Welcome to CS103A!

Turn in:
• Homework Problems 1

Pick up:
• Practice Problems 1
Where We Are Now

- Week 1 covered these key topics:
  - Sets and set theory.
  - Direct proofs.
  - Proof by contradiction.
  - Proof by contrapositive.
  - Categories of statements: universal, existential, and implication.
  - How to negate statements.
- Your goal this week is to get as much practice as you can writing proofs and working around with these concepts. The practice problems for this week will help with this.
Things You Should Be Doing

- **Attending lecture in person.** You will get *so much more* out of this class if you do, and it is significantly harder to fall behind.

- **Taking your own notes in lecture.** This forces you to create your own independent understanding of the material.

- **Working on the problem set.** Do not put this off! You need to work on it gradually over the course of the week.
  - *... and doing so in a pair.* Working alone is harder and makes it trickier to get help when you need it.

- **Reviewing the resources mentioned on PS1.** We've given out a bunch of resources and recommended that you read through them before starting the problem set. Ideally you've done that by now; if not, make sure to go do that!

- **Reading solution sets and asking questions.** You now have solutions to the PS1 checkpoint. Make sure that you have a complete understanding of how to solve each of those problems!
Attendance Problems 1

- Take out your completed Attendance Problems 1
- Discuss in your groups! Go around in a circle and have each person explain one problem.
- Questions? Ask each other! Ask us! Make sure to turn in your completed sheets by the end of class.
Set Theory and Proofwriting
Some Incorrect Set Theory Proofs
**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

⚠ **Incorrect!** ⚠ **Proof:** Consider the arbitrary sets $A = \{a\}$, $B = \{b, c\}$, and $C = \{c\}$. Notice that $A \cup B = \{a, b, c\}$ so $C \subseteq A \cup B$ because every element of $\{c\}$ can be found in $\{a, b, c\}$.

We can also see that $\{c\} \subseteq \{b, c\}$, which means that $C \subseteq B$. Since we’ve shown that either one of $C \subseteq A$ or $C \subseteq B$ is true, the theorem holds. ■
Claim: If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

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What’s wrong with this proof?
Claim: If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

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Thus we can't prove this claim by just giving an example.

This is a universal statement, meaning that it is making a claim about any sets $A$, $B$, and $C$.
Universal vs. Existential Statements

- A **universal statement** is a statement of the form
  
  **For all** $x$, [some-property] **holds for** $x$.

- To prove a universal statement, pick an arbitrary $x$ and show that it has the desired property.

- An **existential statement** is a statement of the form
  
  **There is some** $x$ **where** [some-property] **holds for** $x$.

- To prove an existential statement, find an $x$ that has the desired property, then show why your choice is correct.
Claim: If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

⚠ Incorrect! ⚠ Proof: Consider arbitrary sets $A$, $B$, and $C$ where $C \subseteq A \cup B$.

This means that every element of $C$ is in either $A$ or $B$. If all elements of $C$ are in $A$, then $C \subseteq A$. Alternately, if everything in $C$ is in $B$, then $C \subseteq B$. In either case, everything inside of $C$ has to be contained in at least one of these sets, so the theorem is true. ■
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This means that every element of $C$ is in either $A$ or $B$. If all elements of $C$ are in $A$, then $C \subseteq A$. Alternately, if everything in $C$ is in $B$, then $C \subseteq B$. In either case, everything inside of $C$ has to be contained in at least one of these sets, so the theorem is true. ■

This is just repeating definitions and not making specific claims about specific variables.
Claim: If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

⚠ Incorrect! ⚠ Proof: Consider arbitrary sets $A$, $B$, and $C$ where $C \subseteq A \cup B$.

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Why is this bad?
Claim: If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

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This means that every element of $C$ is in either $A$ or $B$. If all elements of $C$ are in $A$, then $C \subseteq A$. Alternately, if everything in $C$ is in $B$, then $C \subseteq B$. In either case, everything inside of $C$ has to be contained in at least one of these sets, so the theorem is true. ■

While this claim is true, it does not imply the theorem is true. In fact, this theorem is actually false.
Let’s Draw Some Pictures!

