CS231A

Computer Vision: From 3D Reconstruction to Recognition

Optimal Estimation Cont'

Recap

- Recursive Filter
- Kalman Filter
- Extended Kalman Filter

The Bayes Filter

- Recursive filter for estimating x_t only from x_{t-1}, z_t and u_t and not from the ever-growing history $z_{1:t}, u_{1:t}$

Algorithm Bayes_filter($bel(x_{t-1}), u_t, z_t$):
  for all x_t do
    $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx$ **Predict Step**
    $bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t)$ **Update Step**
  endfor
  return $bel(x_t)$

Transition/Dynamics model: $p(x_t \mid u_t, x_{t-1})$

Measurement Model: $p(z_t \mid x_t)$

The Kalman Filter Algorithm

Algorithm Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

$\bar{\mu}_t = A_t \, \mu_{t-1} + B_t \, u_t$

$\bar{\Sigma}_t = A_t \, \Sigma_{t-1} \, A_t^T + R_t$

$K_t = \bar{\Sigma}_t \, C_t^T (C_t \, \bar{\Sigma}_t \, C_t^T + Q_t)^{-1}$

$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \, \bar{\mu}_t)$

$\Sigma_t = (I - K_t \, C_t) \, \bar{\Sigma}_t$

return μ_t, Σ_t

Uncertainty increases

K = Kalman Gain $K \approx \frac{R}{Q}$

Uncertainty decreases

Algorithm Bayes_filter($bel(x_{t-1}), u_t, z_t$):

for all x_t do

$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}) \, bel(x_{t-1}) \, dx$ **Predict Step**

$bel(x_t) = \eta \, p(z_t \mid x_t) \, \overline{bel}(x_t)$ **Update Step**

endfor

return $bel(x_t)$

If R large, then K is large. Update dominated by innovation.

If Q large, then K is small. Update dominated by prediction.

The Extended Kalman Filter Algorithm

1: **Algorithm Extended_Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)**:**
2: $\bar{\mu}_t = g(u_t, \mu_{t-1})$
3: $\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + R_t$
4: $K_t = \bar{\Sigma}_t \, H_t^T (H_t \, \bar{\Sigma}_t \, H_t^T + Q_t)^{-1}$
5: $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$
6: $\Sigma_t = (I - K_t \, H_t) \, \bar{\Sigma}_t$
7: return μ_t, Σ_t

Predict (lines 2–3)

Update (lines 4–6)

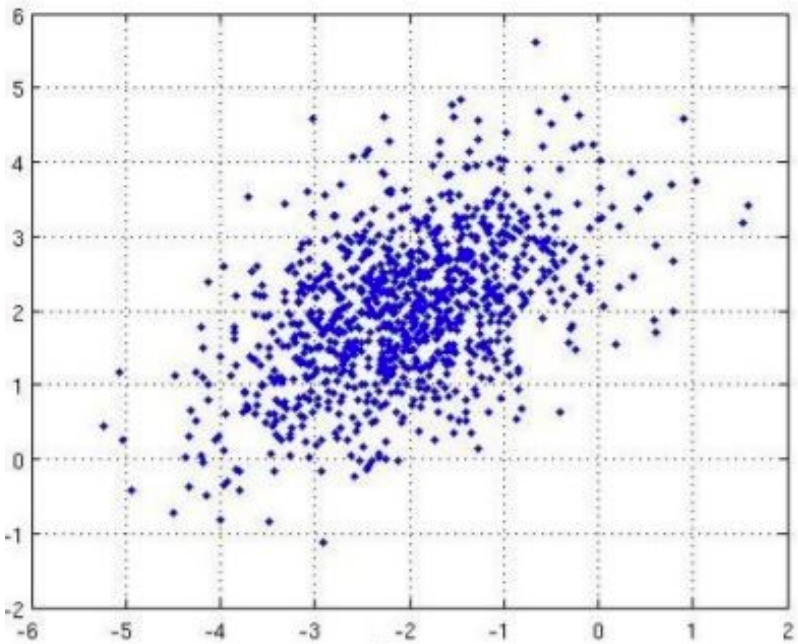
	Kalman filter	EKF
state prediction (Line 2)	$A_t \, \mu_{t-1} + B_t \, u_t$	$g(u_t, \mu_{t-1})$
measurement prediction (Line 5)	$C_t \, \bar{\mu}_t$	$h(\bar{\mu}_t)$

Nonparametric filters

- No fixed functional form of the posterior – can capture multimodality
- Instead: finite numbers of values

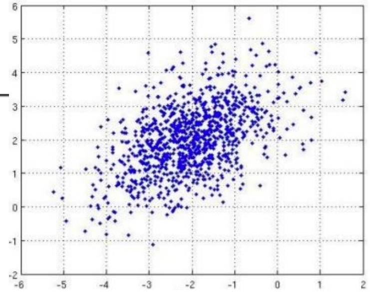
- Histogram filter: State = finitely many regions
- Particle filter: Distribution represented by samples

Particle Filter



$$p(x_t|z_{t:1}, u_{t:1}, \boldsymbol{x}_0) \rightarrow X_t = \{x_t^0, \ldots, x_t^N\}$$

