

Review 1

1. Night Sight

Light that hits a camera comes as a Poisson, where the average photons of light that hit a sensor in one second is r .

- A photo is taken for t seconds. What is the variance of N ?
- If $N > 100$, the pixel will be over exposed (and will be fully white). If you take a photo for $t = 5$ seconds, what is the probability that a given pixel will be over exposed?
- If you take several photos and average them, you can reduce the variance in N . You take k photos, each with exposure length of t seconds. Let N_i be the number of photons in photo i . The average number of photons across the k photos is $N = \frac{1}{k}(N_1 + N_2 + \dots + N_k)$. What is the variance of N ?

2. Birthday Paradox and Tricky Events

- There are n people in a room. What is the probability that at least one pair of people share a birthday?
- 4 toy bunnies and 3 toy foxes are in a toy box. 3 toys are drawn. What is $P(1 \text{ bunny and } 2 \text{ fox drawn})$?

3. Wisdom of the Crowds

It doesn't take many experts for the collective to make a good decision! This phenomena is called the Wisdom of the Crowds and is heavily leveraged in crowd sourcing algorithms. Consider this example. In the game Who Wants to be a Millionaire, when contestants "ask the audience" each of the 200 members of the audience get to vote on the answer. Assume: There are two answers for each audience member to chose from, a Correct answer and an Incorrect answer. 10% of the audience are knowledgeable about the problem (call them experts). An expert votes for the Correct answer with a probability of 0.7, otherwise they vote for the Incorrect answer. 90% of the audience are not knowledgeable (call them non-experts). A non-expert votes randomly with equal likelihood between the Correct answer and the Incorrect answer.

- What is the probability that exactly k of the experts vote for the Correct answer? You may assume that k is a number between 0 and 20 inclusive.
- If exactly k of the experts vote for the Correct answer, what is the probability that the Correct answer will get at least 101 votes? (hint: the Correct answer needs at least $101 - k$ more votes from the non-experts).
- Write an expression for the exact probability that the Correct answer will get at least 101 votes.

- d. Use an approximation to estimate the probability that the Correct answer gets at least 101 votes. You may leave your answer in terms of roots and/or values that could be looked up from the ϕ table. For full credit your approximation calculation should *not* include a summation or integral.

4. **What to Buy?**

You are deciding which of two books to purchase.

Book 1 has the ratings:

Rating	1	2	3	4	5
Count	0	0	0	1	4

Book 2 has the ratings:

Rating	1	2	3	4	5
Count	0	1	2	20	200

Assume your rating of each book is drawn from the same distribution as the folks who rated the book in the past. What is the probability you would prefer book 2 to book 1?