



Reinforcement 2

Chris Piech

CS109, Stanford University

Pset 7: Review

PS7 Shazam

Shazam is trying to detect which song is playing in a noisy bar. We will explore how "wisdom of the crowds" effects allow it to accurately identify the song even when there is far more noise than music.



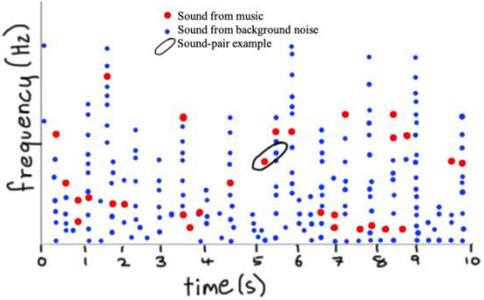
Name any song in seconds

Shazam will identify any music playing around you

When Shazam runs its acoustic fingerprinting algorithm on an audio recording, it extracts many short "notes", represented as (frequency, time) tuples, from the recording.

You are in a really loud restaurant. In the current audio recording, there are 5000 notes from background noise and **only** 25 notes from the song. Is it still possible to identify the song?

Shazam lets every pair of notes vote for which song it thinks is playing. There are (5025 choose 2) such pairs.



53 Previous Question Next Question

PS7 Calibrating ChatGPT

Imagine you use ChatGPT (though you could use any prediction for that matter) to make a prediction for a datapoint with features x and ChatGPT responds to your prompt with a probability that the label is a 1, $P(Y = 1|X = x)$. You hesitate. Are those probabilities trustworthy? In a situation like this, it might be really important to also know how *calibrated* the probabilities are. For example, if ChatGPT says that $P(Y = 1|X = x) = 0.8$ does that mean there really is an 80% chance the label is a 1?



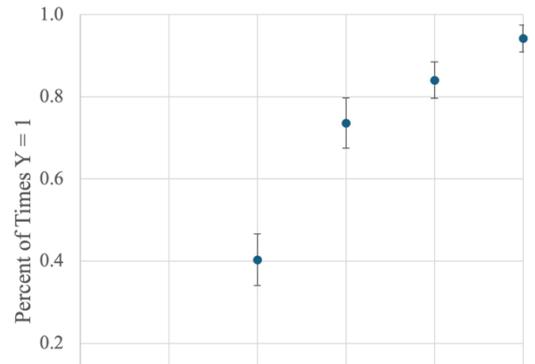
```
ChatGPT 5.1
```

What is the probability that this is spam? return just a probability as json: "Hey Chris. I have a lot of gold to send you. You just need to send me your bank account details"

```
json
```

```
{ "probability_spam": 0.99 }
```

More formally, how can we check if the probabilities output from a model are "calibrated"? To evaluate if a model is calibrated we first group datapoints together based on the $P(Y = 1|X = x)$ probability output given by the model. Then, among those groups we check how often the label is 1. Here is an example of calibration curve which plots probability groupings on the x-axis and fraction of times that the label was 1 on the y-axis:



53 Previous Question Next Question

PS7 Bayesian Flo

As we saw in section Flo models each user's cycle lengths! In this problem we are going to revisit the same task, but this time we are going to approach parameter estimation from a Bayesian perspective.



We will slightly simplify the formula to make the math less long. Assume that the length of someones period X is given by the following pdf which is based off a single parameter β :

$$f(X = x | B = \beta) = \beta(x - 27)^{\beta-1}e^{-(x-27)^\beta}$$

For a new user, you only observe **two** cycle lengths $X_1 = 28$ and $X_2 = 29$, which means that you don't have enough data to estimate β using MLE (in section we used MLE, but we had many more cycles)! How can we estimate β in this low data situation? Inference. Just like in the Beta distribution derivation.

Good news! Based on historical data, we have a strong prior on the values that β can take! For thousands of past users, we have learned individual values for β . We then treat those values as samples from a random variable B . After analysing all the data, we found that B is a Gamma. We then used MLE to fit the Gamma distribution which resulted in the following nice equation for our prior belief in B :

$$P(B = \beta) = \frac{1}{2} \beta^2 e^{-\beta}$$

Given the two observations, use inference to compute your updated belief in B

$$P(B = \beta | X_1 = 28, X_2 = 29)$$

Allow for a normalization constant in your answer. You can assume that the two observations (X_1 and X_2) are independent if you know the value B .

