Lecture 8: MDPs II

- In the previous lecture, you probably had some model of the world (how far Mountain View is, how long biking, driving, and Caltrain take). But now, you should have no clue what’s going on. This is the setting of reinforcement learning. Now, you just have to try things and learn from your experience - that’s life!

- Last time, we talked about MDPs, which we can think of as graphs, where each node is either a state or a chance node \((s, o)\). Actions take us from states to chance nodes (which we choose), and transitions take us from chance nodes to states (which nature chooses according to the transition probabilities).

Review: MDPs

- Following a policy \(\pi\) produces a path (episode)
  \(s_0; a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n\)

- Value function \(V_\pi(s)\): expected utility if follow \(\pi\) from state \(s\)
  \[V_\pi(s) = \begin{cases} 
  0 & \text{if IsEnd}(s) \\
  \sum_a Q_\pi(s, a) & \text{otherwise.}
\end{cases}\]

- Q-value function \(Q_\pi(s, a)\): expected utility if first take action \(a\) from state \(s\) and then follow \(\pi\)
  \[Q_\pi(s, a) = \sum_a T(s, a, s') [\text{Reward}(s, a, s') + \gamma V_\pi(s')]\]
Given a policy \( \pi \) and an MDP, we can run the policy on the MDP yielding a sequence of states, action, rewards \( s_0, a_1, r_1, s_1, a_2, r_2, s_2, \ldots \). Formally, for each time step \( t \), \( a_t = \pi(s_{t-1}) \), and \( s_t \) is sampled with probability \( T(s_{t-1}, a_t, s_t) \). We call such a sequence an episode (a path in the MDP graph). This will be a central notion in this lecture.

Each episode (path) is associated with a utility, which is the discounted sum of rewards: \( u_t = r_1 + \gamma r_2 + \gamma^2 r_3 + \ldots \). It’s important to remember that the utility \( u_t \) is a random variable which depends on how the transitions were sampled.

The value of the policy (from state \( s_0 \)) is \( V_\pi(s_0) = E[u_t] \), the expected utility. In the last lecture, we worked with the values directly without worrying about the underlying random variables (but that will soon no longer be the case). In particular, we defined recurrences relating the value \( V_\pi(s) \) and Q-value \( Q_\pi(s,a) \), which represents the expected utility from starting at the corresponding nodes in the MDP graph.

Given these mathematical recurrences, we produced algorithms: policy evaluation computes the value of a policy, and value iteration computes the optimal policy.

In this lecture, we assume that we have an MDP where we neither know the transitions nor the reward functions. We are still trying to maximize expected utility, but we are in a much more difficult setting called reinforcement learning.

To put yourselves in the shoes of a reinforcement learner, try playing the game. You can either push the A button or the B button. Each of the two actions will take you to a new state and give you some reward.

This simple game illustrates some of the challenges of reinforcement learning: we should take good actions to get rewards, but in order to know which actions are good, we need to explore and try different actions.

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**Unknown transitions and rewards**

**Definition: Markov decision process**

- States: the set of states
- \( s_{\text{start}} \in \text{States} \): starting state
- Actions(s): possible actions from state \( s \)
- IsEnd(s): whether at end of game
- \( 0 \leq \gamma \leq 1 \): discount factor (default: 1)

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**Mystery game**

**Example: mystery buttons**

- For each round \( r = 1, 2, \ldots \):
  - You choose A or B.
  - You move to a new state and get some rewards.

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**Roadmap**

- Reinforcement learning
  - Monte Carlo methods
  - Bootstrapping methods
  - Covering the unknown

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**Summary**
From MDPs to reinforcement learning

Markov decision process (offline)
- Have mental model of how the world works.
- Find policy to collect maximum rewards.

Reinforcement learning (online)
- Don’t know how the world works.
- Perform actions in the world to find out and collect rewards.

