

CS109: Probability for Computer Scientists

Lecture 11 — Inference

Name to Age

Let B be the year that someone was born and let N be their name. From the social security dataset, I am able to give you the joint distribution $P(B = b, N = n) \approx \frac{\text{count}(b,n)}{\text{len}(\text{dataset})}$.

- (a) How would you compute $P(B = 1964|N = \text{Michael})$ using the joint distribution?

- (b) Describe how you would compute the full PMF $P(B = b|N = \text{Michael})$, over all birth years b , using code.

Hidden Chambers

- a. Imagine the entire 100 meter path is limestone. In that case, the rate of muons arriving per month on the detection plate is

$$100 \cdot e^{-100/40} = 8.2.$$

Assume each muon arrives independently of any other muon and at a constant rate. What is the probability that in one month you would observe 12 muons?

- b. Let X be your belief in the meters of limestone above the detection plate. Your prior belief is that any number of meters from 0 to 100 is equally likely:

$$X \sim \text{Uni}(0, 100).$$

After one month, your detection plate has been hit by 12 muons. What is your updated belief in X ? You may leave your answer with integrals or sums.

Stanford Eye Test

Let $A \in \{0.00, 0.01, \dots, 0.99, 1.00\}$ be the (discretized) ability that someone can see. Our prior belief in A is given to you as a dictionary for each possible ability. In this eye test, we show a user a letter at a certain font size and we observe if they get it right or wrong. Then we update our belief in their ability to see.

We observe that a user gets the first letter we show them incorrect. We define $Y = 0$ as the event that the user gets the first letter we show them incorrect.

- a. Write an expression in math for the posterior $P(A = a | Y = 0)$.
- b. Is this expression going to result in a number or a dictionary?
- c. Describe in code how you would solve this expression. Assume you have access to a function **calc_likelihood**. (The likelihood function is super neat - a bit outside of the scope of today's class but happy to talk about it in OH).

