Recommender Systems

**Customer X**
- Buys CD of Mozart
- Buys CD of Haydn

**Customer Y**
- Does search on Mozart
- Recommender system suggests Haydn from data collected about customer X
Recommendations

Products, web sites, blogs, news items, …

Examples:

amazon.com

YouTube

Google News

Pandora

Netflix

Pinterest

Search

Recommendations

Items
From Scarcity to Abundance

**Shelf space is a scarce commodity for traditional retailers**
- Also: TV networks, movie theaters, ...

**Web enables near-zero-cost dissemination of information about products**
- From scarcity to abundance
The Long Tail

Source: Chris Anderson (2004)

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Sources: Erik Brynjolfsson and Jeffrey Hu, MIT, and Michael Smith, Carnegie Mellon; Barnes & Noble; Netflix; RealNetworks
More choice requires:

Recommendation engines!
Knowing how these work

Isn't relevant only for building practical news or product recommenders....
We all need to understand how they work!

QAnon Supporters And Anti-Vaxxers Are Spreading A Hoax That Bill Gates Created The Coronavirus

It has no basis in reality, but that hasn’t slowed its spread across Facebook and Twitter.

Las Vegas survivors furious as YouTube promotes clips calling shooting a hoax


'Fiction is outperforming reality': how YouTube's algorithm distorts truth

THE WALL STREET JOURNAL.

How YouTube Drives People to the Internet’s Darkest Corners

Google’s video site often recommends divisive or misleading material, despite recent changes designed to fix the problem
Types of Recommendations

**Editorial and hand curated**
- List of favorites
- Lists of “essential” items

**Simple aggregates**
- Top 10, Most Popular, Recent Uploads

**Tailored to individual users**
- Amazon, Netflix, ...
Formal Model

\[ X = \text{set of Customers} \]

\[ S = \text{set of Items} \]

Utility function \( u: X \times S \rightarrow R \)

\( R = \text{set of ratings} \)

\( R \) is a totally ordered set

\( \text{e.g., 0-5 stars, real number in } [0,1] \)
Utility Matrix

<table>
<thead>
<tr>
<th></th>
<th>Avatar</th>
<th>LOTR</th>
<th>Matrix</th>
<th>Pirates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice</td>
<td>1</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bob</td>
<td>0.5</td>
<td></td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Carol</td>
<td>0.2</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>David</td>
<td></td>
<td></td>
<td></td>
<td>0.4</td>
</tr>
</tbody>
</table>

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Key Problems

(1) Gathering “known” ratings for matrix
   ◦ How to collect the data in the utility matrix

(2) Extrapolate unknown ratings from known ones
   ◦ Mainly interested in high unknown ratings
      ◦ We are not interested in knowing what you don’t like but what you like

(3) Evaluating extrapolation methods
   ◦ How to measure success/performance of recommendation methods
(1) Gathering Ratings

Explicit
- Ask people to rate items
- Doesn’t work well in practice – people can't be bothered
- Crowdsourcing: Pay people to label items

Implicit
- Learn ratings from user actions
  - E.g., purchase (or finish watching YouTube video) implies high rating
(2) Extrapolating Utilities

**Key problem:** Utility matrix $U$ is sparse
- Most people have not rated most items

- The "Cold Start" Problem:
  - New items have no ratings
  - New users have no history
(2) Extrapolating Utilities

Three approaches to recommender systems:

1. Content-based
2. Collaborative Filtering
3. Latent factor based

} This lecture! CS246!
Content-based Recommender Systems
Content-based Recommendations

**Main idea:** Recommend items to customer $x$ similar to previous items rated highly by $x$

*Example:*

**Movie recommendations**
- Recommend movies with same actor(s), director, genre, ...

**Websites, blogs, news**
- Recommend other sites with “similar” content
Item Profiles

For each item, create an **item profile**

**Profile is a set (vector) of features**

- **Movies**: author, genre, director, actors, year...
- **Text**: Set of “important” words in document

