CS 103X: Discrete Structures
Final Exam

March 24, 2006

THE STANFORD UNIVERSITY HONOR CODE

- The Honor Code is an undertaking of the students, individually and collectively:
  - that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;
  - that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.
- The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid as far as practicable, academic procedures that create temptations to violate the Honor Code.
- While the faculty alone has the right and obligation to set academic requirements, the students and faculty will work together to create optimal conditions for honorable academic work.

Exams are to be done individually and must represent original work—it is a violation of the honor code to copy or derive exam question solutions from other students or anyone at all, textbooks, or previous instances of this course.

I acknowledge and accept the honor code:

Signature: ______________________________

Name (print): ______________________________

EXAM RULES

- You have 3 hours to complete this exam.
- Do not include your scratch work with your exam. Please work the solutions out on another sheet of paper and then write your solutions neatly on the exam.
- You may use Prof. Koltun's lecture notes and two double-sided cheat-sheets. You may *not* use any other material, such as your own course notes, other sets of lecture notes, books, computers, cell phones, crystal balls, Tarot cards, etc.
- Write clearly and neatly.
- Stagger your seats.

GOOD LUCK!

Exercise 1 (30 points)**.** A function $f : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, m\}$ is called monotone nondecreasing if $1 \le i < j \le n \;\Rightarrow\; f(i) \le f(j)$.

(a) How many such functions are there?

(b) How many such functions are there that are surjective?

(c) How many such functions are there that are injective?

Exercise 2 (20 points)**.** Recall that two integers a, b are called coprime if $\gcd(a, b) = 1$. Given $a, b \in \mathbb{Z}$ and $k, l \in \mathbb{N}^+$, prove that a and b are coprime if and only if a^k and b^l are coprime.

Exercise 3 (20 points)**.**

(a) How many simple undirected graphs on the vertex set $\{1, 2, \ldots, n\}$ are there?

(b) How many simple undirected graphs on the vertex set $\{1, 2, \ldots, n\}$ are there that do not have vertices of degree 0?

Note that vertices are distinct and isomorphic graphs are not the same.

Exercise 4 (20 points)**.** Given a graph $G = (V, E)$, an edge $e \in E$ is said to be a bridge if the graph $G' = (V, E \setminus \{e\})$ has more connected components than G. Let G be a bipartite k-regular graph for $k \geq 2$. Prove that G has no bridge.

Exercise 5 (10 points)**.** Find a set with two elements, such that every element of the set is also a subset of it.