Network Effects and Cascading Behavior (1)
Spreading Through Networks

- Spreading through networks:
  - Cascading behavior
  - Diffusion of innovations
  - Network effects
  - Epidemics

- Behaviors that cascade from node to node like an epidemic

- Examples:
  - Biological:
    - Diseases via contagion
  - Technological:
    - Cascading failures
    - Spread of information
  - Social:
    - Rumors, news, new technology
    - Viral marketing
Information Diffusion: Media

Obscure tech story

Small tech blog

Engadget

Slashdot

Wired

BBC

NYT

CNN

Twitter & Facebook post sharing

Lada Adamic shared a link via Erik Johnston.
January 16, 2013

When life gives you an almost empty jar of nutella, add some ice cream... (and other useful tips)

50 Life Hacks to Simplify your World
twistedsifter.com

Life hacks are little ways to make our lives easier. These low-budget tips and trick can help you organize and de-clutter space; prolong and preserve your products; or teach you...
Timeline Photos

Back to Album · I fucking love science's Photos · I fucking love science's Page

Thickness \( a \) —— Radius \( z \)

\[
V = \pi z^2 a \\
V = Pi(z*\pi) a
\]

I fucking love science

Seriously. If you have a pizza with radius "\( z \)" and thickness "\( a \)", its volume is \( \pi(z^2) \).

Lina von Delsteg, Iman Khallaf, 周明佳 and 73,191 others like this.

27,761 shares

Album: Timeline Photos
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Product adoption:
- Senders and followers of recommendations
Spread of Diseases (e.g., Ebola)
Network Cascades

- Contagion that spreads over the edges of the network
- It creates a propagation tree, i.e., cascade

Terminology:
- Stuff that spreads: Contagion
- “Infection” event: Adoption, infection, activation
- We have: Infected/active nodes, adopters
How Do We Model Diffusion?

- **Decision based models (today!):**
  - Models of product adoption, decision making
    - A node observes decisions of its neighbors and makes its own decision
  - **Example:**
    - You join demonstrations if $k$ of your friends do so too

- **Probabilistic models (on Tuesday):**
  - Models of influence or disease spreading
    - An infected node tries to “push” the contagion to an uninfected node
  - **Example:**
    - You “catch” a disease with some prob. from each active neighbor in the network
Decision Based Model of Diffusion
Based on 2 player coordination game

- 2 players – each chooses technology A or B
- Each person can only adopt one “behavior”, A or B
- You gain more payoff if your friend has adopted the same behavior as you

Local view of the network of node v
Example: VHS vs. BetaMax
Example: BlueRay vs. HD DVD
The Model for Two Nodes

- **Payoff matrix:**
  - If both \( v \) and \( w \) adopt behavior \( A \), they each get payoff \( a > 0 \)
  - If \( v \) and \( w \) adopt behavior \( B \), they reach get payoff \( b > 0 \)
  - If \( v \) and \( w \) adopt the opposite behaviors, they each get 0

- **In some large network:**
  - Each node \( v \) is playing a copy of the game with each of its neighbors
  - **Payoff:** sum of node payoffs per game
Let \( v \) have \( d \) neighbors.

Assume fraction \( p \) of \( v \)'s neighbors adopt \( A \).

- **Payoff** \( v \) = \( a \cdot p \cdot d \) if \( v \) chooses \( A \)
  
  = \( b \cdot (1-p) \cdot d \) if \( v \) chooses \( B \)

Thus: \( v \) chooses \( A \) if: \( p > \frac{b}{a + b} = q \)

Threshold:

- \( v \) chooses \( A \) if \(\frac{b}{a + b} = q\)

\( p \)… frac. \( v \)'s nbrs. with \( A \)

\( q \)… payoff threshold
Scenario:

- Graph where everyone starts with all $B$
- Small set $S$ of early adopters of $A$
  - Hard-wire $S$ – they keep using $A$ no matter what payoffs tell them to do

- Assume payoffs are set in such a way that nodes say:
  If more than $q = 50\%$ of my friends take $A$
  I’ll also take $A$.