**How to pick important features?**

- **TF-IDF** (Term frequency * Inverse Doc Frequency)
  - **Term** ... **Feature**
  - **Document** ... **Item**
Content-based Item Profiles

<table>
<thead>
<tr>
<th>Melissa McCarthy</th>
<th>Actor A</th>
<th>Actor B</th>
<th>...</th>
<th>Johnny Depp</th>
<th>Comic Genre</th>
<th>Spy Genre</th>
<th>Pirate Genre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Movie X</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Movie Y</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- If everything is 1 or 0 (indicator features)
- But what if we want to have real or ordinal features too?
**Content-based Item Profiles**

<table>
<thead>
<tr>
<th></th>
<th>Melissa McCarthy</th>
<th>Actor A</th>
<th>Actor B</th>
<th>...</th>
<th>Johnny Depp</th>
<th>Comic Genre</th>
<th>Spy Genre</th>
<th>Pirate Genre</th>
<th>Avg Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Movie X</strong></td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Movie Y</strong></td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

- Maybe we want a scaling factor $\alpha$ between binary and numeric features
• Maybe we want a scaling factor $\alpha$ between binary and numeric features
• Or maybe $\alpha=1$

\[
\text{Cosine}(\text{Movie X, Movie Y}) = \frac{2+12\alpha^2}{\sqrt{5+9\alpha^2}\sqrt{5+16\alpha^2}}
\]
User Profiles

Want a vector with the same components/dimensions as items
- Could be 1s representing user purchases
- Or arbitrary numbers from a rating

User profile is aggregate of items:
- Average(weighted?) of rated item profiles
Sample user profile

- Items are movies
- Utility matrix has 1 if user has seen movie
- 20% of the movies user U has seen have Melissa McCarthy
- $U[“Melissa McCarthy”] = 0.2$

<table>
<thead>
<tr>
<th>User U</th>
<th>Melissa McCarthy</th>
<th>Actor A</th>
<th>Actor B</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.2</td>
<td>.005</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
### Prediction

- Users and items have the same dimensions!

<table>
<thead>
<tr>
<th>Movie i</th>
<th>Melissa McCarthy</th>
<th>Actor A</th>
<th>Actor B</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Movie i</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

| User x | 0.2 | .005 | 0 | 0 | 0 |

- So just recommend the items whose vectors are most similar to the user vector!

- Given user profile $x$ and item profile $i$,
  - estimate $u(x, i) = \cos(x, i) = \frac{x \cdot i}{\|x\| \cdot \|i\|}$
Pros: Content-based Approach

+: No need for data on other users
  ◦ No cold-start or sparsity problems

+: Able to recommend to users with unique tastes

+: Able to recommend new & unpopular items
  ◦ No first-rater problem

+: Able to provide explanations
  ◦ Just list the content-features that caused an item to be recommended
Cons: Content-based Approach

- Finding the appropriate features is hard
  - E.g., images, movies, music

- Recommendations for new users
  - How to build a user profile?

- Overspecialization
  - Never recommends items outside user’s content profile
  - People might have multiple interests
  - Unable to exploit quality judgments of other users
Collaborative Filtering
Harnessing quality judgments of other users
Collaborative Filtering
Version 1: "User-User" Collaborative Filtering

Consider user $x$

Find set $N$ of other users whose ratings are “similar” to $x$’s ratings

Estimate $x$’s ratings based on ratings of users in $N$
Finding Similar Users

Let $r_x$ be the vector of user $x$'s ratings

$$r_x = [*, _, _, *, **]$$
$$r_y = [*, _, **, **, _]$$

Cosine similarity measure

$$\text{sim}(x, y) = \cos(r_x, r_y) = \frac{r_x \cdot r_y}{||r_x|| \cdot ||r_y||}$$

Problem: Treats missing ratings as “negative”

- What do I mean?
9.3.1 Measuring Similarity

The first question we must deal with is how to measure similarity of users or items from their rows or columns in the utility matrix. We have produced Fig. 9.1 here as Fig. 9.4. This data is too small to draw any reliable conclusions, but its small size will make clear some of the pitfalls in picking a distance measure. Observe specifically the users A and C. They rated two movies in common, but they appear to have almost diametrically opposite opinions of these movies. We would expect that a good distance measure would make them rather far apart. Here are some alternative measures to consider.