The Particle filter algorithm



Algorithm Particle_filter$(\mathcal{X}_{t-1}, u_t, z_t)$**:**

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $m = 1$ to M do

sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$ — **Process Model**

$w_t^{[m]} = p(z_t \mid x_t^{[m]})$ — **Measurement Model**

$\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

endfor

for $m = 1$ to M do

draw i with probability $\propto w_t^{[i]}$

add $x_t^{[i]}$ to $\mathcal{X}_t$

endfor

return $\mathcal{X}_t$

Importance Sampling

Before resampling → $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx$

After resampling → $bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t)$

Particle Filter - Process Model

$$p(x_{t-1}|z_{t-1:1}, u_{t-1:1}, \boldsymbol{x}_0) \rightarrow X_{t-1} = \{x_{t-1}^0, \ldots, x_{t-1}^N\}$$

$$x_{t-1}^n \rightarrow p(x_t|x_{t-1}^n, u_t, \boldsymbol{x}_0) \rightarrow \hat{x}_t^n$$

$$\hat{X}_t = \{\hat{x}_t^0, \ldots, \hat{x}_t^N\}$$

Particle Filter – Measurement Model

$$w_t^{[i]} = \frac{target}{proposal} \propto p(z_t \mid x_t, m)$$

- Draw sample i with probability $w_t^{[i]}$. Repeat M times.
- Informally: "Replace unlikely samples by more likely ones"
- Survival of the fittest
- "Trick" to avoid that many samples cover unlikely states
- Needed as we have a limited number of samples

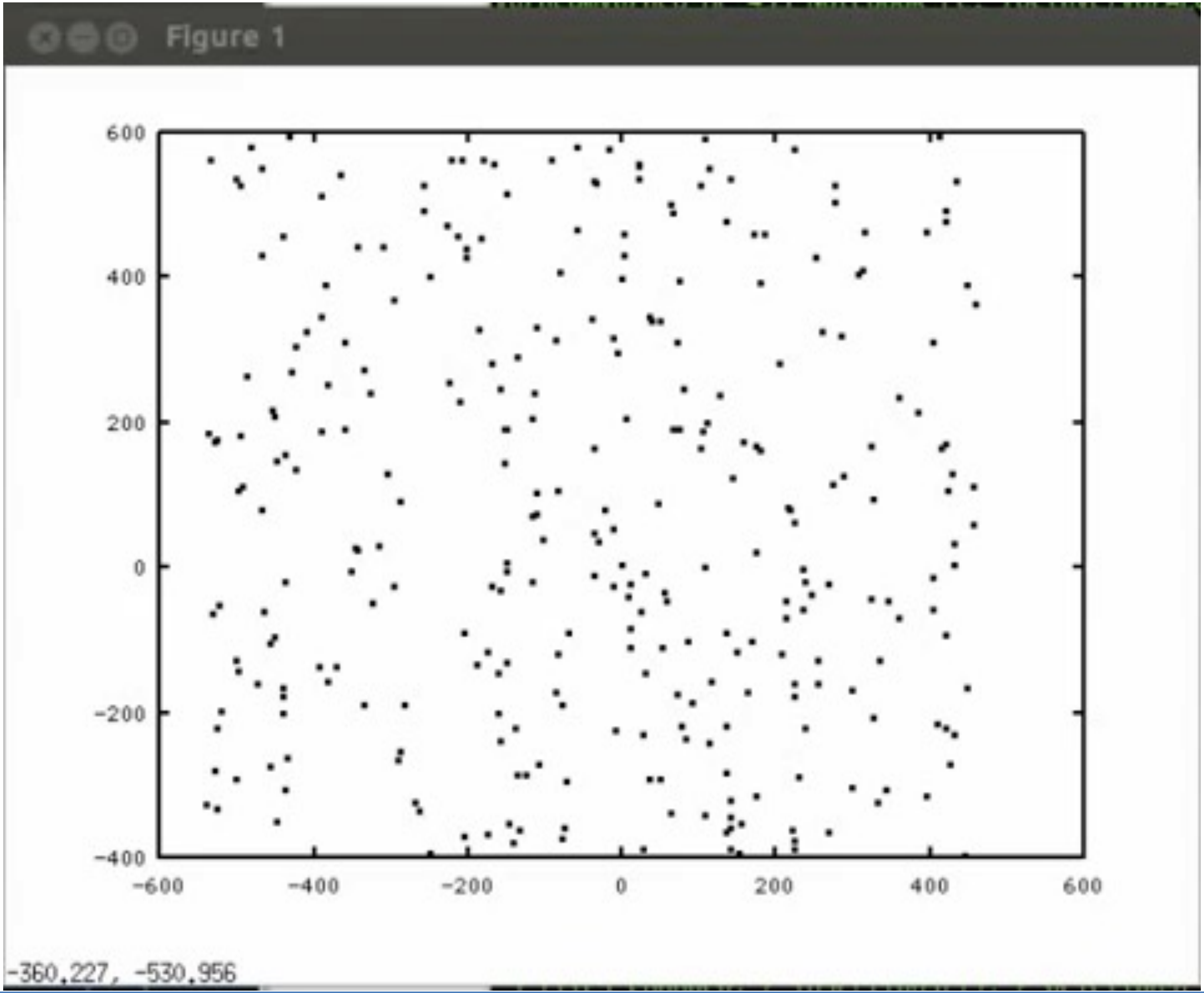
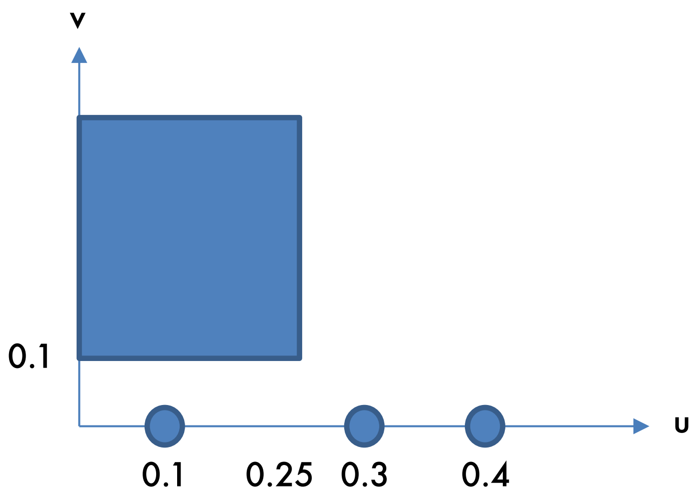
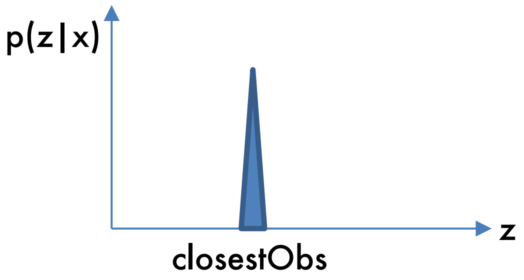


