Transformational Systems

Observable only at the beginning and the end of their execution ("black box")

$$\xrightarrow{\text{input}} \boxed{\text{system}} \xrightarrow{\text{output}}$$

with no interaction with the environment.

- specified by

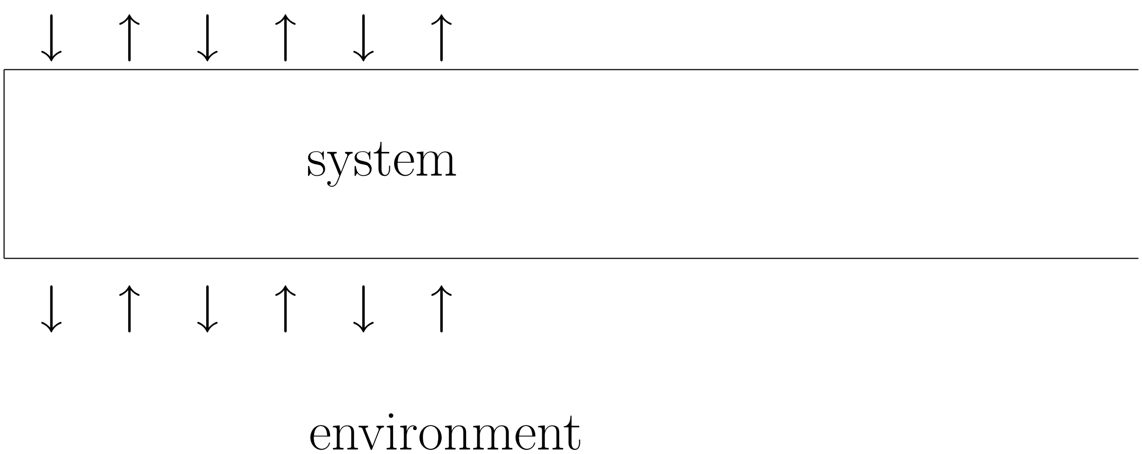
  input-output relations
  $\Downarrow$
  state formulas (assertions)
  First-Order Logic

- typically

  terminating sequential programs
  e.g., input $x \geq 0 \rightarrow$ output $z = \sqrt{x}$

Reactive Systems

Observable throughout their execution
("black cactus")



| ⟶ time

Interaction with the environment

- specified by

their on-going behaviors
(histories of interactions with their environment)
$\Downarrow$
sequence formulas
Temporal Logic

- Typically
  - Airline reservation systems
  - Operating systems
  - Process control programs
  - Communication networks

SPL Semantics (Con't)

accessible configuration –
appears as value of π in some accessible state

`Example:`
$\{[\ell_0], [m_1]\}$ does not appear in any accessible state

Is a given configuration accessible?

Undecidable

The Mutual-Exclusion Problem

$$\begin{array}{lcl} \textbf{loop forever do} & & \textbf{loop forever do} \\ \quad \left[\begin{array}{l} \textbf{noncritical} \\ \ \cdots\cdots\cdots \\ \textbf{critical} \\ \ \cdots\cdots\cdots \end{array}\right] & \| & \quad \left[\begin{array}{l} \textbf{noncritical} \\ \ \cdots\cdots\cdots \\ \textbf{critical} \\ \ \cdots\cdots\cdots \end{array}\right] \end{array}$$

Requirements:

- <u>Exclusion</u>

  While one of the processes is in its critical section, the other is not

- <u>Accessibility</u>

  Whenever a process is at the noncritical section exit, it must eventually reach its critical section

`Example:` mutual exclusion by semaphores Fig. 0.7

Expressibility

There are properties that cannot be specified by a quantifier-free temporal logic formula.

Example:

Specify the property
"x assumes the value 0 only, if ever, at even positions"
i.e., "at positions $0, 2, 4, \ldots$"

- cannot be expressed in quantifier-free TL

- can be expressed in (quantified) TL

  Quantifying over flexible boolean variable b:
  $\exists b[b \ \wedge \ \square(b \leftrightarrow \neg \bigcirc b) \ \wedge \ \square(x = 0 \rightarrow b)]$.
  $\forall b[b \ \wedge \ \square(b \leftrightarrow \neg \bigcirc b) \ \rightarrow \square(x = 0 \rightarrow b)]$.

  Why not

  $$x = 0 \wedge \square[x = 0 \rightarrow \bigcirc \bigcirc (x = 0)]?$$

Temporal vs First-Order

TL formula

$$\Box(p \rightarrow \Diamond[r \wedge \Diamond q])$$

can be transformed into FOL formula

$$(\forall t_1 \geq 0)\left[\ p(t_1) \rightarrow (\exists t_2)\left[\begin{array}{c} t_1 \leq t_2 \ \wedge \ r(t_2) \wedge \\ (\exists t_3)(t_2 \leq t_3 \ \wedge \ q(t_3)) \end{array}\right]\ \right]$$

where t_1, t_2, t_3 are integers.