Please read all instructions (including these) carefully.

There are 5 questions on the exam, all with multiple parts.

This exam is designed to take 2 hours, but you have the full 3 hours to complete the exam.

The exam is open note. You may use laptops, phones and e-readers to read electronic notes, but not for computation or access to the internet for any reason.

Please write your answers in the space provided on the exam, and clearly mark your solutions. Do not write on the back of exam pages or other pages.

Solutions will be graded on correctness and clarity. Each problem has a relatively simple and straightforward solution. You may get as few as 0 points for a question if your solution is far more complicated than necessary. Partial solutions will be graded for partial credit.

NAME: __________________________________________

In accordance with both the letter and spirit of the Honor Code, I have neither given nor received assistance on this examination.

SIGNATURE: _______________________________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>Max points</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td></td>
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<tr>
<td>4</td>
<td>20</td>
<td></td>
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<tr>
<td>5</td>
<td>20</td>
<td></td>
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<tr>
<td>TOTAL</td>
<td>120</td>
<td></td>
</tr>
</tbody>
</table>
1. **Language Design** (30 points)

Consider extending COOL with exceptions. There are two new expressions. The first throws an exception:

```
throw e
```

The expression `e` must evaluate to a string, and that string is propagated as an exception value.

Except for the try-catch expression described below, for any other kind of expression `e`, if a subexpression of `e` throws an exception then `e` throws that same exception. This is described further below.

Exception values can be caught using a try-catch statement:

```
try e1 catch x in e2
```

In this statement, expression `e1` is evaluated first. If it completes normally with a value `v` (that is, without throwing an exception), then the whole try-catch expression completes normally with value `v`. Otherwise, if `e1` throws an uncaught exception, the variable `x` is bound to the exception string, and `e2` is evaluated. As examples, the expression `try 1 catch x in 2` evaluates to `Int(1)`, and `try (throw "a") catch x in x` evaluates to the object `String(1,a)`.

To give operational semantics to exceptions, we need to give the operational semantics for the two new statements. However, we also need to define the exception propagation behavior for existing constructs in the language. We introduce `Exc(m)` (where `m` is a string object) as a new kind of value that expressions can evaluate to—i.e., expressions can evaluate to objects (as usual) or exceptions. The behavior of exceptions is different from the behavior of objects. For example, consider an arithmetic operation `e1 + e2`: If the first operand `e1` evaluates to `Exc(m)`, then the whole expression evaluates to `Exc(m)`. If `e1` completes normally with a value `v` and `e2` evaluates to `Exc(m)`, then the whole expression evaluates to `Exc(m)`. Otherwise (i.e., both expressions evaluate normally), the usual rule from the COOL manual applies. Formally, the rules for the propagation of exceptions in addition expressions are:

\[
\begin{align*}
& \text{so}, S_1, E \vdash e_1 \mapsto \text{Exc}(m), S_2 \\
& \text{so}, S_1, E \vdash e_1 + e_2 \mapsto \text{Exc}(m), S_2 \\
& \text{so}, S_1, E \vdash e_1 \mapsto \text{Int}(i_1), S_2 \\
& \text{so}, S_2, E \vdash e_2 \mapsto \text{Exc}(m), S_3 \\
& \text{so}, S_1, E \vdash e_1 + e_2 \mapsto \text{Exc}(m), S_3
\end{align*}
\]
(a) Give the operational semantics for throw and try-catch expressions. Make sure you cover both exception propagation as well as regular executions.

\[
\begin{align*}
so, S_1, E &\vdash e \mapsto \text{Exc}(m), S_2 \\
so, S_1, E &\vdash \text{throw } e \mapsto \text{Exc}(m), S_2 \\
so, S_1, E &\vdash e \mapsto v, S_2 \\
so, S_1, E &\vdash \text{throw } e \mapsto \text{Exc}(v), S_2 \\
so, S_1, E &\vdash e_1 \mapsto v, S_2 \\
so, S_1, E &\vdash \text{try } e_1 \text{ catch } x \text{ in } e_2 \mapsto v, S_2 \\
so, S_1, E &\vdash e_1 \mapsto \text{Exc}(m), S_2 \\
l &= \text{newloc}(S_2) \\
so, S_2[m/l], E[l/x] &\vdash e_2 \mapsto v, S_3 \\
so, S_1, E &\vdash \text{try } e_1 \text{ catch } x \text{ in } e_2 \mapsto \text{Exc}(m), S_3
\end{align*}
\]
(b) Give sound type-checking rules for throw and try-catch expressions. Hint: Make sure that your type-checking rules can correctly type-check the following program (you don’t have to show the derivation):

