09: Continuous RVs

Lisa Yan
April 24, 2020
Quick slide reference

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09a_continuous_rvs
09b_uniform
09c_exponential
LIVE
09e_extra
Continuous RVs
Not all values are discrete

```python
import numpy as np
np.random.random()
```
People heights

You are volunteering at the local elementary school.

• To choose a t-shirt for your new buddy Jordan, you need to know their height.

1. What is the probability that your buddy is \(54.0923857234\) inches tall?

2. What is the probability that your buddy is between \(52\)–\(56\) inches tall?

\[P(52 < X \leq 56) = P(52 < X \leq 54) + P(54 < X \leq 56)\]
Integrals

Integral = area under a curve

Loving, not scary
Continuous RV definition

A random variable $X$ is **continuous** if there is a **probability density function** $f(x) \geq 0$ such that for $-\infty < x < \infty$:

$$P(a \leq X \leq b) = \int_a^b f(x) \, dx$$

Integrating a PDF must always yield valid probabilities, and therefore the PDF must also satisfy

$$\int_{-\infty}^{\infty} f(x) \, dx = P(-\infty < X < \infty) = 1$$

Also written as: $f_X(x)$
Today’s main takeaway, #1

Integrate $f(x)$ to get probabilities.

PDF Units: probability per units of $X$

$$P(52 \leq X \leq 56) = \int_{52}^{56} f(x) \, dx$$
PMF vs PDF

**Discrete** random variable $X$

Probability mass function (PMF):

$$p(x)$$

To get probability:

$$P(X = x) = p(x)$$

$$P(a \leq X \leq b) = \sum_{x=a}^{b} p(x)$$

Both are measures of how **likely** $X$ is to take on a value.

**Continuous** random variable $X$

Probability density function (PDF):

$$f(x)$$

To get probability:

$$P(a \leq X \leq b) = \int_{a}^{b} f(x)dx$$
Computing probability

Let $X$ be a continuous RV with PDF:

$$f(x) = \begin{cases} \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

What is $P(X \geq 1)$?
Computing probability

Let $X$ be a continuous RV with PDF:

$$f(x) = \begin{cases} \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

What is $P(X \geq 1)$?

**Strategy 1:** Integrate

$$P(1 \leq X < \infty) = \int_{1}^{\infty} f(x)dx = \int_{1}^{2} \frac{1}{2}x dx$$

$$= \frac{1}{2} \left[ \frac{1}{2} x^2 \right]_1^2 = \frac{1}{2} \left[ 2 - \frac{1}{2} \right] = \frac{3}{4}$$

Wait...is this even legal?

**Strategy 2:** Know triangles

$$\frac{1}{2} \times b \times h = A$$

$$1 - \frac{1}{2} \left(\frac{1}{2}\right) = \frac{3}{4} = P(1 \leq X < \infty)$$

$P(0 \leq X < 1) = \int_{0}^{1} f(x)dx$ ??
Today’s main takeaway, #2

For a continuous random variable $X$ with PDF $f(x)$, $P(X = c) = \int_c^c f(x)\,dx = 0$.

Contrast with PMF in discrete case: $P(X = c) = p(c)$
PDF Properties

For a continuous RV $X$ with PDF $f$,

$$P(a \leq X \leq b) = \int_a^b f(x) \, dx$$

True/False:

1. $P(X = c) = 0$

2. $P(a \leq X \leq b) = P(a < X < b) = P(a \leq X < b) = P(a < X \leq b)$

3. $f(x)$ is a probability

4. In the graphed PDF above,
   $P(x_1 \leq X \leq x_2) > P(x_2 \leq X \leq x_3)$
PDF Properties

For a continuous RV $X$ with PDF $f$,\[ P(a \leq X \leq b) = \int_a^b f(x) \, dx \]

True/False:

1. $P(X = c) = 0$
2. $P(a \leq X \leq b) = P(a < X < b) = P(a \leq X < b) = P(a < X \leq b)$
3. $f(x)$ is a probability
4. In the graphed PDF above, $P(x_1 \leq X \leq x_2) > P(x_2 \leq X \leq x_3)$

Interval width $dx \to 0$

Support: set of $x$ where $f(x) > 0$

Compare area under the curve $f$
Uniform RV
**Uniform Random Variable**

**Definition**: An **Uniform** random variable $X$ is defined as follows:

$$f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{if } \alpha \leq x \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

