Recall: Inside Triangle Test

2D Bounding Boxes

```
rasterize( vert v[3] )
{
    bbox b; bound3(v, b);
    line l0, l1, l2;
    makeline(v[0],v[1],l2);
    makeline(v[1],v[2],l0);
    makeline(v[2],v[0],l1);
    for( y=b.ymin; y<b.ymax, y++ ) for( x=b.xmin; x<b.xmax, x++ ){
        e0 = l0.A * x + l0.B * y + l0.C;
        e2 = l2.A * x + l2.B * y + l2.C;
        if( inside(e0,l0)&inside(e1,l1)&inside(e2,l2) )
            fragment(x,y);
    }
}
```
Bounding Volumes

- Enclose a complex object within a simpler piece of geometry
  - Sphere
  - Box (axis-aligned, object-aligned)

- If the ray does not hit the bounding volume, it cannot hit the enclosed complex object
  - Avoid a significantly more expensive intersection test with the complex object
Bounding Sphere

• A sphere enclosing all of the vertices of the object.
• Fastest bounding volume for an intersection test with a ray
• Minimize the radius of the bounding sphere
  – For a triangle or a tetrahedron, the minimal radius is simply the radius of its circumsphere
  – What about more complex objects?
Bounding Sphere

- An straightforward construction:
  - Compute the centroid $P_c$ of all vertices
  - Loop through all vertices and compute the maximal $|P_i - P_c|^2$ to find the farthest vertex $P_{\text{farthest}}$ from $P_c$
  - $|P_{\text{farthest}} - P_c|$ is the minimal radius
  - Poor optimality: for a crowd of points with one discrete point far away, the radius calculated in this way is about twice as big as the minimal one

- A hierarchical construction:
  - Build the bounding sphere for each triangle
  - Recursively merge the neighboring spheres into larger ones until all the spheres are merged into a single sphere
  - The radius of that sphere is more minimal
Pros and Cons

• Advantages:
  – Constructing a bounding sphere is fast
  – Ray-sphere intersection test is fast
  – No need to change the bounding sphere when the object rotates

• Disadvantages:
  – Cannot tightly enclose the object in some cases, e.g. long and thin objects, lead to many false positives
**Axis-Aligned Bounding Box (AABB)**

- An axis-aligned box containing the object
- Constructing an AABB is trivial:
  - Loop through all vertices and find the *min* and *max* values in each dimension
  - Use the *min/max* values as the bottom-left/top-right corners of the bounding box
Ray-AABB Intersection

- A box is the interior of the intersection of 3 pairs of planes
- We detect the intersection between the ray and the box by detecting the intersections between the ray and the three pairs of planes
  - For each pair of planes there are two intersection points $P_{\text{near}}$ and $P_{\text{far}}$
  - 6 intersection points in total
  - If the maximal $P_{\text{near}}$ is before the minimal $P_{\text{far}}$ on the ray, the ray misses the box; otherwise it hits the box
Pros and Cons

• Advantages
  – Constructing an AABB is simple and efficient
    • Scanning the vertices to find min and max values in each dimension takes $O(n)$ time
  – Ray-box intersection test is fast

• Disadvantages
  – Need to recalculate the bounding box any time an object rotates (unless the ray is transformed into object space!!!)
  – Cannot tightly enclose the object (similar to spheres)
Oriented Bounding Box (OBB)

- The orientation of the box depends on the orientation of the object
- Don’t need to recompute the box when the object rotates
- Can pre-compute the OBB in object space and transform it to world space with the object (note: the same is true for spheres)
Constructing an OBB

• Computing the principle orientation of the object
  – Singular Value Decomposition
  – Compute the covariance matrix, and the eigenvalues and corresponding eigenvectors of the 3x3 matrix
  – Using these eigenvectors as the basis of the local coordinate system of the bounding box
  – Similar to moment of inertia calculations for rigid bodies (except with a homogeneous mass density)

• Or just estimate the principal directions with some heuristics...
Constructing an OBB for a Triangle Mesh

• Compute the mean centroid of all triangles:

\[
\mu = \frac{1}{n} \sum_{i=0}^{n} \frac{(p^i + q^i + r^i)}{3}
\]

where \(p^i, q^i,\) and \(r^i\) are the vertices of \(i\)th triangle, and \(n\) is the number of triangles.

• Construct the 3x3 covariance matrix \(C\), the element \(C_{jk}\) is computed as:

\[
C_{jk} = \frac{1}{3n} \sum_{i=0}^{n} (p_j^i p_k^i + q_j^i q_k^i + r_j^i r_k^i)
\]

where \(\bar{p}^i = p^i - \mu\) and \(\bar{p} = (p_1^i, p_2^i, p_3^i)^T\)

• Calculate the eigenvectors \(e_1, e_2, e_3\) of \(C\) and take them as the basis vectors for the local coordinate system of the OBB.

