Recursive Ray Tracing
Color Accumulation at an Intersection Point

- A shadow ray is cast to each light source, and the total contribution from all light sources is accumulated

\[
(k_R, k_G, k_B) \left( \sum_{\text{lights}} V_{\text{light}} I_{\text{light}} \max(0, \cos \theta_{\text{light}}) + I_{\text{ambient}} \prod_{\text{lights}} (1 - V_{\text{light}}) \right)
\]

- \(V_{\text{light}} = 1\) for visible light sources, and \(V_{\text{light}} = 0\) for occluded light sources
- \(I_{\text{ambient}}\) is added to fully shadowed regions where \(\prod_{\text{lights}} (1 - V_{\text{light}}) \neq 0\)
- Can write this in compact form as: \((k_R, k_G, k_B)(L_{\text{diffuse}} + L_{\text{ambient}})\)

- Mirror-like reflective properties can also contribute to the color at an intersection point
- Transparency allows other objects to show through a surface, and thus those objects also contribute color to an intersection point
- In summary: \((k_R, k_G, k_B)(L_{\text{diffuse}} + L_{\text{ambient}}) + L_{\text{reflect}} + L_{\text{transmit}}\)
Additional Light

• Reflection and Transmission add light to a pixel making it brighter
• Thus, scaling coefficients are added in front of every lighting contribution
  \[(k_R, k_G, k_B) (k_a L_{diffuse} + k_a L_{ambient}) + k_r L_{reflect} + k_t L_{transmit}\]
• Typically, coefficients are adjusted relative to each other to get the desired “look”; then, all the coefficients are scaled together to get the appropriate overall brightness/darkness

less reflection (darker)  more reflection (brighter)
Recursion

- \(L_{\text{reflect}}\) and \(L_{\text{transmit}}\) are treated in exactly the same way as pixel color values are treated.
- A ray is constructed for the reflection direction and intersected with scene geometry in exactly the same way as was done for camera rays through pixels (the result is stored in \(L_{\text{reflect}}\)).
- A ray is constructed for the transmission direction and intersected with scene geometry in exactly the same way as was done for camera rays through pixels (the result is stored in \(L_{\text{transmit}}\)).
- Once both \(L_{\text{reflect}}\) and \(L_{\text{transmit}}\) have been computed, the pixel color can be updated.
- \(L_{\text{reflect}}\) and \(L_{\text{transmit}}\) depend on the color computed from whatever object geometry was intersected by their corresponding rays.
- Those intersections will have colors of their own, computed via shadow rays and both diffuse and ambient shading.
- In addition, the color of those intersections can depend on subsequent reflection and transmission, meaning that more rays need to be spawned before a color can be computed.
Termination

• Theoretically, if every subsequently intersected point continued to depend on reflection and/or transmission, one would spawn rays indefinitely (never terminating)
• Eventually one hits the recursion limit (depending on hardware) that prevents stack overflow
• If \( k_d \) and/or \( k_a \) are nonzero frequently enough, then the reflected/transmitted contribution diminishes over time allowing one to terminate the process while adding in a final arbitrary value for \( L_{\text{reflect}} \) and/or \( L_{\text{transmit}} \) without tracing the associated ray(s)

• In some cases (mirrors, bubbles, etc.), there is little to no diffuse/ambient lighting and nearly 100% of the lighting is recursively sought after via reflected/transmitted rays
• In those cases, any arbitrary value will show up as the final pixel color
• This can look terrible, e.g. a black pixel in a mist of bubbles
• One often struggles to choose realistic termination colors, examples include the color of the sky, or background, etc.
Ray Tree Example
Recursion allows for stunning imagery with minimal code, as demonstrated by these 1337 characters printed on the back of a business card.

http://fabiensanglard.net/rayTracing_back_of_business_card/
Reflected Ray

- Given an incoming ray $R(t) = A + Dt$ with direction $D$, and local (outward) unit normal to the geometry $N$, the angle of incidence is defined via $D \cdot N = -\|D\|_2 \cos \theta_i$
- For mirror reflection, the incoming/outgoing rays make the same angle with $N$, i.e. $\theta_o = \theta_i$, and those rays and the normal are all coplanar.
- Thus, the reflected ray direction is $D_{reflect} = D - 2(D \cdot N)N$
- Then, the reflected ray is $R_{reflect}(t) = R(t_{int}) + D_{reflect}t$
Spurious Self-Occlusion