**Support**: $[\alpha, \beta]$ (sometimes defined over $(\alpha, \beta)$)

**Expectation**
$$E[X] = \frac{\alpha + \beta}{2}$$

**Variance**
$$\text{Var}(X) = \frac{1}{12} (\beta - \alpha)^2$$
Quick check

If \( X \sim \text{Uni}(\alpha, \beta) \), the PDF of \( X \) is:

\[
f(x) = \begin{cases} 
\frac{1}{\beta - \alpha} & \text{if } \alpha \leq x \leq \beta \\
0 & \text{otherwise}
\end{cases}
\]

What is \( \frac{1}{\beta - \alpha} \) if the following graphs are PDFs of Uniform RVs \( X \)?

1. \( f(x) \)
2. \( f(x) \)
3. \( f(x) \)

Lisa Yan, CS109, 2020
Quick check

If $X \sim \text{Uni}(\alpha, \beta)$, the PDF of $X$ is:

$$f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{if } \alpha \leq x \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{\alpha}^{\beta} f(x) \, dx = 1$$

What is $\frac{1}{\beta - \alpha}$ if the following graphs are PDFs of Uniform RVs $X$?

1. $f(x)$

   ![Graph 1]

   $$\frac{1}{25}$$

2. $f(x)$

   ![Graph 2]

   2

3. $f(x)$

   ![Graph 3]

   $$\frac{1}{10}$$
Expectation and Variance

**Discrete RV** $X$

$$E[X] = \sum_x x \, p(x)$$

$$E[g(X)] = \sum_x g(x) \, p(x)$$

**Continuous RV** $X$

$$E[X] = \int_{-\infty}^{\infty} x \, f(x) \, dx$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) \, f(x) \, dx$$

Both continuous and discrete RVs

$$E[aX + b] = aE[X] + b$$

$$\text{Var}(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

**TL;DR:** $\sum_{x=a}^{b} \Rightarrow \int_{a}^{b} \, dx$
Uniform RV expectation

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

$$= \int_{\alpha}^{\beta} x \cdot \frac{1}{\beta - \alpha} dx$$

$$= \frac{1}{\beta - \alpha} \cdot \frac{1}{2} x^2 \bigg|_{\alpha}^{\beta}$$

$$= \frac{1}{\beta - \alpha} \cdot \frac{1}{2} (\beta^2 - \alpha^2)$$

$$= \frac{1}{2} \cdot \frac{(\beta + \alpha)(\beta - \alpha)}{\beta - \alpha} = \frac{\alpha + \beta}{2}$$

Interpretation:
Average the start & end
def An **Uniform** random variable $X$ is defined as follows:

$X \sim \text{Uni}(\alpha, \beta)$

**Support:** $[\alpha, \beta]$ (sometimes defined over $(\alpha, \beta)$)

**PDF**

$$f(x) = \begin{cases} 
\frac{1}{\beta - \alpha} & \text{if } \alpha \leq x \leq \beta \\
0 & \text{otherwise}
\end{cases}$$

**Expectation**

$$E[X] = \frac{\alpha + \beta}{2}$$

**Variance**

$$\text{Var}(X) = \frac{(\beta - \alpha)^2}{12}$$
Exponential RV
Grid of random variables

<table>
<thead>
<tr>
<th>One trial</th>
<th>Several trials</th>
<th>Interval of time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ber( (p) )</td>
<td>Bin( (n, p) )</td>
<td>Poi( (\lambda) )</td>
</tr>
<tr>
<td>Geometric( (p) )</td>
<td>Negative Binomial( (r, p) )</td>
<td>Exponential( (\lambda) )</td>
</tr>
</tbody>
</table>

- \( n = 1 \)
- \( r = 1 \)
Consider an experiment that lasts a duration of time until success occurs. An **Exponential** random variable $X$ is the amount of time until success.

**Exponential Random Variable**

$$X \sim \text{Exp}(\lambda)$$

**PDF**

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

**Expectation**

$$E[X] = \frac{1}{\lambda}$$

**(in extra slides)**

**Variance**

$$\text{Var}(X) = \frac{1}{\lambda^2}$$

**(on your own)**

**Examples:**

- Time until next earthquake
- Time for request to reach web server
- Time until end of cell phone contract
Interpreting $\text{Exp}(\lambda)$

**Definition: An Exponential random variable** $X$ is the amount of time until success.

$X \sim \text{Exp}(\lambda)$  

<table>
<thead>
<tr>
<th>Expectation</th>
<th>$E[X] = \frac{1}{\lambda}$</th>
</tr>
</thead>
</table>

Based on the expectation $E[X]$, what are the units of $\lambda$?
Interpreting $\text{Exp}(\lambda)$

def An **Exponential** random variable $X$ is the amount of time until success. 

