Reflection and Transmission
Recall: Ray Tracing

- Generate an image by backwards tracing the path of light through pixels
- Simulate the interaction of light with objects
Recall: Ray Tracing

• Trace a primary ray from the eye through each pixel and detect the first intersection point with the objects in the scene
• Trace secondary shadow rays from the intersection point towards light sources and sum the contributions from each (visible) light:

\[ L = \sum_{j \in \text{lights}} L_i^j (k_a + k_d (\hat{N} \cdot \omega_i^j)_+ + k_s (\hat{N} \cdot \hat{H}_i^j)_+ ) \]

- sum all lights
- ambient
- diffuse
- specular
Recursive Ray Tracing
Recursive Ray Tracing

• Light at a point may not only come directly from light sources:
  – Light can bounce off of objects (reflection)
  – Light can pass through objects (transparency/refraction)
• Trace more rays to look for more lighting information
  – Send secondary rays from the intersection point
  – Recursively compute the color for these secondary rays and sum them onto the primary ray
• Lighting equation becomes:

\[ L = \sum_{j \text{lights}} L^j_i (k_a + k_d (\hat{N} \cdot \omega^j_i)_+ + k_s (\hat{N} \cdot \hat{H}^j_i)_+^s) + k_r L_{reflected} + k_t L_{transmitted} \]

- standard direct illumination
- recursively computed reflection
- recursively computed refraction
Reflection/Transmission Add Light

• Additional reflection/transmission terms add more light to the object making it brighter
• Option 1: Dial down the object color
• Option 2: Dial down reflection/transmission coefficients: $k_r/k_t$
  – but this dims the desired reflections/transmissions
• Recommended approach:
  – First, adjust the ratio between the object color and the reflected/transmitted light to get the desired “look”
  – Then, tune everything up and down together, adjusting brightness while maintaining the ”look”
Ray Tree

- Each reflected or transmitted ray can spawn its own shadow, reflection, and refraction/transmission rays (recursively!)

![Diagram of Ray Tree](image)
Maximum Depth

- If a ray tree is too deep (too many rays), it gets too costly/slow
  - N.B. be aware of the size of the recursion stack (which depends on the hardware) to avoid recursive stack overflow
- If each splitting in the tree has a direct lighting component, then rays deeper in the tree make a smaller and smaller contribution to the total light for a pixel
  - So one can truncate when the contribution is below a threshold
- In some cases (mirrors, bubbles, etc.), there may not be a direct lighting component, and 100% of the light at a split comes from reflected/transmitted rays
- In this case, premature pruning doesn’t return a color (default = 0? black?)
  - Carefully choose a default color for early termination
  - Choose a background color?
  - E.g., choose blue if your scene is under the sky
  - Simply truncating the recursion often behaves as if a black color was set, i.e. multiplying the discarded rays by 0 (depending on the code)
Requires Minimal Implementation

http://fabiensanglard.net/rayTracing_back_of_business_card/
Reflection
Reflective Ray

- Given incident ray \( R(t) = A + tD \), the reflective ray \( R_{ref}(t) = P + tF \) is computed via:
  - \( P \) is where \( R(t) \) intersects the surface
  - \( F \) is the reflected vector with respect to the normal \( N \)
    - The incident angle \( \theta_i \) equals the reflection angle \( \theta_o \)

\[
F = D - 2(D \cdot N)N
\]
Spurious Self-Occlusion

- Add $\varepsilon$ to the starting point of a reflected ray to avoid re-intersection with the original surface
  - This often fails for grazing rays near the objects silhouette
- Offset in the normal direction from the surface
  - Just like shadow rays, except here we preserve the direction of the reflected ray
- Avoid placing the new starting point too close to or inside other nearby objects

The perturbed point may still be inside the object.
Shading Reflections

• Shade recursively:
  – Treat reflected rays like primary rays from the camera
  – Shade the reflected ray and return its color
    • Multiply by the reflection coefficient \( k_r \)
  – Add the resulting color to the shading at the original point
Question 1 (short/long)