Reinforcement learning framework

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<th>environment</th>
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<td>action $a$</td>
<td>reward $r$, new state $s'$</td>
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Algorithm: reinforcement learning template

For $t = 1, 2, 3, \ldots$
- Choose action $a_t = \pi_{act}(s_{t-1})$ (how?)
- Receive reward $r_t$ and observe new state $s_t$
- Update parameters (how?)

Volcano crossing

Run (or press ctrl-enter)

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Utility: 2

- An important distinction between solving MDPs (what we did before) and reinforcement learning (what we will do now) is that the former is offline and the latter is online.
- In the former case, you have a mental model of how the world works. You go lock yourself in a room, think really hard, come up with a policy. Then you come out and use it to act in the real world.
- In the latter case, you don’t know how the world works, but you only have one life, so you just have to go out into the real world and learn how it works from experiencing it and trying to take actions that yield high rewards.
- At some level, reinforcement learning is really the way humans work: we go through life, taking various actions, getting feedback. We get rewarded for doing well and learn along the way.

- To make the framework clearer, we can think of an agent (the reinforcement learning algorithm) that repeatedly chooses an action $a_t$ to perform in the environment, and receives some reward $r_t$, and information about the new state $s_t$.
- There are two questions here: how to choose actions (what is $\pi_{act}$) and how to update the parameters. We will first talk about updating parameters (the learning part), and then come back to action selection later.

- Recall the volcano crossing example from the previous lecture. Each square is a state. From each state, you can take one of four actions to move to an adjacent state: north (N), east (E), south (S), or west (W). If you try to move off the grid, you remain in the same state. The starting state is (2,1), and the end states are the four marked with red or green rewards. Transitions from $(s, a)$ lead where you expect with probability $1 - \text{slipProb}$ and to a random adjacent square with probability $\text{slipProb}$.
- If we solve the MDP using value iteration (by setting $\text{sMax}$ to 10), we will find the best policy (which is to head for the 20). Of course, we can’t solve the MDP if we don’t know the transitions or rewards.
- If you set $\text{sMax}$ to zero, we start off with a random policy. Try pressing the Run button to generate fresh episodes. How can we learn from this data and improve our policy?
Roadmap

- Reinforcement learning
- Monte Carlo methods
- Bootstrapping methods
- Covering the unknown

Summary

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Model-based Monte Carlo

Data: \(s_0, a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n\)

Key idea: model-based learning

Estimate the MDP: \(T(s, a, s')\) and \(\text{Reward}(s, a, s')\)

Transitions:

\[
\hat{T}(s, a, s') = \frac{\text{# times } (s, a, s') \text{ occurs}}{\text{# times } (s, a) \text{ occurs}}
\]

Rewards:

\[
\hat{\text{Reward}}(s, a, s') = r \text{ in } (s, a, r, s')
\]

- The first idea is called model-based Monte Carlo, where we try to estimate the model (transitions and rewards) using Monte Carlo simulation.
- Monte Carlo is a standard way to estimate the expectation of a random variable by taking an average over samples of that random variable.
- Here, the data used to estimate the model is the sequence of states, actions, and rewards in the episode.
- Note that the samples being averaged are not independent (because they come from the same episode), but they do come from a Markov chain, so it can be shown that these estimates converge to the expectations by the ergodic theorem (a generalization of the law of large numbers for Markov chains).
- But there is one important caveat...

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Problem

Data (following policy \(\pi(s) = \text{stay}\)):

\[\text{[in; stay, 4, end]}\]

- Estimates converge to true values (under certain conditions)
- With estimated MDP \((\hat{T}, \hat{\text{Reward}})\), compute policy using value iteration

Problem: won’t even see \((s, a)\) if \(a \neq \pi(s)\) \((a = \text{quit})\)

Key idea: exploration

To do reinforcement learning, need to explore the state space.

Solution: need \(\pi\) to explore explicitly (more on this later)

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From model-based to model-free

\[ \hat{Q}_{opt}(s,a) = \sum_{s'} \hat{T}(s,a,s') \left[ \hat{R}(s,a,s') + \gamma \hat{V}_{opt}(s') \right] \]

All that matters for prediction is (estimate of) \( \hat{Q}_{opt}(s,a) \).

