Welcome Back!

Where We Are Now

- Week 1 covered these key topics:
  - Sets and set theory.
  - Direct proofs.
  - Proof by contradiction.
  - Proof by contrapositive.
  - Categories of statements: universal, existential, and implication.
  - How to negate statements.
  - Rational and irrational numbers.
- Your goal this week is to get as much practice as you can writing proofs and working around with these concepts. The practice problems for this week will help with this.

Where We're Going

- Week 2 is about mathematical logic:
  - Propositional logic. (yesterday)
  - First-order logic. (tomorrow / Friday)
  - Translating into and out of logic. (Friday)
  - Negating and simplifying statements in logic. (Friday)
- From yesterday, you should be able to list all seven propositional connectives and their truth tables. You should also know how to negate the $\rightarrow$ connective and what de Morgan's laws are.
- Use the online truth table to help solidify your understanding.

Things You Should Be Doing

- ***Attending lecture in person***. You will get *so much more* out of this class if you do, and it is significantly harder to fall behind.
- ***Taking your own notes in lecture***. This forces you to create your own independent understanding of the material.
- ***Working on the problem set***. Do not put this off! You need to work on it gradually over the course of the week.
  - ***… and doing so in a team***. Working alone is harder and makes it trickier to get help when you need it.
- ***Going through the checklist on the front of PS1***. We've given out a bunch of resources and recommended that you read through them before starting the problem set. Ideally you've done that by now; if not, make sure to go do that!
- ***Reading solution sets and asking questions***. You now have solutions to the PS1 checkpoint. Make sure that you have a complete understanding of how to solve each of those problems!

Things You Should Do Today

- (Optionally) Read Chapter Two of the course notes and work through some of the exercises.
- Complete Q1 – Q5 of the problem set by the end of the evening. Try to complete Q6 if possible. Start playing around with Q7 – Q10; those problems require some thought.
- Make sure you know all the truth tables for the propositional connectives. You will need them for Wednesday's lecture to make sense. (Today's packet of problems will help with that.)
- Find a problem set group if you don't already have one. (And hey – aren't you in a room full of people who might be good people to work with?)

Things You Should Do Tomorrow

- Look over your feedback on the PS1 checkpoint and make sure you understand all the feedback you get ***completely*** and ***unambiguously***. Ask the course staff for help, either on Piazza or in office hours, if you don't.
- Continue working on PS1. Try to have at least half of Q7 – Q10 completed by then and have drafts of all your answers written up.
- Start reviewing your teammates' answers and compiling a single, definitive set of answers that you're going to turn in.
- Stop by office hours to get feedback on your proofs and take that feedback seriously.

Start working through the packet of problems we've provided. We'll reconvene as a group after a while.

A Few Choice Problems

Calling Back to Definitions

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

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This is some *really* dense notation. Before we begin, let's write down what these terms mean. What do we know about them?

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How do you prove that one set is a subset of another?

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Can we say
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at this point?

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What do we need to prove about x to prove that x ∈ A ∩ B?

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What do we already know about S?

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