\textbf{Claim:} If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$. 
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**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

Recall the intuition of a subset being “something I can circle.”
Let’s Draw Some Pictures!

**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

Recall the intuition of a subset being “something I can circle.”

So $C \subseteq A$ would mean that $C$ is something I can circle in this region.

A

\[ C \]

B
Let’s Draw Some Pictures!

**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

Recall the intuition of a subset being “something I can circle.”

Likewise, $C \subseteq B$ would mean that $C$ is something I can circle in this region.
Let’s Draw Some Pictures!

Claim: If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$. 

\[
A \cup B
\]
Let’s Draw Some Pictures!

**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

But when I look at $A \cup B$, I can draw $C$ as a circle containing elements from both $A$ and $B$. 

![Venn Diagram with overlapping circles for $A \cup B$ and $C$]
Let’s Draw Some Pictures!

**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

But when I look at $A \cup B$, I can draw $C$ as a circle containing elements from both $A$ and $B$.

Do you see why this circle is in neither $A$ nor $B$?
Let’s Draw Some Pictures!

**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

Using this visual intuition, come up with a counterexample to this claim and write it up as a disproof.
Claim: If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$. 
**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

**Disproof:** We will show that there are sets $A$, $B$, and $C$ where $C \subseteq A \cup B$, but $C \not\subseteq A$ and $C \not\subseteq B$. 
**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

**Disproof:** We will show that there are sets $A$, $B$, and $C$ where $C \subseteq A \cup B$, but $C \not\subseteq A$ and $C \not\subseteq B$. Consider the sets $A = \{a\}$.
Claim: If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

Disproof: We will show that there are sets $A$, $B$, and $C$ where $C \subseteq A \cup B$, but $C \not\subseteq A$ and $C \not\subseteq B$. Consider the sets $A = \{a\}$, $B = \{b\}$.
**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

**Disproof:** We will show that there are sets $A$, $B$, and $C$ where $C \subseteq A \cup B$, but $C \not\subseteq A$ and $C \not\subseteq B$. Consider the sets $A = \{a\}$, $B = \{b\}$, and $C = \{a, b\}$.
**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

**Disproof:** We will show that there are sets $A$, $B$, and $C$ where $C \subseteq A \cup B$, but $C \not\subseteq A$ and $C \not\subseteq B$. Consider the sets $A = \{a\}$, $B = \{b\}$, and $C = \{a, b\}$. Now notice that $\{a, b\} \subseteq A \cup B$ so $C \subseteq A \cup B$. 

![Venn Diagram](image)

In the diagram, $A$ and $B$ are two overlapping circles, with $C$ being a subset of both $A$ and $B$. The elements $a$ and $b$ are placed within the respective circles, illustrating the subset relationship.
**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

**Disproof:** We will show that there are sets $A$, $B$, and $C$ where $C \subseteq A \cup B$, but $C \not\subseteq A$ and $C \not\subseteq B$. Consider the sets $A = \{a\}$, $B = \{b\}$, and $C = \{a, b\}$. Now notice that $\{a, b\} \subseteq A \cup B$ so $C \subseteq A \cup B$, but $C \not\subseteq A$ because $b \in C$ but $b \notin A$.
**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

**Disproof:** We will show that there are sets $A$, $B$, and $C$ where $C \subseteq A \cup B$, but $C \not\subseteq A$ and $C \not\subseteq B$. Consider the sets $A = \{a\}$, $B = \{b\}$, and $C = \{a, b\}$. Now notice that $\{a, b\} \subseteq A \cup B$ so $C \subseteq A \cup B$, but $C \not\subseteq A$ because $b \in C$ but $b \notin A$, and $C \not\subseteq B$ because $a \in C$ but $a \notin B$. 

![Venn Diagram](image)
**Claim:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$.

**Disproof:** We will show that there are sets $A$, $B$, and $C$ where $C \subseteq A \cup B$, but $C \nsubseteq A$ and $C \nsubseteq B$. Consider the sets $A = \{a\}$, $B = \{b\}$, and $C = \{a, b\}$. Now notice that $\{a, b\} \subseteq A \cup B$ so $C \subseteq A \cup B$, but $C \nsubseteq A$ because $b \in C$ but $b \notin A$, and $C \nsubseteq B$ because $a \in C$ but $a \notin B$.