**Key idea: model-free learning**

Try to estimate \( \hat{Q}_{opt}(s,a) \) directly.

---

Model-free Monte Carlo

Data (following policy \( \pi \)):

\[ s_0; a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n \]

Recall:

\( Q_{\pi}(s,a) \) is expected utility starting at \( s \), first taking action \( a \), and then following policy \( \pi \)

Utility:

\[ u_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \cdots \]

Estimate:

\[ \hat{Q}_{\pi}(s,a) = \text{average of } u_t \text{ where } s_{t-1} = s, a_t = a \]

(and \( s,a \) doesn’t occur in \( s_0, \ldots, s_{t-2} \))

---

- Taking a step back, if our goal is to just find good policies, all we need is to get a good estimate of \( \hat{Q}_{opt} \).
- From that perspective, estimating the model (transitions and rewards) was just a means towards an end.
- Why not just cut to the chase and estimate \( \hat{Q}_{opt} \) directly? This is called **model-free** learning, where we don’t explicitly estimate the transitions and rewards.

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Model-free Monte Carlo (equivalences)

Data (following policy \( \pi(s) = \text{stay} \)):

\[ \text{in; stay, 4, in; stay, 4, in; stay, 4, in; stay, 4, end} \]

Note: we are estimating \( Q_{\pi} \) now, not \( \hat{Q}_{opt} \)

**Definition: on-policy versus off-policy**

- **On-policy**: estimate the value of data-generating policy
- **Off-policy**: estimate the value of another policy

---

Model-free Monte Carlo (equivalences)

Data (following policy \( \pi \)):

\[ s_0; a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n \]

- **Original formulation**

\[ \hat{Q}_{\pi}(s,a) = \text{average of } u_t \text{ where } s_{t-1} = s, a_t = a \]

- **Equivalent formulation** (convex combination)

\[ \hat{Q}_{\pi}(s,a) \leftarrow (1-\eta) \hat{Q}_{\pi}(s,a) + \eta u \]

[whiteboard: \( u_1, u_2, u_3 \)]
• Over the next few slides, we will interpret model-free Monte Carlo in several ways. This is the same algorithm, just viewed from different perspectives. This will give us some more intuition and allow us to develop other algorithms later.

• The first interpretation is thinking in terms of interpolation. Instead of thinking of averaging as a batch operation that takes a list of numbers (realizations of \( u \)) and computes the mean, we can view it as an iterative procedure for building the mean as new numbers are coming in.

• In particular, it’s easy to work out for a small example that averaging is equivalent to just interpolating between the old value \( \hat{Q}(s,a) \) (current estimate) and the new value \( u \) (data). The interpolation ratio \( \eta \) is set carefully so that \( u \) contributes exactly the right amount to the average.

• Advanced: in practice, we would constantly improve the policy \( \pi \) constantly over time. In this case, we might want to set \( \eta \) to something that doesn’t decay as quickly (for example, \( \eta = 1/\sqrt{t} \) updates to \( \pi \)). This rate implies that a new example contributes more than an old example, which is desirable so that \( \hat{Q}(s,a) \) reflects the more recent policy rather than the old policy.

• The second equivalent formulation is making the update look like a stochastic gradient update. Indeed, if we think about each \((s,a,u)\) as an example (where \(s,a\) is the input and \(u\) is the output), then the model-free Monte Carlo is just performing stochastic gradient descent on a least squares regression problem, where the weight vector is \( Q \) (which has dimensionality \( |S||A| \)) and there is one feature template \((s,a)\).

• The stochastic gradient descent view will become particularly relevant when we use non-trivial features on \((s,a)\).

Let’s run model-free Monte Carlo on the volcano crossing example. \texttt{slipProb} is zero to make things simpler. We are showing the Q-values: for each state, we have four values, one for each action.

Here, our exploration policy is one that chooses an action uniformly at random.

Try pressing “Run” multiple times to understand how the Q-values are set.

