Optics
How Light “Works”...

When light hits a material, three types of interactions can occur:
- Reflection
- Absorption
- Transmission

From conservation of energy:
light incident at surface = light reflected + light absorbed + light transmitted

**Opaque object** - the majority of incident light is either reflected or absorbed (transmitted light ≈ 0)

**Translucent object** - significant light transmission
Modeling How Light “Works”

- The **BRDF** (Bidirectional Reflectance Distribution Function) models how much light is reflected when light makes contact with a certain material.
- The **BTDF** (Bidirectional Transmittance Distribution Function) models how much light is transmitted.
- The **BSSRDF** (Bidirectional Surface Scattering Reflectance Distribution Function) is a combined model for both reflection and transmission.
Opaque (BRDF) vs. Translucent (BSSRDF)
Paint (BRDF) vs. Milk (BSSRDF)
Reflection Only (BRDF)
Subsurface Scattering (BSSRDF)

Modeled by Stephen Stahlberg
BRDF Dependencies

BRDF(\(\lambda, \omega_i, \omega_o, u, v\))

How much light is reflected depends on:

- The wavelength of the light \(\lambda\)
- The 2D incoming light direction \(\omega_i = (\theta_i, \phi_i)\)
- The 2D outgoing reflected direction \(\omega_o = (\theta_o, \phi_o)\)
- Spatial position on the surface \((u, v)\)
  - Many real world materials (e.g. wood) consist of sub-materials with spatially varying density and stochastic characteristics
BRDF Simplifying Assumptions

Spatial invariance

- Modulating the BRDF result by a texture is usually a good enough approximation to spatial variance.

Three channels: R, G, B

- The variation in the BRDF throughout all visible wavelengths is typically approximated by three BRDFs.

Thus we write 3 separate BRDF’s (one Red, one Green, and one Blue), each of the form:

$$\text{BRDF}(\omega_i, \omega_o)$$

This is still a function of 4 variables, i.e. a 4D function!
Measuring/Approximating BRDFs

The 4D BRDF data can be measured using a gonioreflectometer to obtain a 4D table of values.

Often, people resort to simple analytical models of BRDFs:

- **Blinn-Phong Model** - simplest and general purpose (plastic)
- **Cook-Torrance Model** - better specular (metal)
- **Ward Model** - anisotropic (brushed metal, hair)
- **Oren-Nayar Model** - non-Lambertian (concrete, plaster, the moon)
Incoming Light
Incoming Light

- Light doesn’t just come from light sources
- Light also comes from all visible objects in the world
  - We can see the tree’s reflection on the car because light from the tree (with the color and brightness of the tree) impacts the car and is reflected towards the camera
Incoming Light

- Using incoming light from all directions makes an image appear more realistic, and also captures various physical phenomena such as color bleeding.
Incoming Light

using only the light source

using light from all directions
Measuring Incoming Light

- Place and photograph a small chrome sphere (light probe) to collect and record the intensities of the light incoming from all directions.
Measuring Incoming Light

- Technically this should be done with a very tiny sphere and at every point on the object of interest.
- But one often approximates with a single finite size sphere for each object of interest.
- The incoming light information and a BRDF can be used to render a new synthetic object in the original scene.
Summary

- Given a point on an object:
  - Light from every possible incoming direction $\omega_i$ hits that point
  - And for each of these incoming directions $\omega_i$, light is reflected outwards in every direction $\omega_o$
  - The BRDF tells us how much light is reflected in each of the outgoing directions $\omega_o$ given an incoming direction $\omega_i$

- Since light is reflected in all the outgoing directions $\omega_o$, we can all view the same point on the same object
- But it looks different to all of us, since we all see different light
- To render a synthetic scene, we need to figure out what light each pixel of the camera’s film sees
The Lighting Equation

The amount of light reflected in a single outgoing direction is the integral of the amount of light reflected in that direction due to light from every incoming direction:

\[
L_o = \sum_{i \in \text{in}} L_{o \text{ due to } i} (\omega_i, \omega_o)
\]

- The BRDF tells us \( L_{o \text{ due to } i} (\omega_i, \omega_o) \)

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Incoming light

Outgoing light

Surface
The Lighting Equation

For each pixel, integrate the BRDF across all incoming directions for every point in the projected area.

\[
L_o = \int_{i \in \text{in}} BRDF(\omega_i, \omega_o) dE_i(\omega_i)
\]

All incoming directions
The Lighting Equation

For each pixel, integrate the BRDF across all incoming directions for every point in the projected area.

\[ L_o = \int_{\omega} \int_{E_i}^{\infty} B_{i,\omega} dE_i \]

Vanishingly small surface element within the projected area.
Math ☹
Solid Angle

A solid angle is a two-dimensional angle in 3D defined by a point and a surface patch, measured by a dimensionless unit called a steradian (sr)

Angle: \( \theta = \frac{l_{\text{arc}}}{r} \) (a circle has \( 2\pi \) radians) \( C = 2\pi r \)

Solid angle: \( \omega = \frac{A_{\text{on sphere}}}{r^2} \) (a sphere has \( 4\pi \) steradians) \( A = 4\pi r^2 \)
Radiant Intensity of a Light

Power per unit solid angle: \[ I(\omega) = \frac{d\Phi}{d\omega} \]

where \(\Phi\) is the total power of the light.

