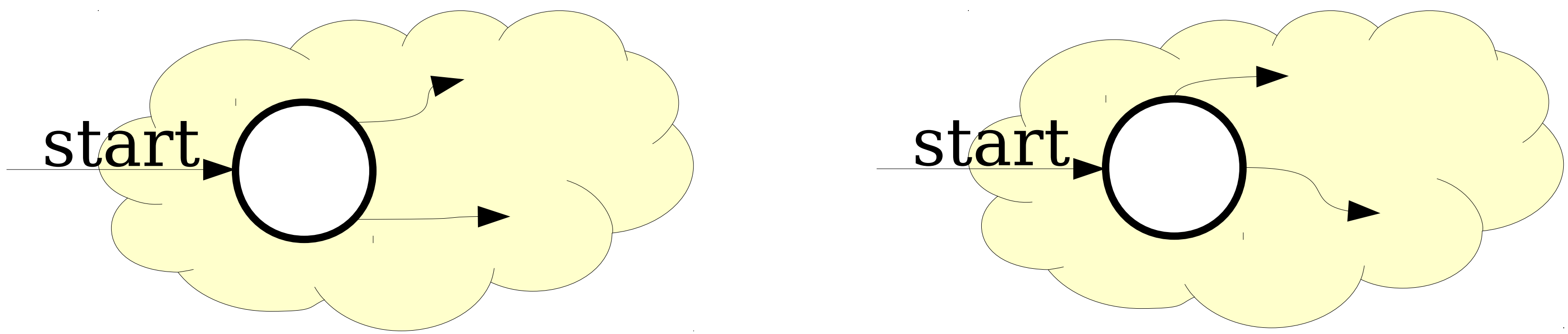
Machine for L_1



Machine for L_2

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?



Machine for L_1 Machine for L_2

b	**o**	**o**	**k**	**k**	**e**	**e**	**p**	**e**	**r**

Concatenating Regular Languages

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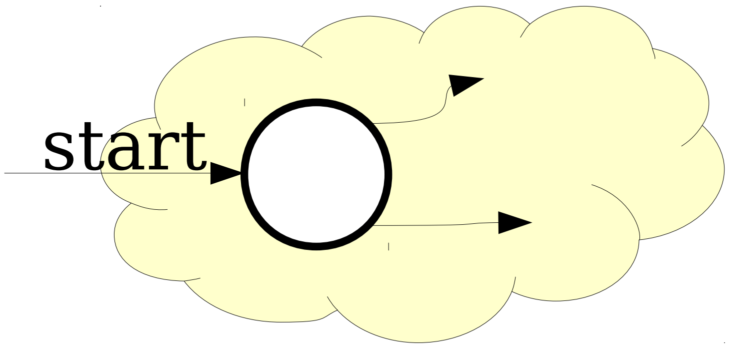

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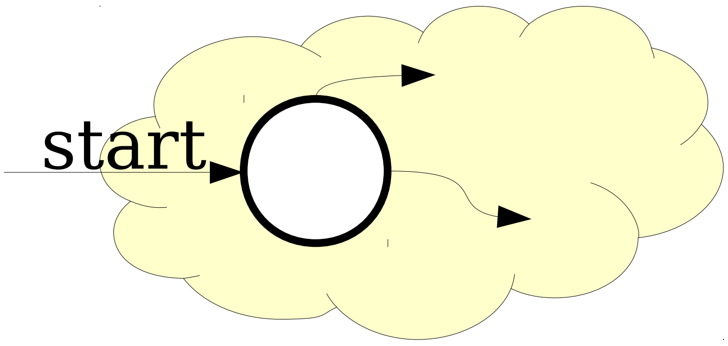
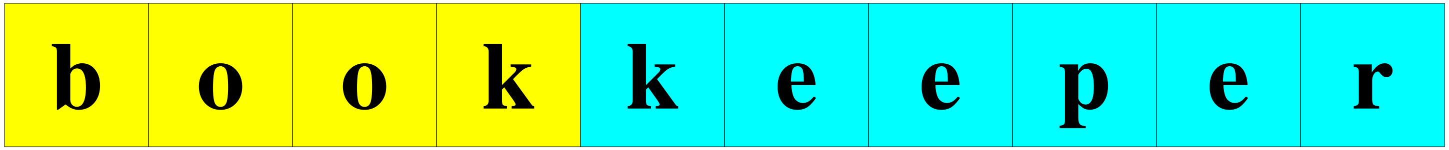


Machine for L_2



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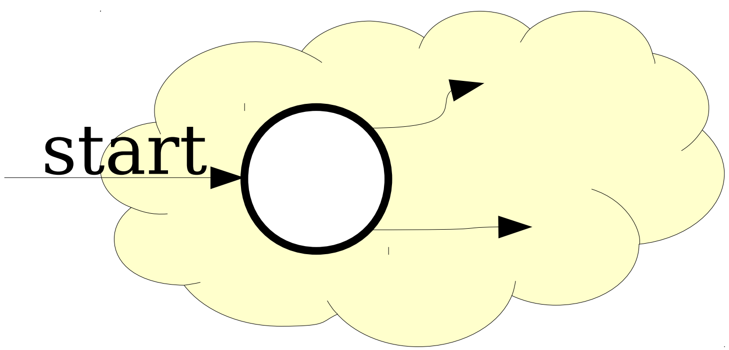
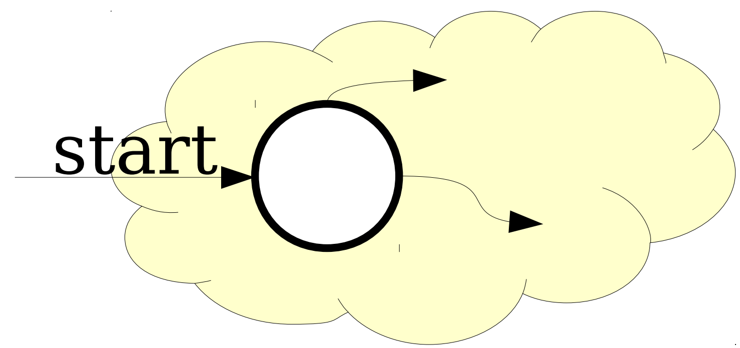


Machine for L_1



Machine for L_2

b	**o**	**o**	**k**

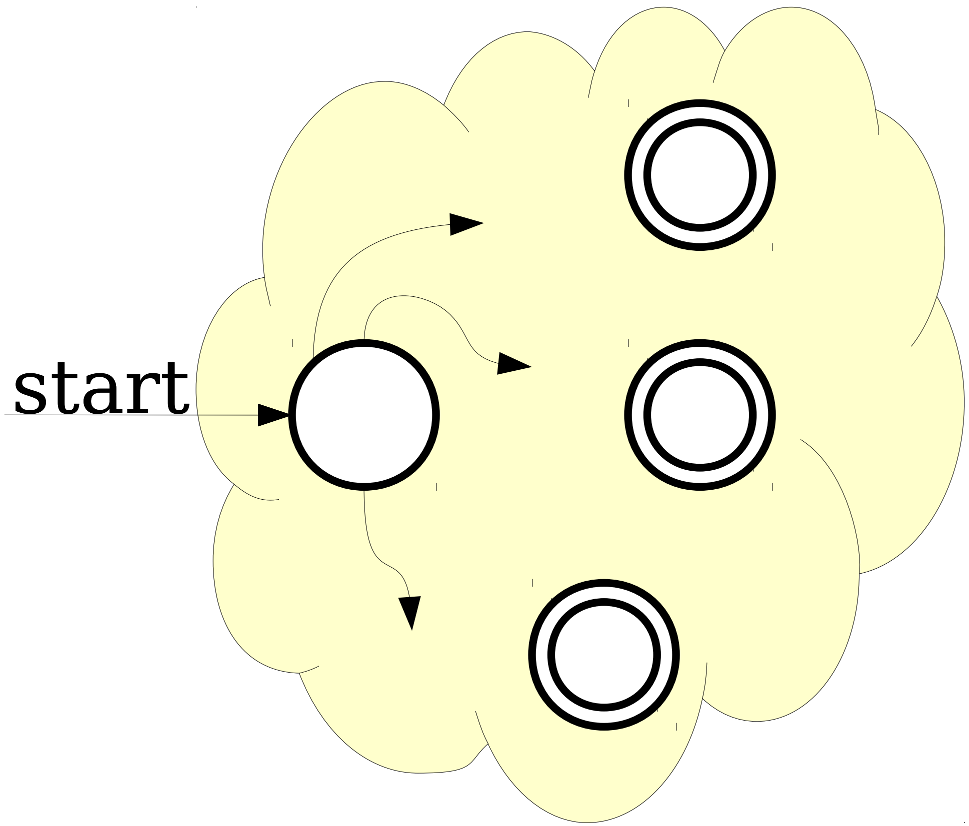
k	**e**	**e**	**p**	**e**	**r**

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?
- ***Idea***: Run the automaton for L_1 on w, and whenever L_1 reaches an accepting state, optionally hand the rest off w to L_2.
  - If L_2 accepts the remainder, then L_1 accepted the first part and the string is in L_1L_2.
  - If L_2 rejects the remainder, then the split was incorrect.