$X \sim \text{Exp}(\lambda)$

| Expectation | $E[X] = \frac{1}{\lambda}$ |

Based on the expectation $E[X]$, what are the units of $\lambda$?

$$\frac{1}{\lambda} = \frac{\text{time}}{\text{event}} \quad \Rightarrow \quad \lambda : \frac{\text{event}}{\text{time}}$$

$Y \sim \text{Poi}(\lambda), \ E[Y] = \lambda \frac{\text{event}}{\text{time}}$

e.g., average # of successes per second

For both Poisson and Exponential RVs, $\lambda = \# \text{ successes/time.}$
Earthquakes

1906 Earthquake
Magnitude 7.8
Earthquakes

Major earthquakes (magnitude 8.0+) occur once every 500 years.*

1. What is the probability of a major earthquake in the next 30 years?

We know on average:

$$E[X] = \frac{1}{\lambda} = 500 \text{ years earthquake}$$

$$\frac{1}{500} = 0.002 \text{ earthquakes year}$$

$$1 \text{ earthquakes} \frac{500 \text{ years}}{500 \text{ years}}$$

$$X \sim \text{Exp}(\lambda)$$

$$E[X] = \frac{1}{\lambda}$$

$$f(x) = \lambda e^{-\lambda x} \text{ if } x \geq 0$$

$$E[X] = \frac{1}{0.002} = 500$$

$$\geq 1 \text{ event within 30 yrs} \iff \text{first event within 30 years}$$

$$P(X < 30)$$

*In California, according to historical data from USGS, 2015
Earthquakes

Major earthquakes (magnitude 8.0+) occur once every 500 years.*

1. What is the probability of a major earthquake in the next 30 years?

Define events/ RVs & state goal

$X$: when next earthquake happens (years)

$X \sim \text{Exp}(\lambda = 0.002)$

$\lambda$: year$^{-1} = 1/500$

Want: $P(X < 30)$

Solve

$P(X < 30) = P(0 < X < 30)$

$=$ $\int_{0}^{30} \lambda e^{-\lambda x} dx$

$=$ $\lambda \left[\frac{1}{-\lambda} e^{-\lambda x}\right]_{0}^{30}$

$=$ $\left[\frac{-e^{-30\lambda}}{-\lambda} - \frac{1}{-\lambda}ight]$

$=$ $1 - e^{-30\lambda} \approx 0.058$

(Recall $\int e^{cx} dx = \frac{1}{c} e^{cx}$)

*In California, according to historical data from USGS, 2015
Earthquakes

Major earthquakes (magnitude 8.0+) occur once every 500 years.*

1. What is the probability of a major earthquake in the next 30 years?
2. What is the standard deviation of years until the next earthquake?

Define events/RVs & state goal

\[ X: \text{when next earthquake happens} \]

\[ X \sim \text{Exp}(\lambda = 0.002) \]

\[ \lambda: \text{year}^{-1} \]

Want: \( P(X < 30) \)

Solve

\[
\frac{1}{\lambda} = 500 \\
\text{SD}(X) = \sqrt{\text{Var}(X)} = \sqrt{\frac{1}{\lambda^2}} \\
= \frac{1}{\lambda} = 500 \text{ years}
\]
09: Continuous RVs

Lisa Yan
April 24, 2020

(live)
Today’s main takeaway, #1

Integrate $f(x)$ to get probabilities.

$P(a \leq X \leq b) = \int_{a}^{b} f(x) \, dx$
Today’s main takeaway, #2

For a continuous random variable $X$ with PDF $f(x)$, 

$$P(X = c) = \int_{c}^{c} f(x) \, dx = 0.$$ 

Implication: $P(a \leq X \leq b) = P(a < X < b)$
Think

Think by yourself: 2 min

Slide 35 has a matching question to go over by yourself. We’ll go over it together afterwards.

Post any clarifications here!

https://us.edstem.org/courses/109/discussion/39083
Determining valid PDFs

Which of the following functions are valid PDFs?