Then try increasing \texttt{slipProb}, and seeing how the Q-values of this policy become more accurate.

You will notice that a random policy has a very hard time reaching the 20.
Using the utility

Data (following policy $\pi(s) = \text{stay}$):

\[
\begin{align*}
\text{[in; stay, 4, end]} & \quad u = 4 \\
\text{[in; stay, 4, in; stay, 4, end]} & \quad u = 8 \\
\text{[in; stay, 4, in; stay, 4, in; stay, 4, end]} & \quad u = 12 \\
\text{[in; stay, 4, in; stay, 4, in; stay, 4, in; stay, 4, end]} & \quad u = 16
\end{align*}
\]

Algorithm: model-free Monte Carlo

On each $(s, a, u)$:

\[
\hat{Q}_\pi(s, a) \leftarrow (1 - \eta) \hat{Q}_\pi(s, a) + \eta u
\]

Using the reward + Q-value

Current estimate: $\hat{Q}_\pi(s, \text{stay}) = 11$

Data (following policy $\pi(s) = \text{stay}$):

\[
\begin{align*}
\text{[in; \text{stay, 4, in; } \text{stay, 4, end}] } & \quad 4 + 0 \\
\text{[in; \text{stay, 4, in; stay, 4, end}] } & \quad 4 + 11 \\
\text{[in; \text{stay, 4, in; stay, 4, in; stay, 4, end}] } & \quad 4 + 11 \\
\text{[in; \text{stay, 4, in; stay, 4, in; stay, 4, in; stay, 4, end}] } & \quad 4 + 11
\end{align*}
\]

Algorithm: SARSA

On each $(s, a, r, s', a')$:

\[
\hat{Q}_\pi(s, a) \leftarrow (1 - \eta) \hat{Q}_\pi(s, a) + \eta \left( r + \gamma \hat{Q}_\pi(s', a') \right)
\]

Model-free Monte Carlo versus SARSA

Key idea: bootstrapping

SARSA uses estimate $\hat{Q}_\pi(s, a)$ instead of just raw data $u$.

- The main advantage that SARSA offers over model-free Monte Carlo is that we don’t have to wait until the end of the episode to update the Q-value.
- If the estimates are already pretty good, then SARSA will be more reliable since $u$ is based on only one path whereas $\hat{Q}_\pi(s, a')$ is based on all the ones that the learner has seen before.
- Advanced: We can actually interpolate between model-free Monte Carlo (all rewards) and SARSA (one reward). For example, we could update towards $r + \gamma \hat{Q}_\pi(s, a') + \hat{Q}_\pi(s', a)$ (two rewards). We can even combine all of these updates, which results in an algorithm called SARSA($\lambda$), where $\lambda$ determines the relative weighting of these targets. See the Sutton/Barto reinforcement learning book (chapter 7) for an excellent introduction.
- Advanced: There is also a version of these algorithms that estimates the value function $V_{\pi}$ instead of $Q_{\pi}$. Value functions aren’t enough to choose actions unless you actually know the transitions and rewards. Nonetheless, these are useful in game playing where we actually know the transition and rewards, but the state space is just too large to compute the value function exactly.

Question

Which of the following algorithms allows you to estimate $Q_{\text{opt}}(s, a)$ (select all that apply)?

- model-based Monte Carlo
- model-free Monte Carlo
- SARSA
• Model-based Monte Carlo estimates the transitions and rewards, which fully specifies the MDP. With the MDP, you can estimate anything you want, including computing $Q_{\pi}(s,a)$.
• Model-free Monte Carlo and SARSA are on-policy algorithms, so they only give you $\hat{Q}_\pi(s,a)$, which is specific to a policy $\pi$. These will not provide direct estimates of $Q_{\text{opt}}(s,a)$.

