Problems 1 & 2

These two problems are the same as those in the last year’s assignment 3. Please refer to the supplement handout www.stanford.edu/class/cs206/hw5-sol2.pdf.
Problem 3

In general, computing the outcome of an auction requires exponential time. In this problem, we make use of the fact that only a bid of consecutive months is accepted. The following is the simulation of the operation of the O(n^2) algorithm mentioned in the class:

Step 1: Bids for (Jan)

Bid 1: (Jan, $50) = $50
Max(Jan) = $50

Step 2: Bids for [Jan – Feb]

Max(Jan) + Bid 4: (Feb, $40) = $90
Bid 2: (Jan – Feb, $80) = $80
Max(Jan – Feb) = $90

Step 3: Bids for (Jan – Mar)

Max(Jan) + Bid 5: (Feb – Mar, $110) = $160
Max(Jan – Feb) + Bid 6: (Mar, $20) = $110
Bid 3: (Jan – Mar, $150) = $150
Max(Jan – Mar) = $160

Hence, the winning bids are Bid 1: (Jan, $50) and Bid 5: (Feb – Mar, $110).
Problem 4

a. The LP constraint for this problem in matrix form is as follows:

\[
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
1 & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix}
\leq
\begin{bmatrix}
1 \\
1
\end{bmatrix}
\]

where \(a\) and \(b\) (binary: \(1 = \text{yes}, 0 = \text{no}\)) determine whether Alice or Bob (or both) gets their bids respectively. Notice that the order of the rows of the bid matrix does not matter in this case.

b. A matrix is TU if and only if the determinants of all the square sub-matrices are \(-1, 0, \) or \(1\). The bid matrix in the problem is TU because:

1. the determinants of all the \(1 \times 1\) sub-matrices are either \(1\) or \(0\).
2. the determinant of the upper \(2 \times 2\) sub-matrix is \(1\) while that of the lower \(2 \times 2\) sub-matrix is \(-1\).