Natural Language Processing with Deep Learning
CS224N/Ling284

Christopher Manning
Lecture 2: Word Vectors, Word Senses, and Neural Classifiers
Lecture 2: Word Vectors, Word Senses, and Neural Network Classifiers

1. Course organization (3 mins)
2. Finish looking at word vectors and word2vec (15 mins)
3. Can we capture the essence of word meaning more effectively by counting? (10 mins)
4. Evaluating word vectors (10 mins)
5. Word senses (8 mins)
6. Review of classification and how neural nets differ (14 mins)
7. Introducing neural networks (10 mins)

Key Goal: To be able to read word embeddings papers by the end of class
1. Course Organization

• **Come to office hours/help sessions!**
  • They started yesterday
  • **Today, after class: 6:00–8:50pm in 260-012, Pigott Hall** (or Zoom link on Canvas)
  • Come to discuss **final project ideas** as well as the assignments
  • Try to come early, often and off-cycle!

• **TA office hours: 3-hour blocks Mon–Fri (and hopefully Sat), with multiple TAs**
  • Just show up! Our friendly course staff will be on hand to assist you!

• **Chris’s office hours:**
  • Mon 2:45–5pm. In person only. You can come along this coming Monday!
  • Book 15 min slot on **Calendly** (coming soon – I’ll send an Ed post)
GPUs for Deep Learning Projects

- We hope to get people access to Azure GPUs to help with projects, etc.
- But it’s gotten really difficult to get free GPUs; there’s a worldwide GPU shortage
- We will send more info about this soon, asking people to sign up for Azure accounts.
- Please read and do this! ASAP!
- A lot of lead time is required – like several weeks
  - Last year, lots of people put it off because they didn’t need it right now, and then GPUs just weren’t available to them when they needed them
- Also consider if there are other GPUs that you can use for the class
2. Review: Main idea of word2vec

• Start with random word vectors
• Iterate through each word position in the whole corpus
• Try to predict surrounding words using word vectors: $P(o|c) = \frac{\exp(u^T_w v_c)}{\sum_{w \in V} \exp(u^T_w v_c)}$

- Learning: Update vectors so they can predict actual surrounding words better
- Doing no more than this, this algorithm learns word vectors that capture well word similarity and meaningful directions in a word space!
Word2vec parameters ... and computations

\[ U \begin{bmatrix} \bullet & \bullet & \bullet & \bullet \end{bmatrix} \]

\[ V \begin{bmatrix} \bullet & \bullet & \bullet & \bullet \end{bmatrix} \]

\[ U. v_4^T \]

outside center 

dot product probabilities

\[ \text{softmax}(U. v_4^T) \]

“Bag of words” model! The model makes the same predictions at each position

We want a model that gives a reasonably high probability estimate to all words that occur in the context (at all often)
Word2vec maximizes objective function by putting similar words nearby in space.
The skip-gram model with negative sampling (HW2)

• The normalization term is computationally expensive (when many output classes):

\[
P(o|c) = \frac{\exp(u_o^T v_c)}{\sum_{w \in V} \exp(u_w^T v_c)}
\]

• Hence, in standard word2vec and HW2 you implement the skip-gram model with negative sampling

• Main idea: train binary logistic regressions to differentiate a true pair (center word and a word in its context window) versus several “noise” pairs (the center word paired with a random word)
Word2vec algorithm family (Mikolov et al. 2013): More details

Why two vectors? → Easier optimization. Average both at the end
  • But can implement the algorithm with just one vector per word ... and it helps a bit

Two model variants:
  1. Skip-grams (SG)
     Predict context (“outside”) words (position independent) given center word
  2. Continuous Bag of Words (CBOW)
     Predict center word from (bag of) context words

We presented: Skip-gram model

Loss functions for training:
  1. Naïve softmax (simple but expensive loss function, when many output classes)
  2. More optimized variants like hierarchical softmax
  3. Negative sampling

So far, we explained naïve softmax
The skip-gram model with negative sampling (HW2)

- Introduced in: “Distributed Representations of Words and Phrases and their Compositionality” (Mikolov et al. 2013)
- Overall objective function (they maximize):
  \[ J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J_t(\theta) \]
  \[ J_t(\theta) = \log \sigma \left( u_o^T v_c \right) + \sum_{i=1}^{k} \mathbb{E}_{j \sim P(w)} \left[ \log \sigma \left( -u_j^T v_c \right) \right] \]

