Reflection and Transmission
Recall: Ray Tracing

- Generate an image by backwards tracing the path of light through pixels on an image plane
- Simulate the interaction of light with objects
Recall: Basic Ray Tracer

• Trace a primary ray from the eye through each pixel and detect the first intersection point with the objects in the scene
• Trace secondary shadow rays from the intersection point towards light sources and sum the contributions from each (visible) light

• The lighting equation:

\[ L = \sum_{j \in \text{lights}} L^j_i (k_a + k_d (\hat{N} \cdot \omega^j_i)_+ + k_s (\hat{N} \cdot \hat{H}^j_i)_+) \]

sum all lights  ambient  diffuse  specular
Embarrassingly Parallel

• Can easily implement on a GPU
• Can easily use all the cores on a CPU
• Can easily use many machines across a network, or on the cloud

http://www.youtube.com/watch?v=-P28LKWzrI
Recursive Ray Tracing
Recursive Ray Tracing

• Light at a point may not just come directly from light sources:
  – Light can bounce off other objects (reflection)
  – Light can pass through objects (transparency/refraction)

• Trace more rays to look for more lighting information
  – Send secondary rays from the intersection point
  – Recursively compute the color for these secondary rays and sum them onto the primary ray

• Lighting equation becomes:

\[ L = \sum_{j\text{lights}} \begin{array}{c}
L_i^j(k_a + k_d(\hat{N} \cdot \omega_i^j)_+ + k_s(\hat{N} \cdot \hat{H}_i^j)_+^s) + k_r L_{reflected} + k_t L_{transmitted}
\end{array} \]

- standard direct illumination
- recursively computed reflection
- recursively computed refraction
Reflection/Transmission Adds Light

• Adding additional reflection/transmission terms adds more light to the object making it **brighter**
• Can offset this by dialing down the object color
• Can offset this by dialing down the reflection/transmission coefficients, $k_r/k_t$, but this can overly dim the reflections/transmissions

• Recommended approach: adjust the ratio between the object color and the reflected/transmitted light first to get the desired “look,” and then tune everything up and down to adjust brightness while maintaining this ratio
Ray Tree

- Each reflected or transmitted ray may spawn its own shadow, reflection, and refraction/transmission rays, which is a recursive process.
Maximum Depth

• If a ray tree is too deep (too many rays), it gets too costly/slow
  – N.B. be aware of the size of the recursion stack (which depends on the hardware) to avoid recursive stack overflow
• If each splitting in the tree has a direct lighting component, then rays deeper in the tree make a smaller and smaller contribution to the total light for a pixel
  – So one can truncate when the contribution is below a threshold

• In some cases (mirrors, etc.), there may not be a direct lighting component and 100% of the light at a split comes from reflected/transmitted rays
• In this case premature pruning will adversely affect the image
  – One typically carefully chooses a default color for the early termination point
  – Often this default color is the background color of the scene (used when a ray does not hit anything in the scene)
  – E.g., the background color could be blue if your scene is under the sky
  – If a default color is not set, simply truncating the rays behaves as if a black color was set, i.e. multiplying the discarded rays by 0
Requires Minimal Implementation

#include <stdlib.h>   // card > aek.ppm
#include <stdio.h>
#include <math.h>
typedef int i;typedef float f;struct v{
f x,y,z;v operator+(v r){return v(x+r.x,
y+r.y,z+r.z);}v operator*(f r){return
v(x*r,y*r,z*r);}f operator%(v r){return
x*x+y*y+z*z+r.x;};v& operator^(v r){return
v(y*r.z-z*r.y,z*r.x-x*r.z,x*r.
y-y*r.x);}v(f a,f b,f c){x=a;y=b;z=c;}v
operator!(){return*this*(1/sqrt(*this*%this));};i G[]={(247570,280596,280600,
249748,18578,18577,231184,16,16};f R(){return(f)rand()/(RAND_MAX);}v T(v o,v d,f
&t,v&n){t=le9;i m=0;f p=0.o.d.z;if(.01
Eight 381x258
720.0x540.0
http://fabiensanglard.net/rayTracing_back_of_business_card/
10
Reflection
Reflective Ray

- Given an incident ray $R(t) = A + tD$, the reflective ray $R_{ref}(t) = P + tF$ is computed as follows:
  - $P$ is the point where $R(t)$ intersects the surface
  - $F$ is the reflected vector of $D$ with respect to the surface normal $N$
  - The incident angle $\theta_i$ equals the reflection angle $\theta_o$

$$F = D - 2(D \cdot N)N$$
Spurious Self-Occlusion

- Add $\varepsilon$ to the starting point of a reflected ray to avoid re-intersection with the original surface.
- This often fails for grazing rays near the object's silhouette.
- Or offset in the normal direction from the surface.
  - Just like shadow rays, except here we preserve the direction of the reflected ray.
- Avoid placing the new starting point too close to or inside other nearby objects.

