Welcome to CS103A!

Turn in:
• Homework Problems 2

Pick up:
• Practice Problems 2
• Graded work from last week
Recap: Attendance Problems 1
Mathematical Logic
Where We Are Now

- Our coverage of logic focused on these topics:
  - Propositional variables.
  - Propositional connectives.
  - Propositional equivalences.
  - Predicates, functions, and constant symbols.
  - Objects and propositions.
  - Quantifiers.
  - Evaluating first-order formulas relative to a world.
  - Translating into first-order logic.
  - Negating and simplifying first-order formulas.
- Your goal this week is to keep your proof skills sharp while mastering the ins and outs of first-order logic.
Things You Should Do Today

- Review the solutions for Problem Set One and make sure you *completely* and *unambiguously* understand the answers. Ask for help if this isn't the case!
- Read the “Guide to Negating Formulas” and “Guide to First-Order Translation” on the course website to get more exposure and practice with those skills.
- Continue working through PS2.
Things You Should Do Tomorrow

- Look over your feedback on PS1 and make sure you understand all the feedback you get *completely* and *unambiguously*. Ask the course staff for help, either on Piazza or in office hours, if you don't.
- Continue working on PS2.
- Start reviewing your partner's answers and compiling a single, definitive set of answers that you're going to turn in.
- Stop by office hours to get feedback on your proofs and take that feedback seriously.
Quantifiers

You love someone.

\( \exists x. (\text{Loves}(\text{You}, x)) \)

\( \exists \) is the **existential quantifier** and says “for some choice of \( x \), the following is true.”
Quantifiers

Everyone loves you.

\( \forall x. (Loves(x, You)) \)

\( \forall \) is the **universal quantifier** and says "for any choice of \( x \), the following is true."
Quantifiers

- How do we check whether a statement with a quantifier is true or false?
  - Universally-quantified statements are true unless there's a counterexample.

$$\forall x. \ (Loves(x, You))$$
Quantifiers

- How do we check whether a statement with a quantifier is true or false?
  - Universally-quantified statements are true unless there's a counterexample.

\[ \forall x. (Loves(x, You)) \]

Treat this like a for loop:
```java
for (elem x : universe) {
   // look for a counterexample
}
```
Quantifiers

• How do we check whether a statement with a quantifier is true or false?
  – Existentially-quantified statements are false unless there's an actual example.

  \[ \exists x. (Loves(You, x)) \]
Quantifiers

- How do we check whether a statement with a quantifier is true or false?
  - Existentially-quantified statements are false unless there's an actual example.

\[ \exists x. \text{Loves(You, x)} \]