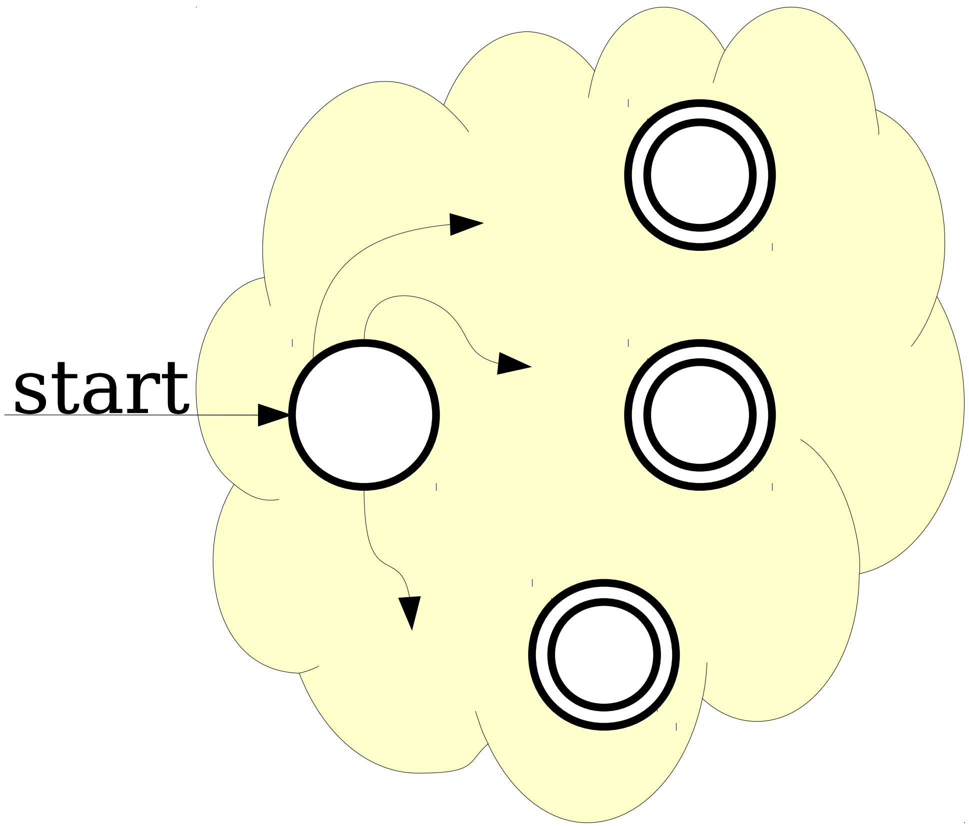
Concatenating Regular Languages

Concatenating Regular Languages

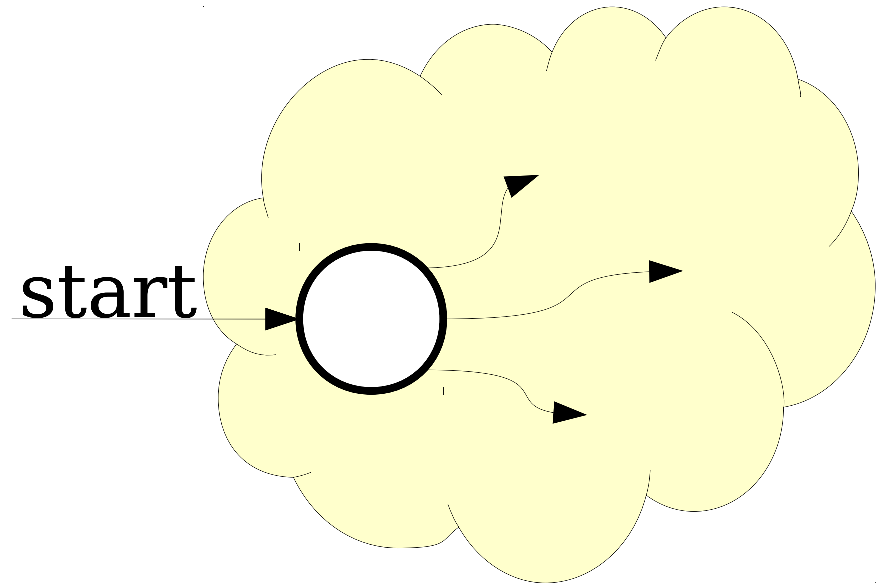


Machine for
L_1

Concatenating Regular Languages

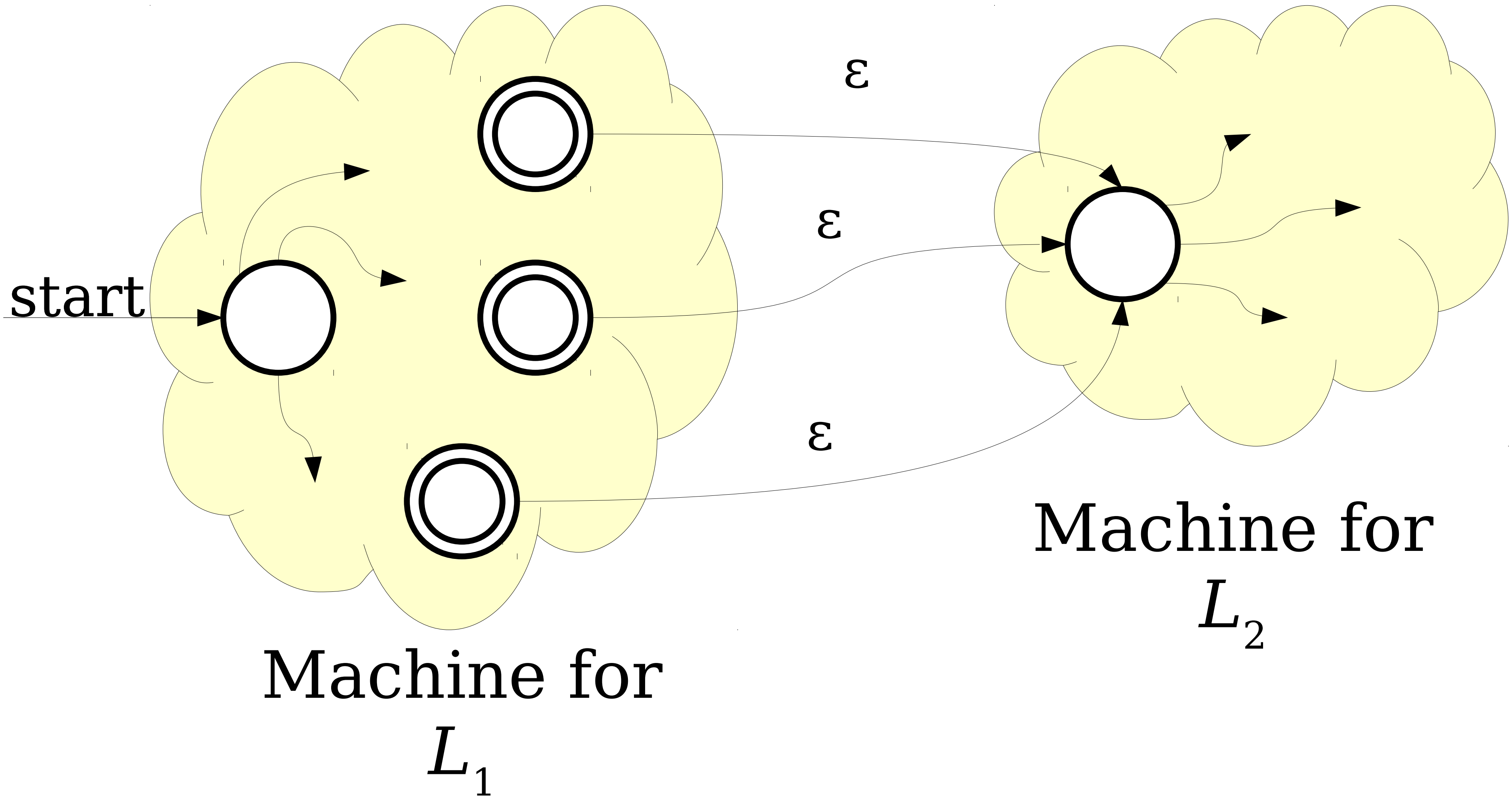


Machine for L_1

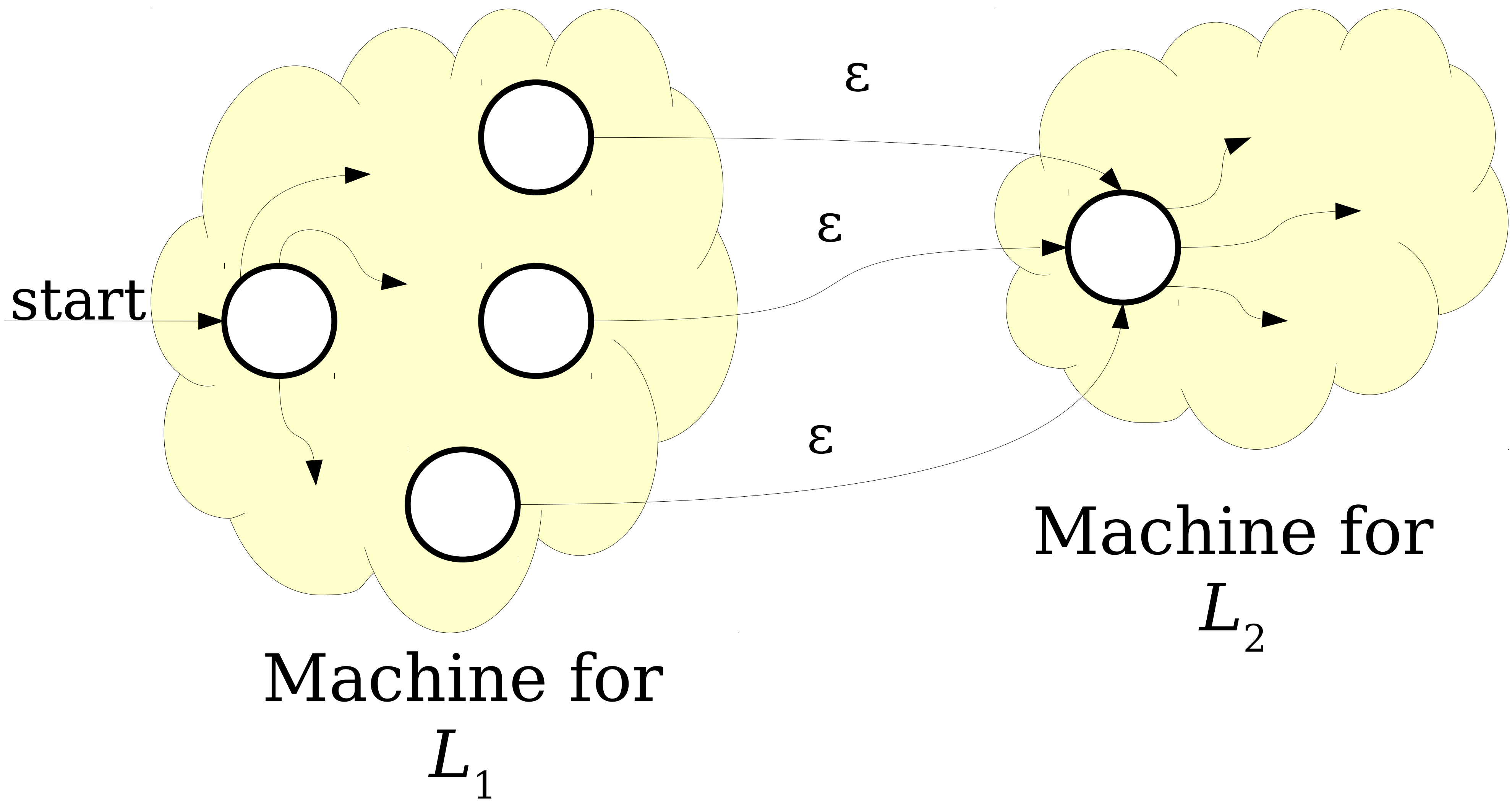


Machine for L_2

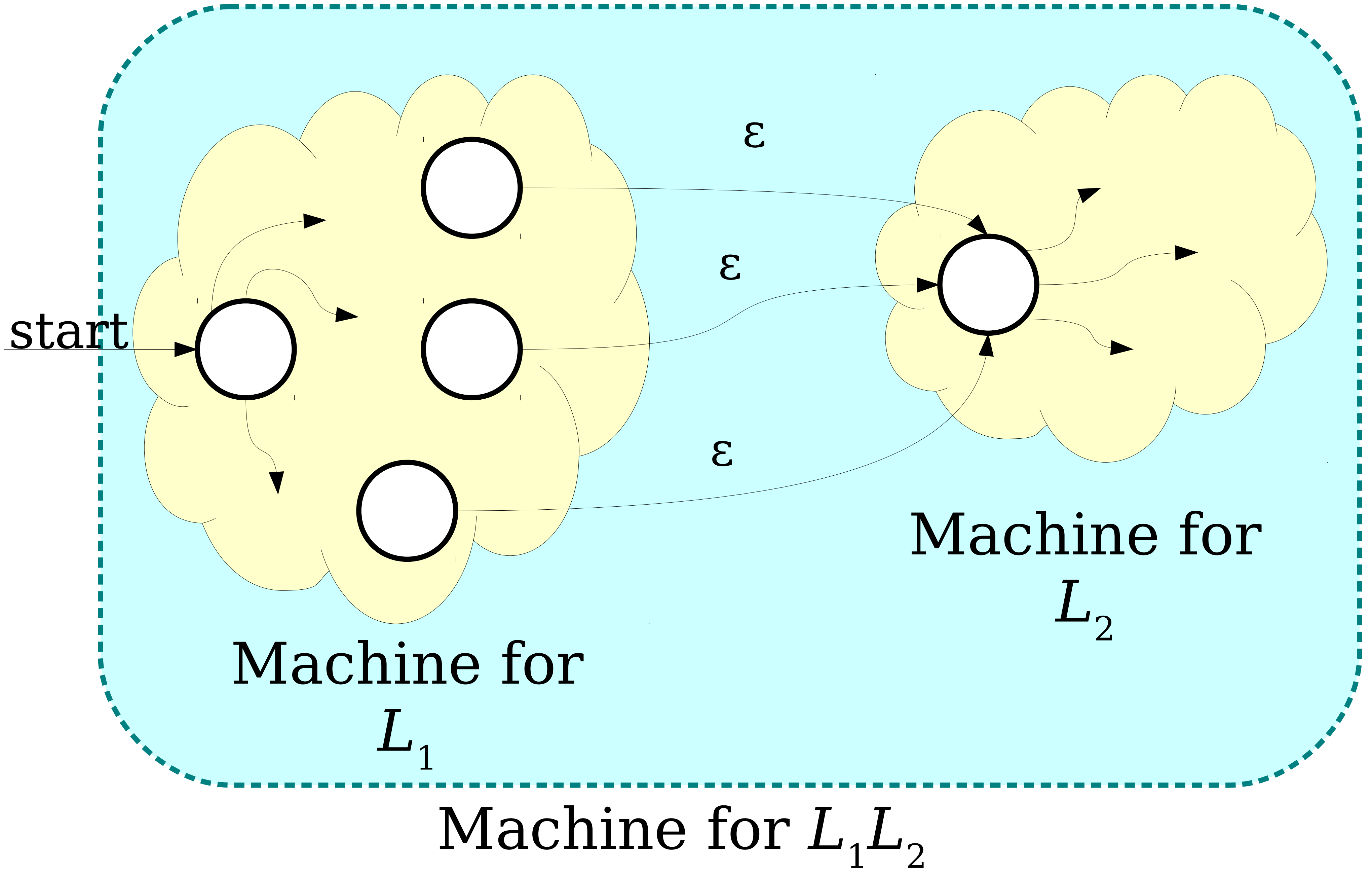
Concatenating Regular Languages



Concatenating Regular Languages



Concatenating Regular Languages



Lots and Lots of Concatenation

- Consider the language L = { aa, b }
- LL is the set of strings formed by concatenating pairs of strings in L.

{ aaaa, aab, baa, bb }

- LLL is the set of strings formed by concatenating triples of strings in L.

{ aaaaaa, aaaab, aabaa, aabb, baaaa, baab, bbaa, bbb}

- $LLLL$ is the set of strings formed by concatenating quadruples of strings in L.

{ aaaaaaaa, aaaaaab, aaaabaa, aaaabb, aabaaaa, aabaab, aabbaa, aabbb, baaaaaa, baaaab, baabaa, baabb, bbaaaa, bbaab, bbbaa, bbbb}

Language Exponentiation

- We can define what it means to "exponentiate" a language as follows:
- $L^0 = \{\varepsilon\}$
  - The set containing just the empty string.
  - Idea: Any string formed by concatenating zero strings together is the empty string.
- $L^{n+1} = LL^n$