The perturbed point may still be inside the object.

offset in the ray direction

offset in the normal direction
Shading Reflections

• Shade recursively
  – **Treat reflected rays like primary rays from the camera**
  – Shade the reflected ray and return its color
    • Multiply by the reflection coefficient $k_r$
  – Add the resulting color to the shading at the original point
Question 1

List 10 objects that can make use of reflected rays
Transmission
Shading Transmission

- If the object is transparent, trace a transmitted ray
  - Treat transmitted rays like primary rays
    - Add $\varepsilon$ to avoid self intersection, or offset in the negative normal direction (respecting collisions, other geometry, etc.)
  - Shade the transmitted ray and return its color
  - Multiply by the refraction coefficient $k_t$
  - Add the resulting color to the shading at the original point
Snell’s Law

- The relationship between the angle of incidence and angle of refraction/transmission for light passing through a boundary between two different isotropic media is:

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}
\]

- \(\theta_1\) and \(\theta_2\) are incoming/outgoing angles
- \(v_1\) and \(v_2\) are the phase velocities in the two materials
- \(n_1\) and \(n_2\) are the indices of refraction in the two materials
**Transmitted Ray**

- **D** - incident ray direction
- **N** - unit surface normal
- **T** - unit surface tangent
  (in the plane of **N** and **D**)
- **R** - refracted ray direction
- (N, T, D, and R are all normalized)

\[
N \cos \theta_1 + T \sin \theta_1 + D = 0 \\
N \cos \theta_2 + T \sin \theta_2 + R = 0 \\
n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{(Snell)}
\]
Transmitted Ray

\[
R = -T \sin \theta_2 - N \cos \theta_2 \\
= (D + N \cos \theta_1) \frac{n_1}{n_2} \sin \theta_2 - N \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_1 \right)^2} \\
= (D + N \cos \theta_1) \frac{n_1}{n_2} - N \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_1 \right)^2} \\
= D \frac{n_1}{n_2} + N \left( \frac{n_1}{n_2} \cos \theta_1 - \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_1 \right)^2} \right) \\
= D \frac{n_1}{n_2} - N \left( \frac{n_1}{n_2} D \cdot N + \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_1 \right)^2} \right)
\]

- If the number under the square root is negative, there is no refracted/transmitted ray and all of the light is reflected (total internal reflection)
- This equation works regardless of which of \( n_1 \) or \( n_2 \) is larger
Question 2

List 10 objects that can make use of transmitted rays
Total Internal Reflection
Total Internal Reflection

- When light goes from a material with a higher refraction index \((n_2)\) to a material with a lower refraction index \((n_1)\), there is no refraction/transmission when the incident angle exceeds a critical angle (in this case, all the light reflects)

\[
\theta_2 < \theta_{2,\text{max}}, \text{ both refraction/transmission and reflection}
\]

\[
\theta_2 > \theta_{2,\text{max}}, \text{ only reflection}
\]
Critical Angle

• Compute the critical angle using Snell’s Law: \[ \frac{\sin \theta_1}{\sin \theta_2} = \frac{\nu_1}{\nu_2} = \frac{n_2}{n_1} \]

• Assume \( n_1 < n_2 \), then

\[ \theta_1 = \frac{\pi}{2} \quad \Rightarrow \quad \theta_{2,\text{max}} = \arcsin \left( \frac{n_1}{n_2} \right) \]
Total Internal Reflection

- Responsible for much of the rich appearance of glass and water
Snell’s Window

$n_1 < n_2$

$\theta_{2,\text{max}}$

$2\theta_{2,\text{max}}$
Reflection vs. Transmission

- The proportion of reflection versus transmission gradually increases as the viewing angle goes from perpendicular to a parallel (grazing)

Perpendicular view: high transmission, low reflection

Parallel (grazing) view: high reflection, low transmission
Reflection vs. Transmission

• Reflection similarly increases for non-transparent (opaque) objects as the viewing direction becomes more parallel/grazing