<table>
<thead>
<tr>
<th></th>
<th>HP1</th>
<th>HP2</th>
<th>HP3</th>
<th>TW</th>
<th>SW1</th>
<th>SW2</th>
<th>SW3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td>2</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

**Utility Matrix**

**Intuitively we want:** \( \text{sim}(A, B) > \text{sim}(A, C) \)

**Cosine similarity:** Yes, 0.386 > 0.322

But only barely works...

Considers missing ratings as “negative”

HP = Harry Potter, TW = Twilight, SW = Star Wars
9.3.1 Measuring Similarity

The first question we must deal with is how to measure similarity of users or items from their rows or columns in the utility matrix. We have produced Fig. 9.1 here as Fig. 9.4. This data is too small to draw any reliable conclusions, but its small size will make clear some of the pitfalls in picking a distance measure. Observe specifically the users $A$ and $C$. They rated two movies in common, but they appear to have almost diametrically opposite opinions of these movies. We would expect that a good distance measure would make them rather far apart. Here are some alternative measures to consider.

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<th>SW1</th>
<th>SW2</th>
<th>SW3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>4</td>
<td></td>
<td></td>
<td>5</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

- **Problem with cosine**: 0 acts like a negative review
  - $C$ really loves SW
  - $A$ hates SW
  - $B$ just hasn’t seen it
- **Another problem**: we’d like to normalize for raters
  - $D$ rated everything the same; not very useful
### Mean-Centered Utility Matrix: subtract the means of each row

<table>
<thead>
<tr>
<th></th>
<th>HP1</th>
<th>HP2</th>
<th>HP3</th>
<th>TW</th>
<th>SW1</th>
<th>SW2</th>
<th>SW3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td></td>
<td></td>
<td>5</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

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<th>HP3</th>
<th>TW</th>
<th>SW1</th>
<th>SW2</th>
<th>SW3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2/3</td>
<td></td>
<td></td>
<td>5/3</td>
<td>-7/3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1/3</td>
<td>1/3</td>
<td>-2/3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td>-5/3</td>
<td>1/3</td>
<td>4/3</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

- Now a 0 means no information
- And negative ratings means viewers with opposite ratings will have vectors in opposite directions!
Modified Utility Matrix: subtract the means of each row

<table>
<thead>
<tr>
<th></th>
<th>HP1</th>
<th>HP2</th>
<th>HP3</th>
<th>TW</th>
<th>SW1</th>
<th>SW2</th>
<th>SW3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2/3</td>
<td></td>
<td></td>
<td>5/3</td>
<td>-7/3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1/3</td>
<td>1/3</td>
<td>-2/3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td>-5/3</td>
<td>1/3</td>
<td>4/3</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

\[\text{Cos}(A,B) = \frac{(2/3) \times (1/3)}{\sqrt{(2/3)^2 + (5/3)^2 + (-7/3)^2} \sqrt{(1/3)^2 + (1/3)^2 + (-2/3)^2}} = 0.092\]

\[\text{Cos}(A,C) = \frac{(5/3) \times (-5/3) + (-7/3) \times (1/3)}{\sqrt{(2/3)^2 + (5/3)^2 + (-7/3)^2} \sqrt{(-5/3)^2 + (1/3)^2 + (4/3)^2}} = -0.559\]

Now A and C are (correctly) way further apart than A,B
Terminological Note: subtracting the mean is mean-centering, not normalizing
(normalizing is dividing by a norm to turn something into a probability), but the textbook (and common usage) sometimes overloads the term “normalize”
Finding similar users with mean-centering