- Recall our goal is to get an optimal policy, which means estimating $Q_{\text{opt}}$.
- The situation is as follows: Our two methods (model-free Monte Carlo and SARSA) are model-free, but only produce estimates $Q_{\pi}$. We have one algorithm, model-based Monte Carlo, which can be used to produce estimates of $Q_{\text{opt}}$, but is model-based. Can we get an estimate of $Q_{\text{opt}}$ in a model-free manner?
- The answer is yes, and Q-learning is an off-policy algorithm that accomplishes this.
- One can draw an analogy between reinforcement learning algorithms and the classic MDP algorithms. MDP algorithms are offline, RL algorithms are online. In both cases, algorithms either output the Q-values for a fixed policy or the optimal Q-values.

- To derive Q-learning, it is instructive to look back at the MDP recurrence for $Q_{\text{opt}}$. There are several changes that take us from the MDP recurrence to Q-learning. First, we don’t have an expectation over $s'$, but only have one sample $s'$.
- Second, because of this, we don’t want to just replace $Q_{\text{opt}}(s,a)$ with the target value, but want to interpolate between the old value (prediction) and the new value (target).
- Third, we replace the actual reward $\text{Reward}(s,a,s')$ with the observed reward $r$ (when the reward function is deterministic, the two are the same).
- Finally, we replace $V_{\text{opt}}(s')$ with our current estimate $\hat{V}_{\text{opt}}(s')$.
- Importantly, the estimated optimal value $\hat{V}_{\text{opt}}(s')$ involves a maximum over actions rather than taking the action of the policy. This max over $a'$ rather than taking the $a'$ based on the current policy is the principle difference between Q-learning and SARSA.
Volcanic SARSA and Q-learning

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Utility: 2

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Roadmap

- Reinforcement learning
- Monte Carlo methods
- Bootstrapping methods
- Covering the unknown

Summary

Exploration

Algorithm: reinforcement learning template

For \( t = 1, 2, 3, \ldots \)

- Choose action \( a_t = \pi_{act}(s_{t-1}) \) (how?)
- Receive reward \( r_t \) and observe new state \( s_t \)
- Update parameters (how?)

\( s_0; a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n \)

Which exploration policy \( \pi_{act} \) to use?

No exploration, all exploitation

Attempt 1: Set \( \pi_{act}(s) = \arg \max_{a \in \text{Actions}(s)} \hat{Q}_{opt}(s, a) \)

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Average utility: 1.95

Problem: \( \hat{Q}_{opt}(s, a) \) estimates are inaccurate, too greedy!

- Let us try SARSA and Q-learning on the volcanic example.
- If you increase numEpisodes to 1000, SARSA will behave very much like model-free Monte Carlo, computing the value of the random policy.
- However, note that Q-learning is computing an estimate of \( \hat{Q}_{opt}(s, a) \), so the resulting Q-values will be very different. The average utility will not change since we are still following and being evaluated on the same random policy. This is an important point for off-policy methods: the online performance (average utility) is generally a lot worse and not representative of what the model has learned, which is captured in the estimated Q-values.

- We have so far given many algorithms for updating parameters (i.e., \( \hat{Q}_t(s, a) \) or \( \hat{Q}_{opt}(s, a) \)). If we were doing supervised learning, we’d be done, but in reinforcement learning, we need to actually determine our exploration policy \( \pi_{act} \) to collect data for learning. Recall that we need to somehow make sure we get information about each \((s, a)\).
- We will discuss two complementary ways to get this information: (i) explicitly explore \((s, a)\) or (ii) explore \((s, a)\) implicitly by actually exploring \((s', a')\) with similar features and generalizing.
- These two ideas apply to many RL algorithms, but let us specialize to Q-learning.
The naive solution is to explore using the optimal policy according to the estimated Q-value $\hat{Q}_{opt}(s, a)$.

But this fails horribly. In the example, once the agent discovers that there is a reward of 2 to be gotten by going south that becomes its optimal policy and it will not try any other action. The problem is that the agent is being too greedy.

In the demo, if multiple actions have the same maximum Q-value, we choose randomly. Try clicking “Run” a few times, and you’ll end up with minor variations.

