More Geometric Modeling
Implicit Surfaces
Implicit Surfaces

• Implicit surfaces define a function $\phi(x)$ over the entire 3D space
• The inside region $\Omega^-$ is defined by $\phi(x) < 0$, the outside region $\Omega^+$ is defined by $\phi(x) > 0$, and the surface itself $\partial \Omega$ is defined by the isocontour $\phi(x) = 0$
  • We have already seen planes/spheres (and lines-rays/circles in 2D) defined by implicit surfaces
  • It’s easy to check if a point is inside/outside simply by evaluating $\phi$ at the point
  • Straightforward Constructive Solid Geometry (CSG) operations (Union, Difference, Intersection, etc.)
• Importantly, ray tracing is quite easy to apply to implicit objects!
Topological Change

- Greatly superior to triangle meshes for topological change!
Bloffies

• Each blob is defined as a density function around a particle
• For each pixel, the aggregate function is created by summing the density function values of every blob
  • Blob kernels can be: 2D ellipses, 2D diamonds, 3D spheres, etc.
• Bloffies were proposed contemporaneously with Metaballs (in Japan) and Soft objects (in Canada and New Zealand)
  • Slightly different density kernel functions
  • Metaballs and Soft Objects have a finite influence around each particle, so each pixel only needs to query nearby density functions (computationally cheaper)
Topological Change

- Two blobby particles easily merging into one blob
Blobby Modeling
Marching Cubes

- Turns an implicit surface into triangles
- Define the implicit surface on a 3D grid
- Then for each grid cell, use the topology of the volume in order to reconstruct the surface with triangles
Computer Vision
Range Scanning

• A range scanner senses 3D positions on an object’s surface and returns an $m \times n$ grid of distances that describe the surface ($m$ points per laser sheet, $n$ laser sheets)
  • This grid is called a range image
  • In case of multiple range images, a rigid transformation is found for each image to align them together
  • This transformation is found using an iterative closest point alignment (ICP), which minimizes the least squared distance between nearest points in two range images
Range Scanning

• Each sample point in the $m \times n$ range image is a potential vertex in the triangle mesh
• Special care is taken to avoid inadvertently joining portions of the surface together that are separated by depth discontinuities
  • Zero, one or two triangles can be created from four points of a range image that are in adjacent rows and column
  • Shortest of the two diagonals between the points is used to identify the two triplets of points that may become triangles
  • Each of these point triples is made into a triangle if the edge lengths fall below a distance threshold
• These meshes corresponding to each range image are combined using the zippering algorithm given the rigid transformation between each image

![Figure 3: Building triangle mesh from range points.](image)
Mobile 3D Scanning

Structure Sensor for iPad

Autodesk 123D Catch
Voxel Carving

- Construct a voxelized 3D model given multiple images of an object, from calibrated cameras, taken from different directions
  - A silhouette is computed for each image
  - The silhouette for each image is back projected onto the grid
  - The voxels that lie in the back-projection of every image correspond to the final 3D model
  - Once the model is acquired, colors can also be back projected

[Diagram showing discretized scene volume and input images]
Voxel Carving

Original image

Extracted silhouettes

Carved out voxels

Back projecting the colors
Reconstruction from Large Photo Collections

- Construct a 3D model from a large number of photos (say from google images)
- Computer vision algorithms are used to predict relative camera position/orientation for each image, and at the same time to obtain a sparse point cloud representation of the object
- The position of a point that is visible in multiple images can be determined
- Many dense reconstruction algorithms can be used to get denser points given the camera parameters and this initial point cloud
Trees, Drones, Etc.
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Denoising

• Computer Vision algorithms construct virtual geometry from real world geometry
• This requires collecting data from the real world, and this data collection is highly sensitive to noise
• In fact, noise is the biggest problem with such methods
• Thus, various methods have been devised to denoise/smooth high problematic geometry with spurious high frequency features
Laplacian Smoothing

• Similar to differential coordinates, one can compute a Laplacian estimate using the one ring of vertices about a point.

• For example, on a curve (see below), one might use

\[ L(p_i) = \frac{1}{2} \left( (p_{i+1} - p_i) + (p_{i-1} - p_i) \right) \]

• Then, update

\[ p_i^{new} = p_i + \lambda L(p_i) \]

where \( \lambda \in (0, 1) \).

