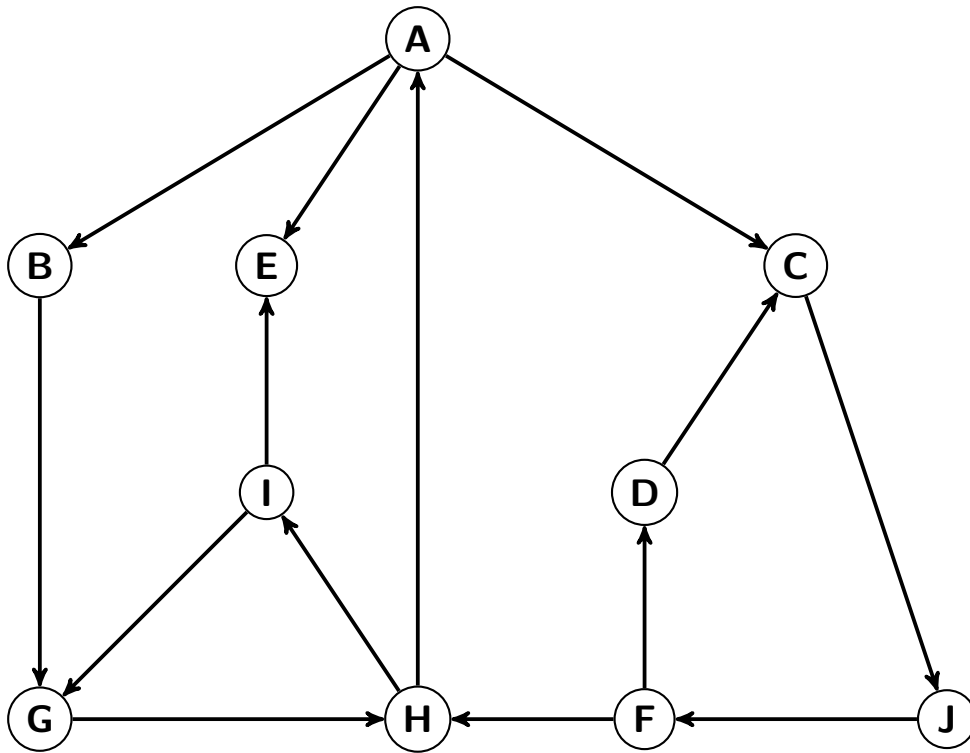


CS 161 Fall '18 Section 3

October 2018

1 Depth First Search



1. What are all the strongly connected components? (i.e. groups of vertices such that there exists a path between any two vertices in the group)
2. Perform DFS on the graph above starting from vertex A. Use lexicographical ordering to break vertex ties. As you go, label each node with the start time and the finish time. Highlight the edges in the tree generated from the search.
3. Perform BFS on the graph above starting from vertex A. Use lexicographical ordering to break vertex ties. As you go, label each node with the discovery order. Highlight the edges in the tree generated from the search.

2 True or False

1. If (u, v) is an edge in an undirected graph and during DFS, $finish(v) < finish(u)$, then u is an ancestor of v in the DFS tree.
2. In a directed graph, if there is a path from u to v and $start(u) < start(v)$ then u is an ancestor of v in the DFS tree.

3 Russian Boxes

You have n boxes. The i -th box has dimensions $w_i \times h_i$. Box i can fit inside box j iff $w_i < w_j$ and $h_i < h_j$. A sequence of boxes b_1, b_2, \dots, b_k form a chain if box b_i fits inside box b_{i+1} for each $1 \leq i < k$. Design an algorithm which takes as input a list of dimensions $w_i \times h_i$ and returns a longest possible chain of boxes. You must construct a directed graph as part of your solution.

4 Random Walks

Suppose you have an undirected, connected graph $G = (V, E)$ and two nodes $s, t \in V$. You then take a *random walk* on G : start at node $v_0 = s$, and at every step v_i , pick an edge coming out of v_i uniformly at random, and set v_{i+1} to be the other node on that edge.

Prove that with probability 1, your random walk will hit t : there exists some i such that $v_i = t$.

5 Bipartite Graphs

A Bipartite Graph is a graph whose vertices can be divided into two independent sets, U and V such that every edge (u, v) either connects a vertex from U to V or a vertex from V to U . A bipartite graph is possible if the graph coloring is possible using two colors such that vertices in a set are colored with the same color.

1. Design an algorithm using DFS to determine whether or not a graph is bipartite.
2. Design an algorithm using BFS to determine whether or not a graph is bipartite.

6 Source Vertices

A source vertex in a graph $G = (V, E)$ is a vertex v such that all other vertices in G can be reached by a path from v . Say we have a directed, connected graph that has at least one source vertex.

1. Describe a naive algorithm to find a source vertex
2. Describe an algorithm that operates in $O(V + E)$ time to find a source vertex.