Even if you increase numEpisodes to 10000, nothing new gets learned.

We can go to the other extreme and use an exploration policy that always chooses a random action. It will do a much better job of exploration, but it doesn’t exploit what it learns and ends up with a very low utility.

It is interesting to note that the value (average over utilities across all the episodes) can be quite small and yet the Q-values can be quite accurate. Recall that this is possible because Q-learning is an off-policy algorithm.

Epsilon-greedy

Algorithm: epsilon-greedy policy

$$
\pi_{act}(s) = \begin{cases} 
\arg \max_{a \in \text{Actions}} \hat{Q}_{opt}(s, a) & \text{probability } 1 - \epsilon, \\
\text{random from Actions}(s) & \text{probability } \epsilon.
\end{cases}
$$

No exploitation, all exploration

Attempt 2: Set $\pi_{act}(s) = \text{random from } \text{Actions}(s)$

Run (or press ctrl-enter)

Problem: average utility is low because exploration is not guided

Exploration/exploitation tradeoff

Key idea: balance

Need to balance exploration and exploitation.

Examples from life: restaurants, routes, research

The natural thing to do when you have two extremes is to interpolate between the two. The result is the epsilon-greedy algorithm which explores with probability $\epsilon$ and exploits with probability $1 - \epsilon$.

It is natural to let $\epsilon$ decrease over time. When you’re young, you want to explore a lot ($\epsilon = 1$). After a certain point, when you feel like you’ve seen all there is to see, then you start exploiting ($\epsilon = 0$).

For example, we let $\epsilon = 1$ for the first third of the episodes, $\epsilon = 0.5$ for the second third, and $\epsilon = 0$ for the final third. This is not the optimal schedule. Try playing around with other schedules to see if you can do better.
Generalization

Problem: large state spaces, hard to explore

Q-learning

Stochastic gradient update:

\[ \hat{Q}_{\text{opt}}(s, a) \leftarrow \hat{Q}_{\text{opt}}(s, a) - \eta \left[ \hat{Q}_{\text{opt}}(s, a) - (r + \gamma \hat{V}_{\text{opt}}(s')) \right] \]

This is rote learning: every \( \hat{Q}_{\text{opt}}(s, a) \) has a different value

Problem: doesn’t generalize to unseen states/actions

Function approximation

Key idea: linear regression model

Define features \( \phi(s, a) \) and weights \( w \):

\[ \hat{Q}_{\text{opt}}(s, a; w) = w \cdot \phi(s, a) \]

Example: features for volcano crossing

\[ \begin{align*}
\phi_1(s, a) &= 1[a = W] \\
\phi_2(s, a) &= 1[a = E] \\
\phi_3(s, a) &= 1[s = (5, \ast)] \\
\phi_4(s, a) &= 1[s = (\ast, 6)] \\
\end{align*} \]
**Function approximation**

**Algorithm: Q-learning with function approximation**

On each \((s, a, r, s')\):

\[
w \leftarrow w - \eta \left[ \hat{Q}_{opt}(s, a; w) \right] \phi(s, a) - (r + \gamma \hat{V}_{opt}(s')) \phi(s, a)
\]

Implied objective function:

\[
\left( \hat{Q}_{opt}(s, a; w) - (r + \gamma \hat{V}_{opt}(s')) \right)^2
\]

**Covering the unknown**

**Epsilon-greedy**: balance the exploration/exploitation tradeoff

**Function approximation**: can generalize to unseen states

- We now turn our linear regression into an algorithm. Here, it is useful to adopt the stochastic gradient view of RL algorithms, which we developed a while back.
- We just have to write down the least squares objective and then compute the gradient with respect to \(w\) now instead of \(Q_{opt}\). The chain rule takes care of the rest.