• Repeat several iterations.
Taubin Smoothing

- Laplacian smoothing eventually shrinks a closed curve/surface to a single point
- Taubin smoothing periodically performs an extra inflation step to counteract the shrinkage due to Laplacian smoothing: $p_i^{new} = p_i - \mu L(p_i)$ where $\mu > 0$
Simulation
Netwon’s Second Law

- **Kinematics** describe position $X(t)$ and velocity $V(t)$ as function of time $t$
  - $\frac{dX(t)}{dt} = V(t)$ or $X'(t) = V(t)$

- **Dynamics** describe responses to external stimuli
  - Newton’s second law $F(t) = MA(t)$ is a dynamics equation
  - $V'(t) = A(t)$ implies $V'(t) = \frac{F(t)}{M}$ as well as $\frac{d^2X(t)}{dt^2} = X''(t) = \frac{F(t)}{M}$

- Combining kinematics and dynamics gives: 
  $$\begin{pmatrix} X'(t) \\ V'(t) \end{pmatrix} = \begin{pmatrix} \frac{V(t)}{F(t)} \\ \frac{V(t)}{M} \end{pmatrix}$$

- Note the (potential) dependency of forces on position and velocity

- Much of the physical world can be simulated with computational mechanics (FEM) and computational fluid dynamics (CFD) using Newton’s second law
  - One merely needs to create degrees of freedom, specify forces, and solve the resulting ordinary differential equations (ODEs)
  - For most materials, the forces have spatial interdependence making this a partial differential equation (PDE)
Computational Mechanics (FEM)
Computational Fluid Dynamics (CFD)
Computational Biomechanics
Procedural Methods
Procedural Geometry

• Whereas simulation uses algorithms to animate existing geometry, one can also use an algorithm to create the geometry itself
  • Typically used for complex/tedious models (e.g. terrain, plants/foliage, buildings/cities)
  • Easy to perturb the algorithm to make variations of the geometry
• Start with a small set of data or rules to describe high level properties of the desired models
  • E.g. tree: branching and leaf shape, building: room subdivision and door/window placement
• The rest of the model is algorithmically constructed (add randomness and use recursion)
L-Systems

- Typically used to model plants
  - Developed by biologist Lindenmayer to study algae growth
- A recursive formal grammar:
  - An alphabet of symbols (terminal and non-terminal)
  - A collection of production rules
  - Non-terminal symbols create new symbols or sequences of symbols recursively
- The process starts with an initial string (axiom) to which the production rules are applied
- Finally, a translator turns both terminals and non-terminals into geometric structures

Nonterminals: A, B: mean “draw forward”
Terminals: +/-: Turn right/left by 60 degrees
Initial Axiom: A; Rules: A → B + A + B, B → A – B – A

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A

B+A+B

A-B-A + B+A+B + A-B-A


Etc.

Sierpinski Triangle
L-System + Stack = Branches

- Nonterminals: X: (no action)  F: draw forward
- Terminals: +/- : Turn right/left by 25 degrees
  - [ : store current state on the stack
  - ] : load state from stack
- Initial Axiom: X
- Rules: \( X \rightarrow F - [ [X]+X] + F [+FX] - X, F \rightarrow FF \)
L-Systems

- Easily extended to 3D
- Model the trunk and branches as cylinders
- As recursion proceeds:
  - Shrink cylinder size
  - Vary color from brown to green
- Add more variety with a stochastic L-system
  - Multiple rules for each symbol
  - At each symbol, randomly choose one rule to replace
- L-system is a relatively abstract specification
  - Requires experience to model a specific and given form
Fractals

- Initiator: start with a shape
- Generator: replace subparts with scaled copy of original
- Apply generator repeatedly
Statistical Fractal Generator

• Add randomness in fractal generation
• Can be used to model an irregular “random” 2D silhouette or terrain
• Random midpoint displacement

Start with single horizontal line segment. Repeat for sufficiently large number of times
{
  Repeat over each line segment in scene
  {
    Find midpoint of line segment.
    Displace midpoint in Y by random amount.
    Reduce range for random numbers.
  }
}

Result: Mountain Range
Generating Height Fields

- Start with a 2D fractal (or any 2D grey-scale image)
- Lay a rectangular grid on the ground
- For each vertex of the grid, vary the height based on pixel intensity
Generating 3D Landscapes

• General procedure:
  • Initiator: start with a shape
  • Generator: random subparts with a self-similar random pattern
• Use this to generate entire terrains
• Similar to subdivision, but with much more interesting rules for setting vertex positions
Fractal Worlds
Fractal Worlds
Machine Learning
Machine Learning

- Interactive Example-Based Terrain Authoring with Conditional Generative Adversarial Networks, Siggraph Asia 2017