This means: $a = b - \varepsilon$ ($\varepsilon > 0$, small positive constant) and then $q = 1/2$
If more than q=50% of my friends are red
I’ll also be red

$S = \{u, v\}$
If more than q=50% of my friends are red, I’ll also be red

\[ S = \{u, v\} \]
If more than \(q=50\%\) of my friends are red
I’ll also be red

\[S = \{u, v\}\]
If more than \( q = 50\% \) of my friends are red
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If more than \( q = 50\% \) of my friends are red, I’ll also be red.
If more than $q=50\%$ of my friends are red, I’ll also be red.
Application: Modeling protest recruitment on social networks

The Dynamics of Protest Recruitment through an Online Network
Bailon et al. Nature Scientific Reports, 2011
The Spanish ‘Indignados’ Movement

- **Anti-austerity protests** in Spain May 15-22, 2011 as a response to the financial crisis
- Twitter was used to organize and mobilize users to participate in the protest
Researchers identified 70 hashtags that were used by the protesters.
70 hashtags were crawled for 1 month period

- Number of tweets: 581,750

- **Relevant users:** Any user who tweeted any relevant hashtag and its followers and followees
  - Number of users: 87,569

- **Created two undirected follower networks:**
  1. **Full network:** with all Twitter follow links
  2. **Symmetric network** with only the reciprocal follow links ($i \rightarrow j$ and $j \rightarrow i$)
    - This network represents “strong” connections only.
Definitions

- **User activation time**: Moment when user starts tweeting protest messages
- $k_{in} = \text{The total number of neighbors when a user became active}$
- $k_a = \text{Number of active neighbors when a user became active}$
- **Activation threshold** $= \frac{k_a}{k_{in}}$
  - The fraction of active neighbors at the time when a user becomes active
Recruitment & Activation Threshold

- If $k_a/k_{in} \approx 0$, then the user joins the movement when very few neighbors are active $\Rightarrow$ no social pressure
- If $k_a/k_{in} \approx 1$, then the user joins the movement after most of its neighbors are active $\Rightarrow$ high social pressure

\[ k_a/k_{in} = \frac{0}{4} = 0.0 \]

No social pressure for middle node to join

\[ k_a/k_{in} = \frac{2}{4} = 0.5 \]

Non-zero social pressure for middle node to join

Already active node
Mostly uniform distribution of activation threshold in both networks, except for two local peaks.

- 0 activation threshold users: Many self-active users.
- 0.5 activation threshold users: Many users who join after half their neighbors do.
Effect of neighbor activation time

- **Hypothesis:** If several neighbors become active in a short time period, then a user is more likely to become active.
- **Method:** Calculate the burstiness of active neighbors as
  \[
  \frac{\Delta k_a}{k_a} = \frac{(k_a^{t+1} - k_a^t)}{k_a^{t+1}}
  \]

Low threshold users are insensitive to recruitment bursts.

High threshold users join after sudden bursts in neighborhood activation.
No cascades are given in the data

So cascades were identified as follows:

- If a user tweets a message at time $t$ and one of its followers tweets a message in $(t, t+\Delta t)$, then they form a cascade.
- E.g., $1 \rightarrow 2 \rightarrow 3$ below form a cascade:
Size of information cascades

- **Size** = number of nodes in the cascade
- **Most cascades are small:**

![Graph showing the size distribution of cascades.](image)

Fraction of cascades with size at least $S$
Who starts successful cascades?

- **Are starters of successful cascades more central in the network?**
- **Method:** $k$-core decomposition
  - $k$-core: every node in the graph has at least degree $k$
  - Method: repeatedly remove all nodes with degree less than $k$
  - Higher $k$-core number of a node means it is more central

Example:

- Peripheral nodes
- Central nodes
Who starts the successful cascades?

- K-core decomposition of follow network
  - Red nodes start successful cascades
  - Red nodes have higher $k$-core values
    - So, successful cascades starters are central and connected to equally well connected users

Successful cascade starters are central (higher $k$-core number)
Summary: Cascades on Twitter

- Uniform activation threshold for users, with two local peaks
- Most cascades are short
- Successful cascades are started by central (more core) users
So far:

Decision Based Models
- Utility based
- Deterministic
- “Node” centric: A node observes decisions of its neighbors and makes its own decision

Next: Extending decision based models to multiple contagions
Extending the Model: Allow People to Adopt A and B
Extending the model

- **So far:**
  - Behaviors $A$ and $B$ compete
  - Can only get utility from neighbors of same behavior: $A-A$ get $a$, $B-B$ get $b$, $A-B$ get $0$

