Welcome Back!
Where We Are Now

• Week 1 covered these key topics:
  • Sets and set theory.
  • Direct proofs.
  • Proof by contradiction.
  • Proof by contrapositive.
  • Categories of statements: universal, existential, and implication.
  • How to negate statements.

• Your goal this week is to get as much practice as you can writing proofs and working around with these concepts. The practice problems for this week will help with this.
Things You Should Be Doing

- **Attending lecture in person.** You will get so much more out of this class if you do, and it is significantly harder to fall behind.

- **Taking your own notes in lecture.** This forces you to create your own independent understanding of the material.

- **Working on the problem set.** Do not put this off! You need to work on it gradually over the course of the week.
  - ... and doing so in a pair. Working alone is harder and makes it trickier to get help when you need it.

- **Reviewing the resources mentioned on PS1.** We've given out a bunch of resources and recommended that you read through them before starting the problem set. Ideally you've done that by now; if not, make sure to go do that!

- **Reading solution sets and asking questions.** You now have solutions to the PS1 checkpoint. Make sure that you have a complete understanding of how to solve each of those problems!
Things You Should Do Today

- Read Chapter Two of the course notes and (optionally) work through some of the exercises.
- Complete Q1 – Q6 of the problem set by the end of the evening. Try to complete Q7 if possible. Start playing around with Q8 – Q10; those problems require some thought.
- Find a problem set partner if you don't already have one. (And hey – aren't you in a room full of people who might be good people to work with?)
Things You Should Do Tomorrow

• Look over your feedback on the PS1 checkpoint and make sure you understand all the feedback you get completely and unambiguously. Ask the course staff for help, either on Piazza or in office hours, if you don't.

• Continue working on PS1. Try to have at least half of Q8 – Q10 completed by then and have drafts of all your answers written up.

• Start reviewing your partner's answers and writing up a single, definitive set of answers that you're going to turn in.

• Stop by office hours to get feedback on your proofs and take that feedback seriously.
Start working through the packet of problems we've provided. We'll reconvene as a group after a while.
A Few Choice Problems
Calling Back to Definitions
Theorem: If $A$ and $B$ are sets, then
\[
\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B).
\]
**Theorem:** If \( A \) and \( B \) are sets, then \( \mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B) \).

This is some *really* dense notation.
Before we begin, let's write down what these terms mean. What do we know about them?
**Theorem:** If $A$ and $B$ are sets, then $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

**Proof:** Let $A$ and $B$ be sets.
**Theorem:** If $A$ and $B$ are sets, then $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

**Proof:** Let $A$ and $B$ be sets. We need to show that $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.
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**Proof:** Let $A$ and $B$ be sets. We need to show that $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

Consider an arbitrary $S \in \mathcal{P}(A) \cap \mathcal{P}(B)$. This means that $S \in \mathcal{P}(A)$ and $S \in \mathcal{P}(B)$, so $S \subseteq A$ and $S \subseteq B$. We need to prove that $S \in \mathcal{P}(A \cap B)$, meaning that we need to prove that $S \subseteq A \cap B$.

To prove that $S \subseteq A \cap B$, consider any $x \in S$. Since $x \in S$ and $S \subseteq A$, we know that $x \in A$. Similarly, since $x \in S$ and $S \subseteq B$, we know that $x \in B$. Collectively, we've shown that $x \in A$ and $x \in B$, so we see that $x \in A \cap B$. This means that every $x \in S$ satisfies $x \in A \cap B$, so $S \subseteq A \cap B$, which is what we needed to show. 

$\blacksquare$

How do you prove that one set is a subset of another?
**Theorem:** If $A$ and $B$ are sets, then \( \mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B) \).

**Proof:** Let $A$ and $B$ be sets. We need to show that \( \mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B) \).

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To prove that $S \subseteq A \cap B$, consider any $x \in S$. Since $x \in S$ and $S \subseteq A$, we know that $x \in A$. Similarly, since $x \in S$ and $S \subseteq B$, we know that $x \in B$. Collectively, we've shown that $x \in A$ and $x \in B$, so we see that $x \in A \cap B$. This means that every $x \in S$ satisfies $x \in A \cap B$, so $S \subseteq A \cap B$, which is what we needed to show.

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Consider an arbitrary $S \in \mathcal{P}(A) \cap \mathcal{P}(B)$. Can we say anything about $S$ at this point?
**Theorem:** If $A$ and $B$ are sets, then $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

**Proof:** Let $A$ and $B$ be sets. We need to show that $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

Consider an arbitrary $S \in \mathcal{P}(A) \cap \mathcal{P}(B)$. This means that $S \in \mathcal{P}(A)$ and $S \in \mathcal{P}(B)$, so $S \subseteq A$ and $S \subseteq B$. Therefore, every element of $S$ is in both $A$ and $B$, implying $S \subseteq A \cap B$.

This shows that $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$, as required.
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To prove that $S \subseteq A \cap B$, consider any $x \in S$.

What do we need to prove about $x$ to prove that $x \in A \cap B$?
**Theorem:** If $A$ and $B$ are sets, then $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

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To prove that $S \subseteq A \cap B$, consider any $x \in S$.

What do we already know about $S$?
**Theorem:** If $A$ and $B$ are sets, then $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

**Proof:** Let $A$ and $B$ be sets. We need to show that $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

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To prove that $S \subseteq A \cap B$, consider any $x \in S$. Since $x \in S$ and $S \subseteq A$, we know that $x \in A$. 

**Theorem:** If \( A \) and \( B \) are sets, then \( \mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B) \).

**Proof:** Let \( A \) and \( B \) be sets. We need to show that \( \mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B) \).

Consider an arbitrary \( S \in \mathcal{P}(A) \cap \mathcal{P}(B) \). This means that \( S \in \mathcal{P}(A) \) and \( S \in \mathcal{P}(B) \), so \( S \subseteq A \) and \( S \subseteq B \). We need to prove that \( S \in \mathcal{P}(A \cap B) \), meaning that we need to prove that \( S \subseteq A \cap B \).

To prove that \( S \subseteq A \cap B \), consider any \( x \in S \). Since \( x \in S \) and \( S \subseteq A \), we know that \( x \in A \). Similarly, since \( x \in S \) and \( S \subseteq B \), we know that \( x \in B \).
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**Theorem:** If $A$ and $B$ are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

**Proof:** Let $A$ and $B$ be sets. We need to show that $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

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