- The logistic/sigmoid function: \( \sigma(x) = \frac{1}{1 + e^{-x}} \) (we’ll become good friends soon)
- We maximize the probability of two words co-occurring in first log and minimize probability of noise words in second part
The skip-gram model with negative sampling (HW2)

• Using notation consistent with this class and HW2:

\[
J_{\text{neg-sample}}(u_o, v_c, U) = -\log \sigma(u_o^T v_c) - \sum_{k \in \{K \text{ sampled indices}\}} \log \sigma(u_k^T v_c)
\]

• We take \(k\) negative samples (using word probabilities)
• Maximize probability that real outside word appears; minimize probability that random words appear around center word

• Sample with \(P(w) = U(w)^{3/4}/Z\), the unigram distribution \(U(w)\) raised to the 3/4 power (We provide this function in the starter code).
• The power makes less frequent words be sampled more often
Stochastic gradients with negative sampling [aside]

- We iteratively take gradients at each window for SGD
- In each window, we only have at most $2m + 1$ words plus $2km$ negative words with negative sampling, so $\nabla_{\theta} J_t(\theta)$ is very sparse!

$$\nabla_{\theta} J_t(\theta) = \begin{bmatrix} 0 \\ \vdots \\ \nabla_{u_{like}} \\ \vdots \\ 0 \\ \nabla_{u_I} \\ \vdots \\ \nabla_{u_{learning}} \\ \vdots \end{bmatrix} \in \mathbb{R}^{2dV}$$
Stochastic gradients with with negative sampling [aside]

• We might only update the word vectors that actually appear!

• Solution: either you need sparse matrix update operations to only update certain rows of full embedding matrices $U$ and $V$, or you need to keep around a hash for word vectors

• If you have millions of word vectors and do distributed computing, it is important to not have to send gigantic updates around!

This is also a particular issue with more advanced optimization methods in the Adagrad family!
3. Why not capture co-occurrence counts directly?

There’s something weird about iterating through the whole corpus (perhaps many times); why don’t we just accumulate all the statistics of what words appear near each other?!?

Building a co-occurrence matrix $X$

- 2 options: windows vs. full document
- Window: Similar to word2vec, use window around each word $\rightarrow$ captures some syntactic and semantic information (“word space”)
- Word-document co-occurrence matrix will give general topics (all sports terms will have similar entries) leading to “Latent Semantic Analysis” (“document space”)

Example: Window based co-occurrence matrix

- Window length 1 (more common: 5–10)
- Symmetric (irrelevant whether left or right context)

- Example corpus:
  - I like deep learning
  - I like NLP
  - I enjoy flying

<table>
<thead>
<tr>
<th>counts</th>
<th>I</th>
<th>like</th>
<th>enjoy</th>
<th>deep</th>
<th>learning</th>
<th>NLP</th>
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</tbody>
</table>
Co-occurrence vectors

• Simple count co-occurrence vectors
  • Vectors increase in size with vocabulary
  • Very high dimensional: require a lot of storage (though sparse)
  • Subsequent classification models have sparsity issues → Models are less robust

• Low-dimensional vectors
  • Idea: store “most” of the important information in a fixed, small number of dimensions: a dense vector
  • Usually 25–1000 dimensions, similar to word2vec
  • How to reduce the dimensionality?
Classic Method: Dimensionality Reduction on $X$ (HW1)

Singular Value Decomposition of co-occurrence matrix $X$

Factorizes $X$ into $U\Sigma V^T$, where $U$ and $V$ are orthonormal

$$X^k = U \Sigma V^T$$

Retain only $k$ singular values, in order to generalize.

$\hat{X}$ is the best rank $k$ approximation to $X$, in terms of least squares.

Classic linear algebra result. Expensive to compute for large matrices.
Hacks to X (several used in Rohde et al. 2005 in COALS)

• Running an SVD on raw counts doesn’t work well!!!

• Scaling the counts in the cells can help a lot
  • Problem: function words (the, he, has) are too frequent → syntax has too much impact. Some fixes:
    • log the frequencies
    • min(X, t), with t ≈ 100
    • Ignore the function words

• Ramped windows that count closer words more than further away words
• Use Pearson correlations instead of counts, then set negative values to 0
• Etc.
Interesting semantic patterns emerge in the scaled vectors

COALS model from Rohde et al. ms., 2005. An Improved Model of Semantic Similarity Based on Lexical Co-Occurrence
**GloVe** [Pennington, Socher, and Manning, EMNLP 2014]:

Encoding meaning components in vector differences

Q: How can we capture ratios of co-occurrence probabilities as linear meaning components in a word vector space?

