Guide to the Subset Construction
Hi everybody!
In lecture, we talked about the subset construction, which turns NFAs into DFAs.
We did one example of the construction in class together, but that example didn’t hit all cases.
For example, it didn’t talk about ε-transitions, about NFAs that die, or about multiple accepting states.
Here’s a worked example that talks through the reasoning behind the subset construction. Hope this helps!
Because there's going to be a lot of content on these slides, I'm going to change how I look from the normal "Guide to X" setup.
I know that I now look like a super formal piece of text captioning something, but I promise that this is still me and that I’m acting as the narrator for this section. 😊
Once More, With Epsilons!

Here’s the NFA that we’re going to convert into a DFA. (Or rather, we’re going to build a new automaton that’s a DFA that has the same language as this NFA. But you can think of it as performing some sort of conversion if you’d like.)
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We’ll represent this new DFA as a table, since that’s usually much easier than drawing a state transition diagram!
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There are more rows here than states, and that’s normal. The subset construction usually makes a DFA with more states than the NFA.
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There’s no general way to predict how many rows you’ll need. We’ve scouted ahead here and made this table exactly the right size. 😃
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Let’s start off by thinking about what the start state of our DFA is going to be.
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To do that, let’s see what happens when we power on the NFA. We’ll begin in the start state, as usual.
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But... there's a twist. When this NFA powers on, if we use the "massive parallel" intuition, we'd also end up in state $q_3$ because there's an $\varepsilon$-transition from state $q_0$ to state $q_3$. 
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Note that we *don’t* include state $q_4$ here. The transition from $q_3$ to $q_4$ is a $\Sigma$-transition rather than an $\varepsilon$-transition. Remember that $\Sigma$ means “you can take this transition when reading any character,” whereas an $\varepsilon$-transition means “you can take this transition whenever you’d like.”
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Now, we ask – what would this NFA do in this collection of states if it read a particular character?

Let’s start off by seeing what the NFA would do on an a.
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State $q_0$ only has one option – go to state $q_1$. We could alternatively take the $\varepsilon$-transition to state $q_3$ and then try to follow a transition there, but we don’t need to consider that here. We’re already going to be looking at state $q_3$ in a second anyway, so following the $\varepsilon$ transition here first would be redundant.
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State $q_3$ can take its $\Sigma$-transition to state $q_4$. That's another option.
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So here’s where we’d end up if we saw an $a$ in these combination of states.
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So we’ll write down this combo of states in the column for $a$. 

\[
\begin{array}{c|c|c}
\text{a} & \text{b} \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} \\
\hline
\end{array}
\]

So we’ll write down this combo of states in the column for $a$. 

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Now, let’s do the same thing for the character $b$. 

\[
\begin{array}{c|c|c}
   & a & b \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \\
\end{array}
\]

Now, let’s do the same thing for the character $b$. 

\[
\begin{array}{c|c|c}
   & a & b \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \\
\end{array}
\]
Once More, With Epsilons!

State $q_0$ has no transitions on $b$, so that state will die off. (We denote this by having an arrow pointing into the abyss.)
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State $q_3$ can only go to $q_4$. 
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So overall we’re in a combination of just one state...
Once More, With Epsilons!

\[
\begin{array}{c}
\text{start} \\
q_0 \\
\varepsilon \\
q_3 \\
q_2 \ \ \ \Sigma \\
q_1 \ \ \ \ b \\
q_4 \ \ \ \ \Sigma \\
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{a} & \text{b} \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} \\
\hline
\{q_4\} & \hline
\end{array}
\]

... so that’s what we’ll put into our table.
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At this point, we’re done processing that combination of states. Great!

From this point forward, we’ll repeat a simple process: pick a combination of states that’s an entry in the table but not yet a row in the table, then apply this same process.
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Let's try this combination.

What happens if we read a?
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Well, state $q_1$ has nowhere to go, so it’ll die off.
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And similarly, state $q_4$ has nowhere to go, so it’ll die off as well.
Once More, With Epsilons!

That means that we’re in this combination of states – nothing is active, since everything died off. (Uh oh).

So what goes in the table here? Make a guess, then move on to the next slide.
Once More, With Epsilons!

