Max Flow Potpurri

(a) Suppose that instead of having a single source and sink $s, t$ respectively, we have multiple sources $S = \{s_1, s_2 \ldots s_k\}$ and multiple sinks $T = \{t_1, t_2 \ldots t_l\}$. We wish to still find the max flow in the graph from sources to sinks.

(b) Suppose that in addition to edges having max flow capacities, vertices also have a limit to their capacity; that is, each vertex $v_i$ has capacity $c_i$. We wish to find the max flow from a source $s$ to sink $t$ in this graph.

(c) Given a solution to max-flow, verify that it is correct in linear time.

Expense Settling

You’ve gone on a trip with $k$ friends, where friend $i$ paid $c_i$ for the group’s expenses. You would like to develop an algorithm to ensure that everyone gets paid back fairly, but without going through one person (that is, each person would either pay or receive money, but not both).

Project Selection

Suppose you have a set of $k$ tasks $t_1 \ldots t_k$. There are certain tasks such that $t_i$ is a prerequisite of $t_j$. Each task also has a reward $r_i$, which may be negative. Find an optimal subset of tasks to complete to maximize your reward.