Answer Editor

Numeric Answer:

Explanation:

Block LaTeX

Pset 7: Review

The screenshot shows a web browser window with the URL `cs109psets.netlify.app/fall25/pset7/createaproblem`. The page is titled "PS7 Imagine Your Own Problem". On the left, a vertical navigation menu shows steps 1 through 9, with step 9 highlighted. The main content area contains the following text:

For this problem, devise a CS109 pset problem or exam question using any material covered in class.



You can pick any topic that is covered by or related to CS109! It might be tempting to think up a problem that's really simple, like "what is the probability of two heads in five coin flips", but we're looking for creativity and a level of challenge above that. As a guiding principle your problem should be of the difficulty of at least a "medium-difficulty" exam problem or pset question. To get full credit, submit both the question you invent and a written solution for it in the text box to the right.

One path is to build off a problem you have already seen in a previous problem set, practice exam or section. If you go down that path please make sure to explain why your take on the problem is novel.

Do:

- try to come up with something that exemplifies how well you understand a particular piece of content from class and is unique to you.
- use this as a chance to go deeper into something you are curious about.
- use this as a chance to learn more about the structure of CS109 and the relationship between word problems and math formalizations

At the bottom of the page, there are buttons for "Previous Question" and "Next Question". On the right side of the browser window, there is an "Answer Editor" and "Solution" section with an "Explanation:" label and a toolbar containing "Block LaTeX", "Inline LaTeX", "Python", and "Image" options.

Today

What to Buy?

Poker Information
Theory

Quant Interview
(with cupcakes)

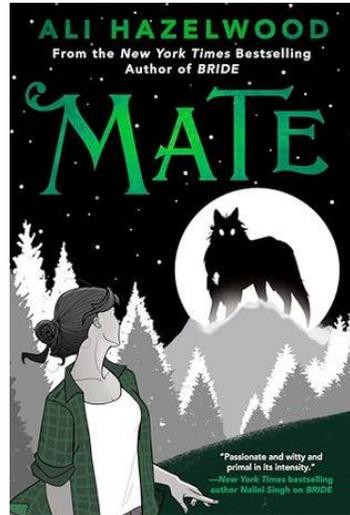
Child word
identification

Summarize
Medical Record

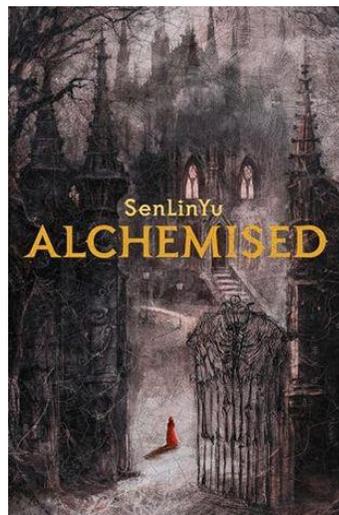
Beta Binomial

Rating Preference

Which Book Would you Prefer?



Rating	1	2	3	4	5
Count	0	0	0	1	4



Rating	1	2	3	4	5
Count	0	1	2	20	200

MLE Estimate

Which Book Would you Prefer? MLE Solution

```
import numpy as np
```

```
# MLE probabilities for each book  
# Order: ratings 1, 2, 3, 4, 5
```

```
p1 = np.array([0, 0, 0, 1/5, 4/5])          # Book 1  
p2 = np.array([0, 1/223, 2/223, 20/223, 200/223]) # Book 2
```

```
prob = 0.0  
for i in range(5):          # rating for Book 1 (0-index → rating i+1)  
    for j in range(i+1, 5): # rating for Book 2 strictly greater  
        prob += p1[i] * p2[j]
```

```
print("P(Book 2 > Book 1) =", prob)
```

$$P(R_2 > R_1) = \sum_{i=1}^5 \sum_{j=i+1}^5 P_1(i) P_2(j)$$

Another Perspective: Beta, but Better

Which Book Would you Prefer? Better Beta Solution

Assume: Rating probabilities are distributed as a Dirchlet with the following joint PDF:

