1. Write regular expressions for the following languages over the alphabet \( \Sigma = \{0, 1\} \):

(a) The set of all strings in which no two 1’s appear together.

\[ 0^*(10^+)^*1 ? \]

(b) The set of all strings representing a binary number which is a multiple of \( 8_{10} \) (8 in base-10).

\[ (1(0|1)^*000)|0 \]

(c) The set of all strings representing a binary number which is greater than \( 8_{10} \) (8 in base-10).

\[ 1((0|1)^*(0|1)(0|1)(0|1)(0|1)|1(0|1)(0|1)|(0|1)(0|1)|(0|1)(0|1)) \]
2. Draw DFA's for each of the languages from question 1.

(a) The set of all strings in which no two 1’s appear together.

(b) The set of all strings representing a binary number which is a multiple of $8_{10}$ ($8$ in base-$10$).

(c) The set of all strings representing a binary number which is greater than $8_{10}$ ($8$ in base-$10$).
3. Using the techniques covered in class, transform the following NFAs with $\epsilon$-transitions over the given alphabet $\Sigma$ into DFAs. Note that a DFA must have a transition defined for every state and symbol pair, whereas a NFA need not. You must take this fact into account for your transformations. Hint: Is there a subset of states the NFA transitions to when fed a symbol for which the set of current states has no explicit transition?

(a) Original NFA, $\Sigma = \{a, b, c\}$:

DFA:

(b) Original NFA, $\Sigma = \{a, b, c\}$:

DFA:

(c) Original NFA, $\Sigma = \{a, b\}$:
DFA:
4. Draw the NFA for the set of strings over the alphabet $\Sigma = \{a, b\}$ which contain either a pair of b’s separated by exactly $2n + 2$ consecutive a’s (for some $n \geq 1$), or a pair of a’s separated by exactly $2m + 1$ consecutive b’s (for some $m \geq 1$). The NFA should have a single final state. You can use $\epsilon$-transitions.

Examples of strings that should be accepted by this NFA: aaabbba, baaaaba, ababaaaaab. Examples of strings that should not be accepted: abbbba, ababaaab, aba.
5. Consider the following tokens and their associated regular expressions, given as a flex scanner specification:

```%
(01|10) printf("snake")
0(01)*1 printf("badger")
(1010*1|0101*0) printf("mushroom")
%
```

Give an input to this scanner such that the output string is (`badger^11mushroom^2^4 snake^3`, where `A^i` denotes `A` repeated `i` times. (And, of course, the parentheses are not part of the output.) You may use similar shorthand notation in your answer.

```((0011)^110100^2^4)011001```
6. Recall from the lecture that, when using regular expressions to scan an input, we resolve conflicts by taking the largest possible match at any point. That is, if we have the following `flex` scanner specification:

```flex
%%
do { return T_Do; }
[A-Za-z_][A-Za-z0-9_]* { return T_Identifier; }
```

and we see the input string “dot”, we will match the second rule and emit T_Identifier for the whole string, not T_Do.

However, it is possible to have a set of regular expressions for which we can tokenize a particular string, but for which taking the largest possible match will fail to break the input into tokens. Give an example of a set of regular expressions and an input string such that: a) the string can be broken into substrings, where each substring matches one of the regular expressions, b) our usual lexer algorithm, taking the largest match at every step, will fail to break the string in a way in which each piece matches one of the regular expressions. Explain how the string can be tokenized and why taking the largest match won’t work in this case.

**Answer:** Consider the following scanner:

```flex
%%
a { return A; }
aba { return B; }
bab { return C; }
```

and the string “abab”. This can be broken into ‘a’, followed by ‘bab’. However, the largest possible match strategy will first consume the begining of the string as ‘aba’, then stop when it finds that the remainder of the input is just ‘b’, which can’t be matched to any token.