Figure 1
600
400
200
0
−200
−400
−600
−400
−200
0
200
400
600
-360.227, -530.956

Particle Filter Example





$x_{t+1}=x_t + (0.1, 0)$ $z=0.1$

When to Use Each?

Bayes Filter

General Framework
No implementation!

Kalman Filter

Linear Models
Gaussian Distributions

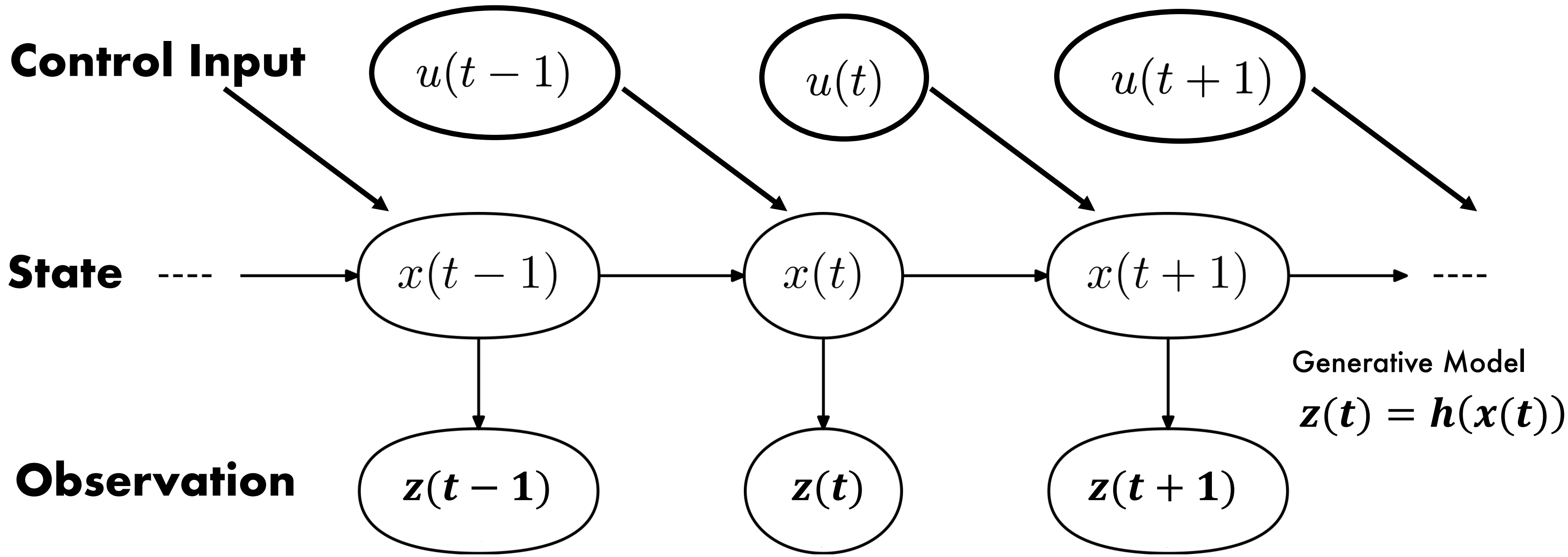
Extended Kalman Filter

Non-Linear Models (linearizable)
Gaussian Distributions

Particle Filter

Any Model
Any Distribution
Low Dimensional State Space

Graphical Model of System to Estimate



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  for all x_t do
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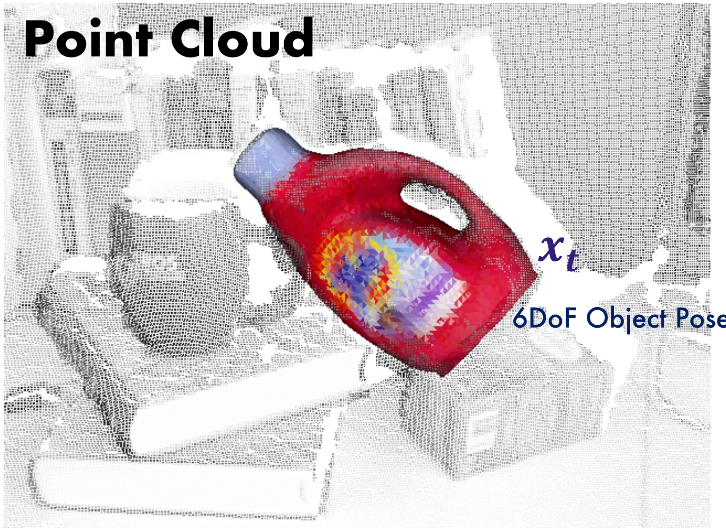
    $bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t)$
  endfor
  return $bel(x_t)$