  - Idea: Concatenating (n+1) strings together works by concatenating n strings, then concatenating one more.
- ***Question:*** Why define $L^0 = \{\varepsilon\}$?

The Kleene Closure

- An important operation on languages is the ***Kleene Closure***, which is defined as

$$L^* = \{ w \in \Sigma^* \mid \exists n \in \mathbb{N}.\ w \in L^n \}$$

- Mathematically:

$$w \in L^* \quad \text{iff} \quad \exists n \in \mathbb{N}.\ w \in L^n$$

- Intuitively, all possible ways of concatenating zero or more strings in L together, possibly with repetition.

The Kleene Closure

If L = { **a**, **bb** }, then L^* = {

ε,

a, **bb**,

aa, **abb**, **bba**, **bbbb**,

aaa, **aabb**, **abba**, **abbbb**, **bbaa**, **bbabb**, **bbbba**, **bbbbbb**,

…

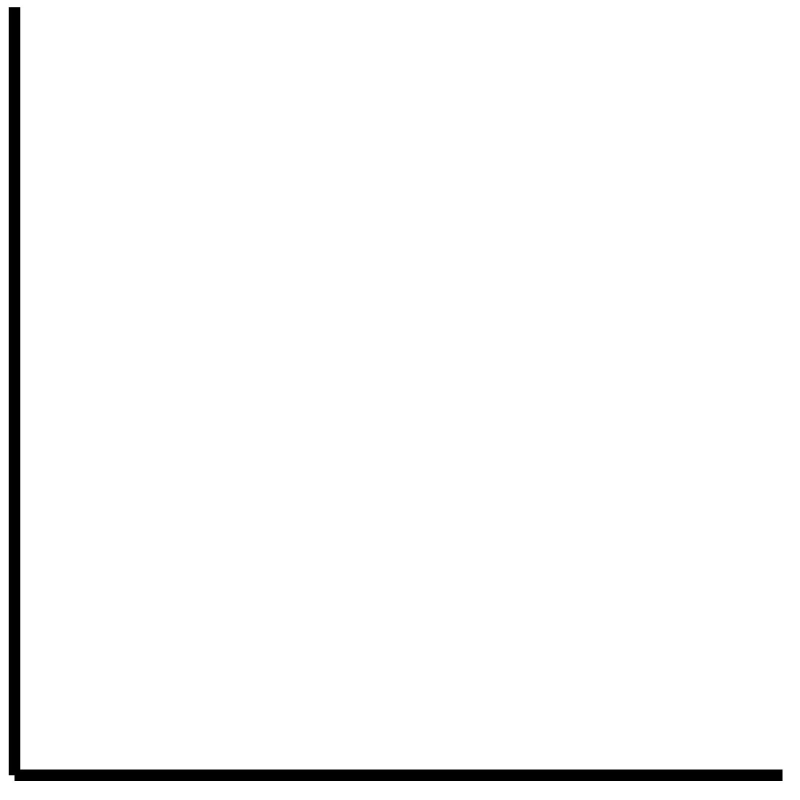
}

Think of L* as the set of strings you can make if you have a collection of stamps – one for each string in L – and you form every possible string that can be made from those stamps.

Reasoning about Infinity

- If L is regular, is L^* necessarily regular?
- **⚠ A Bad Line of Reasoning: ⚠**
  - $L^0 = \{ \varepsilon \}$ is regular.
  - $L^1 = L$ is regular.
  - $L^2 = LL$ is regular
  - $L^3 = L(LL)$ is regular
  - …
  - Regular languages are closed under union.
  - So the union of all these languages is regular.