• Notice the increase in reflectivity of the table from left to right

[Lafortune et al. 1997]
Fresnel Equations

- Light can be polarized into 2 parts based on whether the plane containing the incident, reflected, and refracted rays is parallel or perpendicular to the electric field of the light.
- We denote the light as **p-polarized** if it is parallel and by **s-polarized** if it is perpendicular (to the electric field).
- The **Fresnel equations** give the fraction of light reflected off the interface between two materials as:

\[
R_p = \left| \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i} \right|^2 \\
R_s = \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right|^2
\]

- It is assumed that any light which is not reflected is instead transmitted through the interface from one material to the other, so:

\[
T_p = 1 - R_p \\
T_s = 1 - R_s
\]

- For **unpolarized** light (a typical assumption in ray tracing), the overall reflection and transmission coefficients are assumed to be:

\[
R = \frac{R_s + R_p}{2} \\
T = 1 - R
\]
Fresnel Equations

Light entering a denser material, e.g. from air into water

Light leaving a denser material, e.g. from water into air
Conductors vs. Dielectrics

- **Conductors**
  - Objects that conduct electricity (e.g. metal)
  - Have reflection and absorption, but no transmission

- **Dielectrics**
  - Objects that don’t conduct electricity (e.g. glass)
  - Have both reflection and transmission

http://renderman.pixar.com/resources/current/rms/tutorialRayTracing.html
Conductors

• Conductors don’t transmit light
• Most of the incident light is reflected, and some of it is absorbed
  – The reflection term changes relatively slowly with the incident angle (as compared to dielectric materials)
    • E.g., for aluminum, the reflection changes from about 90% to 100% as the incident angle changes from 0° to 90°
• Can typically treat $k_r$ as a constant independent of viewing direction (and $k_t = 0$)
Question 3

List 5 conductors
Dielectrics

• Strongly depend on the Fresnel equations to get the proper reflection vs. transmission of light based on viewing angle
Schlick’s Approximation

- Approximates the reflection term in the Fresnel Equation as:

\[ R(\theta_i) = R_0 + (1 - R_0)(1 - \cos\theta_i)^5 \quad \text{where} \quad R_0 = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2 \]

- \( R_0 \) controls the Fresnel reflections of different materials
  - \( R_0 \) ranges from 0 to 1
  - Reference values are available for many real-world materials
  - Same equation can be used for both dielectrics and conductors

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\( R(\theta_i) \) part of light is reflected

1-\( R(\theta_i) \) part of light is refracted or absorbed
Schlick’s Approximation

Schlick’s approximation (dotted lines) compared to the Fresnel equations (solid lines)
Question 4

List 5 dielectrics
Attenuation
Attenuation

• Light is attenuated as it travels through media
• The attenuation effect from the media is stronger over longer distances
• Different colors attenuate with different rates
  – Shallow water is clear (almost no attenuation)
  – Deeper water attenuates all the red light and looks blue-green
  – Even deeper water also attenuates all the green light and looks blue
  – Eventually all the light attenuates, and the color ranges from blackish-blue to black
Beer’s Law

• If the media is homogeneous, the attenuation along the ray can be described using Beer’s Law:

\[
\frac{dI}{dx} = -cI
\]

where \( I \) is the light intensity, \( x \) is the distance along the ray, and \( c \) is the attenuation constant (which varies based on color/wavelength)

• Solving this Ordinary Differential Equation (ODE) with the initial value \( I(0) = I_0 \) results in:

\[
I(x) = I_0 e^{-cx}
\]
The color of a transparent object can be described by three independent attenuation components for the three color channels (R, G, B)
Question 5

List 5 objects that attenuate light
Bending Rays
Atmospheric Refraction

- Light is bent into a curved path when it passes through the atmosphere
- Happens continuously, not just at an interface
- This is due to the continuous variations in air density, and thus continuous variations in the refractive index

- Inferior mirage - the mirage is located under the real object
- Superior mirage - the mirage is located above the real object

http://science-at-home.org/mirages/
Inferior Mirage
Superior Mirage

Credit: Timpaananen
Ray Tracing Mirages

• Bend the rays as they go through varying air densities
• Change the light direction between every interval in the vertical direction or along the ray direction
Ray Tracing Mirages
Ray Tracing Curved Paths

http://www.wired.com/2014/10/astrophysics-interstellar-black-hole/
Extra Credit

List two distinct causes for the bending of light
Iridescence
Iridescence

• Color of light changes with the viewing direction
Iridescence

- Various light waves are emitted in the same direction giving constructive and destructive interference

Take Home Extra Credit

List 10 iridescent objects