Thus we’ve found a set $C$ which is a subset of $A \cup B$ but is not a subset of either $A$ or $B$, which is what we needed to show. ■
Proofwriting Advice

• Be **very wary** of proofs that speak generally about “all objects” of a particular type.

• As you’ve just seen, it’s easy to accidentally prove a false statement at this level of detail.

• Making broad, high-level claims often indicates deeper logic errors or conceptual misunderstanding (like *code smell* but for proofs!)
Proofwriting Advice

**A Very Good Idea:** After you’ve written a draft of a proof, run through all of the points on the Proofwriting Checklist.

- This is a *great* exercise that you can do with a partner!
The Proofwriting Checklist

☐ Clearly articulate your start and end points.
☐ Make each sentence “load-bearing.”
☐ Scope and properly introduce variables.
☐ Make specific claims about specific variables.
☐ Don’t repeat definitions; use them instead.
☐ Write in complete sentences and complete paragraphs.
☐ Distinguish between proofs and disproofs.
The Proofwriting Checklist

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Ask yourself: Is it clear what’s being assumed and what’s being proven?

• A proof should make sense outside of the context of the problem it was presented in.
The Proofwriting Checklist

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Go through all of the variables in the proof and ask yourself: What does this variable represent? Is this variable properly introduced?

Variables should be introduced with a verb such as "let", "consider", "pick", etc.
The Proofwriting Checklist

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☐ Clearly articulate your start and end points.
☐ Make each sentence “load-bearing.”
☐ Scope and properly introduce variables.
☐ Make specific claims about specific variables.
☐ Don’t repeat definitions; use them instead.

Go through each sentence and ask yourself: Is this claim true in general, or does it rely on something I’ve assumed or established in the proof?

If it’s the former, either eliminate it or rewrite it to be more concrete.
The Proofwriting Checklist

☐ Clearly articulate your start and end points.

☐ Make each sentence “load-bearing.”

☐ Scope and properly introduce variables.

☐ Make specific claims about specific variables.

☐ Don’t repeat definitions; use them instead.

If you find yourself unable to make specific, precise claims about named variables, this could be a sign that there is a concept you’re having trouble with, or maybe that the claim isn’t even true to begin with.
The Proofwriting Checklist

☐ Clearly articulate your start and end points.
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Read things over again and ask yourself: Am I adhering to standard proofwriting conventions?
Let’s do a proof together!
Theorem: If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$. 
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

Hold on, isn’t this the claim we just disproved?
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

Notice that that's an intersection, not a union! It turns out that this claim is actually true.
Let’s Draw Some Pictures!

**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$. 
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**Theorem:** If \( A, B, \) and \( C \) are sets and \( C \subseteq A \cap B \), then \( C \subseteq A \) and \( C \subseteq B \).

\[
\begin{array}{c}
\text{A} \\
\cap \\
\text{B}
\end{array}
\]

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\cap \\
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Let’s Draw Some Pictures!

**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

Recall the intuition of a subset being “something I can circle.”
Let's Draw Some Pictures!

**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

Recall the intuition of a subset being "something I can circle.

When I look at $A \cap B$, any circle I can draw in this region can be found in both $A$ and $B$. 

[Diagram of Venn diagram with $A$, $B$, and $A \cap B$ intersections, and shaded region $C$]
Let’s Draw Some Pictures!

**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

This is a great visual intuition to see why the theorem is true. Now we have to drill down to the level of individual elements to write the proof.
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

<table>
<thead>
<tr>
<th>What We’re Assuming</th>
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<tbody>
<tr>
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**What We’re Assuming**

- $A$, $B$, and $C$ are sets
- $C \subseteq A \cap B$

**What We Need To Show**

- $C \subseteq A$ and $C \subseteq B$

A great proofwriting strategy is to **write down relevant definitions**. This gives you a better sense of what you need to prove and what tools you have at hand.
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

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Talk with your neighbors and figure out:
- What is the definition of subset?
- How do you prove that one set is a subset of another?
- If you know that one set is a subset of another, what can you conclude?
**Theorem:** If A, B, and C are sets and \( C \subseteq A \cap B \), then \( C \subseteq A \) and \( C \subseteq B \).