Let \( r_x \) be the vector of user \( x \)'s ratings

\[
\begin{align*}
  r_x &= \{1, 0, 0, 1, 3\} \\
  r_y &= \{1, 0, 2, 2, 0\}
\end{align*}
\]

Mean-centering:

- For each user \( x \), let \( \bar{r}_x \) be mean of \( r_x \) (ignoring missing values)
- \( \bar{r}_x = (1 + 1 + 3)/3 = 5/3 \) \( r_y = (1 + 2 + 2)/3 = 5/3 \)
- Subtract this average from each of their ratings
  - (but do nothing to the "missing values"; they stay "null").
  - mean centered \( r_x = \{-2/3, 0, 0, -2/3, 4/3\} \)

Now: Keep only items they both rate (unlike 2 slides ago)

\[
\begin{align*}
  r_x &= \{-2/3, \_\_\, -2/3, \_\_\, \_\} \\
  r_y &= \{-2/3, \_\_\, \_\_\, \_\_\, 1/3\, \_\}
\end{align*}
\]

Now: \( r_x = \{-2/3, -2/3\} \) \( r_y = \{-2/3, 1/3\} \)

Take cosine:

- Now compute cosine between user vectors
- \( \cos([-2/3, -2/3], [-2/3, 1/3]) \)
Finding similar users with mean centering: more formally

Let $r_x$ be the vector of user $x$’s ratings, and $r^-_x$ be its mean (ignoring missing values)

**Cosine similarity measure**

- $\text{sim}(x, y) = \cos(r_x, r_y) = \frac{r_x \cdot r_y}{\|r_x\| \|r_y\|}$
- **Problem:** Treats missing ratings as “negative”

**Mean-centered overlapping-item cosine similarity**

- $S_{xy} = \text{items rated by both users } x \text{ and } y$

$$
\text{sim}(x, y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - r_x)(r_{ys} - r_y)}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - r_x)^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - r_y)^2}}
$$
Rating Predictions

From similarity metric to recommendations:

Let $r_x$ be the vector of user $x$’s ratings

Let $N$ be the set of $k$ users most similar to $x$ who have rated item $i$

Prediction for item $i$ of user $x$:

- Rate $i$ as the mean of what $k$-people-like-me rated $i$

$$r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi}$$

- Even better: Rate $i$ as the mean weighted by their similarity to me ...

$$r_{xi} = \frac{\sum_{y \in N} S_{xy} r_{yi}}{\sum_{y \in N} S_{xy}}$$

**Shorthand:**

$$S_{xy} = sim(x, y)$$

- Many other tricks possible...

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Collaborative Filtering Version 2: Item-Item Collaborative Filtering

So far: **User-user collaborative filtering**

Alternate view that often works better: **Item-item**

- For item $i$, find other similar items
- Estimate rating for item $i$ based on ratings for similar items
- Can use same similarity metrics and prediction functions as in user-user model
- "Rate $i$ as the mean of my ratings for other items, weighted by their similarity to $i$"

$$r_{xi} = \frac{\sum_{j \in N(i;x)} S_{ij} r_{xj}}{\sum_{j \in N(i;x)} S_{ij}}$$

- $S_{ij}$ ... similarity of items $i$ and $j$
- $r_{xj}$ ... rating of user $x$ on item $i$
- $N(i;x)$ ... set of items rated by $x$ similar to $i$
### Item-Item CF ($|N|=2$)

<table>
<thead>
<tr>
<th>users</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
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<td>5</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>6</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>2</td>
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<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

- unknown rating
- rating between 1 to 5

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**Item-Item CF (|N|=2)**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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- estimate rating of movie 1 by user 5

SLIDES ADAPTED FROM JURE LESKOVEC
## Item-Item CF ($|N|=2$)

**Neighbor selection:**
Identify movies similar to movie 1, rated by user 5

Here we use mean centered item-overlap cosine as similarity:
1) Subtract mean rating $m_i$ from each movie $i$
2) Compute (item-overlapping) cosine similarities between rows
**Item-Item CF (|N|=2)**