  – All of the eigenvectors are mutually orthogonal since \(C\) is symmetric.

• Find the extremal vertices along each basis vector direction and resize the bounding box to bound those vertices (similar to the AABB)

[Gottschalk et.al. 96]
Ray-OBB Intersection

• Similar to Ray-AABB intersection
  – Calculate the maximum $t_{\text{near}}$ and the minimum $t_{\text{far}}$
  – The planes are not axis aligned any more
  – Recall how to compute intersection between a ray and an arbitrary plane in 3D space
Or we can transform the ray...

- Transform the ray into the OBB coordinate system and perform the ray-AABB intersection test.

World Coordinates → OBB Coordinates
Pros and Cons

• Advantages
  – Fit the object tighter than an AABB or sphere, so it prunes out more rays

• Disadvantages
  – Extra cost for ray-box intersection
  – Finding an OBB is computationally expensive (but can usually be done as a pre-computation)
  – Finding the minimal OBB can be difficult (but an approximation is usually good enough)
Bounding Volume Hierarchies (BVH)

• Subparts of objects can be contained in their own bounding boxes in a hierarchical fashion
• For example, sub-OBBs can be constructed by recursively splitting an OBB using center points along the longest axis

[Gottschalk et.al. 96] [Kamat and Martinez, 2007]
BVH Tree Structure

- Each leaf represents a bounding volume at the lowest level of the hierarchy.
- Nodes at higher levels of the tree represent bounding volumes that enclose all the bounding volumes of that node’s children.
Constructing BVH

- **Top down**
  - Begin with the bounding volume for the entire object
  - Recursively split the object into smaller pieces according to some criteria, and construct bounding volumes for these sub-pieces
    - E.g., split primitives w.r.t the longest coordinate axis
  - Stop when some tolerance is reached for leaf nodes

- **Bottom up**
  - Begin with a bounding volume for each subpart of an object
  - Recursively merge them into larger volumes according to some criteria
    - e.g., merge nearest neighbors
  - Stop when there is only a single bounding volume at the root
Hierarchical Ray Traversal

- Intersect child bounding volumes only if intersections are found with the parent
- Intersect low level geometry only if a leaf node is reached and the ray intersects its bounding volume
Early Termination

• In this example, every node and triangle needs to be checked, except for the triangle in F which can be skipped since the ray does not intersect sphere F.

However if you sort the intersections based on the ray parameter, and find the intersection with the triangle in G before checking the triangles in H and I, then those intersection tests can be skipped.
Ray-Scene Intersection

Intersection FindIntersection(Ray ray, Scene scene)
{
    min_t = infinity
    minPrimitive = NULL
    For each primitive in scene {
        t = Intersect(ray, primitive);
        if (t > 0 && t < min_t) then
            minPrimitive = primitive
            min_t = t
    }
}
return Intersection(min_t, minPrimitive)

- Traversing all objects in the scene for each ray is inefficient
- Need auxiliary structures to accelerate this process
Two Uses for BVHs

• Create a BVH for an individual object to serve as its bounding volume *(we already considered this)*
  – The BVH is in object space
  – As long as we transform the rays into object space, the BVH can be precomputed and does not need to be updated

• Create a BVH for unioning together all the bounding volumes of all the objects in the scene *(new)*
  – If an object has its own object BVH, just use the root BV of the BVH
  – This global BVH is in world space
  – Must be updated whenever something moves in the scene

• Do both!
Global BVHs don’t always work well

• If many similar sized objects are approximately uniformly distributed in space, the global BVH hierarchy is traversed quite a bit looking for the object the ray intersects
• One can do better...

http://hof.povray.org/shorebir.html

“Warm Up” by Norbert Kern (2001)
http://hof.povray.org/warm_up.html
Partition the **space** instead...

- Partition all of space and register objects with the cells that their bounding volumes overlap
Uniform Grids

• Divide 3D space into $n_x \times n_y \times n_z$ axis-aligned grid cells
• Perform ray-object intersection tests only when the ray hits a grid cell containing an object
• The size of the grid’s cells is crucial to the performance
  – No speedup if the cell size is too big; no pruning, if everything is in one cell
  – A lot of empty cells, if the cell size is too small; the same object may be in too many different cells
  – A practical way to choose cell size is to average the edge lengths of the bounding boxes of all the objects
Constructing a Uniform Grid

• Find a bounding box for the entire scene
• Initialize the grid with that bounding box and a properly chosen cell size
  – Each cell maintains a list (or an array) to store the objects whose bounding volumes overlap that cell
• Insert objects into cells
  – The same object may be inserted into multiple cells
Ray Traversal

