CS 205b / CME 306

Application Track

Homework 1

1. Use conservation of mass to show that the sum of the outward-facing area-weighted normals of a triangle mesh must be the zero vector.

2. The strong form conservation of mass in an Eulerian frame can be written as $\rho_t + \rho_x u + \rho u_x = 0$. For each of the three terms:

   (a) Provide a physical description of what the term means,

   (b) Describe a physical situation in which that term is identically zero in a region while the other two terms remain nonzero, and

   (c) Show that the situation can actually occur by finding ρ and u such that the term is identically zero in the region $x, t \in [0, 1]$ while the other two terms are nonzero throughout the entire region.

3. In this sequence of problems, we will construct a kernel function $W(\mathbf{x}, h)$ for use in the SPH method in 1D, 2D, and 3D.

   (a) Since we would like $W(\mathbf{x}, h)$ to be symmetric about the origin, we take $W(\mathbf{x}, h) = c_d(h) f(\|x\|/h)$, where $c_d(h)$ is a normalization factor that depends on the dimension d and the radius of influence $h > 0$. The function $f(r)$ need not be defined for $r < 0$. Find $c_1(h)$, $c_2(h)$, and $c_3(h)$. (Hint: Use polar coordinates in 2D and spherical coordinates in 3D.)

   (b) We would like the radius of influence of the kernel $W(\mathbf{x}, h)$ to be h. What conditions does this place on $f(r)$?

   (c) We further require that $W(\mathbf{x}, h)$ be have continuous second derivatives everywhere. What conditions does the continuity requirement place on $f(r)$? Be sure the kernel also satisfies this continuity requirement at the origin. (Hint: it is sufficient to look at 1D with $h = 1$.)

   (d) Find a suitable piecewise cubic function $f(r)$ defined for $r \geq 0$ that satisfies all of these requirements.

   (e) Evaluate $c_1(h)$, $c_2(h)$, and $c_3(h)$.