Reasoning about Infinity



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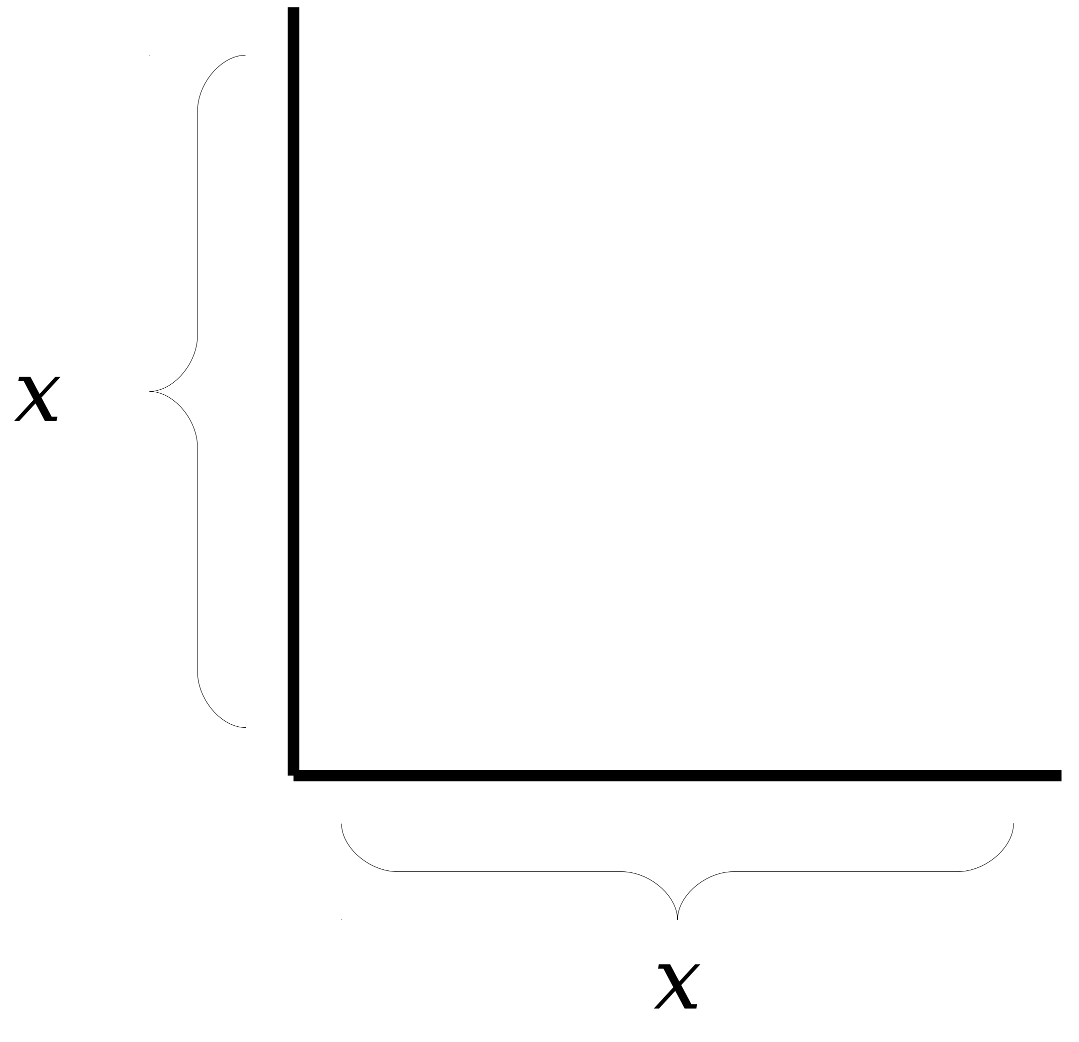

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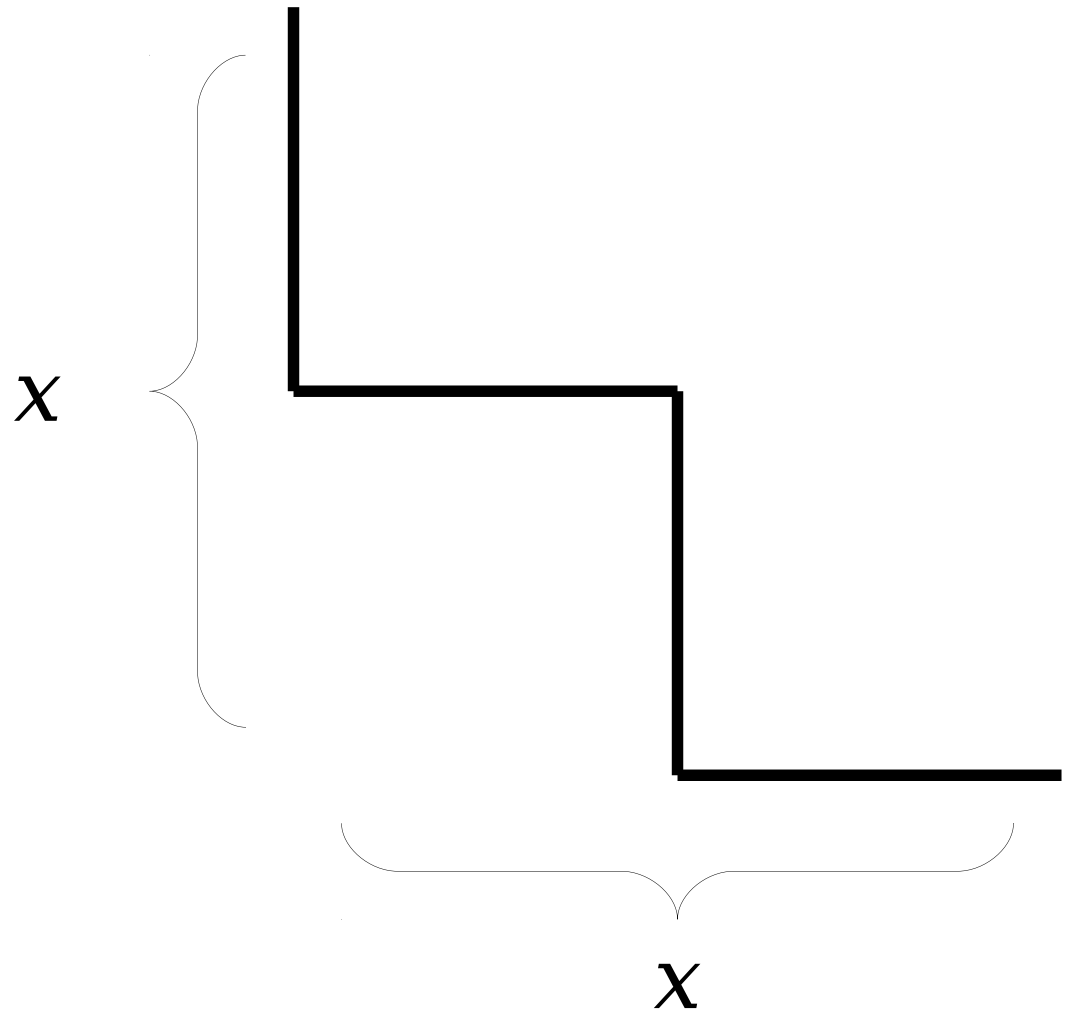

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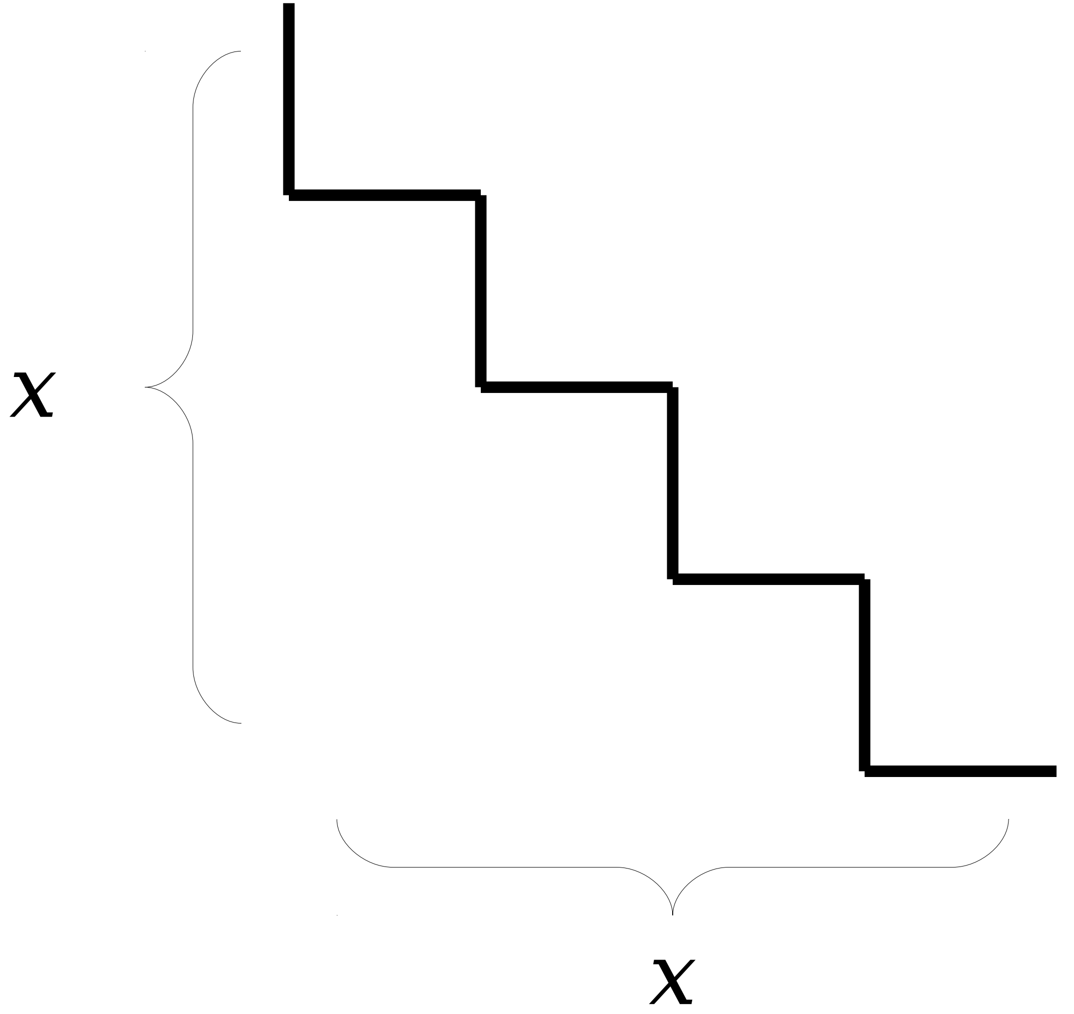