Treat this like a for loop:
```java
for (elem x : universe) {
    // look for an actual example
}
```
\( \forall x \in P. \forall y \in P. (\text{Loves}(x, y) \lor \text{Loves}(y, x)) \)
\( \forall x \in P. \ \forall y \in P. (Loves(x, y) \lor Loves(y, x)) \)
\[ \forall x \in P. \forall y \in P. (Loves(x, y) \lor Loves(y, x)) \]
∀x ∈ P. ∀y ∈ P. (Loves(x, y) ∨ Loves(y, x))

```java
for (elem x : P) {
    for (elem y : P) {
        // check if Loves(x, y)
        // or Loves(y, x)
    }
}
```
∀x ∈ P. ∀y ∈ P. (Loves(x, y) v Loves(y, x))
∀x ∈ P. ∀y ∈ P. (Loves(x, y) ∨ Loves(y, x))
\[ \forall x \in P. \forall y \in P. \ (Loves(x, y) \lor Loves(y, x)) \]
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```java
for (elem x : P) {
    for (elem y : P) {
        // check if Loves(x, y)
        // or Loves(y, x)
    }
}
```
\[ \forall x \in P. \forall y \in P. \ (Loves(x, y) \lor Loves(y, x)) \]

Remember that quantifiers can range over the same objects at the same time!
Your turn! For each of the other formulas, determine whether that formula is true or false.
Mechanics: *Negating Statements*
∀p. (Person(p) →
    ∃q. (Person(q) ∧ p ≠ q ∧
        Loves(p, q)
    )
)
\[ \neg \forall p. \ (\text{Person}(p) \rightarrow \exists q. \ (\text{Person}(q) \land p \neq q \land \text{Loves}(p, q)) \) \]
\[ \neg \forall p. (\text{Person}(p) \rightarrow \\
\exists q. (\text{Person}(q) \land p \neq q \land \\
\text{Loves}(p, q)) 
\]
\[
\neg \forall p. \ (\text{Person}(p) \rightarrow \\
\exists q. \ (\text{Person}(q) \land p \neq q \land \\
\text{Loves}(p, q))
\) 
\]

\[
\neg \forall x. A \\
\exists x. \neg A
\]

\neg \forall p. (\text{Person}(p) \rightarrow
\exists q. (\text{Person}(q) \land p \neq q \land
\text{Loves}(p, q))
)
)
\[\neg \forall p. (\text{Person}(p) \rightarrow \exists q. (\text{Person}(q) \land p \neq q \land \text{Loves}(p, q)))\]
\[ \exists p. \neg (\text{Person}(p) \rightarrow \exists q. (\text{Person}(q) \land p \neq q \land \text{Loves}(p, q)) \) \) \]
\[ \exists p. \neg (Person(p) \rightarrow \exists q. (Person(q) \land p \neq q \land Loves(p, q)) \) \) \]
\[ \exists p. \neg (Person(p) \rightarrow \exists q. (Person(q) \land p \neq q \land Loves(p, q)) ) \]

\[ \neg(A \rightarrow B) \]

\[ \frac{\neg(A \rightarrow B)}{A \land \neg B} \]
\[ \exists p. \neg (\text{Person}(p) \rightarrow \exists q. (\text{Person}(q) \land p \neq q \land \text{Loves}(p, q))) \]

\[ \neg (A \rightarrow B) \]

\[ \frac{\neg (A \rightarrow B)}{A \land \neg B} \]
\[ \exists p. \neg (\text{Person}(p) \rightarrow \exists q. (\text{Person}(q) \land p \neq q \land \text{Loves}(p, q))) \]

\[ \neg (A \rightarrow B) \]

\[ A \land \neg B \]
\( \exists p. \ ( \text{Person}(p) \land \neg \exists q. \ (\text{Person}(q) \land p \neq q \land \text{Loves}(p, q) ) ) \)

\[ \neg (A \rightarrow B) \]
\[ \frac{}{A \land \neg B} \]
\[\exists p. (\text{Person}(p) \land \\
\neg \exists q. (\text{Person}(q) \land p \neq q \land \\
\text{Loves}(p, q))
\]
\[ \exists p. (\text{Person}(p) \land \neg \exists q. (\text{Person}(q) \land p \neq q \land \text{Loves}(p,q))) \]
\[ \exists p. \ ( \text{Person}(p) \land \\
\neg \exists q. \ ( \text{Person}(q) \land p \neq q \land \\
\text{Loves}(p, q) ) ) \]
\[ \exists p. (\text{Person}(p) \land \neg \exists q. (\text{Person}(q) \land p \neq q \land \text{Loves}(p, q)) \) \]
\[ \exists p. \ (Person(p) \land \forall q. \neg(Person(q) \land p \neq q \land Loves(p, q)) \) \]
\[ \exists p. \ (\text{Person}(p) \land \\
\forall q. \ \neg(\text{Person}(q) \land p \neq q \land \\
\text{Loves}(p, q)) \) \]
\[
\exists p. (\text{Person}(p) \land \\
\forall q. \neg(\text{Person}(q) \land p \neq q \land \\
\text{Loves}(p, q))
\]

\[
\neg(A \land B) \\
A \rightarrow \neg B
\]
\[ \exists p. (\text{Person}(p) \land
\forall q. \neg (\text{Person}(q) \land p \neq q \land 
\text{Loves}(p, q)) \]

\[ \neg (A \land B) \]

\[ A \to \neg B \]
\[ \exists p. \ (\text{Person}(p) \land
\forall q. \ \neg(\text{Person}(q) \land p \neq q \land \text{Loves}(p, q)) \]
∃p. (Person(p) ∧
    ∀q. (Person(q) ∧ p ≠ q →
        ¬Loves(p, q)
    )
)
\[\exists p. \ (\text{Person}(p) \land
\forall q. \ (\text{Person}(q) \land p \neq q \rightarrow
\neg \text{Loves}(p, q)\right)\]
\[ \forall p. \,(Person(p) \rightarrow \\
  \exists q. \,(Person(q) \land p \neq q \land \\
  Loves(p, q)) \\
) \\
) \\
) \\
)

\[ \exists p. \,(Person(p) \land \\
  \forall q. \,(Person(q) \land p \neq q \rightarrow \\
  \neg Loves(p, q)) \\
) \\
) \\
) \]
Let’s play a game!
Telephone Pictionary

1. Ninja fighting a pirate
2. Old man fights ninja kid
3. Beating the Russians to the moon
4. Beating an astronaut
5. 3 people dancing for a taco
Telephone Logictionary
How did it go?
Strategies to Keep in Mind

“All $Ps$ are $Q$s.”
\[ \forall x. (P(x) \rightarrow Q(x)) \]

“Some $Ps$ are $Q$s.”
\[ \exists x. (P(x) \land Q(x)) \]

“No $Ps$ are $Q$s.”
\[ \forall x. (P(x) \rightarrow \neg Q(x)) \]

“Some $Ps$ aren't $Q$s.”
\[ \exists x. (P(x) \land \neg Q(x)) \]

- The $\forall$ quantifier *usually* is paired with $\rightarrow$.
  \[ \forall x. (P(x) \rightarrow Q(x)) \]

- The $\exists$ quantifier *usually* is paired with $\land$.
  \[ \exists x. (P(x) \land Q(x)) \]
### Before You Leave...

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