CS 231A PS1 Review

CS231A

Computer Vision: From 3D Reconstruction to Recognition
Winter 2018
Problem Outline

- Q1: Projective Geometry
- Q2: Affine Camera Calibration
- Q3: Single View Geometry
Problem Outline

• Q1: Projective Geometry

• Q2: Affine Camera Calibration

• Q3: Single View Geometry
P1: Reference Frames

Source: http://ycpcs.github.io/cs470-fall2014/
P1: Cross Products

- Lines $k$ and $l$ are parallel
  - $k_1$ and $k_2$ are any two points on $k$
  - $l_1$ and $l_2$ are any two points on $l$
  - by definition of parallel lines:
    \[
    (k_1 - k_2) \times (l_1 - l_2) = 0
    \]

- Given a square $pqrs$,
  - Area = \[ \| (q - p) \times (s - p) \| \]

*Courtesy of last year’s slides*
Problem Outline

• Q1: Projective Geometry

• Q2: Affine Camera Calibration

• Q3: Single View Geometry
P2: Setup

(a) Image formation in an affine camera. Points are projected via parallel rays onto the image plane.

(b) Image of calibration grid at Z=0

(c) Image of calibration grid at Z=150
P2: Perspective Camera Model

\[
\begin{bmatrix}
x \\
y \\
w
\end{bmatrix} =
\begin{bmatrix}
p_{11} & p_{12} & p_{13} & p_{14} \\
p_{21} & p_{22} & p_{23} & p_{24} \\
p_{31} & p_{32} & p_{33} & p_{34}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
u_i \\
v_i
\end{bmatrix} =
\begin{bmatrix}
m_1 P_i \\
m_3 P_i \\
m_2 P_i \\
m_3 P_i
\end{bmatrix}
\]
P2: Affine Camera Model

\[
\begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
= \begin{bmatrix}
  p_{11} & p_{12} & p_{13} & p_{14} \\
  p_{21} & p_{22} & p_{23} & p_{24} \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  X \\
  Y \\
  Z \\
  1
\end{bmatrix}
\]

- Linear
- 8 Unknowns
P2: Solve for $P$

$$Ax = b$$

$$x = (A^T A)^{-1} A^T b$$

Hint: `numpy.linalg.pinv` / `numpy.linalg.lstsq`
Problem Outline

• Q1: Projective Geometry

• Q2: Affine Camera Calibration

• Q3: Single View Metrology
Vanishing Points

*Courtesy of last year’s slides*
Vanishing Points

- In 3D space, points at infinity are defined as the intersection of parallel lines, which have direction $d$.
- In the image plane, parallel lines meet at the vanishing point $v$.
- With camera intrinsic matrix as $K$, we have

\[ v = Kd \]
Vanishing Points

Image Plane

Vanishing Point

Ground Plane

Parallel Lines
Calculating Vanishing Point

(a) Image 1 (1.jpg) with marked pixels

*Courtesy of last year’s slides
Calculating Vanishing Point

• Points in $L_1$: $(x_1, y_1), (x_2, y_2) \rightarrow m_1 = (y_2 - y_1)/(x_2 - x_1)$
• Points in $L_2$: $(x_3, y_3), (x_4, y_4) \rightarrow m_2 = (y_4 - y_3)/(x_4 - x_3)$
• Intersection of $L_1$ and $L_2$: Vanishing Point

*Courtesy of last year’s slides*
Vanishing Points to Compute K

• Course notes “Single View Metrology”
• HZ (2nd edition) Page 223-226
Vanishing Points to Compute $K$
Vanishing Points to Compute K

- Define vanishing line $d_1$ and $d_2$, vanishing points $v_1$ and $v_2$. We have:

\[
\cos \theta = \frac{d_1 \cdot d_2}{\|d_1\| \|d_2\|} = \frac{v_1^T \omega v_2}{\sqrt{v_1^T \omega v_1} \sqrt{v_2^T \omega v_2}}
\]

, where $\omega = (KK^T)^{-1}$
Vanishing Points to Compute K

(a) Image 1 (1.jpg) with marked pixels
Vanishing Points to Compute K

- Define vanishing line $d_1$ and $d_2$, vanishing points $v_1$ and $v_2$. We have:

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\]

, where \( \omega = (KK^T)^{-1} \)

If $d_1$ and $d_2$ are orthogonal, $v_1^T \omega v_2 = 0$
Vanishing Points to Compute K

• $\omega = (K K^T)^{-1}$
  
  – Matrix $\omega$ is the projective transformation in the image plane of an absolute conic in 3D

$$
\omega = \begin{bmatrix}
\omega_1 & \omega_2 & \omega_4 \\
\omega_2 & \omega_3 & \omega_5 \\
\omega_4 & \omega_5 & \omega_6
\end{bmatrix}
$$

*Courtesy of last year’s slides*
Vanishing Points to Compute K

bullet We assume the camera has zero skew and square pixels
  - Zero skew: $\omega_2 = 0$
  - Square pixels: $\omega_1 = \omega_3$

$$w = \begin{bmatrix} \omega_1 & 0 & \omega_4 \\ 0 & \omega_1 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$

*Courtesy of last year’s slides*
Compute Angle Between Planes

- Vanishing lines $L_1$ and $L_2$
- $L_1 = v_1 \times v_2$; $L_2 = v_3 \times v_4$
  - $v_1 \text{ and } v_2$ = vanishing points corresponding to one plane
  - $v_3 \text{ and } v_4$ for the other plane

\[
\cos \theta = \frac{l_1^T \omega^* l_2}{\sqrt{l_1^T \omega^* l_1} \sqrt{l_2^T \omega^* l_2}}
\]

, where $\omega^* = \omega^{-1} = KK^T$

*Courtesy of last year’s slides*
Rotation Matrix using Vanishing Points

• Find corresponding vanishing points from both images \((v_1, v_2, v_3)\) and \((v_1', v_2', v_3')\)

• Calculate directions of vanishing points:

\[ v = K d \rightarrow \quad d = \frac{K^{-1} v}{\|K^{-1} v\|} \]

• \(d_i' = R d_i\), where
  
  – \(d_i'\) = direction of the \(i^{th}\) vanishing point in second image
  
  – \(d_i\) = direction of the \(i^{th}\) vanishing point in first image

*Courtesy of last year’s slides*