Example Observation model for 3D object



Algorithm Particle_filter$(\mathcal{X}_{t-1}, u_t, z_t)$:

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $m = 1$ to M do

sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$

$w_t^{[m]} = p(z_t \mid x_t^{[m]})$

$\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

endfor

for $m = 1$ to M do

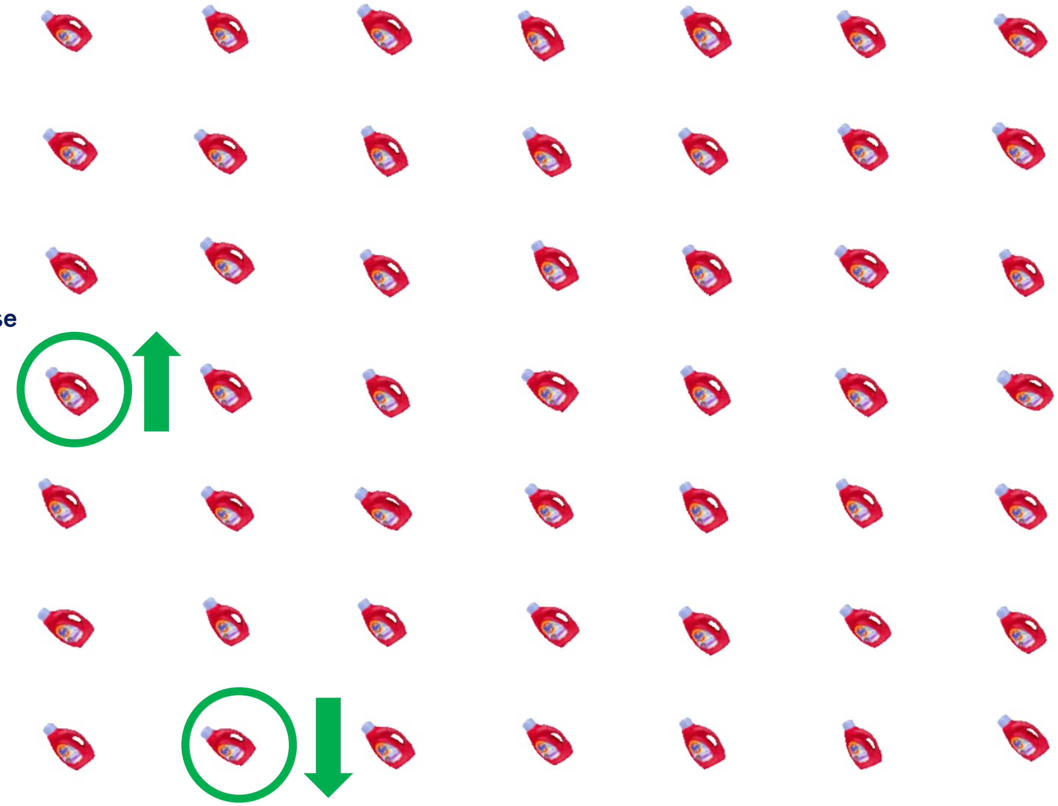
draw i with probability $\propto w_t^{[i]}$