1. $f(x)$
   \[ \int_{-\infty}^{\infty} f(x) \, dx = 0.5 \]

2. $g(x)$
   \[ \int_{-\infty}^{\infty} g(x) \, dx = 1 \]

3. $h(x)$
   \[ \int_{-\infty}^{\infty} h(x) \, dx = 1 \]

4. $w(x)$
   \[ \int_{-\infty}^{\infty} w(x) \, dx = 1 \]

By yourself

\[ P(a \leq X \leq b) = \int_{a}^{b} f(x) \, dx \]
Determining valid PDFs

Which of the following functions are valid PDFs?

1. $f(x)$
   - $\int_{-\infty}^{\infty} f(x) \, dx = 0.5$

2. $g(x)$
   - $\int_{-\infty}^{\infty} g(x) \, dx = 1$
   - $P(a \leq X \leq b) = \int_{a}^{b} f(x) \, dx$

3. $h(x)$
   - $\int_{-\infty}^{\infty} h(x) \, dx = 1$

4. $w(x)$
   - $\int_{-\infty}^{\infty} w(x) \, dx = 1$
Check out the question on the next slide (Slide 38). Post any clarifications here!

https://us.edstem.org/courses/109/discussion/39083

Breakout rooms: 4 min. Introduce yourself!
Riding the Marguerite Bus

You want to get on the Marguerite bus.
• The bus stops at the Gates building at 15-minute intervals (2:00, 2:15, etc.).
• You arrive at the stop uniformly between 2:00-2:30pm.

P(you wait < 5 minutes for bus)?
You want to get on the Marguerite bus.

- The bus stops at the Gates building at 15-minute intervals (2:00, 2:15, etc.).
- You arrive at the stop uniformly between 2:00-2:30pm.

$P(\text{you wait < 5 minutes for bus})$?

1. Define events/RVs & state goal
   
   $X$: time passenger arrives after 2:00
   
   $X \sim \text{Uni}(0,30)$

   Want: $\frac{1}{30}$

2. Solve

   
   \[
   P(10 < X \leq 15) + P(25 < X \leq 30)
   \]
   
   \[
   = \int_{10}^{15} \frac{1}{30} \, dx + \int_{25}^{30} \frac{1}{30} \, dx
   \]
   
   \[
   = \frac{5}{30} + \frac{5}{30} = \frac{1}{3}
   \]
Interlude for jokes/announcements
Announcements

Quiz #1
Time frame: Thursday 4/30 12:00am-11:59pm PT
Covers: Up to end of Week 3 (including Lecture 9)
Review session (Tim): Tuesday 4/28 12-2pm PT
https://stanford.zoom.us/j/92275547392
Info and practice: https://web.stanford.edu/class/cs109/exams/quizzes.html

Python tutorial #2 (Sandra)
When: today 4/24 5:00-6:00PT
https://stanford.zoom.us/j/621852324
Recorded Notes online
Useful for: pset2, pset3

Note: If you have an emergency situation during the quiz, please contact Lisa and Cooper. We will try our best to accommodate.
Interesting probability news

NYC subway math


Distribution of time until the next subway arrival
Probably Beta RV (Week 8)
Cumulative Distribution Function (CDF)

For a random variable $X$, the cumulative distribution function (CDF) is defined as

$$F(a) = F_X(a) = P(X \leq a), \text{ where } -\infty < a < \infty$$

For a discrete RV $X$, the CDF is:

$$F(a) = P(X \leq a) = \sum_{all \ x \leq a} p(x)$$
Cumulative Distribution Function (CDF)

For a random variable \( X \), the cumulative distribution function (CDF) is defined as

\[
F(a) = F_X(a) = P(X \leq a), \text{ where } -\infty < a < \infty
\]

For a discrete RV \( X \), the CDF is:

\[
F(a) = P(X \leq a) = \sum_{x \leq a} p(x)
\]

For a continuous RV \( X \), the CDF is:

\[
F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x) \, dx
\]

CDF is a probability, though PDF is not.

If you learn to use CDFs, you can avoid integrating the PDF.
Slide 46 has a matching question to go over by yourself. We’ll go over it together afterwards.