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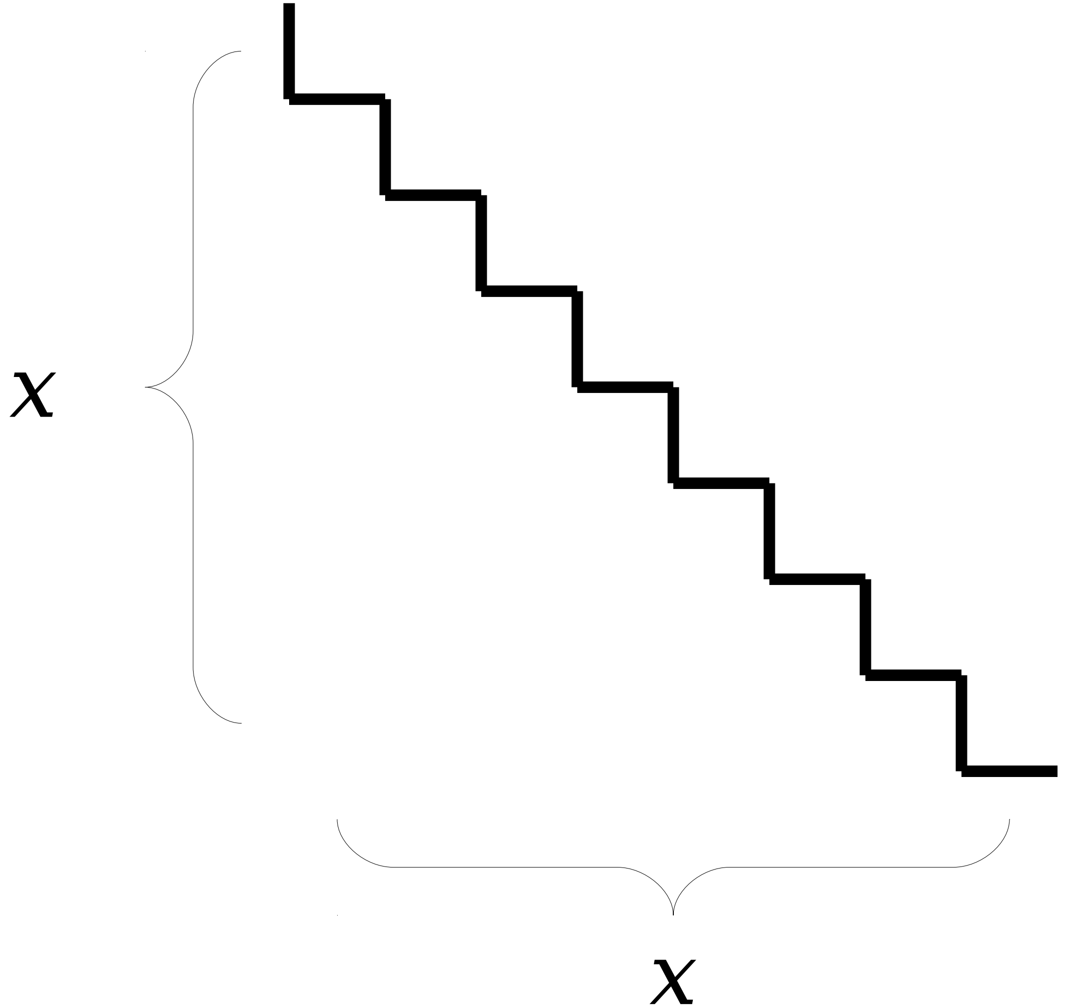

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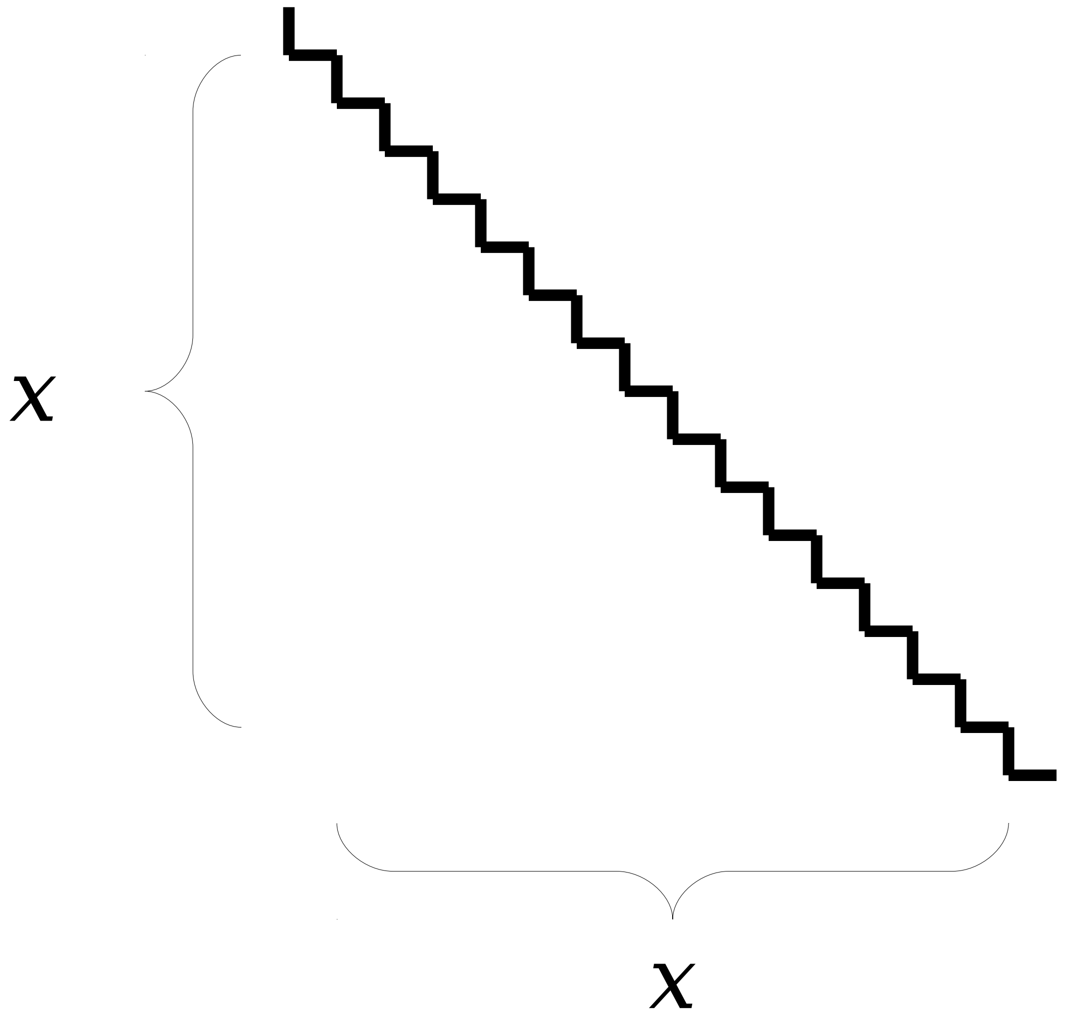

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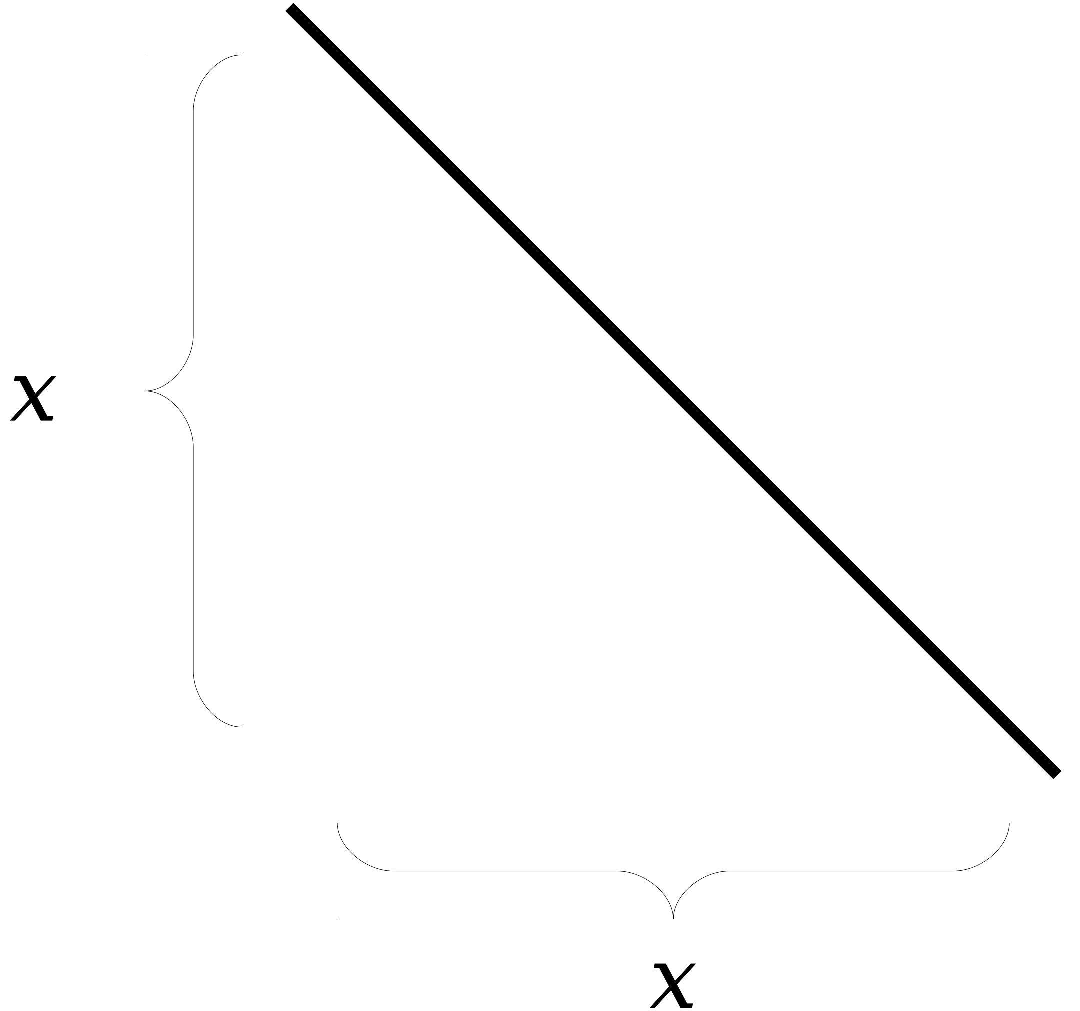