\[
\text{if } 1 < 2 \text{ then throw "error" else } 1 + 2
\]

\[
\frac{O, M, C \vdash e : \text{String}}{O, M, C \vdash \text{throw } e : T}
\]

\[
\frac{O, M, C \vdash e_1 : T_1 \quad O[\text{String}/x], M, C \vdash e_2 : T_2}{O, M, C \vdash \text{try } e_1 \text{ catch } x \text{ in } e_2 : \text{lub}(T_1, T_2)}
\]
2. **Garbage Collection** (20 points)

Assume we are programming in a safe language like Java or python. Consider the following fragment of code:

```java
class Foo { Int i; }

for (Int j = 0; j < 1000000000; j++) {
    // Allocate memory
    Foo data = new Foo;   // Allocates a Foo object on the heap.

    // Generate random number
    data.i = random_number();

    // Print random number
    print("Number is %d\n", data.i);
}
```

(a) Which GC Algorithm (Reference Counting, Mark and Sweep, Stop and Copy) is best suited (gives the fastest start-to-finish execution times) for this code? Justify your choice.

Stop-and-Copy, because the GC cost is proportional to the number of live objects, and there is only 1 live object in this example.

A possible alternative explanation is that reference counting will be fastest because of improved locality.
(b) Assume the same program is run on a machine with sufficient physical memory to hold \( n \) \texttt{Foo} objects. What is the maximum number of objects that can be allocated on the heap at any single time by an implementation using a Mark and Sweep collector? Justify your answer.

\( n \); Mark and Sweep can use all available memory and will not trigger a collection until it is full.

(c) What is the maximum number of objects that can be allocated on the heap at any single time by an implementation using a Reference Counting collector? Justify your answer.

1 or 2; each object becomes dead as soon as a new one is allocated.

Depending on how the student interprets the order in which reference counting operations are done, there will be a most 1 or 2 objects allocated.

(d) What is the maximum number of objects that can be allocated on the heap at any single time by an implementation using a Stop and Copy collector? Justify your answer.

\( n/2 \); Stop and Copy can use only half of the heap to hold objects and will trigger a collection when that half is full.
3. **Global Optimization & Register Allocation** (30 points)

(a) You are given the control-flow graph for a function which uses variables a to g, with a and b being the arguments and g the return value. The program statements have been removed from this graph, and replaced by empty boxes. Below/above every such box we give the set of variables that are live right after/before the corresponding statement.

Fill in program statements in the boxes, so that the given live sets are consistent with the code. Program statements can take one of the following forms: \( x = 1 \), \( x = y \) or \( x = y + z \) (where \( x, y \) and \( z \) can be any one of the variables used by the function).

If more than one statement would work in some position, pick the simplest one (prefer \( x = 1 \) over \( x = y \), and \( x = y \) over \( x = y + z \)).
(b) Give the register interference graph for the above code, by filling in the adjacency matrix below (mark \( X \) on the cell at row \( v_1 \) and column \( v_2 \) iff there’s an edge between nodes \( v_1 \) and \( v_2 \)). You may also want to draw the graph to help with the next questions, but we won’t grade your drawing.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>c</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>e</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Perform register allocation for the above code, using the graph coloring algorithm presented in class. Fill in (and use!) the minimum number of registers \( k \) for which you can color the graph without spilling. If at any point there’s a choice between multiple variables to push on the stack, pick the one which comes first alphabetically. Show the state of the stack right after you’ve removed the last node from the graph:

\[
\begin{align*}
k &= 3 \\
\text{top (pushed last)} &\quad \text{bottom (pushed first)}
\end{align*}
\]

(d) Show your final coloring on the following table, by listing the variables assigned to each register. Use only the first \( k \) registers, where \( k \) is the number of colors you used.