Post any clarifications here!

https://us.edstem.org/courses/109/discussion/27279

Think by yourself: 1 min
Using the CDF for continuous RVs

For a **continuous** random variable $X$ with PDF $f(x)$, the CDF of $X$ is

$$P(X \leq a) = F(a) = \int_{-\infty}^{a} f(x) \, dx$$

Matching (choices are used 0/1/2 times)

1. $P(X < a)$   
   A. $F(a)$
2. $P(X > a)$   
   B. $1 - F(a)$
3. $P(X \geq a)$   
   C. $F(a) - F(b)$
4. $P(a \leq X \leq b)$   
   D. $F(b) - F(a)$

(by yourself)
Using the CDF for continuous RVs

For a continuous random variable $X$ with PDF $f(x)$, the CDF of $X$ is

$$P(X \leq a) = F(a) = \int_{-\infty}^{a} f(x)dx$$

Matching (choices are used 0/1/2 times)

1. $P(X < a)$  
   A. $F(a) = \mathbb{P}(X \leq a)$
2. $P(X > a)$  
   B. $1 - F(a)$
3. $P(X \geq a)$  
   C. $F(a) - F(b)$
4. $P(a \leq X \leq b)$  
   D. $F(b) - F(a)$  
   (next slide)
Using the CDF

For a continuous random variable $X$ with PDF $f(x)$, the CDF of $X$ is

$$F(a) = \int_{-\infty}^{a} f(x)dx$$

4. $P(a \leq X \leq b) = F(b) - F(a)$

Proof:

$$F(b) - F(a) = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx$$

$$= \left(\int_{-\infty}^{a} f(x)dx + \int_{a}^{b} f(x)dx\right) - \int_{-\infty}^{a} f(x)dx$$

$$= \int_{a}^{b} f(x)dx = P(a \leq X \leq b)$$
CDF of an Exponential RV

\[ X \sim \text{Exp}(\lambda) \quad F(x) = 1 - e^{-\lambda x} \quad \text{if } x \geq 0 \]

Proof:

\[
F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y)dy = \int_{0}^{x} \lambda e^{-\lambda y}dy
\]

\[
= \lambda \left[ \frac{1}{-\lambda} e^{-\lambda y} \right]_{0}^{x}
\]

\[
= -1(e^{-\lambda x} - e^{-\lambda 0})
\]

\[
= 1 - e^{-\lambda x}
\]

Recall

\[
\int e^{cx}dx = \frac{1}{c}e^{cx}
\]
PDF/CDF $X \sim \text{Exp}(\lambda = 1)$

- $f(x) = \lambda e^{-\lambda x}$
- $F(x) = 1 - e^{-\lambda x}$

**PDF Normalization**

- $\int_0^2 \lambda e^{-\lambda x} \, dx \approx 0.86$

**CDF for $X \leq 2$**

- $1 - e^{-2\lambda} \approx 0.86$

**CDF for $X > 2$**

- $1 - F(2) = e^{-2\lambda} \approx 0.14$

- $P(X \leq 2)$

- $P(X > 2)$
Breakout Rooms

Check out the question on the next slide (Slide 52). Post any clarifications here!

https://us.edstem.org/courses/109/discussion/39083

Breakout rooms: 4 min. Introduce yourself!
Earthquakes

Major earthquakes (magnitude 8.0+) occur once every 500 years.*

What is the probability of zero major earthquakes next year?

Average: \[
\text{successes per year} = \frac{1}{500}
\]

\[\lambda = \frac{1}{500} \text{ per year}\]

\[X \sim \text{Poisson RV: wait 1 yr, observe } n \text{ events: } P(X=0)\]

\[Y \sim \text{Exponential RV: wait until } 1\text{st event, observe duration of time } P(Y>1)\]

*In California, according to historical data form USGS, 2015
Earthquakes

Major earthquakes (magnitude 8.0+) occur once every 500 years.*
What is the probability of zero major earthquakes next year?

Strategy 1: Exponential RV

Define events/RVs & state goal

\[ X: \text{when first earthquake happens} \]
\[ X \sim \text{Exp}(\lambda = 0.002) \]

Want: \( P(X > 1) = 1 - F(1) \)

Solve

\[
P(X > 1) = 1 - (1 - e^{-\lambda \cdot 1}) = e^{-\lambda}
\]

*In California, according to historical data form USGS, 2015
Earthquakes

Major earthquakes (magnitude 8.0+) occur once every 500 years.*

What is the probability of zero major earthquakes next year?

