1 Lecture 11, 4-29-20: Joint Distributions

1. Given a Normal RV \( X \sim N(\mu, \sigma^2) \), how can we compute \( P(X \leq x) \) from the standard Normal distribution \( Z \) with CDF \( \phi \)?

2. What is a continuity correction and when should we use it?

3. If we have a joint PMF for discrete random variables \( p_{X,Y}(x, y) \), how can we compute the marginal PMF \( p_X(x) \)?

   1. First, we write \( \phi((x - \mu)/\sigma) \). We then look up the value we’ve computed in the Standard Normal Table.

   2. Continuity correction is used when a Normal distribution is used to approximate a Binomial. Since a Normal is continuous and Binomial is discrete, we have to use a continuity correction to discretize the Normal. The continuity correction makes it so that the normal variable is evaluated from + or - 0.5 increments from the desired \( k \) value.

   3. The marginal distribution is \( p_X(x) = \sum_y p_{X,Y}(x, y) \)

2 Lecture 12, 5-1-20: Independent Random Variables

1. What distribution does the sum of two independent binomial RVs \( X + Y \) have, where \( X \sim Bin(n_1, p) \) and \( Y \sim Bin(n_2, p) \)? Include the parameter(s) in your answer. Why is this the case?

2. What distribution does the is of two independent Poisson RVs \( X + Y \) have, where \( X \sim Poi(\lambda_1) \) and \( Y \sim Poi(\lambda_2) \)? Include the parameter(s) in your answer.

3. If \( Cov(X, Y) = 0 \), are \( X \) and \( Y \) independent? Why or why not?

   1. Binomial; \( X + Y \sim Bin(n_1 + n_2, p) \)

   2. Poisson; \( X + Y \sim Poi(\lambda_1 + \lambda_2) \)

   3. Not necessarily. Suppose there are three outcomes for \( X \): let \( X \) take on values in \( \{-1, 0, 1\} \) with equal probability \( 1/3 \). Let \( Y = X^2 \). Then, \( E[XY] = E[X^3] = E[X] = 0 \) (since \( X^3 = X \)) and \( E[X] = 0 \), so \( Cov(X, Y) = E[XY] - E[X]E[Y] = 0 - 0 = 0 \) but \( X \) and \( Y \) are dependent since \( P(Y = 1) = 2/3 \neq 1 = P(Y = 1|X = 1) \).

3 Lecture 13, 5-13-20: Joint Random Variables Statistics

1. True or False? The symbol \( Cov \) is covariance, and the symbol \( \rho \) is Pearson correlation.

   \[
   \begin{array}{c|c|c}
   X \perp Y & Cov(X, Y) = 0 & Var(X + X) = 2Var(X) \\
   \hline
   Cov(X, Y) = 0 & \Rightarrow X \perp Y & X \sim N(0, 1) \land Y \sim N(0, 1) \Rightarrow \rho(X, Y) = 1 \\
   \hline
   Y = X^2 & \rho(X, Y) = 1 & Y = 3X \Rightarrow \rho(X, Y) = 3
   \end{array}
   \]
1. **True or False?**

<table>
<thead>
<tr>
<th>True</th>
<th>False (... = (4\text{Var}(X)))</th>
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</thead>
<tbody>
<tr>
<td>False (antecedent necessary, not sufficient)</td>
<td>False (don’t know how independent (X) &amp; (Y) are)</td>
</tr>
<tr>
<td>False ((Y = X \implies \ldots))</td>
<td>False (... = 1)</td>
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