List 10 objects that can make use of reflected rays
Transmission
Shading Transmission

- If the object is transparent, trace a transmitted ray
  - Treat transmitted rays like primary rays
  - Add $\varepsilon$ to avoid self intersection, or offset in the negative normal direction (respecting collisions, other geometry, etc.)
  - Shade the transmitted ray and return its color
  - Multiply by the refraction coefficient $k_t$
  - Add the resulting color to the shading at the original point
Snell’s Law

• The relationship between the angle of incidence and angle of refraction/transmission for light passing through a boundary between two different isotropic media is:

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}
\]

• $\theta_1$ and $\theta_2$ are incoming/outgoing angles
• $v_1$ and $v_2$ are the phase velocities in the two materials
• $n_1$ and $n_2$ are the indices of refraction in the two materials
Transmitted Ray

- **D** - incident ray direction
- **N** - normal
- **T** - tangent (in the plane of **N** and **D**)
- **R** - refracted ray direction
- Note: **D**, **N**, **T**, **R** are unit vectors

\[ N \cos \theta_1 + T \sin \theta_1 + D = 0 \]
\[ N \cos \theta_2 + T \sin \theta_2 + R = 0 \]
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{(Snell)} \]
$R = -T \sin \theta_2 - N \cos \theta_2$

$= (D + N \cos \theta_1) \frac{n_1}{n_2} - N \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_1\right)^2}$

$= D \frac{n_1}{n_2} + N \left(\frac{n_1}{n_2} \cos \theta_1 - \sqrt{1 - \frac{n_1^2}{n_2^2} (1 - \cos^2 \theta_1)}\right)$

$= D \frac{n_1}{n_2} - N \left(\frac{n_1}{n_2} \mathbf{D} \cdot \mathbf{N} + \sqrt{1 - \frac{n_1^2}{n_2^2} (1 - (\mathbf{D} \cdot \mathbf{N})^2)}\right)$

- If the number under the square root is negative, there is no refracted/transmitted ray and all of the light is reflected (total internal reflection)
- This equation works regardless of whether $n_1$ or $n_2$ is bigger
Question 2 (short/long)

List 10 objects that can make use of transmitted rays
Total Internal Reflection
Total Internal Reflection

- When light goes from a material with a higher refraction index ($n_2$) to a material with a lower refraction index ($n_1$), there is no refraction/transmission when the incident angle exceeds a critical angle (in this case, all the light reflects).

$$\theta_2 < \theta_{2,\text{max}}, \text{ both reflection and refraction/transmission}$$

$$\theta_2 > \theta_{2,\text{max}}, \text{ only reflection}$$
Critical Angle

- Compute the critical angle using Snell’s Law: \( \frac{\sin \theta_1}{\sin \theta_2} = \frac{\nu_1}{\nu_2} = \frac{n_2}{n_1} \)
- For \( n_1 < n_2 \),

\[
\theta_1 = \frac{\pi}{2} \quad \Rightarrow \quad \theta_{2,\text{max}} = \arcsin \left( \frac{n_1}{n_2} \right)
\]
Total Internal Reflection

• Responsible for much of the rich appearance of glass and water
Snell’s Window

\[ n_1 < n_2 \]

\[ \theta_{2,\text{max}} \]

\[ 2\theta_{2,\text{max}} \]
Viewing Angle
Reflection vs. Transmission

- The proportion of reflection versus transmission gradually increases as the viewing angle goes from perpendicular to a parallel (grazing).

Perpendicular view: high transmission, low reflection

Parallel (grazing) view: high reflection, low transmission
Reflection vs. Transmission

• Reflection similarly increases for non-transparent (opaque) objects as the viewing direction becomes more parallel/grazing

• Notice the increase in reflectivity of the table from left to right
• E.g. the increased ability to see the reflection of the book on the table

[Lafortune et al. 1997]
Fresnel Equations

• Light can be polarized into 2 parts based on whether the plane containing the incident, reflected, and refracted rays is parallel or perpendicular to the electric field of the light
• Denoted as p-polarized when parallel and s-polarized when perpendicular
• Fresnel equations give the fraction of light reflected off the interface between two materials as:

\[ R_p = \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_i} \right|^2 \]
\[ R_s = \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_i} \right|^2 \]