<table>
<thead>
<tr>
<th>Register ( r_1 )</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 )</td>
<td>g, e, a</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>f, b</td>
</tr>
<tr>
<td>( r_3 )</td>
<td>d, c</td>
</tr>
<tr>
<td>( r_4 )</td>
<td></td>
</tr>
<tr>
<td>( r_5 )</td>
<td></td>
</tr>
<tr>
<td>( r_6 )</td>
<td></td>
</tr>
<tr>
<td>( r_7 )</td>
<td></td>
</tr>
</tbody>
</table>
4. **Local Optimization** (20 points)

Consider three address programs that consist only of the following types of operations:

- $x = y$ for variable $x$ and variable or constant $y$
- $x = y + z$ for variable $x$ and variables or constants $y$ and $z$.

Answer each of the following questions. For each problem, state which variable(s) are live on exit. For full credit, your solution should be as short as possible.

(a) Give a program where performing local optimizations in the order copy propagation, constant folding and then dead code elimination results in a final program with fewer instructions than performing first constant folding, copy propagation and then dead code elimination.

(This question was removed from the exam)

(b) Give a program for which the opposite is true: performing constant folding, copy propagation and then dead code elimination is better than performing copy propagation, constant folding and then dead code elimination.

\[
y = 1 + 1
\]
\[
x = y + z
\]

(x live out)

Minimum program is of length 2.
(c) Give a program for which both of copy propagation and constant folding must be done at least twice each before performing dead code elimination to achieve the best results.

\[
x = 1 + 1 \\
y = x + 1 \\
z = y + 1
\]

(z live out)

Minimum program is of length 3.
5. **Activation Records** (20 points)

Consider the following COOL class and method definitions:

```coolkotlin
class Main {
    apple(a: Int, b: Int, c: Bool): Int {
        if c then
            if a - b = 0 then
                0
            else
                a + b
            fi
        else
            { a <- a * b; a + b; fi
        }
    }

    banana(x: Int, y: Int): String {
        if x < cherry(y) then
            "Peel"
        else
            "Split"
        fi
    }

    cherry(z: Int): Int {
        let p: Int <- 2 in {
            apple(z, p, false);
        }
    }

    main(): Object {
        let q: Int <- 3, r: Int <- 4 in {
            banana(q, r);
        }
    }
}
```

A compiler with an unknown runtime management strategy is used to compile this code—in particular, the strategy used may not be exactly one that you have seen before. A partial listing of the stack at a certain point in the execution of the compiled program is given below. There are three kinds of entries on the stack: a return address, a stack address, and a reference to an object in the heap (e.g., \texttt{Int(3)}).

(a) Fill in the missing entries in the table.

<table>
<thead>
<tr>
<th>Address</th>
<th>Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>\textit{OS Frame Pointer}</td>
</tr>
<tr>
<td>1004</td>
<td>\textit{OS return address}</td>
</tr>
<tr>
<td>1008</td>
<td>\texttt{Int(3)}</td>
</tr>
<tr>
<td>1012</td>
<td>\texttt{Int(4)}</td>
</tr>
<tr>
<td>1016</td>
<td>1000</td>
</tr>
<tr>
<td>1020</td>
<td>\texttt{Int(3)}</td>
</tr>
<tr>
<td>1024</td>
<td>\texttt{Int(4)}</td>
</tr>
<tr>
<td>1028</td>
<td>\textit{return address of banana(q,r)}</td>
</tr>
<tr>
<td>1032</td>
<td>\texttt{Int(3)}</td>
</tr>
<tr>
<td>1036</td>
<td>1016</td>
</tr>
<tr>
<td>1040</td>
<td>\texttt{Int(4)}</td>
</tr>
<tr>
<td>1044</td>
<td>\texttt{cherry(y)}</td>
</tr>
<tr>
<td>1048</td>
<td>\texttt{Int(2)}</td>
</tr>
<tr>
<td>1052</td>
<td>1036</td>
</tr>
<tr>
<td>1056</td>
<td>\texttt{Int(4)}</td>
</tr>
<tr>
<td>1060</td>
<td>\texttt{Int(2)}</td>
</tr>
<tr>
<td>1064</td>
<td>\texttt{Bool(false)}</td>
</tr>
<tr>
<td>1068</td>
<td>\textit{return address of apple(z,p,false)}</td>
</tr>
</tbody>
</table>

(b) On the previous page, clearly indicate the latest possible point in the execution of the program that could have this stack contents. Just before the return of the call to apple.