$$f(x_1, \dots, x_5) = K \cdot \prod_{i=1}^5 x_i^{c_i}$$

How many times
you saw rating i

Probability
that you will
get rating 1

Probability
that you will
get rating k

You can assume you have access to

take a sample from the joint

`x_list = sample_dirchlet()`

evaluate the joint pdf

`likelihood = pdf(x_list)`

Another Perspective: Latent Like

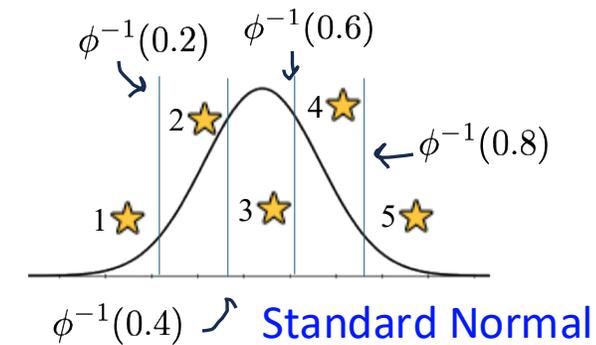
Book vibe

Could fit these using historical data!

$$Y \sim N(\mu = 0.8, \sigma^2 = 1)$$

$$X_i \sim N(\mu = Y, \sigma^2 = 1)$$

Rating	1	2	3	4	5
Count	0	0	0	1	4



Rater i vibe of the book

$$P(\text{Ratings} | Y = y) = P(4 \star | Y = y) \cdot P(5 \star | Y = y)^4$$

Threshold 3 Threshold 4

$$P(\tau_3 < X_i < \tau_4 | Y = y)$$

$$= P\left(\frac{\tau_3 - y}{1} < \frac{X_i - y}{1} < \frac{\tau_4 - y}{1}\right)$$