A: Log-bilinear model:

\[ w_i \cdot w_j = \log P(i|j) \]

with vector differences

\[ w_x \cdot (w_a - w_b) = \log \frac{P(x|a)}{P(x|b)} \]

**Loss:**

\[ J = \sum_{i,j=1}^{V} f \left( X_{ij} \right) \left( w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij} \right)^2 \]

- Fast training
- Scalable to huge corpora
4. How to evaluate word vectors?

- Related to general evaluation in NLP: Intrinsic vs. extrinsic
- Intrinsic:
  - Evaluation on a specific/intermediate subtask
  - Fast to compute
  - Helps to understand that system
  - Not clear if really helpful unless correlation to real task is established
- Extrinsic:
  - Evaluation on a real task
  - Can take a long time to compute accuracy
  - Unclear if the subsystem is the problem or its interaction or other subsystems
  - If replacing exactly one subsystem with another improves accuracy → Winning!
Intrinsic word vector evaluation

• Word Vector Analogies

\[
\begin{align*}
\text{man} : \text{woman} &:: \text{king} : ? \\
\text{a} : \text{b} &:: \text{c} : ?
\end{align*}
\]

• Evaluate word vectors by how well their cosine distance after addition captures intuitive semantic and syntactic analogy questions

• Discarding the input words from the search (!)

• Problem: What if the information is there but not linear?
GloVe Visualization
Meaning similarity: Another intrinsic word vector evaluation

- Word vector distances and their correlation with human judgments

<table>
<thead>
<tr>
<th>Word 1</th>
<th>Word 2</th>
<th>Human (mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>tiger</td>
<td>cat</td>
<td>7.35</td>
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<tr>
<td>tiger</td>
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Correlation evaluation

- Word vector distances and their correlation with human judgments

<table>
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<th>Model</th>
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<th>RG</th>
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<th>RW</th>
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<tbody>
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<tr>
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<td>65.6</td>
<td>68.2</td>
<td>57.0</td>
<td>32.5</td>
</tr>
<tr>
<td>SG‡</td>
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<td>CBOW*</td>
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<td>68.4</td>
<td>79.6</td>
<td>75.4</td>
<td>59.4</td>
<td>45.5</td>
</tr>
</tbody>
</table>

- Some ideas from Glove paper have been shown to improve skip-gram (SG) model also (e.g., average both vectors)
Extrinsic word vector evaluation

- One example where good word vectors should help directly: **named entity recognition**: identifying references to a person, organization or location: Chris Manning lives in Palo Alto.

<table>
<thead>
<tr>
<th>Model</th>
<th>Dev</th>
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<th>MUC7</th>
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<tr>
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<td>78.7</td>
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<td>CW</td>
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<tr>
<td>CBOW</td>
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<tr>
<td>GloVe</td>
<td><strong>93.2</strong></td>
<td>88.3</td>
<td><strong>82.9</strong></td>
<td><strong>82.2</strong></td>
</tr>
</tbody>
</table>

- Subsequent NLP tasks in this class are other examples. So, more examples soon.
5. Word senses and word sense ambiguity

- Most words have lots of meanings!
  - Especially common words
  - Especially words that have existed for a long time

- Example: **pike**

- Does one vector capture all these meanings or do we have a mess?
pike

- A sharp point or staff
- A type of elongated fish
- A railroad line or system
- A type of road
- The future (coming down the pike)
- A type of body position (as in diving)
- To kill or pierce with a pike
- To make one’s way (pike along)
- In Australian English, pike means to pull out from doing something: I reckon he could have climbed that cliff, but he piked!
Improving Word Representations Via Global Context And Multiple Word Prototypes (Huang et al. 2012)

- Idea: Cluster word windows around words, retrain with each word assigned to multiple different clusters bank₁, bank₂, etc.
Linear Algebraic Structure of Word Senses, with Applications to Polysemy (Arora, ..., Ma, ..., TACL 2018)

- Different senses of a word reside in a linear superposition (weighted sum) in standard word embeddings like word2vec
  \[ v_{\text{pike}} = \alpha_1 v_{\text{pike}_1} + \alpha_2 v_{\text{pike}_2} + \alpha_3 v_{\text{pike}_3} \]
  - Where \( \alpha_1 = \frac{f_1}{f_1 + f_2 + f_3} \), etc., for frequency \( f \)
- Surprising result:
  - Because of ideas from *sparse coding* you can actually separate out the senses (providing they are relatively common)!