You’ve got your guess, right? Because of course you wouldn’t peek ahead without doing that. 😃

\[
\begin{array}{c c c}
  & a & b \\
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & & \\
\end{array}
\]
Once More, With Epsilons!

We’re going to put the empty set here. Why?

The general rule is that we take the set of states that we’d end up in, then put that in the table. Stated differently, we’d gather all the active states into a set and write that set down. There are no active states, so the set of active states is $\emptyset$. 

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More generally, if the NFA ever dies off on some branch, the entry for the table will be the empty set, indicating “we are in a collection of zero NFA states right now.”

Remember: DFAs have to have a transition defined for each state/symbol pair. So we have to go somewhere.
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Let’s talk about something happier. What happens if we read $b$ here?
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State $q_1$ goes to $q_2$...

State $q_1$ goes to $q_2$...

\[
\begin{array}{|c|c|c|}
\hline
& a & b \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & \emptyset & \emptyset \\
\hline
\end{array}
\]
Once More, With Epsilons!

... and state $q_4$ goes to state $q_3$. 
Once More, With Epsilons!

So overall we’re now here...

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0, q_3}$</td>
<td>${q_1, q_4}$</td>
<td>${q_4}$</td>
</tr>
<tr>
<td>${q_1, q_4}$</td>
<td>$\emptyset$</td>
<td></td>
</tr>
</tbody>
</table>

So overall we’re now here...
Once More, With Epsilons!

So let’s write this combination down.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0, q_3}</td>
<td>{q_1, q_4}</td>
<td>{q_4}</td>
</tr>
<tr>
<td>{q_1, q_4}</td>
<td>\Ø</td>
<td>{q_2, q_3}</td>
</tr>
</tbody>
</table>

So let’s write this combination down.
Once More, With Epsilons!

Another row down. Wonderful! Let’s pick a new set of states to explore.
Once More, With Epsilons!

Here's a nice one. What are the entries in this row going to be?
First, what happens on $a$?

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0, q_3}$</td>
<td>${q_1, q_4}$</td>
<td>${q_4}$</td>
</tr>
<tr>
<td>${q_1, q_4}$</td>
<td>$\emptyset$</td>
<td>${q_2, q_3}$</td>
</tr>
<tr>
<td>${q_4}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Here's a nice one. What are the entries in this row going to be?
First, what happens on $a$?
Once More, With Epsilons!

Well, \( q_4 \) dies on \( a \)...

\[
\begin{array}{ccc}
\text{start} & \rightarrow & q_0 \\
\epsilon & \rightarrow & q_3 \\
q_0 & \stackrel{\Sigma}{\rightarrow} & q_1 \\
q_2 & \rightarrow & b \\
q_3 & \rightarrow & b \\
q_4 & \rightarrow & q_4 \\
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
& a & b \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & \emptyset & \{q_2, q_3\} \\
\{q_4\} & & \\
\hline
\end{array}
\]
Once More, With Epsilons!

Once More, With Epsilons!

so we end up in no states ...

\[
\begin{array}{c|cc}
 & a & b \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & \emptyset & \{q_2, q_3\} \\
\{q_4\} & & \\
\end{array}
\]
Once More, With Epsilons!

So we write down the empty set here. Done.
Once More, With Epsilons!

What if we read $b$?
Once More, With Epsilons!

That would take us to $q_3$…

That would take us to $q_3$…
Once More, With Epsilons!

\[
\begin{array}{ll}
\{q_0, q_3\} & \{q_1, q_4\} \\
\{q_1, q_4\} & \emptyset \\
\{q_4\} & \emptyset \\
\end{array}
\]

... giving us this set of one active state...
Once More, With Epsilons!

So we write down a singleton set for our result.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0, q_3}</td>
<td>{q_1, q_4}</td>
<td>{q_4}</td>
</tr>
<tr>
<td>{q_1, q_4}</td>
<td>\emptyset</td>
<td>{q_2, q_3}</td>
</tr>
<tr>
<td>{q_4}</td>
<td>\emptyset</td>
<td>{q_3}</td>
</tr>
</tbody>
</table>

So we write down a singleton set for our result.
Once More, With Epsilons!