• Traverse all the cells pierced by the ray until the ray intersects with a primitive or reaches the boundary of the uniform grid
Ray Traversal

• An incremental algorithm similar to line rasterization:
• From the current intersection point \( P \) on the face of cell \((i, j, k)\), perform ray-plane intersection tests with the next 3 grid planes along the ray direction to get the 3 candidate intersection points for the next intersection
• The next intersection point is the nearest one among the 3 candidates
• Update the cell index according to the new intersection point. Perform ray-primitive intersection tests in the new cell
• Repeat the above process until the ray intersects with a primitive or reaches the boundary of the bounding box

Two candidate points for the next ray-cell intersection in 2D

Cell \((i, j)\) transmit to Cell \((i+1, j)\)
Ray Traversal Optimization

- The intersections with the grid planes have the same spacing in each independent dimension
- Use precomputed $\delta t_x$, $\delta t_y$, and $\delta t_z$ in order to avoid ray-plane intersection tests
  - $\delta t_x = C_x / D_x$, $\delta t_y = C_y / D_y$, $\delta t_z = C_z / D_z$, in which $(C_x, C_y, C_z)$ is the cell size and $(D_x, D_y, D_z)$ is the ray direction
Avoid Redundant Intersection Tests

- A primitive may be stored in multiple cells
- An intersection test on that primitive may accidently be performed repeatedly for every cell it is in
- Store a bool with each primitive
  - If the primitive has already been determined to not intersect the ray, or is not the closest intersecting primitive, store false
  - Before an actual ray-primitive intersection test, check the bool to see if the intersection was already performed
Viewing Frustums

• Send rays in “camera space”
  – Create a uniform grid in the frustum space
  – Avoid the traversal steps and plane intersection tests
  – Cache coherent, all the cells are aligned along the ray!
  – Can traverse a bundle of rays at the same time
Optimizing the Uniform Grid

- Optimizing storage
  - Spatial Hashing: use a hash table instead of a 3D array
  - Avoid the extra storage for a large number of empty cells

- Optimizing performance
  - Adaptive grids: rectilinear grid, grid hierarchy, octree
Octree

• Each node has 0 or 8 children
  – Each node can equally subdivide its space (an AABB) into eight subboxes by 3 midplanes
  – Children of a node are contained within the box of the node itself
Constructing an Octree

• Top-down:
  – Find the global bounding box that contains all the primitives and make it the root
  – Recursively partition a node into 8 octants by 3 midplanes
    • If a primitive belongs to multiple octants, put it in all of them
  – Recursion stops when the termination criteria are satisfied.
    • e.g., maximum depth, minimum number of primitives in a node, etc.
Ray Traversal

- Traverse all leaf nodes in the octree passed through by the ray and perform intersection tests for the primitives inside those leaves
- Similar to (the same as) a BVH...
Observation

• For a given ray and parent box, the subboxes that intersect the ray and their order of intersection is determined by the intersection points of the parent box.
Find the Next Subnode

• Build an automaton whose states correspond to the boxes and whose transitions are associated to the movements between neighboring sub-nodes visited sequentially by the ray
Example...
Hybrid Structure

- Start with a uniform grid and subdivide each node
  - Each node of a uniform grid can contain its own octree
  - Improves performance over that of a standard octree: no need to traverse the tree from a single root every time

[Losasso et.al. 2006]
K-d Tree

- Using a hyperplane translated in one dimension
K-d Tree

- A binary tree for space searching:
  - Every non-leaf node can be thought of as implicitly generating a splitting hyperplane that divides space into two parts
  - Points to the left of this hyperplane are represented by the left subtree of that node and points to the right of the hyperplane are represented by the right subtree
Constructing a K-d Tree

- **Top-down:**
  - Begin with the global bounding box containing all primitives.
  - Choose an axis and a splitting plane perpendicular to that axis.
  - Subdivide the primitives on both sides of the plane into two groups.
  - Stop when the number of primitives in each single group is below a threshold.
Ray Traversal

• Traverse all leaf nodes in the k-d tree passed through by the ray
Recursive Traversal

• Similar to recursive traversal in an octree
  – Each interior node has two children, there are 3 cases for subnode intersections:
    • left child only, the right child only, and both
  – The number of intersected subboxes and its sequence can be determined by the intersection points
    • Notice the difference with an octree; the splitting plane is not necessarily in the middle of the box
Non-Recursive Traversal

- Recursion is expensive on the GPU for real-time ray tracing
- Kd-Restart
  - Restart the traversal at the root every time it reaches a leaf
- Kd-Backtrack
  - Return to parent nodes to enable traversing other unvisited nodes
- Neighbor-link
  - Each leaf stores ropes that directly link it to the adjacent node via its 6 faces
  - Octrees typically have these neighbor structures too

[Popov et.al. 07]