<table>
<thead>
<tr>
<th>tie</th>
<th>trousers</th>
<th>blouse</th>
<th>waistcoat</th>
<th>skirt</th>
<th>sleeved</th>
<th>pants</th>
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<th>teams</th>
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<th>league</th>
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<th>contralto</th>
<th>baritone</th>
<th>coloratura</th>
</tr>
</thead>
</table>

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6. Deep Learning Classification: Named Entity Recognition (NER)

• The task: find and classify names in text, by labeling word tokens, for example:

  Last night, Paris Hilton wowed in a sequin gown.

    PER  PER

  Samuel Quinn was arrested in the Hilton Hotel in Paris in April 1989.

    PER  PER    LOC  LOC  LOC  DATE  DATE

• Possible uses:
  • Tracking mentions of particular entities in documents
  • For question answering, answers are usually named entities
  • Relating sentiment analysis to the entity under discussion
  • Often followed by Entity Linking/Canonicalization into a Knowledge Base such as Wikidata
Simple NER: Window classification using binary logistic classifier

- **Idea:** classify each word in its context window of neighboring words
- Train logistic classifier on hand-labeled data to classify center word {yes/no} for each class based on a concatenation of word vectors in a window
  - Really, we usually use multi-class softmax, but we’re trying to keep it simple 😊
- **Example:** Classify “Paris” as +/- location in context of sentence with window length 2:

  ```
  the museums in Paris are amazing to see .
  ```

  \[
  X_{\text{window}} = [ x_{\text{museums}} \ x_{\text{in}} \ x_{\text{Paris}} \ x_{\text{are}} \ x_{\text{amazing}} ]^T
  \]

  - Resulting vector \( x_{\text{window}} = [ x \in \mathbb{R}^{5d} ] \)
  - To classify all words: run classifier for each class on the vector centered on each word in the sentence
Classification review and notation

• Supervised learning: we have a training dataset consisting of samples

\[ \{x_i, y_i\}_{i=1}^N \]

• \( x_i \) are inputs, e.g., words (indices or vectors!), sentences, documents, etc.
  • Dimension \( d \)

• \( y_i \) are labels (one of \( C \) classes) we try to predict, for example:
  • classes: sentiment (+/−), named entities, buy/sell decision
  • other words
  • later: multi-word sequences
Neural classification

- Typical ML/stats softmax classifier: 
  \[ p(y|x) = \frac{\exp(W_y{x})}{\sum_{c=1}^{C} \exp(W_c{x})} \]
- Learned parameters \( \theta \) are just elements of \( W \) (not input representation \( x \), which has sparse symbolic features)
- Classifier gives linear decision boundary, which can be limiting

- A neural network classifier differs in that:
  - We learn both \( W \) and (distributed!) representations for words
  - The word vectors \( x \) re-represent one-hot vectors, moving them around in an intermediate layer vector space, for easy classification with a (linear) softmax classifier
    - Conceptually, we have an embedding layer: \( x = Le \)
  - We use deep networks—more layers—that let us re-represent and compose our data multiple times, giving a non-linear classifier

But typically, it is linear relative to the pre-final layer representation
Softmax classifier

\[ p(y|x) = \frac{\exp(W_y.x)}{\sum_{c=1}^{C} \exp(W_c.x)} \]

Again, we can tease apart the prediction function into three steps:

1. For each row \( y \) of \( W \), calculate dot product with \( x \):
   \[ W_y.x = \sum_{i=1}^{d} W_{yi}x_i = f_y \]

2. Apply softmax function to get normalized probability:
   \[ p(y|x) = \frac{\exp(f_y)}{\sum_{c=1}^{C} \exp(f_c)} = \text{softmax}(f_y) \]

3. Choose the \( y \) with maximum probability
   - For each training example \((x,y)\), our objective is to maximize the probability of the correct class \( y \) or we can minimize the negative log probability of that class:
     \[ -\log p(y|x) = -\log \left( \frac{\exp(f_y)}{\sum_{c=1}^{C} \exp(f_c)} \right) \]
NER: Binary classification for center word being location

- We do supervised training and want high score if it’s a location

\[ J_t(\theta) = \sigma(s) = \frac{1}{1 + e^{-s}} \]

\[ s = u^T h \]

\[ h = f(Wx + b) \]

\[ x \text{ (input)} \]

\[ x = [x_{\text{museums}}, x_{\text{in}}, x_{\text{Paris}}, x_{\text{are}}, x_{\text{amazing}}] \]

\( f = \) Some element-wise non-linear function, e.g., logistic, tanh, ReLU
Training with “cross entropy loss” – you use this in PyTorch!