add $x_t^{[i]}$ to $\mathcal{X}_t$

endfor

return $\mathcal{X}_t$

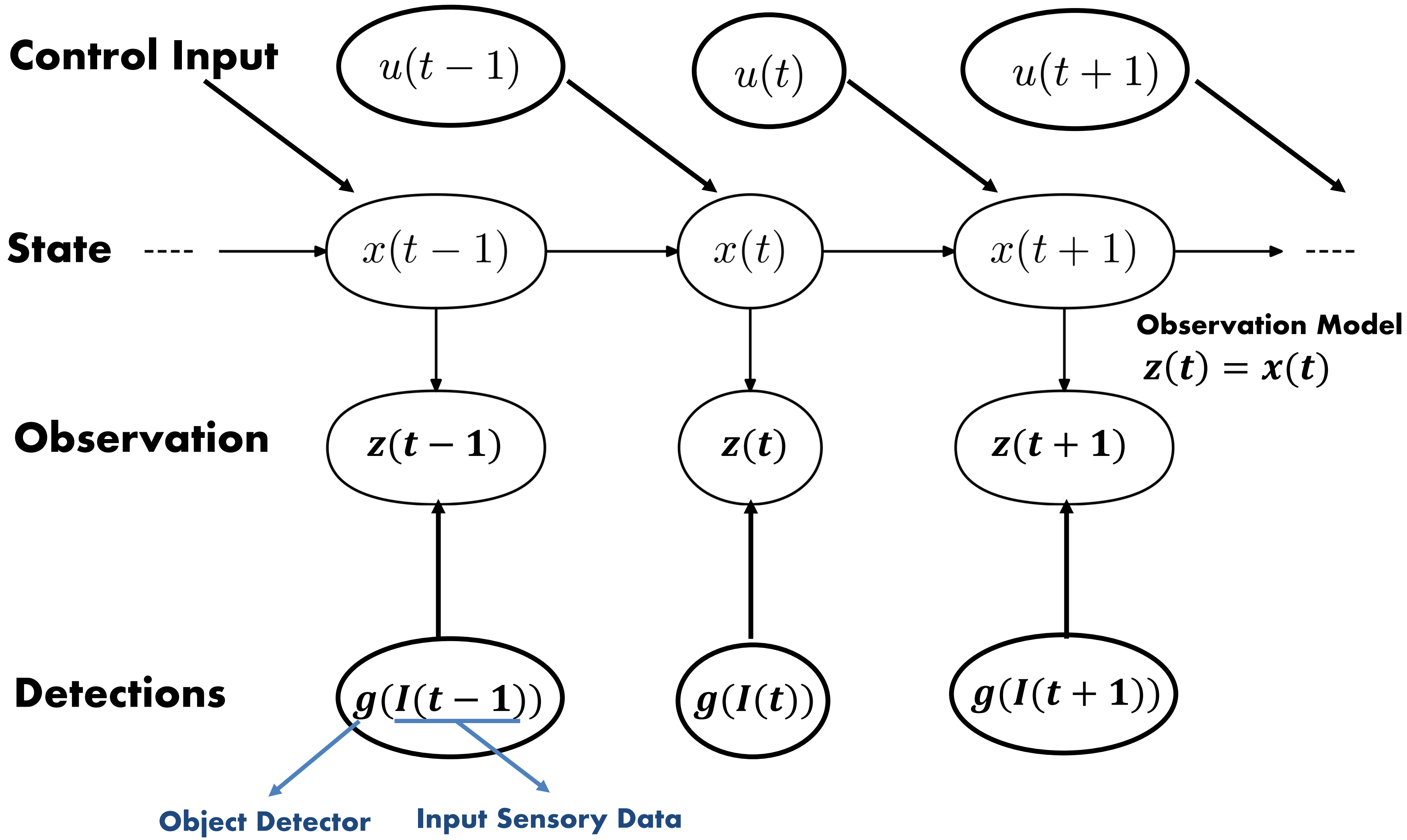
Importance Sampling



Rendered Particles

Changhyun Choi and Henrik I. Christensen. Rgb-d object tracking: A particle filter approach on gpu. In IROS, pages 1084–1091, 2013

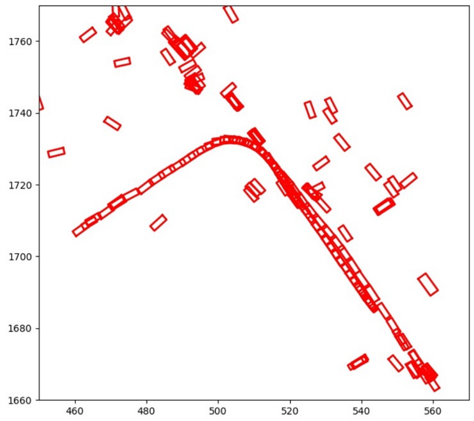
Tracking by Detection



Problem Statement: Input

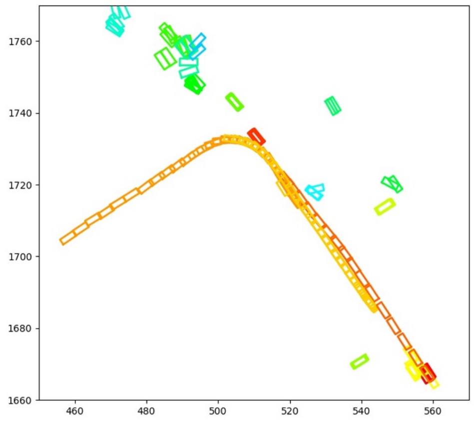
Probabilistic 3d multi-object tracking for autonomous driving. H Chiu, A Prioletti, J Li, J Bohg
arXiv preprint arXiv:2001.05673

- Object detections at each frame in a sequence
- Each detection bounding box is represented by:
    - center position (x, y, z), rotation angle along the z-axis (a), and the scale (l, w, h)
    - category label (car, pedestrian, ...), confidence score (c)