$$= P(\tau_3 - y < Z < \tau_4 - y)$$

$$= \Phi(\tau_4 - y) - \Phi(\tau_3 - y)$$

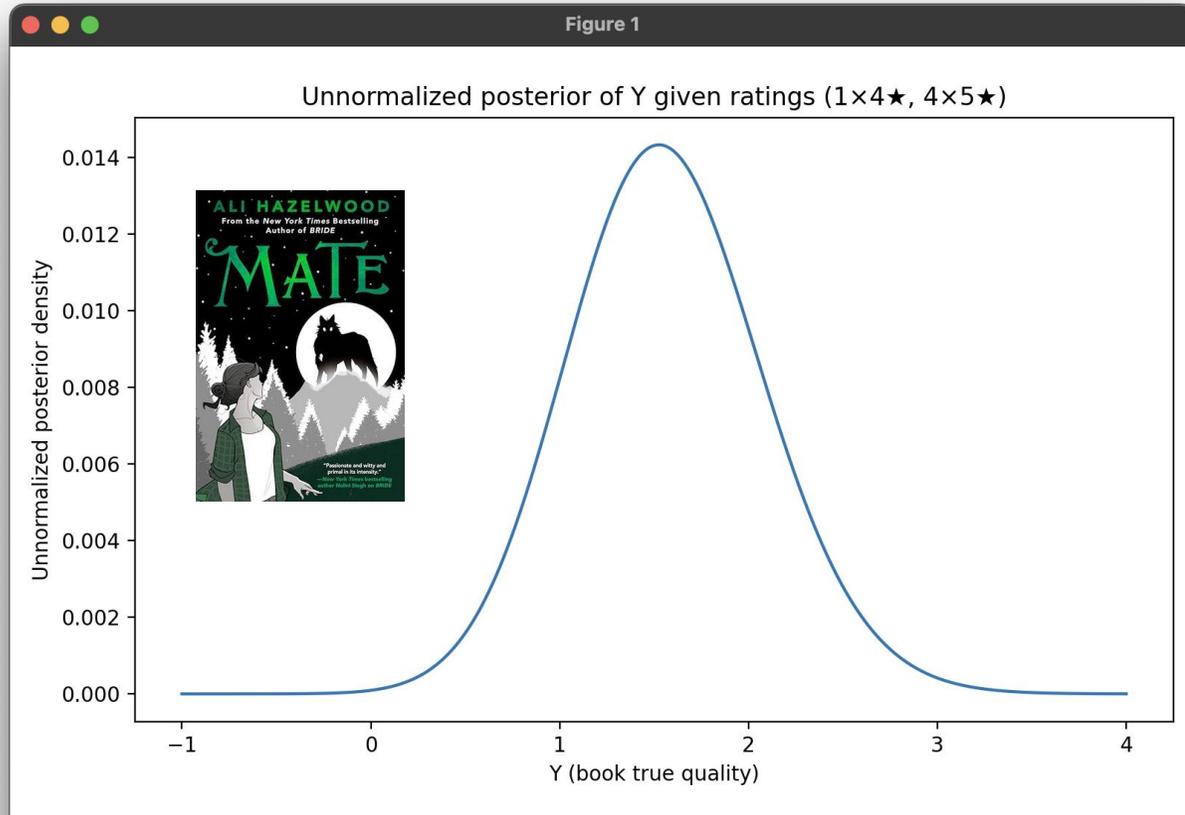
Normal PDF

$$f(Y = y | \text{Ratings}) \propto \exp\left(-\frac{1}{2}(y - 0.8)^2\right) \cdot [\Phi(\tau_4 - y) - \Phi(\tau_3 - y)] \cdot [1 - \Phi(\tau_4 - y)]^4$$

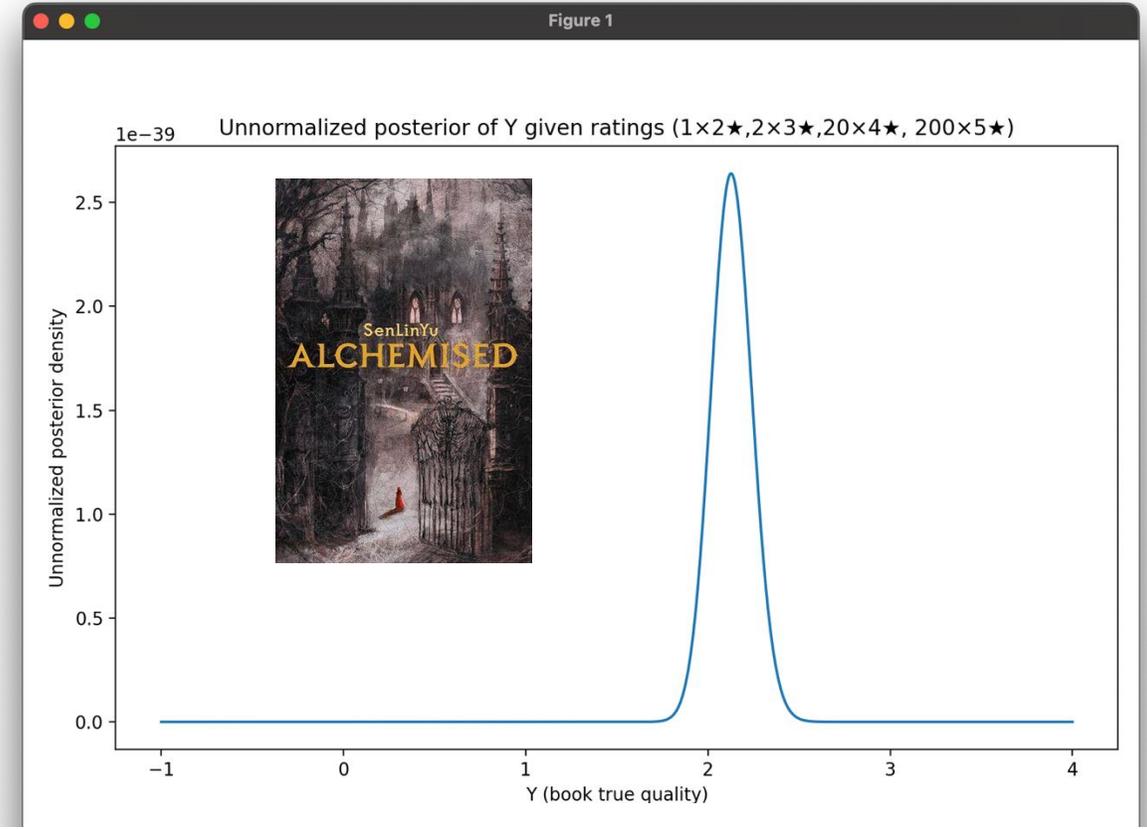
\uparrow \uparrow \uparrow
 $\Phi^{-1}(0.8)$ $\Phi^{-1}(0.6)$ $\Phi^{-1}(0.8)$

Which Book Would you Prefer?