**Neighbor selection:**
Identify movies similar to movie 1, rated by user 5

Here we use mean centered item-overlap cosine as similarity:
1) Subtract mean rating $m_i$ from each movie $i$
2) Compute (item-overlapping) cosine similarities between rows

### Subtract mean rating $m_i$ from each movie $i$

$$m_1 = \frac{(1+3+5+5+4)}{5} = \frac{18}{5}$$

### Showing computation only for #3 and #6

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Movies
## Item-Item CF (|N|=2)

### Neighbor selection:
Identify movies similar to movie 1, rated by user 5

Here we use mean centered item-overlap cosine as similarity:
1) Subtract mean rating $m_i$ from each movie $i$
2) Compute (item-overlapping) cosine similarities between rows

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**sim(1,m): 1.00**
Compute Cosine Similarity:

For rows 1 and 3, they both have values for users 1, 9 and 11.

\[
\text{sim}(1, 3) = \frac{\frac{-13}{5}(-1) + \frac{7}{5}(1) + \frac{2}{5}(2)}{\sqrt{\left(\frac{-13}{5}\right)^2 + \left(\frac{7}{5}\right)^2 + \left(\frac{2}{5}\right)^2} \cdot \sqrt{\left(-1\right)^2 + \left(1\right)^2 + \left(2\right)^2}} \approx 0.658
\]

For rows 1 and 6, they both have values for users 1, 3 and 11.

\[
\text{sim}(1, 6) = \frac{\frac{-13}{5}(-8) + \frac{-3}{5}(2) + \frac{2}{5}(7)}{\sqrt{\left(\frac{-13}{5}\right)^2 + \left(-\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2} \cdot \sqrt{\left(-\frac{8}{5}\right)^2 + \left(\frac{2}{5}\right)^2 + \left(\frac{7}{5}\right)^2}} \approx 0.768
\]
### Item-Item CF ($|N| = 2$)

#### Compute similarity weights:

$s_{1,3} = .658$, $s_{1,6} = .768$ (we compute $s_{1,2}$, $s_{1,4}$, $s_{1,5}$ too; let's assume those are smaller)

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**sim(1,m)**

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|   |   |   |   |   |   |   |   |   |     |     |     |

**.658**

**.768**

---

SLIDES ADAPTED FROM JURE LESKOVEC
### Item-Item CF ($|N|=2$)

Approximate rating with weighted mean

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**Sim (1,m)**

- 1.000
- .658
- 0.768

**Predict by taking weighted average:**

$$r_{1,5} = \frac{\sum_{j \in N(i; x)} s_{ij} r_{jx}}{\sum s_{ij}} = \frac{(0.658 \times 2 + 0.768 \times 3)}{(0.658 + 0.768)} = 2.54$$

Slides adapted from Jure Leskovec
Item-Item vs. User-User

- **In practice, item-item often works better than user-user**
- **Why?** Items are simpler, users have multiple tastes
  - (People are more complex than objects)
Simplified item-item for our homework

First, assume you've converted all the values to

+1 (like), 0 (no rating), -1 (dislike)

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users
Simplified item-item for our homework

First, assume you've converted all the values to

+1  (like),
0   (no rating)
−1  (dislike)

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movies
Simplified item-item for our tiny PA6 dataset

Assume you've **binarized**, i.e. converted all the values to
- +1 (like), 0 (no rating) -1 (dislike)

For this binary case, some tricks that the TAs recommend:
- Don't mean-center users, just keep the raw +1,0,-1
- Don't normalize (i.e. don't divide the product by the sum)
- i.e., instead of this:
  $$r_{xi} = \frac{\sum_{j \in N(i;x)} S_{ij} \ r_{xj}}{\sum_{j \in N(i;x)} S_{ij}}$$

