Notation
Unit 1: Intro

- \(x, y, z\) data inputs/outputs
- \(i = 1, m\) subscript enumerates data (and thus rows of \(A\))
- \(f\) function of the data
- \(\hat{x}, \hat{y}, \hat{z}, \hat{f}, \hat{\phi}\) inference/approximation of same variables
- \(c\) unknown parameters to characterize functions
- \(k = 1, n\) subscript enumerates \(c\) (and thus columns of \(A\))
- A matrix (\(I\) identity), \(b\) right hand side (\(y\) used when it is the actual data \(y\))
- \(a_k\) column of \(A\)
- Quadratic Formula slide: uses standard notation for all variables
- \(\phi\) basis functions
- \(\theta\) pose parameters, \(\varphi\) collection of all vertex positions for a triangle surface mesh
- Cloth Slides: \(S, D\) functions, \(u, v\) texture space, \(n\) normal direction, \(I\) image data, \(h\) interpolation function
Unit 2: Linear Systems

- $a_{ik}$ elements of $A$
- $A^T$ transpose, $A^{-1}$ inverse
- $\hat{e}_i$ standard basis vectors
- Gaussian Elimination slides $m_{ik}$ special column, $M_{ik}, L_{ik}$ special matrices
- $I_{mxm}$ size $mxm$ identity
- $U$ upper triangular matrix, $L$ lower triangular matrix
- $\hat{c}$ transformed version of $c$
- $P$ permutation matrix (with its own special notation)
Unit 3: Understanding Matrices

- $\lambda$ eigenvalue (scalar)
- $v$ eigenvector, $u$ right eigenvector (both column vectors)
- $\alpha$ scalar
- $^*$ superscript is complex conjugate (for imaginary numbers)
- $i = \sqrt{-1}$ when dealing with complex numbers
- $\hat{c}, \hat{b}$ perturbed or transformed $b, c$
- $\hat{A}^{-1}, \hat{I}$ approximate version $A^{-1}, I$
- $U, V$ orthogonal (for SVD)
- $\Sigma$ diagonal (not necessarily square, potentially zeros on diagonal)
- $\sigma$ singular values
Unit 4: Special Matrices

- $\nu, u$ column vectors
- $u_k, v_k$ columns of $U, V$
- $\Lambda$ diagonal matrix of eigenvalues
- $l_{ik}$ element of $L$
- $\hat{A}$ approximation of $A$
Unit 5: Iterative Solvers

- $q$ superscript, integer for sequences/iterations (iterative solvers)
- $\epsilon$ small number
- $t$ time
- $X, V$ position and velocity
- $r, e$ residual and error (column vectors)
- $s$ search direction
- $\bar{S}$ column vector
- $\beta$ scalar
Unit 6: Local Approximations

- $h$ scalar (relatively small)
- $f^{(p)}$ parenthesis (integer) indicate taking $p$ derivatives
- $f'$ and $f''$ one derivative and two derivatives
- Cubic Splines Slide: special notation
- $p$ integer, polynomial degree, order of accuracy, etc.
- $w$ weighting function
Unit 7: Curse of Dimensionality

- $A, V$ area and volume
- $r$ radius
- $N$ integer, number of sample points
- $\vec{x}$ vector of data input to a function
Unit 8: Least Squares

- False Statements (first slide): $a, b$ scalars
- $D, \hat{D}$ diagonal matrices
Unit 9: Basic Optimization

• $F$ system of functions (output is a vector not a scalar)
• $\partial$ partial derivative symbol
• $J$ Jacobian matrix of all first partial derivatives
• $F'$ Jacobian of $F$
• $\nabla f$ gradient of scalar function $f$ (Jacobian transposed)
• $H$ matrix of all second partial derivatives of scalar function $f$ (Jacobian of gradient transposed)
• $c^*$ critical point (special value of $c$)
• $\tilde{A}, \tilde{b}, \tilde{c}$ matrix, and two vectors
Unit 10: Solving Least Squares

- $\Sigma$ diagonal invertible matrix (no zeros on the diagonal)
- $I_{n \times n}$ stresses the size of the identity as $n \times n$
- $\hat{b}_r, \hat{b}_z$ sub-vectors of $\hat{b}$ of shorter length (range and zero abbreviations)
- $Q$ orthogonal matrix
- $q_k$ column of $Q$
- $R$ upper triangular matrix
- $r_{ik}$ entry of $R$
- $\tilde{Q}$ submatrix
- **Householder slides**: $\hat{v}$ normal vector, $H$ householder matrix, $a$ column vector (all this notation is specialized)
Unit 11: Zero Singular Values

- $c_r, c_z$ sub-vectors of $\hat{c}$ of shorter length (range and zero abbreviations)
- $A^+$ pseudo-inverse of $A$
- $T$ matrix (for similarity transforms)
- **Power Method Slides:** $A^q$ and $\lambda^q$ are $A$ and $\lambda$ raised to the $q$ power (not an iteration as is the case for other $q$’s on these slides)
Unit 12: Regularization

• $c^*$ is an initial guess for $c$
• $r$ used in its geometric series capacity (a scalar)
• $\theta$ angle between two vectors
• $C, C^*$ curves (vertices connected by line segments)
Unit 13: Optimization

• $f$ briefly is allowed to be either vector valued (or stay scalar)
• $\hat{f}$ becomes the (scalar) cost function for optimization
• $F$ system of functions (gradient in the case of optimization)
Unit 14: Nonlinear Systems

• $c^*$ is a point to linearize about
• $d$ is for the standard derivative
• $t$ is an arbitrary variable
• $dt$ is a differential
• $\Delta$ finite size difference
• $g$ scalar function (that determines the line search parameter)
Unit 15: Root Finding

- \( \hat{g} \) modified \( g \)
- \( t \) search parameter in 1D, replacing \( \alpha \)
- \( t^* \) converged solution
- \( \hat{t} \) particular \( t \)
- \( C \) scalar
- \( p \) integer (power)
- \( g' \) derivative of \( g \)
- \( t_L, t_R \) interval bounds
- \( t_M \) interval midpoint
Unit 16: 1D Optimization

- $t_{\text{min}}, t_{M1}, t_{M2}$ more $t$ values
- $s$ scalar (interval size)
- $f, \tau$ scalars between 0 and 1
- $H_F$ is a $3^{\text{rd}}$ order tensor of $2^{\text{nd}}$ derivatives
- $OMG_f$ is a $3^{\text{rd}}$ order of $3^{\text{rd}}$ derivatives