Note: \(I\) is a function of \(\omega\) for an anisotropic light emission.

The relation between power and radiant intensity for an isotropic point light source is:

\[ \Phi = \int I \, d\omega = 4\pi I \]
In irradiance on a surface, the power per unit surface area is given by:

\[ E \equiv \frac{d\Phi}{dA} \]

Note how the irradiance decreases as you tilt the object, since less light hits it.
The relationship between solid angle and area is

\[ d\omega = \frac{dA \cos \theta}{r^2} \]

where \( dA \cos \theta \) is the area of the orthogonal cross section.
Summary

No energy can flow through a point; it must go through either a solid angle or an area (interchangeable).

Total light power per unit solid angle $\rightarrow$ Radiant Intensity
- This gives a measure of how strong a (point) light source is

Total light power per unit area $\rightarrow$ Irradiance
- This gives a measure of how much light is hitting a surface
- It varies based on the distance to the light and the angle of the surface (as per the last slide)
- The farther away and more tilted a surface area patch is, the smaller the solid angle of light that hits that surface area patch (as per the last slide)
More Math 😞
Area Lights
Area Lights

• For an area light, a certain amount of light power is emitted per unit area (not from a single point)
• This light goes in various directions, or solid angles

• One could approximate an area light by breaking it up into small area chunks, and for each small area chunk we emit light into each of the solid angle directions (i.e. radiant intensity per area chunk)
• For each direction there is a cosine term similar to irradiance (think of a flashlight)

• Radiance - radiant intensity per area chunk

\[
L = \frac{dI}{dA \cos \theta} \quad \left( = \frac{d^2 \Phi}{d\omega dA \cos \theta} = \frac{dE}{d\omega \cos \theta} \right)
\]
The BRDF relates the incoming light that hits a surface patch \( E_i \) to the outgoing light emitted from the surface patch acting as if it were an area light \( L_o \).

\[
\text{BRDF}(\omega_i, \omega_o) = \frac{\frac{d L_o(\omega_o)}{d E_i(\omega_i)}}
\]

We care about the outwards radiance of a surface patch in the direction \( \omega_o \) of our pixel, given an incoming irradiance from a direction \( \omega_i \) onto that surface patch. (A patch of an "area light" has radiant intensity in every outgoing direction, but we only care about the outgoing light in the direction of our pixel)
The Lighting Equation

Multiplying the BRDF by an incoming irradiance gives a resulting outgoing radiance

\[ L_{o\ due\ to\ i}(\omega_i,\omega_o) = BRDF(\omega_i,\omega_o)\ d\ E_i \]

For more complex lighting we will bounce light all around the scene, and it is tedious to convert between irradiance and radiance --- so we use \( E = L \cos \theta \) (from two slides earlier) to obtain:

\[ L_{o\ due\ to\ i}(\omega_i,\omega_o) = BRDF(\omega_i,\omega_o)L_i \cos \theta_i \]

Finally, the outgoing radiance considering the light coming from all incoming directions is

\[ L_o = \int_{\text{in}} BRDF(\omega_i,\omega_o)L_i \cos \theta_i \ d\omega_i \]

completing the derivation
Pixels
Pixel Color

- Irradiance measures the power per unit area hitting a pixel

\[ E = \int L_i \cos \theta_i \, d\omega_i \]  
(integrating an equation from 2 slides ago)

- Since the pixels are small, we approximate this by

\[ E_{\text{pixel}} \approx L_{\text{pixel,avg}} \cos \theta_{\text{pixel}} \omega_{\text{pixel}} \]

- If the film is small, we may also approximate

\[ \cos \theta_{\text{pixel}} \approx 1, \quad \omega_{\text{pixel}} \approx \frac{\omega_{\text{film}}}{\# \text{pixels}} \]

- Then, the radiance and the irradiance differ only by a spatially global constant (and we may store L instead of E, scaling later)

\[ E_{\text{pixel}} \approx \frac{\omega_{\text{film}}}{\# \text{pixels}} L_{\text{pixel,avg}} \]  
(so we can store L instead of E, and scale later)
Question 1

Summarize...