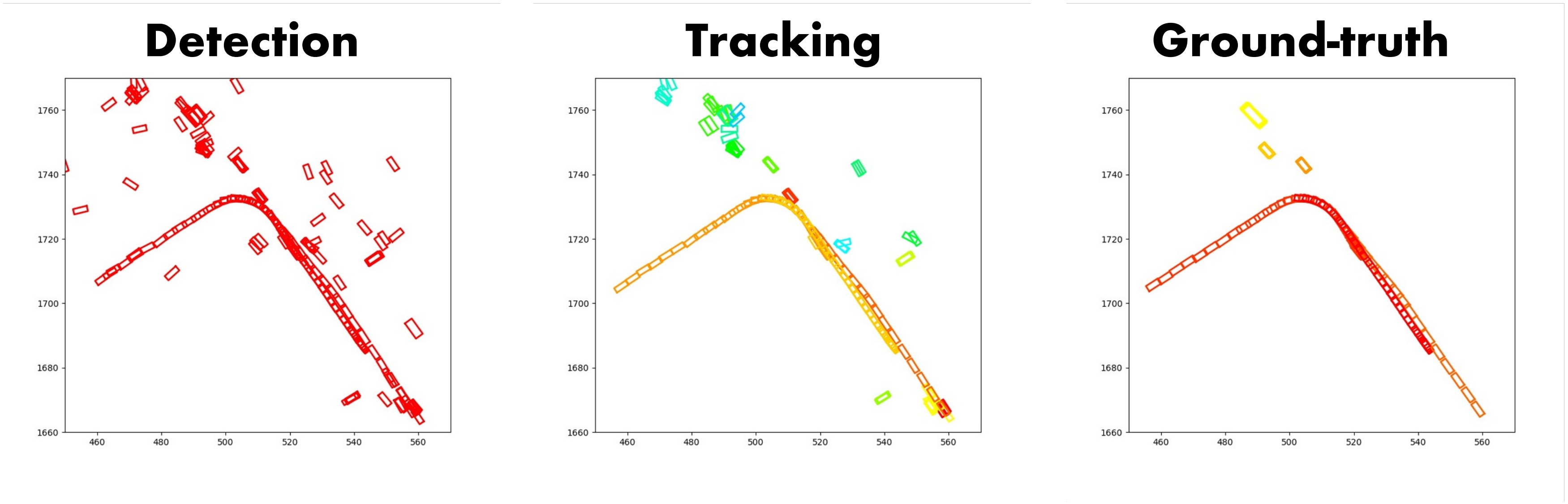
Problem Statement: Output

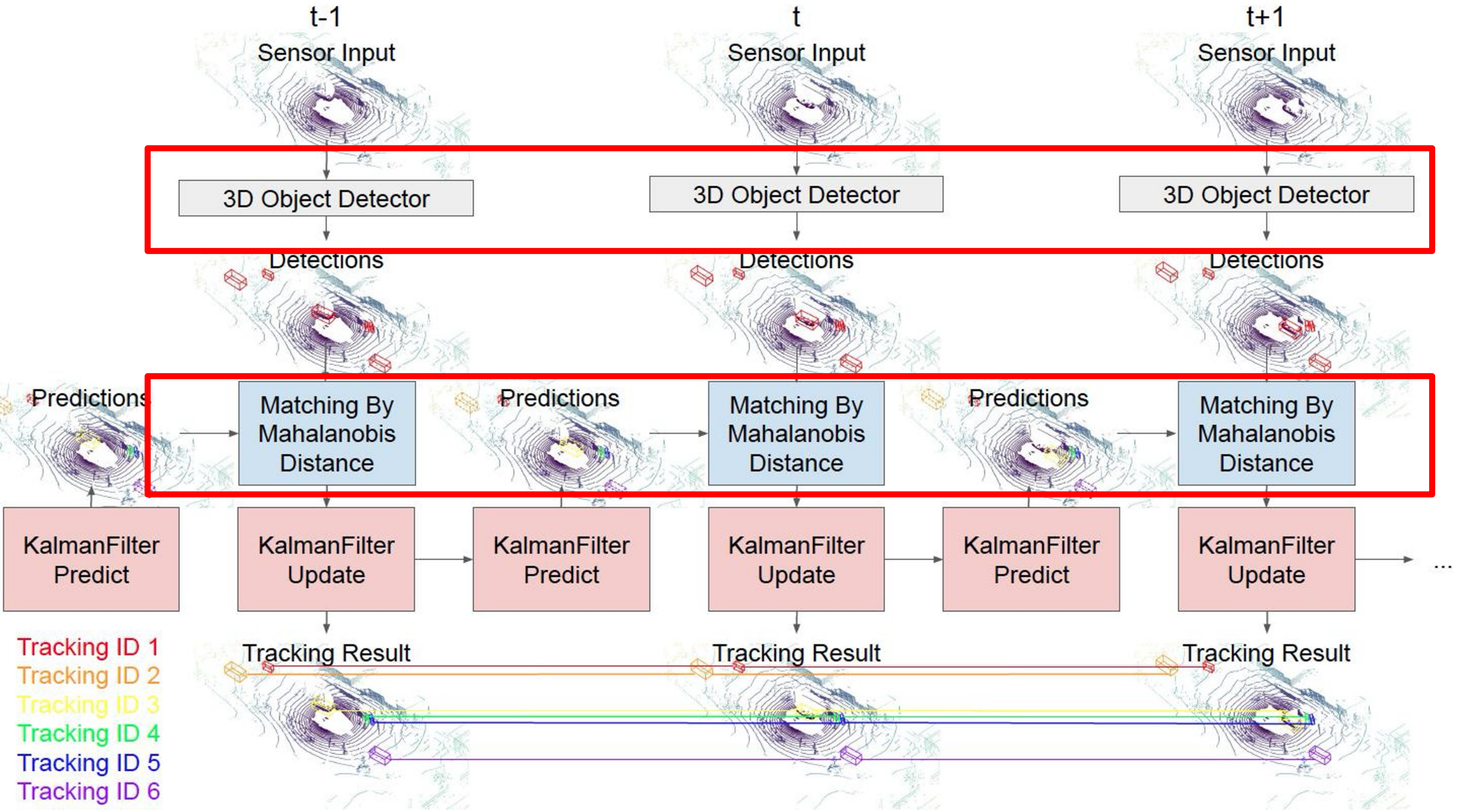
- Tracking object bounding boxes at each frame in a sequence
- Each tracking bounding box is represented by:
  - center position (x, y, z), rotation angle along the z-axis (a), and the scale (l, w, h)
  - category label (car, pedestrian, ...), confidence score (c)
  - tracking id: one unique tracking id for each object instance across frames



Why Tracking?