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**Definition:** If S and T are sets, then \( S \subseteq T \) when for every \( x \in S \), we have \( x \in T \).
**Theorem:** If \( A, B, \) and \( C \) are sets and \( C \subseteq A \cap B \), then \( C \subseteq A \) and \( C \subseteq B \).

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**Definition:** If \( S \) and \( T \) are sets, then \( S \subseteq T \) when for every \( x \in S \), we have \( x \in T \).

*To prove that \( S \subseteq T \):*
Pick an arbitrary \( x \in S \), then prove \( x \in T \).

*If you know that \( S \subseteq T \):*
If you have an \( x \in S \), you can conclude \( x \in T \).
Brief Interlude: Definitions in Proofs

• Typically, you will leverage definitions in your proofwriting in the following two ways:

**Definition:** If $S$ and $T$ are sets, then $S \subseteq T$ when for every $x \in S$, we have $x \in T$.

*To prove that $S \subseteq T$:*
Pick an arbitrary $x \in S$, then prove $x \in T$.

*If you know that $S \subseteq T$:*
If you have an $x \in S$, you can conclude $x \in T$.
Brief Interlude: Definitions in Proofs

- Typically, you will leverage definitions in your proofwriting in the following two ways:
  - **Scaffolding a proof**: You should be using the definition to identify the start and end points of your proof.

**Definition**: If $S$ and $T$ are sets, then $S \subseteq T$ when for every $x \in S$, we have $x \in T$.

**To prove that $S \subseteq T$**: Pick an arbitrary $x \in S$, then prove $x \in T$.

**If you know that $S \subseteq T$**: If you have an $x \in S$, you can conclude $x \in T$. 
Brief Interlude: Definitions in Proofs

- Typically, you will leverage definitions in your proofwriting in the following two ways:

  - **Applying to known objects**: If you know or discover that a definition holds for some particular object(s), you can establish something about those objects and hopefully make progress towards your goal.

**Definition**: If $S$ and $T$ are sets, then $S \subseteq T$ when for every $x \in S$, we have $x \in T$.

*To prove that $S \subseteq T*:
Pick an arbitrary $x \in S$, then prove $x \in T$.

*If you know that $S \subseteq T*:
If you have an $x \in S$, you can conclude $x \in T$. 
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**Definition:** If $S$ and $T$ are sets, then $S \subseteq T$ when for every $x \in S$, we have $x \in T$.

**To prove that $S \subseteq T$:**
Pick an arbitrary $x \in S$, then prove $x \in T$.

**If you know that $S \subseteq T$:**
If you have an $x \in S$, you can conclude $x \in T$. 
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

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---

This reading of the definition is usually helpful for unpacking this column! 

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**What We Need To Show**

- $C \subseteq A$ and $C \subseteq B$
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

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- In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$
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**Our Tools**

• In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$

Now we know that ultimately, we’re going to have to do these two things. Let’s see what tools we have that can get us here!
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

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**What We’re Assuming**

- $A$, $B$, and $C$ are sets
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**To prove that $S \subseteq T$:**
Pick an arbitrary $x \in S$, then prove $x \in T$.

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If you have an $x \in S$, you can conclude $x \in T$. 

This reading of the definition is usually helpful for unpacking this column!
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**Our Tools**

- In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$.

Talk with your neighbors and figure out:

- What is the definition of set intersection?
- How are we going to combine that with what we know about subsets?
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

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**Definition:** The set $S \cap T$ is the set where, for any $x$:

$x \in S \cap T$ when $x \in S$ and $x \in T$
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

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**Definition:** The set $S \cap T$ is the set where, for any $x$:

- $x \in S \cap T$ when $x \in S$ and $x \in T$

**To prove that** $x \in S \cap T$:
Prove both that $x \in S$ and that $x \in T$.

**If you know that** $x \in S \cap T$:
You can conclude both that $x \in S$ and that $x \in T$. 
\textbf{Theorem:} If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

\textbf{What We’re Assuming}

- $A$, $B$, and $C$ are sets
- $C \subseteq A \cap B$

\textbf{Definition:} The set $S \cap T$ is the set where, for any $x$:

$x \in S \cap T$ when $x \in S$ and $x \in T$

\textbf{To prove that $x \in S \cap T$:}

Prove both that $x \in S$ and that $x \in T$.