Probability you prefer book 2 is 86%



Rating	1	2	3	4	5
Count	0	0	0	1	4



Rating	1	2	3	4	5
Count	0	1	2	20	200

Poker Information Theory

Probability for Computer Science

Stanford University

Reference

- Notation Reference
- Core Probability Reference
- Random Variable Reference
- Python Reference
- Calculus Reference
- Calculators
- Language Model Tool

Part 1: Core Probability

- Probability
- Equally Likely Outcomes
- Axioms of Probability
- Probability of **or**
- Conditional Probability
- Law of Total Probability
- Bayes' Theorem
- Independence
- Probability of **and**
- De Morgan's Law
- Log Probabilities
- Many Coin Flips
- Counting
- Combinatorics
- Stories
- Bacteria Evolution
- Google Rain Prediction
- Random Walks
- Binomial with Different Probs
- Netflix Genres
- Poker

Isabel 100

Flopp Turn River

2♠ 2♦ 9♣ 6♥ A♠

Continue

Fabian 100

Jade 100

Emir 100

Nisha 100

You 100

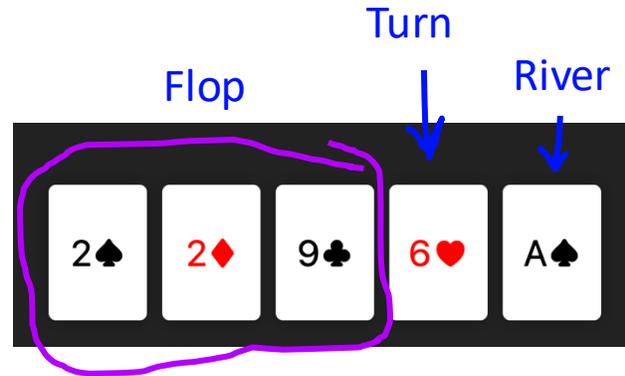
Three of a Kind, 2's

2♣ 10♦

Let X be a random variable for the set of 7 cards at the end. What is $H(X)$ after each event?

There are 52 unique cards. Each card is equally likely.

Poker Entropy



$$H(\text{pre flop}) = 21.014$$

$$H(\text{pre turn}) = 10.078$$

$$H(\text{pre river}) = 5.524$$

$$H(\text{post river}) = 0$$

Quant Interview

Quant Interview



100 cupcakes.

Scenario 1: 51 are blue.

Scenario 2: 49 are blue.

a) You look at one cupcake. It is blue.

b) You look at three cupcakes (with replacement). Two are blue.

c) You look at three cupcakes (without replacement). Two are blue.

Quant Interview: Bonus

- c. A hypergeometric is a random variable for the number of successes if you remove items **without** replacement from a fixed population. If $X \sim \text{Hypergeom}(t, k, n)$, the PMF is

$$P(X = x) = \frac{\binom{k}{x} \binom{t-k}{n-x}}{\binom{t}{n}}$$

Where:

- t is the population size,
- k is the number of “success” items in the population,
- n is the number of draws (without replacement),
- X counts the number of successes observed in those n draws.

Three cupcakes are drawn without replacement and two are blue. What is the probability that the majority were blue?

Did they read that?

Did they read that?

Confident, Independent Reading

E||o



Did they read that?



Medical Summary

2. What an Ideal Classifier Looks Like

We compute the true conditional probability $P(I = i \mid x, y_1, \dots, y_N)$. Using conditional probability,

$$P(I = i \mid x, y_1, \dots, y_N) = \frac{P(x, y_1, \dots, y_N, i)}{P(x, y_1, \dots, y_N)}.$$

Because the distractors Y_j for $j \neq i$ are drawn independently from $P(Y)$ and independently of X ,

$$P(x, y_1, \dots, y_N, i) = \frac{1}{N} P(x, y_i) \prod_{j \neq i} P(y_j).$$

The denominator is a sum over all possible positions of the true record:

$$P(x, y_1, \dots, y_N) = \sum_{k=1}^N \frac{1}{N} P(x, y_k) \prod_{j \neq k} P(y_j).$$

How does
this follow



$$\begin{aligned} P(I = i \mid x, y_1, \dots, y_N) &= \frac{P(x, y_i)/P(y_i)}{\sum_{k=1}^N P(x, y_k)/P(y_k)} \\ &= \frac{P(y_i \mid x)P(y_i)}{\sum_{k=1}^N P(y_k \mid x)/P(y_k)} \end{aligned}$$