This row is now complete. On to the next.

This row is now complete. On to the next.
Once More, With Epsilons!

Things will get interesting now. A quick little time-saving observation: notice that the only transitions out of these states are $\Sigma$-transitions. That means the behavior will be the same on all possible characters. So where do we go if we read something?
Once More, With Epsilons!

State $q_2$ goes to $q_0$.

(There’s an $\varepsilon$-transition leaving state $q_0$ that we’ll need to consider, but for now, let’s ignore that.)
Once More, With Epsilons!

State $q_3$ goes to $q_4$.  

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0, q_3}$</td>
<td>${q_1, q_4}$</td>
<td>${q_4}$</td>
</tr>
<tr>
<td>${q_1, q_4}$</td>
<td>$\emptyset$</td>
<td>${q_2, q_3}$</td>
</tr>
<tr>
<td>${q_4}$</td>
<td>$\emptyset$</td>
<td>${q_3}$</td>
</tr>
<tr>
<td>${q_2, q_3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Once More, With Epsilons!

Once More, With Epsilons!

So that takes us here. Normally, we’d stop and write this combination down, but we aren’t done yet. Importantly, there’s an \( \varepsilon \)-transition leaving state \( q_0 \), and we have to consider that.
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We’ll therefore expand our set of states to include $q_3$. We’re only now considering $\varepsilon$-transitions. Our policy will be to only care about $\varepsilon$-transitions in two cases: first, when determining the start state; second, after considering all transitions we can take by reading characters.
Once More, With Epsilons!

So we'll fill this set of states into our table for both a and b, since, as we saw earlier, the transitions out of $q_2$ and $q_3$ are $\Sigma$-transitions and thus work for all characters.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0, q_3}$</td>
<td>${q_1, q_4}$</td>
<td>${q_4}$</td>
</tr>
<tr>
<td>${q_1, q_4}$</td>
<td>$\emptyset$</td>
<td>${q_2, q_3}$</td>
</tr>
<tr>
<td>${q_4}$</td>
<td>$\emptyset$</td>
<td>${q_3}$</td>
</tr>
<tr>
<td>${q_2, q_3}$</td>
<td>${q_0, q_3, q_4}$</td>
<td>${q_0, q_3, q_4}$</td>
</tr>
</tbody>
</table>

So we’ll fill this set of states into our table for both a and b, since, as we saw earlier, the transitions out of $q_2$ and $q_3$ are $\Sigma$-transitions and thus work for all characters.
Once More, With Epsilons!

That’s convenient, because we’re now done with this row!

From here on out, we just keep applying these same rules. I’m going to pause on the narration until Slide 85. Feel free to go one step at a time until then if you want to see the algorithm at work.
Once More, With Epsilons!

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A non-deterministic finite automaton (NFA) with symbols following the transitions:

- Start state: $q_0$
- Final states: $q_3$, $q_4$
- Transitions:
  - $q_0$: On input $a$, move to $q_1$; On input $\varepsilon$, move to $q_3$
  - $q_1$: On input $a$, move to $q_2$; On input $b$, move to $q_4$
  - $q_3$: On input $a$, move to $q_1$; On input $b$, move to $q_4$
  - $q_4$: On input $a$, move to $q_0$; On input $b$, move to $q_2$

---

Transition table:

<table>
<thead>
<tr>
<th>States</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0, q_3}$</td>
<td>${q_1, q_4}$</td>
<td>${q_4}$</td>
</tr>
<tr>
<td>${q_1, q_4}$</td>
<td>$\emptyset$</td>
<td>${q_2, q_3}$</td>
</tr>
<tr>
<td>${q_4}$</td>
<td>$\emptyset$</td>
<td>${q_3}$</td>
</tr>
<tr>
<td>${q_2, q_3}$</td>
<td>${q_0, q_3, q_4}$</td>
<td>${q_0, q_3, q_4}$</td>
</tr>
<tr>
<td>${q_3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Once More, With Epsilons!