- **For example:**
  - Using Skype vs. WhatsApp
    - Can only talk using the same software
  - Having a VHS vs. BetaMax player
    - Can only share tapes with people using the same type of tape
  - But one can buy 2 players or install 2 programs
So far:
- Behaviors $A$ and $B$ compete
- Can only get utility from neighbors of same behavior: $A-A$ get $a$, $B-B$ get $b$, $A-B$ get $0$

Let’s add an extra strategy “$AB$”
- $AB-A$ : gets $a$
- $AB-B$ : gets $b$
- $AB-AB$ : gets $\max(a, b)$
- Also: Some cost $c$ for the effort of maintaining both strategies (summed over all interactions)
  - Note: a given node can receive $a$ from one neighbor and $b$ from another by playing $AB$, which is why it could be worth the cost $c$
Every node in an infinite network starts with $B$
- Then a finite set $S$ initially adopts $A$
- Run the model for $t=1,2,3,...$
  - Each node selects behavior that will optimize payoff (given what its neighbors did in at time $t-1$)

How will nodes switch from $B$ to $A$ or $AB$?
Example: Path Graph (1)

- **Path graph:** Start with Bs, \( a > b \) (A is better)
- **One node switches to A** – what happens?
  - With just A, B: A spreads if \( a > b \)
  - With A, B, AB: Does A spread?
- **Example:** \( a=3, b=2, c=1 \)

![Diagram of path graph with nodes A and B showing the conditions for A to spread and cascade stops with a specific example.](http://cs224w.stanford.edu)
Example: Path Graph (2)

Example: a=5, b=3, c=1

Cascade never stops!

Hard-wired to adopt A
Let’s solve the model in a general case:
- **Infinite path, start with all Bs**
- **Payoffs for w**: A:a, B:1, AB:a+1-c

**For what pairs (c,a) does A spread?**
- **We need to analyze two cases for node w**: Based on the values of a and c, what would w do?
For what pairs \((c, a)\) does A spread?

- Infinite path, start with Bs
- **Payoffs for** \(w\): A: \(a\), B: 1, AB: \(a + 1 - c\)
- What does node \(w\) in A-w-B do?
For what pairs \((c,a)\) does A spread?

- Infinite path, start with Bs
- **Payoffs for** \(w\): A:a, B:1, AB:a+1-c
- What does node \(w\) in A-w-B do?

\[ \begin{align*}
A & : a, \\
B & : 1, \\
AB & : a+1-c
\end{align*} \]

Since \(a<1\), \(c>1\)

- **A** is big
- **B** is big
- **AB** is optimal for \(w\)

\[ \begin{align*}
a+1-c &= a \\
a+1-c &= 1
\end{align*} \]
Same reward structure as before but now payoffs for $w$ change: $A:a$, $B:1+1$, $AB:a+1-c$

Notice: Now also $AB$ spreads

What does node $w$ in $AB-w-B$ do?
For what pairs \((c,a)\) does A spread?

- Same reward structure as before but now payoffs for \(w\) change: \(A:a, B:1+1, AB:a+1-c\)
- Notice: Now also AB spreads
- What does node \(w\) in AB-w-B do?

\[
\begin{align*}
\text{B vs A:} & \quad a < 2, \ c > 1 \quad \text{then} \ 2b > 2a \\
\text{AB vs A:} & \quad c < 1, \ \text{then} \ a+1-c > a \quad \text{AB is optimal for w} \\
\end{align*}
\]
For what pairs \((c, a)\) does \(A\) spread?

- **Joining the two pictures:**

![Diagram showing the spread of \(A\) from \(B\) through \(AB\).](image-url)
- **B** is the default throughout the network until new/better **A** comes along. What happens?
  - **Infiltration:** If **B** is too compatible then people will take on both and then drop the worse one (**B**)
  - **Direct conquest:** If **A** makes itself not compatible – people on the border must choose. They pick the better one (**A**)
  - **Buffer zone:** If you choose an optimal level then you keep a static “buffer” between **A** and **B**
Models of Cascading Behavior

- **So far:**
  - Decision Based Models
    - Utility based
    - Deterministic
    - “Node” centric: A node observes decisions of its neighbors and makes its own decision
    - Require us to know too much about the data
  
- **Next:** Probabilistic Models
  - Lets you do things by observing data
  - We lose “why people do things”