CS 103X: Discrete Structures
Homework Assignment 6

Due February 24, 2006

Unless specified otherwise, for each of the following questions you should state your answer and give a *brief* and convincing argument to support it. Messy and long-winded explanations will not be accepted.

Exercise 1 (10 points)**.** A group of $m+n$ kids in a theme park are boarding two ferris wheels, which can accommodate m and n kids, respectively. How many distinct seating arrangements are there? (Yes, all kids are different, as any mother will readily testify.)

Exercise 2 (10 points)**.** The 60 cs103x students need to form 20 study groups consisting of 3 students each. How many possibilities are there?

Exercise 3 (10 points)**.** A Silicon Valley question: How many possible six-figure salaries (in whole dollar amounts) are there that contain the lucky digit 7 at least once? (Hint: How about ones that do not contain 7 at all?)

Exercise 4 (20 points)**.** The night of an important deadline, two Stanford computer science professors together drank 6 espresso drinks. (For the sake of concreteness, call these purely fictional characters Vladlen and Dawson.)

(a) How many professor-espresso configurations are there if all the drinks are the same?

(b) How many professor-espresso configurations are there if all the drinks are different (a plain latte, a double espresso, a mocha, etc.)?

(c) How many possibilities are there if 6 different drinks had been bought, but not all of them were necessarily drunk? That is, it is possible that Vladlen drank the double espresso and the mocha, Dawson had the quadruple espresso and the iced latte, and the other two were left untouched.

Exercise 5 (20 points)**.** After successfully completing the cs103x midterm, the 60 cs103x students all go out to pamper themselves in a chocolate boutique that carries four kinds of gourmet truffles: rose-vanilla, honey-lavender, kava kava-coconut, and tangerine-cayenne. The boutique has 100 truffles of each kind. How many possibilities are there for the students to buy truffles if:

(a) Each student buys exactly one truffle.

(b) Each student buys one truffle, and exactly 20 students go for honey-lavender.

(c) How many possible collections of truffles can be left in the boutique after the students buy one truffle each?

Exercise 6 (15 points)**.** A rook on a chessboard is said to put another chess piece under attack if they are in the same row or column.

(a) How many ways are there to arrange 8 rooks on a chessboard (the usual 8×8 one) so that none are under attack?

(b) How many ways are there to arrange k rooks on an $n \times n$ chessboard so that none are under attack?

Give solutions with no summation.

Exercise 7 (15 points)**.** How many ways are there to express a positive integer n as:

(a) A sum of k natural numbers? (For example, if $n = 2$ and $k = 3$ the answer is 6, since $2 = 2+0+0 = 0+2+0 = 0+0+2 = 1+1+0 = 1+0+1 = 0+1+1$.)

(b) A sum of positive integers?

The order of the summands is important. (Imagine the summation written down.)