**Summary so far**

- **Online setting**: learn and take actions in the real world!
- **Exploration/exploitation tradeoff**
- **Monte Carlo**: estimate transitions, rewards, Q-values from data
- **Bootstrapping**: update towards target that depends on estimate rather than just raw data

**Roadmap**

- Reinforcement learning
- Monte Carlo methods
- Bootstrapping methods
- Covering the unknown

**Summary**
### Challenges in reinforcement learning

**Binary classification (sentiment classification, SVMs):**
- Stateless, full feedback

**Reinforcement learning (flying helicopters, Q-learning):**
- Stateful, partial feedback

**Key idea: partial feedback**
- Only learn about actions you take.

**Key idea: state**
- Rewards depend on previous actions ⇒ can have delayed rewards.

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### Deep reinforcement learning

- If we compare simple supervised learning (e.g., binary classification) and reinforcement learning, we see that there are two main differences that make learning harder.
- First, reinforcement learning requires the modeling of state. State means that the rewards across time steps are related. This results in delayed rewards, where we take an action and don’t see the ramifications of it until much later.
- Second, reinforcement learning requires dealing with partial feedback (rewards). This means that we have to actively explore to acquire the necessary feedback.
- There are two problems that move towards reinforcement learning, each on a different axis. Structured prediction introduces the notion of state, but the problem is made easier by the fact that we have full feedback, which means that for every situation, we know which action sequence is the correct one; there is no need for exploration, we just have to update our weights to favor that correct path.
- Multi-armed bandits require dealing with partial feedback, but do not have the complexities of state. One can think of a multi-armed bandit problem as an MDP with unknown random rewards and one state. Exploration is necessary, but there is no temporal depth to the problem.

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### States and information

<table>
<thead>
<tr>
<th>Stateless</th>
<th>State</th>
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</thead>
<tbody>
<tr>
<td>Supervised learning (binary classification)</td>
<td>Supervised learning (structured prediction)</td>
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<td>Partial feedback</td>
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<td>Reinforcement learning</td>
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### Deep reinforcement learning

- **Playing Atari [Google DeepMind, 2013]:**
  - last 4 frames (images) ⇒ 3-layer NN ⇒ keystroke
  - $\epsilon$-greedy, train over 10M frames with 1M replay memory
  - Human-level performance on some games (breakout), less good on others (space invaders)

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### Deep reinforcement learning

- **Policy gradient:** train a policy $\pi(a \mid s)$ (say, a neural network) to directly maximize expected reward

- Andrej Karpathy’s blog post
  
  http://karpathy.github.io/2016/05/31/rl
• One other major class of algorithms we will not cover in this class is **policy gradient**. Whereas Q-learning attempts to estimate the value of the optimal policy, policy gradient methods optimize the policy to maximize expected reward, which is what we care about. Recall that when we went from model-based methods (which estimated the transition and reward functions) to model-free methods (which estimated the Q function), we moved closer to the thing that we want. Policy gradient methods take this farther and just focus on the only object that really matters at the end of the day, which is the policy that an agent follows.

• Policy gradient methods have been quite successful. For example, it was one of the components of AlphaGo, Google DeepMind’s program that beat the world champion at Go. One can also combine value-based methods with policy-based methods in actor-critic methods to get the best of both worlds.

• There is a lot more to say about deep reinforcement learning. If you wish to learn more, Andrej Karpathy’s blog post offers a nice introduction.

• There are many other applications of RL, which range from robotics to game playing to other infrastructural tasks. One could say that RL is so general that anything can be cast as an RL problem. Deep RL — the use of powerful neural networks with increased compute — has vastly expanded the realm of problems which are solvable by RL.

• For a while, RL only worked for small toy problems or settings where there were a lot of prior knowledge / constraints. Deep RL — the use of powerful neural networks with increased compute — has vastly expanded the realm of problems which are solvable by RL.

Applications

Autonomous helicopters: control helicopter to do maneuvers in the air

Backgammon: TD-Gammon plays 1-2 million games against itself, human-level performance

Elevator scheduling; send which elevators to which floors to maximize throughput of building

Managing datacenters; actions: bring up and shut down machine to minimize time/cost

Markov decision processes: against nature (e.g., Blackjack)

Adversarial games: against opponent (e.g., chess)