- Just do this:
  $$r_{xi} = \sum_{j \in N(i;x)} S_{ij} \ r_{xj}$$

- Don't use mean-centered item-overlap cosine to compute $s_{ij}$
- Just use cosine
Simplified item-item for our tiny PA6 dataset

1. binarize, i.e. convert all values to
   - +1 (like), 0 (no rating) −1 (dislike)

2. The user x gives you (say) ratings for 2 movies m1 and m2

3. For each movie i in the dataset
   - $r_{xi} = \sum_{j \in (m1,m2)} s_{ij} r_{xj}$
   - Where $s_{ij}$... cosine between vectors for movies i and j

4. Recommend the movie i with max $r_{xi}$
Pros/Cons of Collaborative Filtering

+ **Works for any kind of item**
  - No feature selection needed

- **Cold Start:**
  - Need enough users in the system to find a match

- **Sparsity:**
  - The user/ratings matrix is sparse
  - Hard to find users that have rated the same items

- **First rater:**
  - Cannot recommend an item that has not been previously rated
  - New items, Esoteric items

- **Popularity bias:**
  - Cannot recommend items to someone with unique taste
  - Tends to recommend popular items
Hybrid Methods

Implement two or more different recommenders and combine predictions
- Perhaps using a linear model

Add content-based methods to collaborative filtering
- Item profiles for new item problem
- Demographics to deal with new user problem
Evaluation

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## Evaluation

The image depicts a matrix used in evaluating data sets. The matrix is labeled with the dimensions of users and movies.

### Users and Movies

<table>
<thead>
<tr>
<th>Users</th>
<th>Movies</th>
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### Test Data Set

The test data set is highlighted in blue and is located on the right side of the matrix.
Evaluating Predictions

Compare predictions with known ratings

- **Root-mean-square error (RMSE)**

  \[
  \sqrt{\frac{\sum_{xi}(r_{xi} - r^*_{xi})^2}{N}}
  \]

  where \(r_{xi}\) is predicted, \(r^*_{xi}\) is the true rating of \(x\) on \(i\)

- **Rank Correlation:**

  Spearman’s *correlation* between system’s and user’s complete rankings
Problems with Error Measures

Narrow focus on accuracy sometimes misses the point
- Prediction Diversity
- Prediction Context

In practice, we care only to predict high ratings:
- RMSE might penalize a method that does well for high ratings and badly for others
There’s No Data like More Data

Leverage all the data
  ◦ Simple methods on large data do best

Add more data
  ◦ e.g., add IMDB data on genres

More data beats better algorithms

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Summary on Recommendation Systems

• The Long Tail
• Content-based Systems
• Collaborative Filtering
State of the Art Example: YouTube

Covington, Adams, Sargin 2016
YouTube's Recommendation Algorithm

Covington, Adams, Sargin 2016. Deep Neural Networks for YouTube Recommendations

1. Represent each video as an embedding
2. Train neural net classifier (softmax over videos) to predict next video
   - Only include videos with many minutes watched
3. Input features:
   - User's watch history
   - Sequence of "video embeddings" for each video they watched
   - User's recent queries (word embeddings)
   - User's location
   - Date, popularity, virality of video
What could go wrong?: ethical and societal implications


Propaganda campaigns

- Russia’s Internet Research Agency (IRA)
  - attack on the United States 2013-2018
  - computational propaganda on YouTube, Facebook, Instagram, to misinform/polarize US voters.
  - Goal: induce African American, Mexican American voters to boycott elections
Ethical and societal implications: Filter bubbles

“I realized really fast that YouTube’s recommendation was putting people into filter bubbles,” Chaslot said. “There was no way out. If a person was into Flat Earth conspiracies, it was bad for watch-time to recommend anti-Flat Earth videos, so it won’t even recommend them.”
“The question before us is the ethics of leading people down hateful rabbit holes full of misinformation and lies at scale just because it works to increase the time people spend on the site – and it does work”
◦— Zeynep Tufekci,
Open research questions

What would algorithms look like that could recommend but also include these social costs?