- Numerical precision issues can cause the reflected ray to incorrectly re-intersect the same object near $R(t_{int})$, similar to the issues for shadow rays.
- Once again, one can use $t \in (\varepsilon, \infty)$ for some $\varepsilon > 0$ large enough to avoid the ray incorrectly re-intersecting the same object.
- However, grazing rays near the object’s silhouette may still incorrectly re-intersect the object.
Spurious Self-Occlusion

- Alternatively, perturb the starting point of the reflected ray to be slightly away from the object, e.g. from $R(t_{int})$ to $R(t_{int}) + \epsilon N$
- Perturbed reflected rays do not have the ray direction modified (unlike shadow rays)
- The new reflected ray is $R_{reflect}(t) = R(t_{int}) + \epsilon N + D_{reflect} t$ with $t \in (0, \infty)$
- This works well, but one needs to be careful that the new starting point does not fall inside (or too close to) nearby geometry
Reflections
Transmission

- The relationship between the angle of incidence and angle of refraction/transmission for light passing through a boundary between two different isotropic media is given by **Snell’s Law**

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \quad \text{where} \quad \theta_1 \text{ and } \theta_2 \text{ are the incoming/outgoing angles, } v_1 \text{ and } v_2 \text{ are the phase velocities and } n_1 \text{ and } n_2 \text{ are the indices of refraction}
\]
Transmitted Ray

- $D$ is the (unit) incoming ray direction, $N$ is the unit normal, and $T$ is the unit tangent in the plane of $D$ and $N$, so that $D + N\cos\theta_1 + T\sin\theta_1 = 0$
- Let $D_{\text{transmit}}$ be the (unit) transmitted ray direction, then $D_{\text{transmit}} + T\sin\theta_2 + N\cos\theta_2 = 0$
Transmitted Ray

- So $D_{transmit} = -T \sin \theta_2 - N \cos \theta_2 = (D + N \cos \theta_1) \frac{\sin \theta_2}{\sin \theta_1} - N \sqrt{1 - \sin^2 \theta_2}$
- Using Snell’s Law, $D_{transmit} = (D + N \cos \theta_1) \frac{n_1}{n_2} - N \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_1\right)^2}$
- $D_{transmit} = D \frac{n_1}{n_2} + N \left(\frac{n_1}{n_2} \cos \theta_1 - \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2} (1 - \cos^2 \theta_1)\right)$
- Using $\cos \theta_1 = -D \cdot N$ gives $D_{transmit} = D \frac{n_1}{n_2} - N \left(\frac{n_1}{n_2} D \cdot N + \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2} (1 - (D \cdot N)^2)\right)$
- If the term in the square root is negative, there is no refracted/transmitted ray, and all the light is reflected (total internal reflection)
- This equation works regardless of whether $n_1$ or $n_2$ is bigger
- Add $\varepsilon$ to avoid self intersection, or offset in the negative normal direction (respecting collisions, other geometry, etc.)
Total Internal Reflection

• When light goes from a higher index of refraction to lower index of refraction, no light is refracted/transmitted when the incident angle exceeds a critical angle

• In that case, all the light reflects

\[ \theta_2 < \theta_{2,\text{max}}, \text{ both reflection and refraction/transmission} \] 

\[ \theta_2 > \theta_{2,\text{max}}, \text{ only reflection} \]
Critical Angle

- When $\theta_1 = \frac{\pi}{2}$, which is the maximum angle for transmission, $\sin \frac{\pi}{2} = 1$ and Snell’s Law becomes $\frac{1}{\sin \theta_2} = \frac{n_2}{n_1}$ or $\theta_2 = \arcsin \frac{n_1}{n_2}$
- So for $n_1 < n_2$, the critical angle is $\theta_2 = \arcsin \frac{n_1}{n_2}$
Total Internal Reflection

- Responsible for many impressive visuals in both glass and water
Snell’s Window

- Yes, fish can see you on the shore!
Snell’s Window
Reflection vs. Transmission

- The amount of transmission vs. reflection decreases as the viewing angle goes from perpendicular (overhead) to parallel (grazing)