• Assume any light not reflected is instead transmitted through the interface:

\[ T_p = 1 - R_p \]
\[ T_s = 1 - R_s \]

• For unpolarized light (a typical assumption in ray tracing), the overall reflection and transmission coefficients are assumed to be:

\[ R = \frac{R_s + R_p}{2} \]
\[ T = 1 - R \]
Fresnel Equations

Light entering a denser material, e.g. from air into water

Light leaving a denser material, e.g. from water into air
Conductors vs. Dielectrics

• Conductors
  – Conduct electricity (e.g. metal)
  – Have reflection and absorption, but no transmission

• Dielectrics
  – Don’t conduct electricity (e.g. glass)
  – Have both reflection and transmission

http://renderman.pixar.com/resources/current/rms/tutorialRayTracing.html
Conductors

- No transmission: $k_t = 0$
- Most incident light is reflected, and some is absorbed
- Reflection amount changes slowly with the incident angle
  - E.g., for aluminum, the reflection changes from about 90% to 100% as the incident angle changes from 0° to 90°
- Approximate $k_r$ as a constant independent of viewing direction

Ray traced conductors

http://download.autodesk.com
http://www.raytracing.co.uk/
Question 3 (short/long)

List 5 conductors
Dielectrics

- Very important to use the Fresnel equations to get the proper reflection vs. transmission based on viewing angle
Schlick’s Approximation

• Approximate reflection in the Fresnel equation via:

\[ R(\theta_i) = R_0 + (1 - R_0)(1 - \cos\theta_i)^5 \quad \text{where} \quad R_0 = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2 \]

• \( R_0 \) controls the Fresnel reflections of different materials
  – \( R_0 \) ranges from 0 to 1
  – Reference values are available for many real-world materials
  – Same equation can be used for both dielectrics and conductors
Schlick’s Approximation

Schlick’s approximation (dotted lines)
Fresnel equations (solid lines)
Question 4 (short/long)

List 5 dielectrics
Attenuation
Attenuation

• Light is attenuated as it travels through media
• The attenuation effect is stronger over longer distances
• Different colors attenuate with different rates
  – Shallow water is clear (almost no attenuation)
  – Deeper water attenuates all the red light and looks bluish-green
  – Even deeper water attenuates all the green light too, and looks blue
  – Eventually all the light attenuates, and the color ranges from blackish-blue to black
Beer’s Law

• If the media is homogeneous, attenuation along a ray can be described by Beer’s Law:

\[
\frac{dI}{dx} = -cI
\]

where \( I \) is the light intensity, \( x \) is the distance along the ray, and \( c \) is the attenuation constant (which varies based on color/wavelength)

• Given initial value \( I(0) = I_0 \), this Ordinary Differential Equation (ODE) has the solution:

\[
I(x) = I_0 e^{-cx}
\]
The color of a transparent object can be described by three independent attenuation components for the three color channels (R, G, B)
Question 5 (short/long)

List 5 objects that attenuate light
Bending Rays
Atmospheric Refraction

- Light *continuously* bends in a curved path when it passes through the atmosphere.
- This is due to continuous variations in air density, and thus continuous variations in the refractive index.

- **Inferior mirage** - mirage located under the real object.
- **Superior mirage** - mirage located above the real object.

http://science-at-home.org/mirages/
Inferior Mirage
Superior Mirage
Ray Tracing Mirages

- Bend the rays as they go through varying air densities
- Change the light direction between every interval in the vertical direction or along the ray direction
Ray Tracing Curved Paths

http://www.wired.com/2014/10/astrophysics-interstellar-black-hole/
Extra Credit (short, due at end of class)

List two distinct causes for the bending of light
Iridescence
Iridescence

- Color of light changes with the viewing direction
Iridescence

• Various light waves are emitted in the same direction giving constructive and destructive interference

Take Home Extra Credit (long)

List 10 iridescent objects