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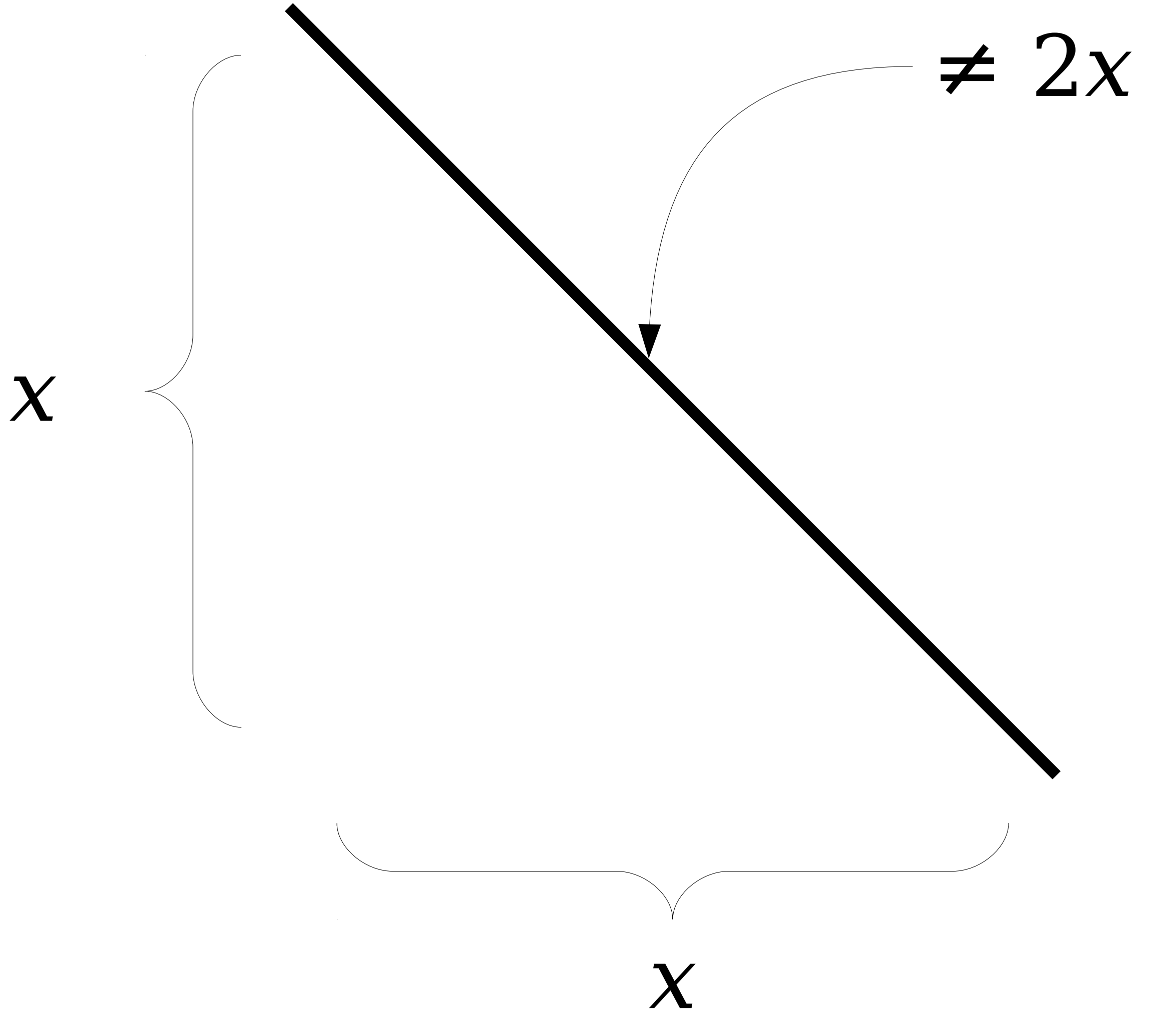


Reasoning about Infinity

$$0.9 < 1$$

Reasoning about Infinity

$$0.99 < 1$$

Reasoning about Infinity

$$0.999 < 1$$

Reasoning about Infinity

$$0.9999 < 1$$

Reasoning about Infinity

$$0.9999\overline{9} < 1$$

Reasoning about Infinity

$$0.9999\overline{9} \not< 1$$

Reasoning about Infinity

0 is finite

Reasoning about Infinity

1 is finite

Reasoning about Infinity

2 is finite

Reasoning about Infinity

3 is finite

Reasoning about Infinity

4 is finite

Reasoning about Infinity

∞ is finite

Reasoning about Infinity

∞ is finite

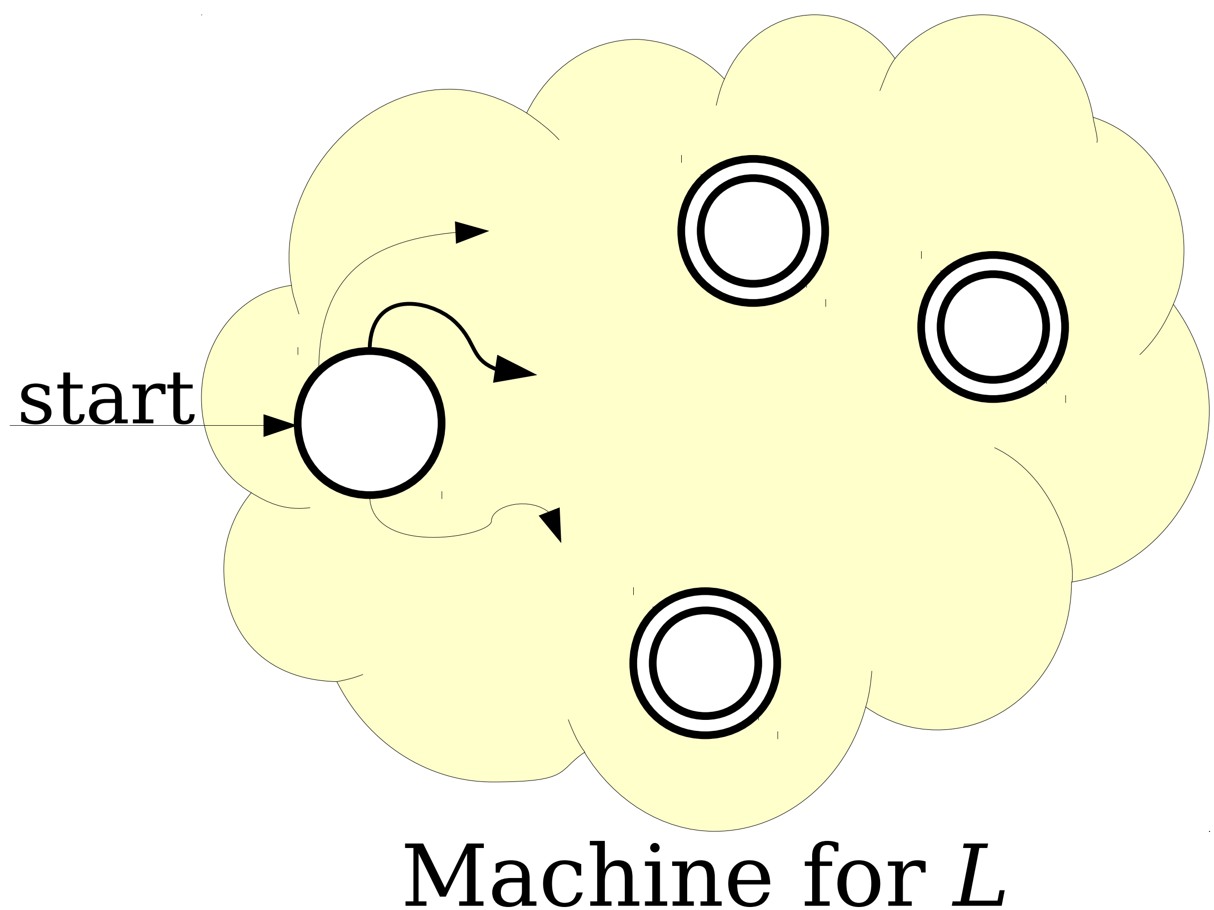
^ not

Reasoning About the Infinite

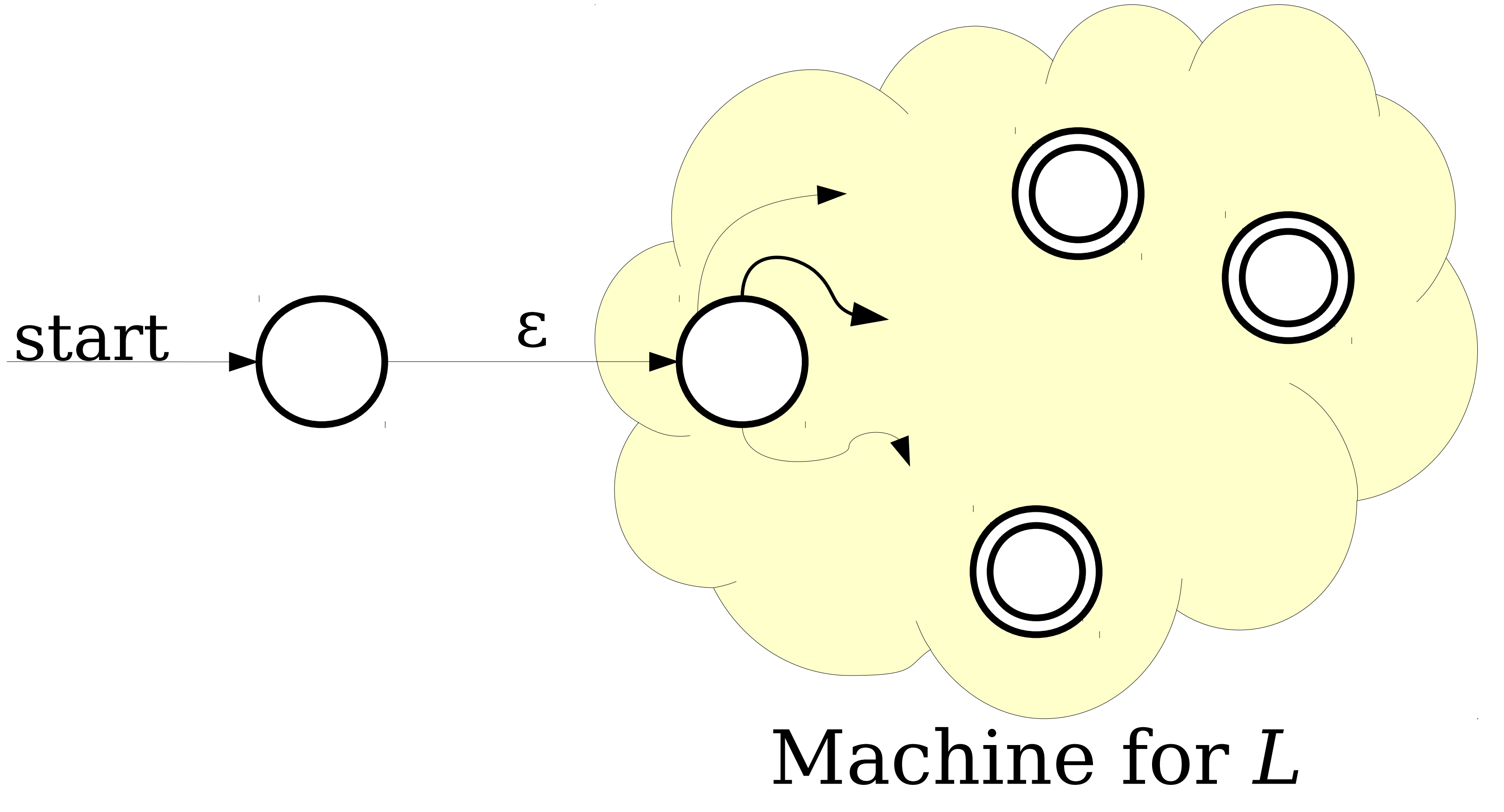
- If a series of finite objects all have some property, the "limit" of that process *does not* necessarily have that property.
- In general, it is not safe to conclude that some property that always holds in the finite case must hold in the infinite case.
  - (This is why calculus is interesting).