\[
\begin{array}{ccc}
q_0 & \xrightarrow{a} & q_1 \\
\varepsilon & \xrightarrow{} & \Sigma \\
\xrightarrow{\Sigma} & \xrightarrow{b} & q_2 \\
\xrightarrow{b} & \xrightarrow{} & q_3 \\
\xrightarrow{} & \xrightarrow{\Sigma} & q_4 \\
\end{array}
\]

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0, q_3}</td>
<td>{q_1, q_4}</td>
<td>{q_4}</td>
</tr>
<tr>
<td>{q_1, q_4}</td>
<td>\emptyset</td>
<td>{q_2, q_3}</td>
</tr>
<tr>
<td>{q_4}</td>
<td>\emptyset</td>
<td>{q_3}</td>
</tr>
<tr>
<td>{q_2, q_3}</td>
<td>{q_0, q_3, q_4}</td>
<td>{q_0, q_3, q_4}</td>
</tr>
<tr>
<td>{q_3}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Once More, With Epsilons!

![Diagram of a finite automaton with states and transitions labeled with symbols ε and Σ, and a transition matrix showing state transitions for inputs a and b.]
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\[
\begin{array}{ccc}
\text{start} & q_0 & \text{q} \\
\epsilon & \Sigma & \Sigma \\
q_0 & q_1 & q_2 \\
\Sigma & a & b \\
q_3 & \text{q} & q_4 \\
b & \Sigma & \Sigma \\
q_3 & \text{q} & q_4 \\
q_4 & \text{q} & \text{q} \\
q_4 & \text{q} & \text{q} \\
q_4 & \text{q} & \text{q} \\
q_4 & \text{q} & \text{q} \\
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Input} & \text{a} & \text{b} \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & \emptyset & \{q_2, q_3\} \\
\{q_4\} & \emptyset & \{q_3\} \\
\{q_2, q_3\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} \\
\{q_3\} & \{q_4\} & \{q_4\} \\
\hline
\end{array}
\]
Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!

\[
\begin{array}{ccc}
\text{start} & \varepsilon & \Sigma \\
q_0 & a & b \\
\qquad \quad q_1 & \Sigma & \quad \quad \quad \Sigma \\
q_3 & b & q_4 \\
q_2 & \Sigma & b \\
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & \emptyset & \{q_2, q_3\} \\
\{q_4\} & \emptyset & \{q_3\} \\
\{q_2, q_3\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} \\
\{q_3\} & \{q_4\} & \{q_4\} \\
\{q_0, q_3, q_4\} & \emptyset & \emptyset \\
\hline
\end{array}
\]
Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!

\[
\begin{array}{c}
\text{start} \quad \epsilon \\
q_0 \quad a \quad q_1 \\
q_3 \quad \Sigma \quad b \\
q_2 \quad b \\
q_4
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
 & a & b \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & \emptyset & \{q_2, q_3\} \\
\{q_4\} & \emptyset & \{q_3\} \\
\{q_2, q_3\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} \\
\{q_3\} & \{q_4\} & \{q_4\} \\
\{q_0, q_3, q_4\} & \{q_1, q_4\} & \\
\hline
\end{array}
\]
Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!

The diagram and table illustrate a finite automaton with transitions for inputs 'a' and 'b'. The start state is $q_0$, and the values for each transition are:

- $q_0 \xrightarrow{a} q_1$
- $q_0 \xrightarrow{b} q_3$
- $q_3 \xrightarrow{b} q_4$
- $q_2 \xrightarrow{\Sigma} q_1$
- $q_2 \xrightarrow{\Sigma} q_3$
- $q_3 \xrightarrow{\Sigma} q_4$

The table specifies the next states for each combination of inputs and states, including:

- For 'a': $\{q_0, q_3\} \rightarrow \{q_1, q_4\}$
- For 'b': $\{q_0, q_3, q_4\} \rightarrow \{q_1, q_4\}$