- Until now, our objective was stated as to maximize the probability of the correct class $y$ or equivalently we can minimize the negative log probability of that class.

- Now restated in terms of cross entropy, a concept from information theory.

- Let the true probability distribution be $p$; let our computed model probability be $q$.

- The cross entropy is:

$$H(p, q) = -\sum_{c=1}^{C} p(c) \log q(c)$$

- Assuming a ground truth (or true or gold or target) probability distribution that is 1 at the right class and 0 everywhere else, $p = [0, \ldots, 0, 1, 0, \ldots, 0]$, then:

- Because of one-hot $p$, the only term left is the negative log probability of the true class $y_i$: $-\log p(y_i | x_i)$

Cross entropy can be used in other ways with a more interesting $p$, but for now just know that you’ll want to use it as the loss in PyTorch.
Classification over a full dataset

- Cross entropy loss function over full dataset \( \{x_i, y_i\}_{i=1}^{N} \)

\[
J(\theta) = \frac{1}{N} \sum_{i=1}^{N} - \log \left( \frac{e^{f_{y_i}}}{\sum_{c=1}^{C} e^{f_c}} \right)
\]
Remember: Stochastic Gradient Descent

Update equation:

$$
\theta^{new} = \theta^{old} - \alpha \nabla_\theta J(\theta)
$$

\(\alpha = \text{step size or learning rate}\)

i.e., for each parameter: 

$$
\theta_j^{new} = \theta_j^{old} - \alpha \frac{\partial J(\theta)}{\partial \theta_j^{old}}
$$

In deep learning, \(\theta\) includes the data representation (e.g., word vectors) too!

How can we compute \(\nabla_\theta J(\theta)\)?

1. By hand
2. Algorithmically: the backpropagation algorithm (next lecture!)
7. Neural computation
A binary logistic regression unit is a bit similar to a neuron

\[ f = \text{nonlinear activation function (e.g. sigmoid)}, \ w = \text{weights}, \ b = \text{bias}, \ h = \text{hidden}, \ x = \text{inputs} \]

\[ h_{w,b}(x) = f(w^T x + b) \]

\[ f(z) = \frac{1}{1 + e^{-z}} \]

\( b \): We can have an “always on” bias feature, which gives a class prior, or separate it out, as a bias term.

\( w, b \) are the parameters of this neuron i.e., this logistic regression model.
A neural network
= running several logistic regressions at the same time

If we feed a vector of inputs through a bunch of logistic regression functions, then we get a vector of outputs ...

But we don’t have to decide ahead of time what variables these logistic regressions are trying to predict!
A neural network = running several logistic regressions at the same time

... which we can feed into another logistic regression function, giving composed functions

It is the loss function that will direct what the intermediate hidden variables should be, so as to do a good job at predicting the targets for the next layer, etc.
A neural network = running several logistic regressions at the same time

Before we know it, we have a multilayer neural network....

This allows us to re-represent and compose our data multiple times and to learn a classifier that is highly non-linear in terms of the original inputs

(but typically is linear in terms of the pre-final layer representations)
Matrix notation for a layer

We have

\[ a_1 = f(W_{11}x_1 + W_{12}x_2 + W_{13}x_3 + b_1) \]
\[ a_2 = f(W_{21}x_1 + W_{22}x_2 + W_{23}x_3 + b_2) \]

etc.

In matrix notation

\[ z = Wx + b \]
\[ a = f(z) \]

Activation \( f \) is applied element-wise:

\[ f([z_1, z_2, z_3]) = [f(z_1), f(z_2), f(z_3)] \]
Non-linearities (like $f$ or sigmoid): Why they’re needed

- Neural networks do function approximation, e.g., regression or classification
  - Without non-linearities, deep neural networks can’t do anything more than a linear transform
  - Extra layers could just be compiled down into a single linear transform: $W_1 W_2 x = Wx$
  - But, with more layers that include non-linearities, they can approximate more complex functions!

![Graphs showing function approximation](image)