Idea: Can we directly convert an NFA for language L to an NFA for language L^*?

The Kleene Star



The Kleene Star



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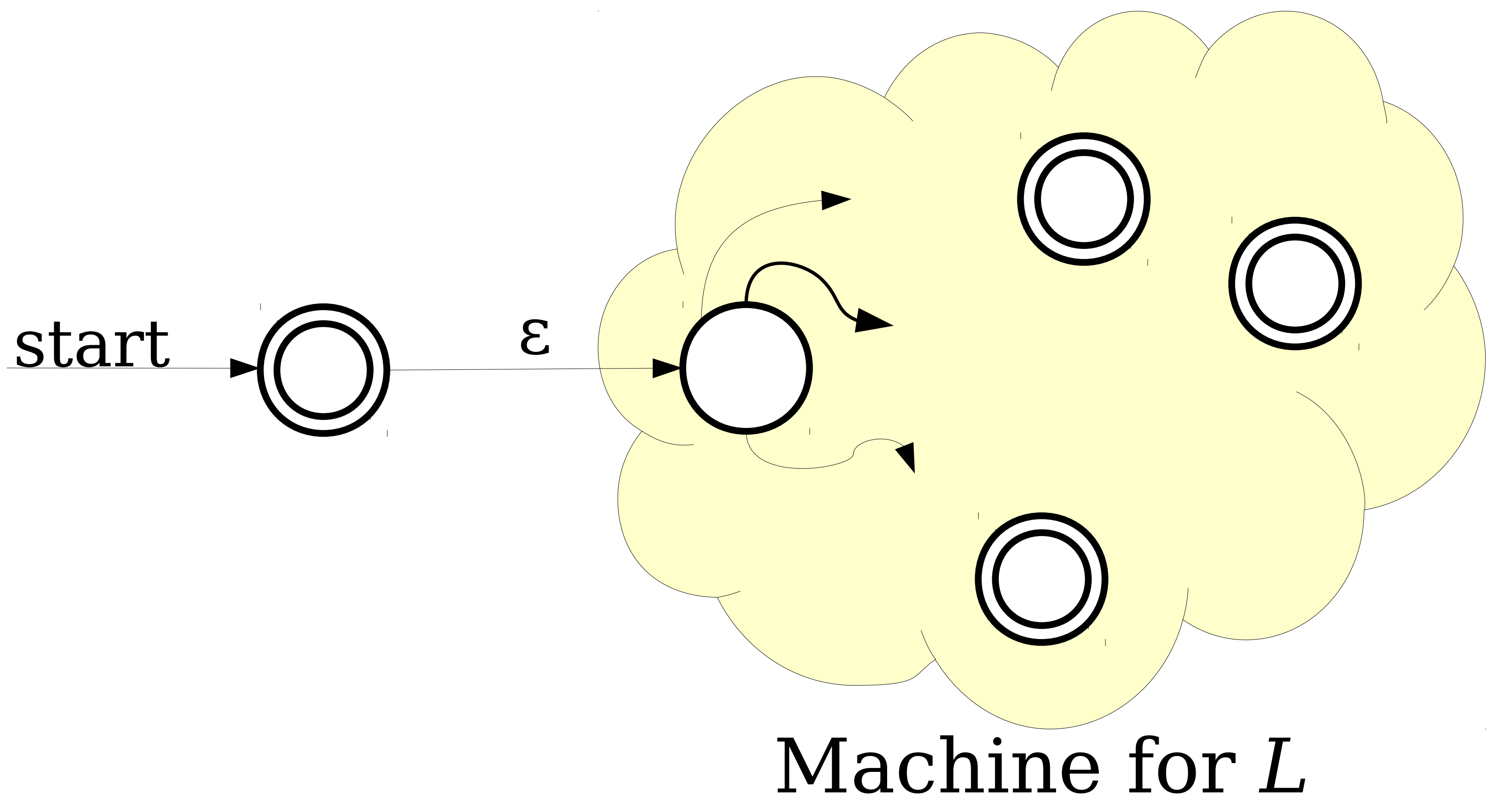

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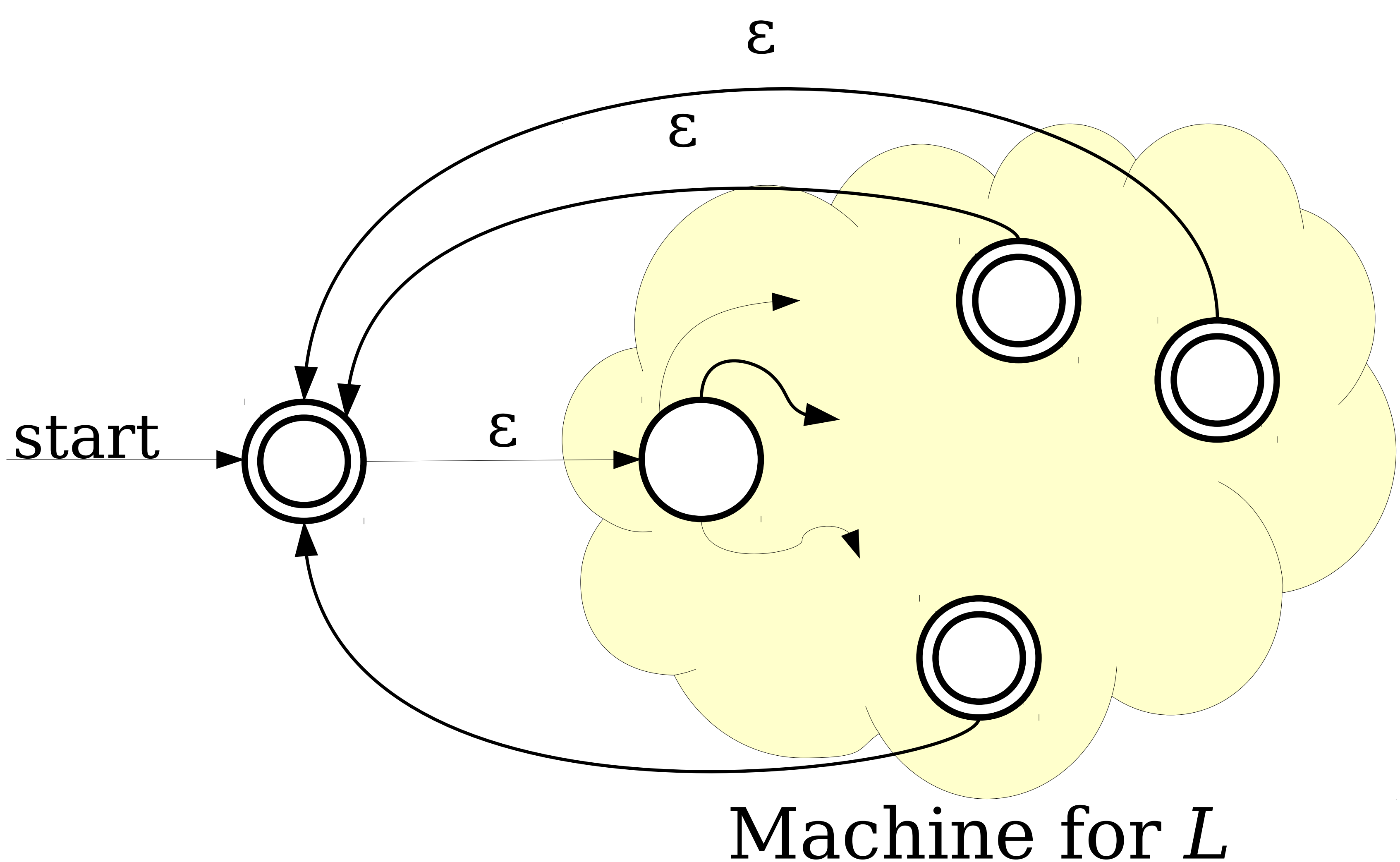