This automaton includes epsilon transitions, indicated by $\varepsilon$.
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\[
\begin{align*}
\text{start} & \quad \xrightarrow{\varepsilon} q_0 \\
q_0 & \quad \xrightarrow{a} q_1 \\
q_0 & \quad \xrightarrow{\varepsilon} q_3 \\
q_1 & \quad \xrightarrow{b} q_4 \\
q_2 & \quad \xrightarrow{\Sigma} q_3 \\
q_3 & \quad \xrightarrow{b} q_4 \\
q_4 & \quad \xrightarrow{\Sigma} q_3 \\
\end{align*}
\]

\[
\begin{array}{|c|c|c|}
\hline
& a & b \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & \emptyset & \{q_2, q_3\} \\
\{q_4\} & \emptyset & \{q_3\} \\
\{q_2, q_3\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} \\
\{q_3\} & \{q_4\} & \{q_4\} \\
\{q_0, q_3, q_4\} & \{q_1, q_4\} & \{q_3, q_4\} \\
\{q_3, q_4\} & & \\
\hline
\end{array}
\]
Once More, With Epsilons!
Once More, With Epsilons!

Once More, With Epsilons!

### NFA Diagram

- **States:** \( q_0, q_1, q_2, q_3, q_4 \)
- **Transition Function:**
  - \( q_0 \) on \( \epsilon \) to \( q_3 \)
  - \( q_0 \) on \( \Sigma \) to \( q_1 \)
  - \( q_1 \) on \( a \) to \( q_0 \)
  - \( q_1 \) on \( b \) to \( q_4 \)
  - \( q_2 \) on \( \Sigma \) to \( q_0 \)
  - \( q_3 \) on \( \Sigma \) to \( q_4 \)
  - \( q_4 \) on \( \epsilon \) to \( q_2 \)

### Transition Table

<table>
<thead>
<tr>
<th>State ( \leftarrow ) ( { \epsilon } )</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( { q_0, q_3 } )</td>
<td>( { q_1, q_4 } )</td>
<td>( { q_4 } )</td>
</tr>
<tr>
<td>( { q_1, q_4 } )</td>
<td>( \emptyset )</td>
<td>( { q_2, q_3 } )</td>
</tr>
<tr>
<td>( { q_4 } )</td>
<td>( \emptyset )</td>
<td>( { q_3 } )</td>
</tr>
<tr>
<td>( { q_2, q_3 } )</td>
<td>( { q_0, q_3, q_4 } )</td>
<td>( { q_0, q_3, q_4 } )</td>
</tr>
<tr>
<td>( { q_3 } )</td>
<td>( { q_4 } )</td>
<td>( { q_4 } )</td>
</tr>
<tr>
<td>( { q_0, q_3, q_4 } )</td>
<td>( { q_1, q_4 } )</td>
<td>( { q_3, q_4 } )</td>
</tr>
<tr>
<td>( { q_3, q_4 } )</td>
<td>( { q_4 } )</td>
<td>( \emptyset )</td>
</tr>
</tbody>
</table>
Once More, With Epsilons!

![Diagram of a finite automaton with states and transitions.

Transition Table:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>{q_0, q_3}</td>
<td>{q_1, q_4}</td>
<td>{q_4}</td>
</tr>
<tr>
<td>{q_1, q_4}</td>
<td>\emptyset</td>
<td>{q_2, q_3}</td>
</tr>
<tr>
<td>{q_4}</td>
<td>\emptyset</td>
<td>{q_3}</td>
</tr>
<tr>
<td>{q_2, q_3}</td>
<td>{q_0, q_3, q_4}</td>
<td>{q_0, q_3, q_4}</td>
</tr>
<tr>
<td>{q_3}</td>
<td>{q_4}</td>
<td>{q_4}</td>
</tr>
<tr>
<td>{q_0, q_3, q_4}</td>
<td>{q_1, q_4}</td>
<td>{q_3, q_4}</td>
</tr>
<tr>
<td>{q_3, q_4}</td>
<td></td>
<td>{q_4}</td>
</tr>
</tbody>
</table>

\(\Sigma\) is the alphabet with symbols \(a\) and \(b\).
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\[
\begin{array}{cccc}
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & \emptyset & \{q_2, q_3\} \\
\{q_4\} & \emptyset & \{q_3\} \\
\{q_2, q_3\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} \\
\{q_3\} & \{q_4\} & \{q_4\} \\
\{q_0, q_3, q_4\} & \{q_1, q_4\} & \{q_3, q_4\} \\
\{q_3, q_4\} & \{q_4\} & \\
\end{array}
\]
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\[ \begin{array}{c|cc}
\text{a} & \text{b} \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} \\
\{q_1, q_4\} & \emptyset & \{q_2, q_3\} \\
\{q_4\} & \emptyset & \{q_3\} \\
\{q_2, q_3\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} \\
\{q_3\} & \{q_4\} & \{q_4\} \\
\{q_0, q_3, q_4\} & \{q_1, q_4\} & \{q_3, q_4\} \\
\{q_3, q_4\} & \{q_4\} & \\
\end{array} \]
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Once More, With Epsilons!
Once More, With Epsilons!
Once More, With Epsilons!