- Filter out the out-liners from the detection results
- Continue estimating object states even if occluded
- Forecast the future based on past trajectories and motion patterns
- Make autonomous driving decisions




t-1
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Sensor Input
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3D Object Detector
3D Object Detector
3D Object Detector
Detections
Detections
Detections
Predictions
Matching By Mahalanobis Distance
Predictions
Matching By Mahalanobis Distance
Predictions
Matching By Mahalanobis Distance
KalmanFilter Predict
KalmanFilter Update
KalmanFilter Predict
KalmanFilter Update
KalmanFilter Predict
KalmanFilter Update
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Tracking Result
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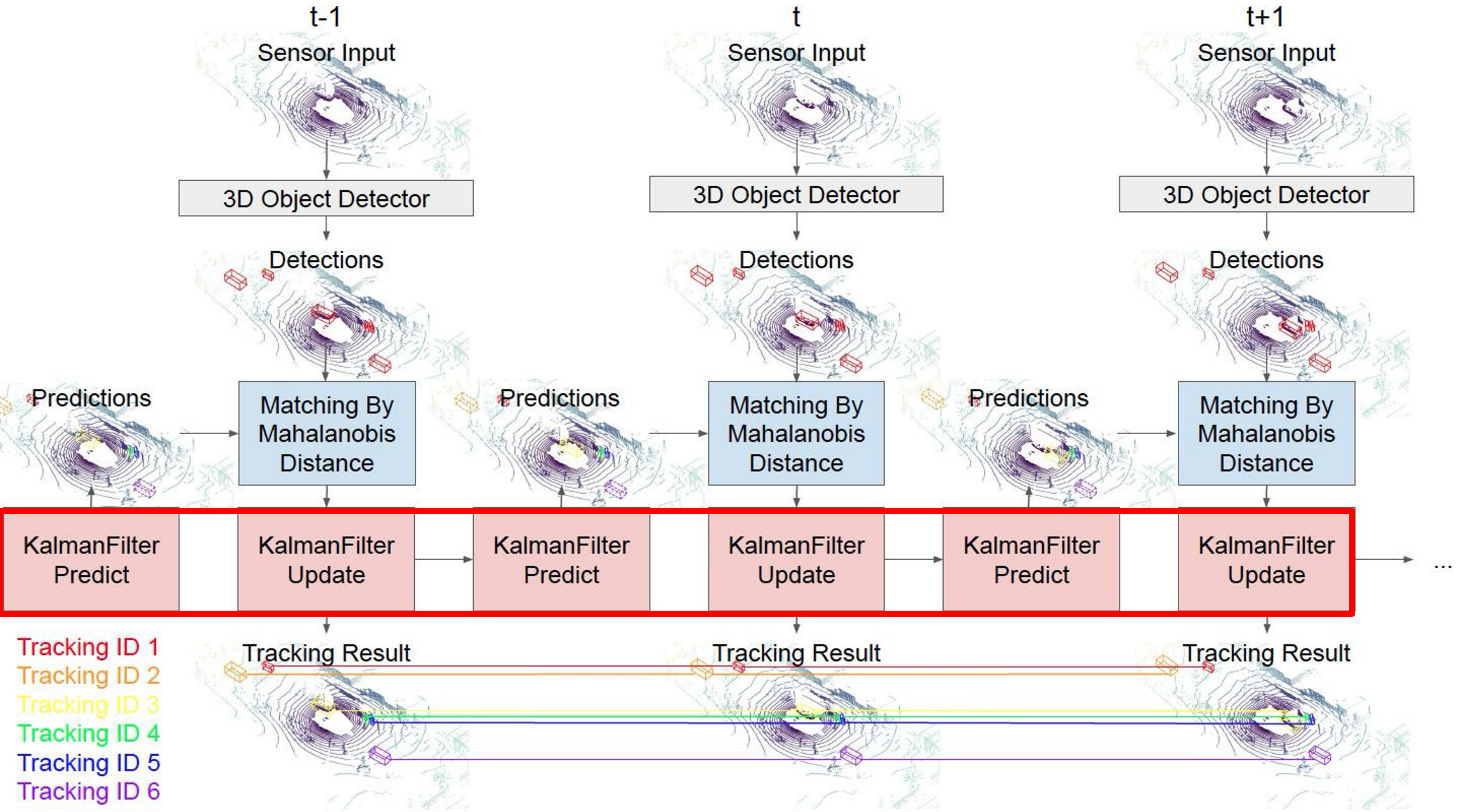
Tracking Result
Tracking ID 1
Tracking ID 2
Tracking ID 3
Tracking ID 4
Tracking ID 5
Tracking ID 6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Predictions
Matching By Mahalanobis Distance
KalmanFilter Predict
KalmanFilter Update
Tracking Result
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3D Object Detector
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KalmanFilter Update
Tracking Result
t+1
Sensor Input
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KalmanFilter Predict
KalmanFilter Update
Tracking Result
...
Tracking ID 1
Tracking ID 2
Tracking ID 3
Tracking ID 4
Tracking ID 5
Tracking ID 6