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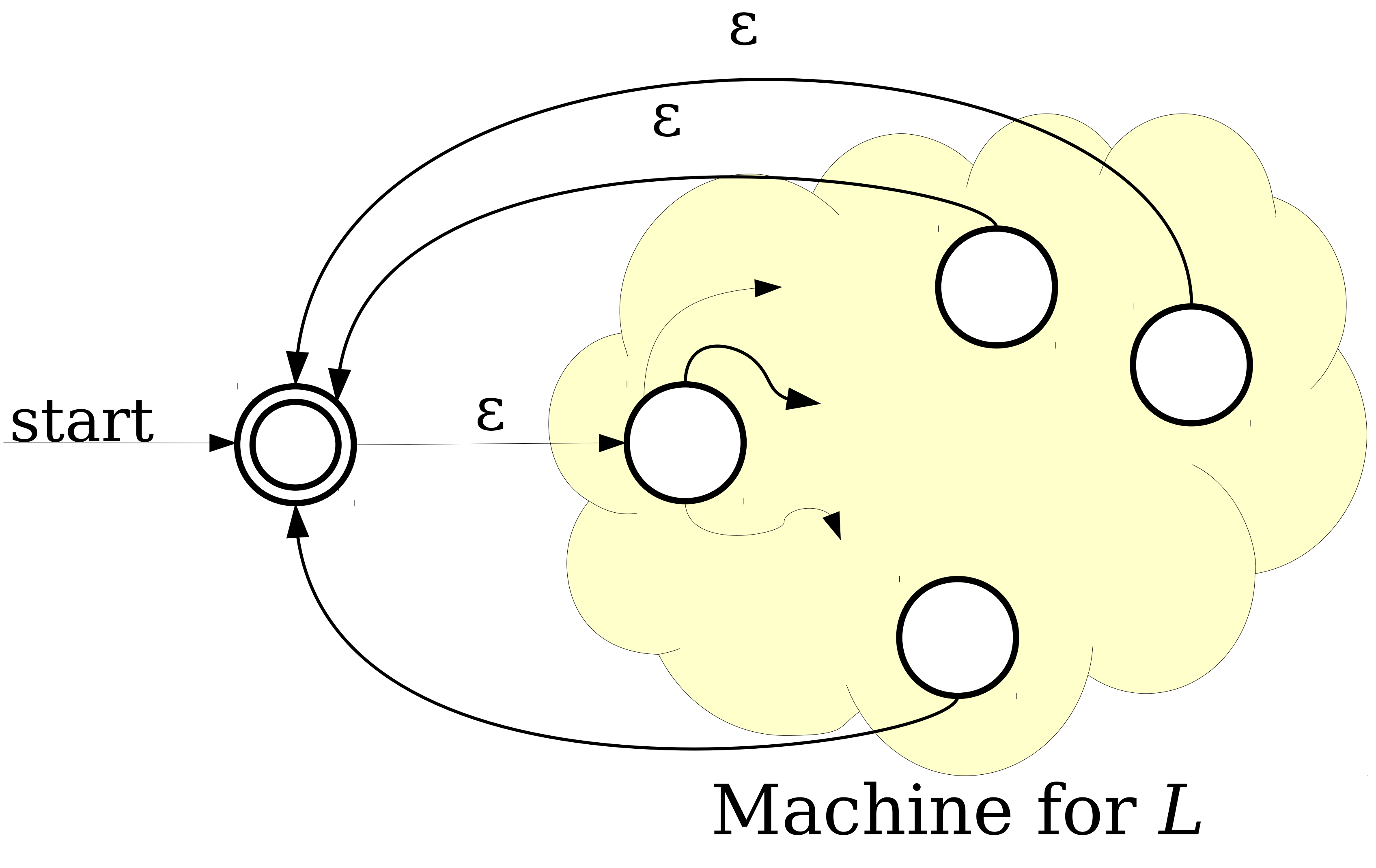

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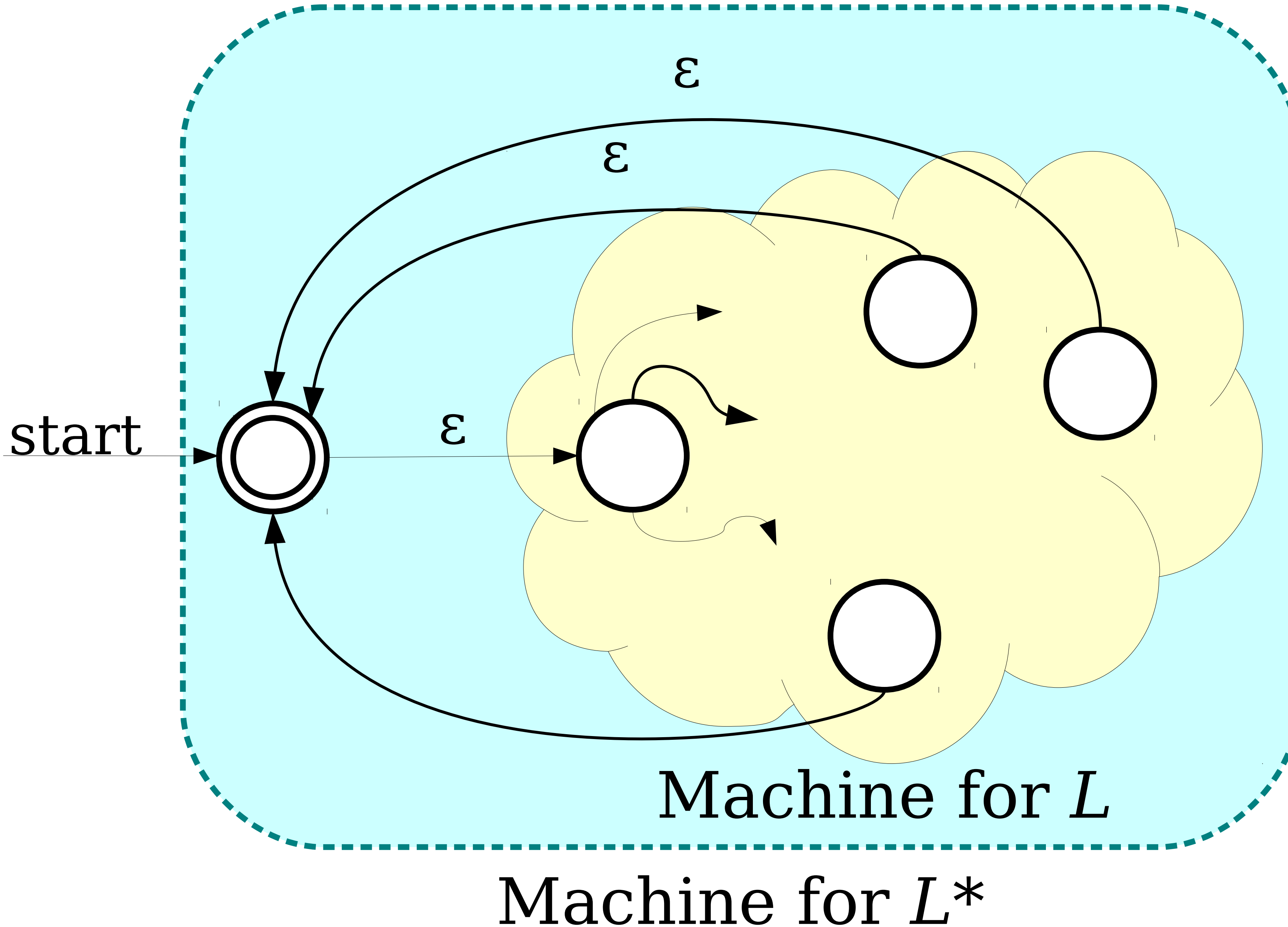
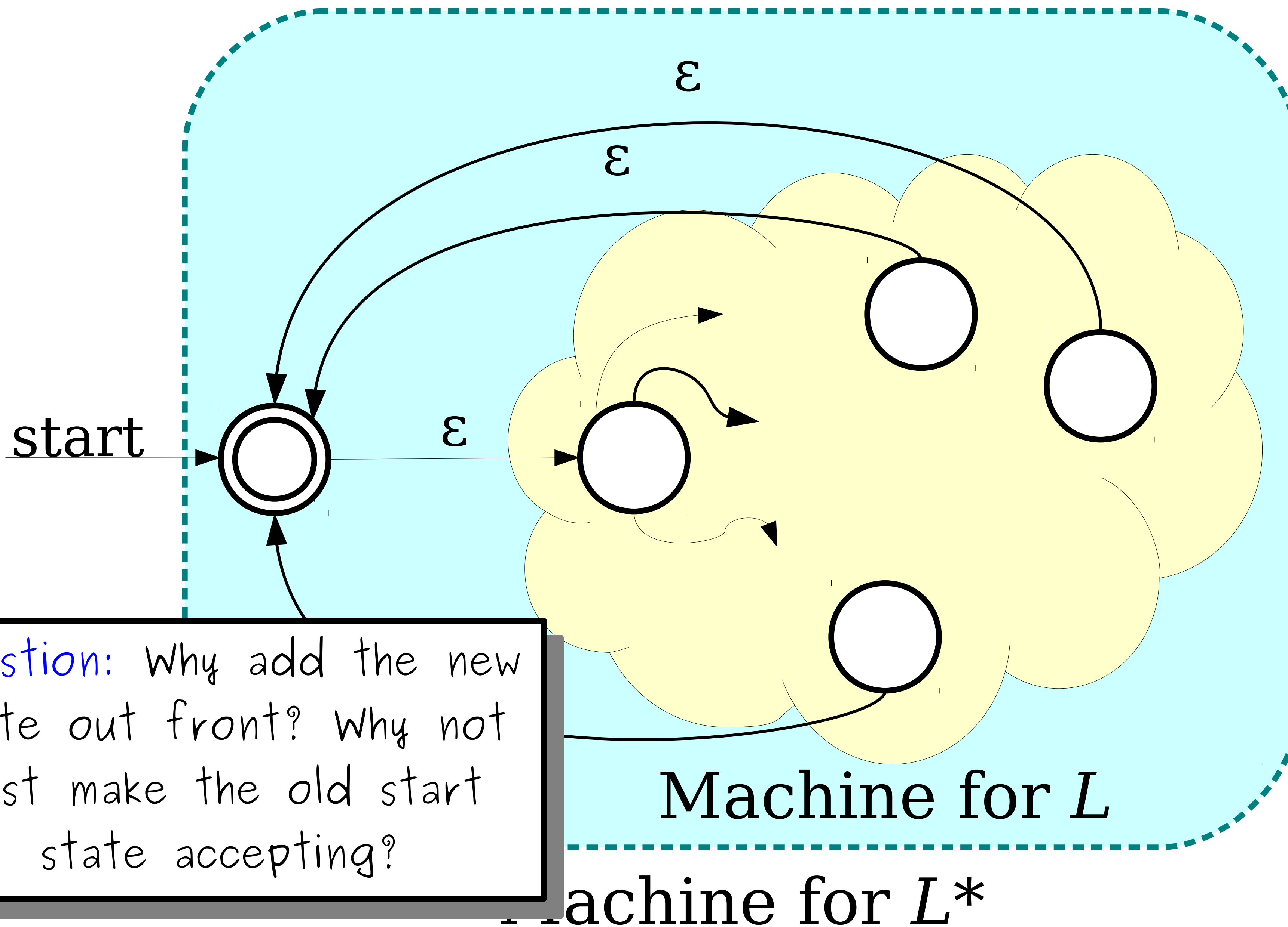


The Kleene Star



Question: Why add the new state out front? Why not just make the old start state accepting?

Closure Properties

- ***Theorem:*** If L_1 and L_2 are regular languages over an alphabet Σ, then so are the following languages:
  - $\overline{L_1}$
  - $L_1 \cup L_2$
  - $L_1 \cap L_2$
  - L_1L_2
  - L_1^*
- These properties are called ***closure properties of the regular languages***.