Perpendicular (overhead) view: high transmission, low reflection

Parallel (grazing) view: high reflection, low transmission
Reflection vs. Transmission

• For opaque objects (without transmission) reflection still behaves similarly

As the viewing angle changes from overhead to a more grazing angle (from left to right), the amount of reflection off of the table increases (and one can better see the book’s reflection)
Reflection vs. Transmission

- For curved objects, the viewing angle varies (from perpendicular to parallel) across the object
- Capturing the right amounts of reflection vs. transmission is important for realism

Correct reflection vs. transmission based on viewing angle

Incorrect reflection vs. transmission with no dependence on viewing angle
Conductors vs. Dielectrics

- Conductors of electricity (e.g. metals) mostly reflect light, having low absorption and no transmission.
- The amount reflected doesn’t change much with viewing angle (e.g. aluminum varies from 90% to 100% as the viewing angle changes from overhead to grazing).
- Thus, can approximate $k_r$ independent of viewing direction.
- In contrast, dielectrics (which don’t conduct electricity, e.g. glass) have significant variance in reflection vs. transmission with viewing angle.
Fresnel Equations

- The proportion of reflection gradually increases as the viewing angle goes from perpendicular (coincident with the normal) to parallel (orthogonal to the normal)

Light entering a denser material (e.g. from air into water)

Light leaving a denser material (e.g. exiting water into air)
Fresnel Equations

- Light is polarized into 2 parts based on whether the plane containing the incident, reflected, refracted rays is parallel (p-polarized) or perpendicular (s-polarized) to the electric field.
- The Fresnel equations give the fraction of light reflected as:

\[ R_p = \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_i} \right|^2 \quad \text{and} \quad R_s = \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_i} \right|^2 \]

- For unpolarized light (a typical assumption in ray tracing), the overall reflection coefficients is assumed to be:

\[ R = \frac{R_p + R_s}{2} \]

- Transmission (if applicable) is typically calculated as the remaining light: either \( T_p = 1 - R_p \) and \( T_s = 1 - R_s \) for polarized light or \( T = 1 - R \) for unpolarized light.
Schlick’s Approximation

- Approximate reflection (for both conductors and dielectrics) via:

\[ R(\theta_i) = R_0 + (1 - R_0)(1 - \cos \theta_i)^5 \]

\[ R_0 = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2 \]

- In the figure, solid lines are Fresnel and dotted lines are Schlick
Attenuation

• Light is attenuated as it travels through media (as light is absorbed and scattered)
• The attenuation effect is stronger over longer distances
• Different colors are attenuated at different rates
• For example:
  • Shallow water is clear (almost no attenuation)
  • Deeper water attenuates all the red light and looks bluish-green
  • Even deeper water attenuates all the green light too, and looks blue
  • Eventually all the light attenuates, and the color ranges from blackish-blue to black
Beer’s Law

- If the media is homogeneous, attenuation along a ray can be described by Beer’s Law
- Light with intensity $I$ is attenuated over a distance $x$ via the ordinary differential equation (ODE): $\frac{dl}{dx} = -cl$ where $c$ is the coefficient of attenuation
- The exact solution to this ODE is $I(x) = I_o e^{-cx}$ where $I_o$ is the original unattenuated amount of light
Beer’s Law

• The color of a transparent object is described by three independent attenuation coefficients, one for each color channel (i.e. $c_R, c_G, c_B$)
• Shadow rays are also attenuated
Atmospheric Refraction

• Light continuously bends in a curved path when it passes through varying temperature gases, such as the atmosphere
• This is due to variations in density, and thus variations in the refractive index
Inferior Mirage
Superior Mirage
Atmospheric Refraction

• Bend the rays as they go through varying air densities
• Change the light direction between every interval in the vertical direction (left) or along the ray direction (right)
Gravity can bend light too!

http://www.wired.com/2014/10/astrophysics-interstellar-black-hole/
Iridescence

• Surface can gradually change color as the viewing angle or the lighting change
Iridescence

- Various light waves are emitted in the same direction giving constructive and destructive interference