HW1 (Due: Wed Jan 13)

1.1 [10 points each]

1.1 (PL validity & satisfiability). For each of the following PL formulae, identify whether it is valid or not. If it is valid, prove it with a truth table or semantic argument; otherwise, identify a falsifying interpretation. Recall our conventions for operator precedence and associativity from Section 1.1.

(*e*) $\neg(P \land Q) \rightarrow R \rightarrow \neg R \rightarrow Q$

(*f*) $P \land Q \lor \neg P \lor (\neg Q \rightarrow \neg P)$

1.2 [10 points each]

1.2 (Template equivalences). Use the truth table or semantic argument method to prove the following template equivalences.

(*s*) $F_1 \rightarrow F_2 \Leftrightarrow \neg F_2 \rightarrow \neg F_1$

(*x*) $(F_1 \rightarrow F_2) \land (F_1 \rightarrow F_3) \Leftrightarrow F_1 \rightarrow F_2 \land F_3$

1.3 Note typo: the last disjunction (or) symbol should be a conjunction (and) symbol [30 points]

1.3 (Redundant logical connectives). Given $\top$, $\land$, and $\neg$, prove that $\bot$, $\lor$, $\rightarrow$, and $\leftrightarrow$ are redundant logical connectives. That is, show that each of $\bot$, $F_1 \lor F_2$, $F_1 \rightarrow F_2$, and $F_1 \leftrightarrow F_2$ is equivalent to a formula that uses only F_1, F_2, $\top$, $\lor$, and $\neg$.

1.5d (15 points) Convert this formula into NNF, DNF, and CNF. Also, convert it into an equisatisfiable formula in CNF using the efficient method, and decide whether it is satisfiable.

1.5 (Normal forms). Convert the following PL formulae to NNF, DNF, and CNF via the transformations of Section 1.6.

(*d*) $\neg(Q \rightarrow R) \land P \land (Q \lor \neg(P \land R))$

1.8b (15 points)

1.8 (DPLL). Describe the execution of DPLL on the following formulae.

(*b*) $(P \lor Q \lor R) \land (\neg P \lor \neg Q \lor \neg R) \land (\neg P \lor Q \lor R) \land (\neg Q \lor R) \land (Q \lor \neg R)$