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CS109

Section #1
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Section 1: Core Probability

1 Warm-Up with Music Preferences

In one prior offering of CS109, the distribution of students by year and their likelihood of liking the song *We Are the Champions* by Queen is shown in the table below. Here, "Graduate +" includes graduate students, CGOE students, coterminal students (everyone not in the other partitions). Let L_1 denote the event that a student likes the song.

Year	**% of Students**	$P(L_1 \mid \text{Year})$
Freshman	20%	0.23
Sophomore	25%	0.42
Junior	25%	0.39
Senior	10%	0.74
Graduate +	20%	0.89

What is the probability that a randomly chosen student likes *We Are the Champions* by Queen?

We are going to use the Law of Total Probability (LOTP) to solve this problem! The probability that a randomly chosen student likes this song is:

$$\begin{aligned} P(L_1) = &\ P(L_1 \mid \text{Freshman})P(\text{Freshman}) \\ &+ P(L_1 \mid \text{Sophomore})P(\text{Sophomore}) \\ &+ P(L_1 \mid \text{Junior})P(\text{Junior}) \\ &+ P(L_1 \mid \text{Senior})P(\text{Senior}) \\ &+ P(L_1 \mid \text{Graduate})P(\text{Graduate}) \end{aligned}$$

$$= (0.23)(0.20) + (0.42)(0.25) + (0.39)(0.25) + (0.74)(0.10) + (0.89)(0.20)$$

$$= 0.046 + 0.105 + 0.0975 + 0.074 + 0.178$$

$$= 0.5005.$$

Answer: $P(L_1) \approx 0.50$.

2 WebMD mini

In this problem, we will compute the probability that a person has a particular disease given that they present with the symptom of a fever. A person may have either *Influenza (Flu)*, *Streptococcal Pharyngitis (Strep)*, or *No Disease*. Assume these outcomes are mutually exclusive, so each person belongs to exactly one of the three groups: Flu, Strep, or No Disease.

In the general population, 8% of people have Influenza (Flu), 4.1% have Streptococcal Pharyngitis (Strep), and the remaining 87.9% have no disease. If a person has the flu, the probability they have a fever is 0.92. If they have strep, the probability they have a fever is 0.86. If they have no disease, the probability they have a fever is 0.01.

For each disease (Flu, Strep, No Disease), compute the probability that a person has that disease given that they present with a **fever**.

Define:

- F: the event the person has a fever.
- I: the event the person has Influenza (*Flu*).
- R: the event the person has Streptococcal Pharyngitis (*Strep*).
- N: the event the person has No Disease.

We want to compute $P(I \mid F)$, $P(R \mid F)$, and $P(N \mid F)$. By Bayes' Rule:

$$P(\text{Disease} \mid F) = \frac{P(F \mid \text{Disease})\, P(\text{Disease})}{P(F)}.$$

First, compute $P(F)$ using the Law of Total Probability:

$$P(F) = P(F \mid I)P(I) + P(F \mid R)P(R) + P(F \mid N)P(N).$$

$$P(F) = (0.92)(0.080) + (0.86)(0.041) + (0.01)(0.879) \approx 0.118.$$

Now compute each posterior:

$$P(I \mid F) = \frac{(0.92)(0.080)}{0.118} \approx 0.624.$$

$$P(R \mid F) = \frac{(0.86)(0.041)}{0.118} \approx 0.299.$$

$$P(N \mid F) = \frac{(0.01)(0.879)}{0.118} \approx 0.074.$$

Answer: Given that a person has a fever,

$$P(\text{Flu} \mid \text{Fever}) \approx 0.624, \quad P(\text{Strep} \mid \text{Fever}) \approx 0.299, \quad P(\text{No Disease} \mid \text{Fever}) \approx 0.074.$$

You can have a fever without having a disease due to hormone changes, exercise, heat stroke and stress.

2mm

3 The Birthday Problem

When solving a counting problem, it can often be useful to come up with a generative process, a series of steps that "generates" examples. A correct generative process to count the elements of set A will (1) *generate every element of A* and (2) *not generate any element of A more than once*. If our process has the added property that (3) *any given step always has the same number of possible outcomes*, then we can use the product rule of counting.

Assume that birthdays happen on any of the 365 days of the year with equal likelihood (we'll ignore leap years).

a. **Warmup:** How many ways can you choose birthdays for two (distinct) people?

> There are 365 choices for one person's birthday. If we break down the generative process of choosing two birthdays into the steps 1) choose the first person's birthday and 2) choose the second person's birthday, then we can use the step/product rule of counting to say that the total number of ways to assign two birthdays is $365 \cdot 365 = 365^2$.
> Note: because the two people are distinct, we don't need to do any correcting for overcounting – "person 1's birthday is 1/1, person 2's birthday is 2/1" and "person 1's birthday is 2/1, person 2's birthday is 1/1" are two different outcomes.

b. What is the probability that of the n people in class, at least two people share the same birthday?

> For a more complete explanation of this problem, check out this chapter in the course reader: https://probabilityforcs.firebaseapp.com/book/birthday_paradox
>
> Computing $P(\text{at least 2 people share a birthday})$ is difficult. We realize that this can be thought of as
>
> $P(\text{exactly 2 people share birthday} \cup \text{exactly 3} \cup \text{exactly 4} \cup \ldots \cup \text{exactly } n \text{ people share birthday})$
>
> Using the additivity axiom of probability, we realize that this can be split up because the events are mutually exclusive.
>
> $P(\text{exactly 2 people share birthday}) + P(\text{exactly 3}) + P(\text{exactly 4}) + \ldots + P(\text{exactly } n \text{ people share birthday})$
>
> However, this is very tedious!
>
> It is much easier to calculate $1 - P(\text{no one shares a birthday})$. Let our sample space, S be the set of all possible assignments of birthdays to the students in section. By the assumptions of this problem, each of those assignments is equally likely, so this is a good choice of sample space. We can use the product rule of counting to calculate $|S|$:
>
> $$|S| = (365)^n$$

Our event E will be the set of assignments in which there are no matches (i.e. everyone has a different birthday). We can think of this as a generative process where there are 365 choices of birthdays for the first student, 364 for the second (since it can't be the same birthday as the first student), and so on. Verify for yourself that this process satisfies the three conditions listed above. We can then use the product rule of counting:

$$|E| = (365) \cdot (364) \cdot \cdots \cdot (365 - n + 1)$$

$$\begin{aligned} P(\text{birthday match}) &= 1 - P(\text{no matches}) \\ &= 1 - \frac{|E|}{|S|} \\ &= 1 - \frac{(365) \cdot (364) \ldots (365 - n + 1)}{(365)^n} \end{aligned}$$

A common misconception is that the size of the event E can be computed as $|E| = \binom{365}{n}$ by choosing n distinct birthdays from 365 options. However, outcomes in this event (n unordered distinct dates) cannot recreate any outcomes in the sample space $|S| = 365^n$ (n distinct dates, one for each distinct person). However if we compute the size of event E as $|E| = \binom{365}{n} n!$ (equivalent to the number above), then we can assign the n birthdays to each person in a way consistent with the sample space. The expression $\binom{365}{n} n!$ is equivalent to $\frac{365!}{(365-n)!}$ which is known as a "falling factorial" and also as "365 permute n" outside of this class.

Interesting values: $(n = 13 : p \approx 0.19)$, $(n = 23 : p \approx 0.5)$, $(n = 70 : p \geq 0.99)$.