\textbf{If you know that $x \in S \cap T$:}

You can conclude both that $x \in S$ and that $x \in T$.

\textbf{What We Need To Show}

- Pick an $x \in C$, show that $x \in A$
- Pick an $x \in C$, show that $x \in B$

This is the one we want!
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

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• In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$.
• If you know that $S \subseteq T$ and you have an $x \in S$, you can conclude $x \in T$.
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**Our Tools**

• In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$.

• If you know that $S \subseteq T$ and you have an $x \in S$, you can conclude $x \in T$.

• If you know that $x \in S \cap T$, we can conclude that $x \in S$ and $x \in T$.

Let’s go and try and do the proof with what we’ve got here!
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

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**Rough Outline**

- **Assume** $C \subseteq A \cap B$
- **Proving** $C \subseteq A$
  - **Pick an** $x \in C$
  - **Conclude** $x \in A$

**Relevant Definitions**

- In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$
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What goes here?
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

**Rough Outline**

- **Assume** $C \subseteq A \cap B$
- **Proving** $C \subseteq A$
  - Pick an $x \in C$
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**Rough Outline**

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- Proving $C \subseteq A$
  - Pick an $x \in C$
  - $x \in A \cap B$
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  - Pick an $x \in C$
  - $x \in A \cap B$
  - $x \in A$ and $x \in B$
  - Conclude $x \in A$

We also need to prove that $C \subseteq B$.

Notice that if you take the outline here and literally swap the variable $A$ for the variable $B$, you get a working proof.
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

**Rough Outline**

- Assume $C \subseteq B \cap A$
- Proving $C \subseteq B$
  - Pick an $x \in C$
  - $x \in B \cap A$
  - $x \in B$ and $x \in A$
  - Conclude $x \in B$

In a case like this where your proof would have two completely symmetric branches, it’s fine to write up just one and say “by symmetry, [the other branch] is also true.”
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

**Rough Outline**

- Assume $C \subseteq A \cap B$
- **Proving $C \subseteq A$**
  - Pick an $x \in C$
  - $x \in A \cap B$
  - $x \in A$ and $x \in B$
  - Conclude $x \in A$

Take a few minutes and write up a proof of the theorem using this outline.

Then swap proofs with a neighbor and critique each other!
**Theorem:** If $A$, $B$, and $C$ are sets and $C \subseteq A \cap B$, then $C \subseteq A$ and $C \subseteq B$.

**Proof:** Let $A$, $B$, and $C$ be arbitrary sets where $C \subseteq A \cap B$. We need to show that $C \subseteq A$ and $C \subseteq B$. Because the roles of $A$ and $B$ in this proof are symmetric, we can just prove that $C \subseteq A$.

Choose any element $x \in C$. Since $C \subseteq A \cap B$, we know that $x \in A \cap B$. This tells us that $x \in A$ and $x \in B$. In particular, this means that $x \in A$, thus completing the proof. ■
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Are you making specific claims about specific variables? Your proof should NOT have statements of the form "every element of $C$".
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Are all variables properly introduced and scoped? You should be able to point at every variable and say that it is either:

1) an arbitrarily chosen value
2) an existentially instantiated value
3) an explicitly chosen value
Proofwriting Strategies

- **Articulate a Clear Start and End Point**
  - What are you assuming? What are you trying to prove?

- **Write Down Relevant Terms and Definitions**
  - Identify existing tools to help you get from your starting point to your ending point

- **Work Backwards**
  - Use your end goal to figure out intermediate steps
Before You Leave...

Turn in:
• Attendance Problems 1

Pick up:
• Homework Problems 2
• Attendance Problems 2
• Solutions to Practice Problems 1