**Strategy 1: Exponential RV**

Define events/RVs & state goal

- \( X: \) when first earthquake happens
- \( X \sim \text{Exp}(\lambda = 0.002) \)

Want: \( P(X > 1) = 1 - F(1) \)

Solve

\[
P(X > 1) = 1 - (1 - e^{-\lambda \cdot 1}) = e^{-\lambda}
\]

**Strategy 2: Poisson RV**

Define events/RVs & state goal

- \( X: \) # earthquakes next year
- \( X \sim \text{Poi}(\lambda = 0.002) \)

Want: \( P(X = 0) \)

Solve

\[
P(X = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda} \approx 0.998
\]

*In California, according to historical data form USGS, 2015
Extra

1. $E[X] = \frac{1}{\lambda}$

2. after lecture
   $CDF = \frac{x}{\lambda}$
Expectation of the Exponential

\[ X \sim \text{Exp}(\lambda) \]

Expectation

\[ E[X] = \frac{1}{\lambda} \]

Proof:

\[
E[X] = \int_{-\infty}^{\infty} xf(x) \, dx = \int_{0}^{\infty} x\lambda e^{-\lambda x} \, dx
\]

\[
= -xe^{-\lambda x} \bigg|_{0}^{\infty} + \int_{0}^{\infty} e^{-\lambda x} \, dx
\]

\[
= -xe^{-\lambda x} \bigg|_{0}^{\infty} - \frac{1}{\lambda} e^{-\lambda x} \bigg|_{0}^{\infty}
\]

\[
= [0 - 0] + \left[ 0 - \left( \frac{-1}{\lambda} \right) \right]
\]

\[
= 0 - 0 + \left( \frac{1}{\lambda} \right)
\]

\[
= \frac{1}{\lambda}
\]

Integration by parts

\[
\int x\lambda e^{-\lambda x} \, dx = \int u \cdot dv
\]

\[
u = x \quad dv = \lambda e^{-\lambda x} \, dx
\]

\[
du = dx \quad v = -e^{-\lambda x}
\]

\[
\int u \cdot dv = uv - \int v \cdot du
\]

\[
-xe^{-\lambda x} - \int -e^{-\lambda x} \, dx
\]

\[
= -xe^{-\lambda x} + \frac{1}{\lambda} e^{-\lambda x}
\]

\[
= \frac{1}{\lambda}
\]
Suppose a visitor to your website leaves after $X$ minutes.

- On average, visitors leave the site after 5 minutes.
- The length of stay, $X$, is exponentially distributed.

1. $P(X > 10)$?

2. $P(10 < X < 20)$?
Website visits

Suppose a visitor to your website leaves after $X$ minutes.

- On average, visitors leave the site after 5 minutes.
- The length of stay, $X$, is exponentially distributed.

1. $P(X > 10)$?

   Define
   
   $X$: when visitor leaves
   $X \sim \text{Exp}(\lambda = 1/5 = 0.2)$

   Solve
   
   $P(X > 10) = 1 - F(10)$
   
   $= 1 - (1 - e^{-10/5}) = e^{-2} \approx 0.1353$

2. $P(10 < X < 20)$?

   Solve
   
   $P(10 < X < 20) = F(20) - F(10)$
   
   $= (1 - e^{-4}) - (1 - e^{-2}) \approx 0.1170$
Replacing your laptop

Let $X = \#$ hours of use until your laptop dies.

- $X$ is distributed as an Exponential RV, where
- On average, laptops die after 5000 hours of use.
- You use your laptop 5 hours a day.

What is $P$ (your laptop lasts 4 years)?

$X \sim \text{Exp}(\lambda)$  
$E[X] = \frac{1}{\lambda}$  
$F(x) = 1 - e^{-\lambda x}$
Replacing your laptop

Let $X = \#$ hours of use until your laptop dies.
- $X$ is distributed as an Exponential RV, where
- On average, laptops die after 5000 hours of use.
- You use your laptop 5 hours a day.

What is $P(\text{your laptop lasts 4 years})$?

**Define**

- $X$: # hours until laptop death
- $X \sim \text{Exp}(\lambda = 1/5000)$

**Want:** $P(X > 5 \cdot 365 \cdot 4)$

**Solve**

$$P(X > 7300) = 1 - F(7300) = 1 - (1 - e^{-7300/5000}) = e^{-1.46} \approx 0.2322$$

Better plan ahead if you’re co-terming!

- 5-year plan:
  $$P(X > 9125) = e^{-1.825} \approx 0.1612$$
- 6-year plan:
  $$P(X > 10950) = e^{-2.19} \approx 0.1119$$