Hello again! Time for the narration to resume.
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Now that we have your attention...
We are almost done filling in this table. There's one set of states that's missing from here. Can you spot what it is?
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It’s the empty set. What do we do here?
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Well, the rule is to look at what happens if those states are active and to ask what the NFA would do in that configuration.
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In this case, if the NFA is not in any states, the NFA has died off. And reading any characters isn’t going to change that! The NFA is still dead.
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So, if we start in set of states $\emptyset$, we stay in that set of states regardless of what we read. And that’s how we’ll fill the table in.
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We’ll put $\emptyset$ in both columns. This really is a dead state – once we’re here, there’s no turning back! We can never leave.
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We’ve just about finished doing the subset construction. We now have all the DFA states and transitions. But something’s missing… accepting states!
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As a reminder, the rule is to look at the states in the NFA that are accepting and to mark each DFA state as accepting if it contains at least one NFA accepting state. The idea here is that if the NFA ends up in a combination of states with an accepting state, we accept, so the DFA has to simulate that.
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Based on that, which states should be accepting? Make a guess, then move on.
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\[ \begin{array}{cccc|ccc}
q_0 & q_1 & q_2 & q_3 & q_4 \\
\text{start} & a & b & \epsilon & \Sigma & \Sigma & \Sigma \\
\hline
\{q_0, q_3\} & \{q_1, q_4\} & \{q_4\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} \\
\{q_1, q_4\} & \emptyset & \{q_2, q_3\} & \emptyset & \{q_3\} & \emptyset & \{q_3\} \\
\{q_4\} & \emptyset & \{q_3\} & \emptyset & \{q_3\} & \emptyset & \{q_3\} \\
\{q_2, q_3\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} & \{q_0, q_3, q_4\} \\
\{q_3\} & \{q_4\} & \{q_3\} & \{q_3\} & \{q_3\} & \{q_3\} & \{q_3\} \\
\{q_0, q_3, q_4\} & \{q_1, q_4\} & \{q_3, q_4\} & \{q_3, q_4\} & \{q_3, q_4\} & \{q_3, q_4\} & \{q_3, q_4\} \\
\{q_3, q_4\} & \{q_4\} & \{q_3, q_4\} & \{q_3, q_4\} & \{q_3, q_4\} & \{q_3, q_4\} & \{q_3, q_4\} \\
\emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\
\end{array} \]
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Here’s the answer. Most of these states are accepting!
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And that’s it, we’re done!
The next slide just has a summary of the subset construction on it. It’s really formal and is designed to combine together everything we’ve talked about so far. If you need to code up the subset construction for some reason, the next slide is a great reference! If not, think of it as a way of deciding what to do in super tricky edge cases. 😃
The Subset Construction

- Each state in the DFA is associated with a set of states in the NFA.
- The start state in the DFA corresponds to the start state of the NFA, plus all states reachable via ε-transitions.
- If a state \( q \) in the DFA corresponds to a set of states \( S \) in the NFA, then the transition from state \( q \) on a character \( a \) is found as follows:
  - Let \( S' \) be the set of states in the NFA that can be reached by following a transition labeled \( a \) from any of the states in \( S \). *(This set may be empty.)*
  - Let \( S'' \) be the set of states in the NFA reachable from some state in \( S' \) by following zero or more epsilon transitions.
  - The state \( q \) in the DFA transitions on \( a \) to a DFA state corresponding to the set of states \( S'' \).
So there you have it: how the subset construction works in trickier cases.
Did you find this useful?
If so, let us know and we can make more guides like these for other topics.