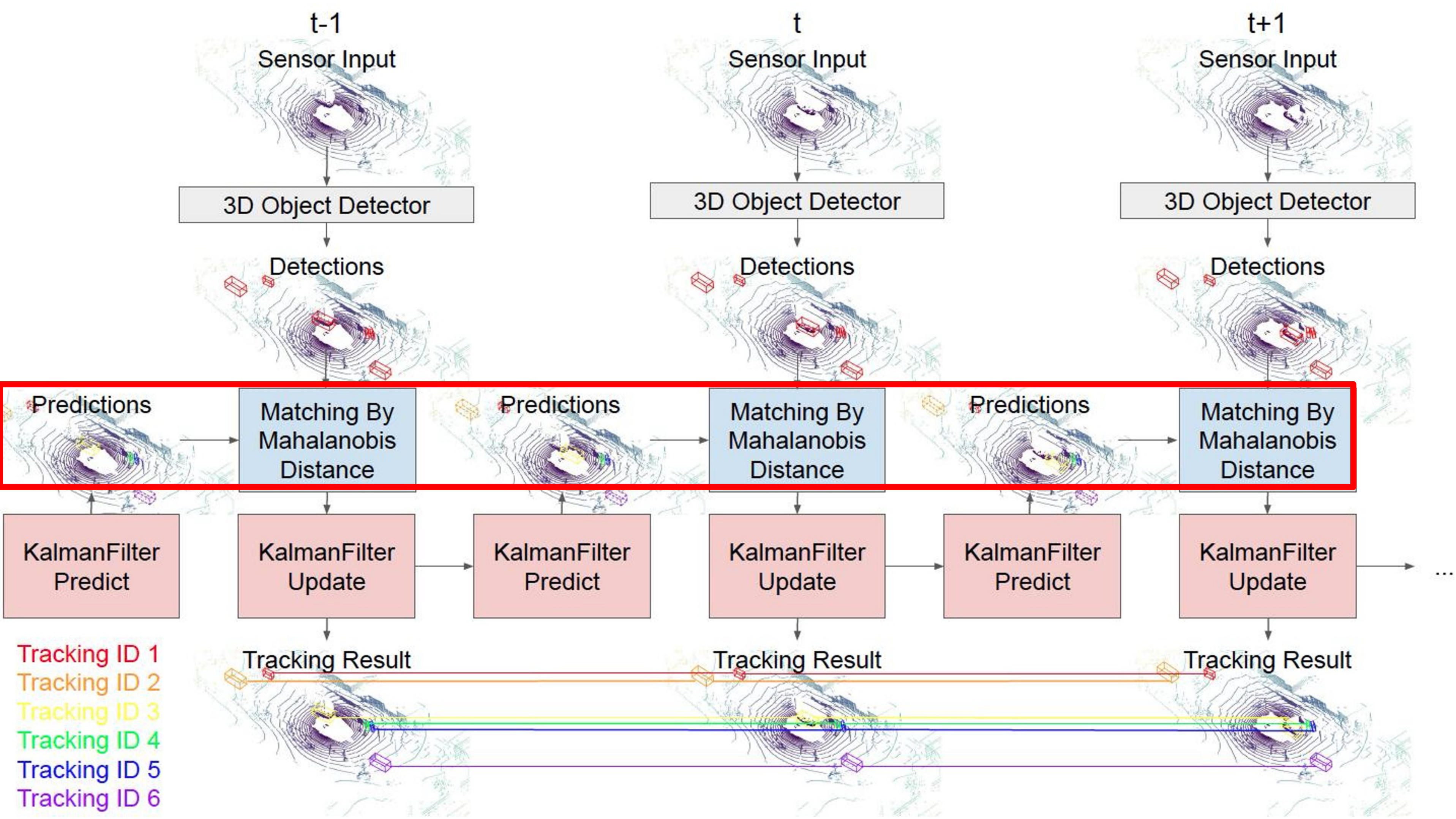
Kalman Filter for Tracking

Define the object **state** using a vector of random variables including the position, the rotation, the scale, linear velocity, and the angular velocity.

$$\mathbf{s}_t = (x, y, z, a, l, w, h, d_x, d_y, d_z, d_a)^T$$

Define the **Process Model** for prediction based on the constant velocity motion:

$$\begin{aligned}
\hat{x}_{t+1} &= x_t + d_{x_t} + q_{x_t}, & \hat{d}_{x_{t+1}} &= d_{x_t} + q_{d_{x_t}} & \hat{l}_{t+1} &= l_t \\
\hat{y}_{t+1} &= y_t + d_{y_t} + q_{y_t}, & \hat{d}_{y_{t+1}} &= d_{y_t} + q_{d_{y_t}} & \hat{w}_{t+1} &= w_t \\
\hat{z}_{t+1} &= z_t + d_{z_t} + q_{z_t}, & \hat{d}_{z_{t+1}} &= d_{z_t} + q_{d_{z_t}} & \hat{h}_{t+1} &= h_t \\
\hat{a}_{t+1} &= a_t + d_{a_t} + q_{a_t}, & \hat{d}_{a_{t+1}} &= d_{a_t} + q_{d_{a_t}} & &
\end{aligned}$$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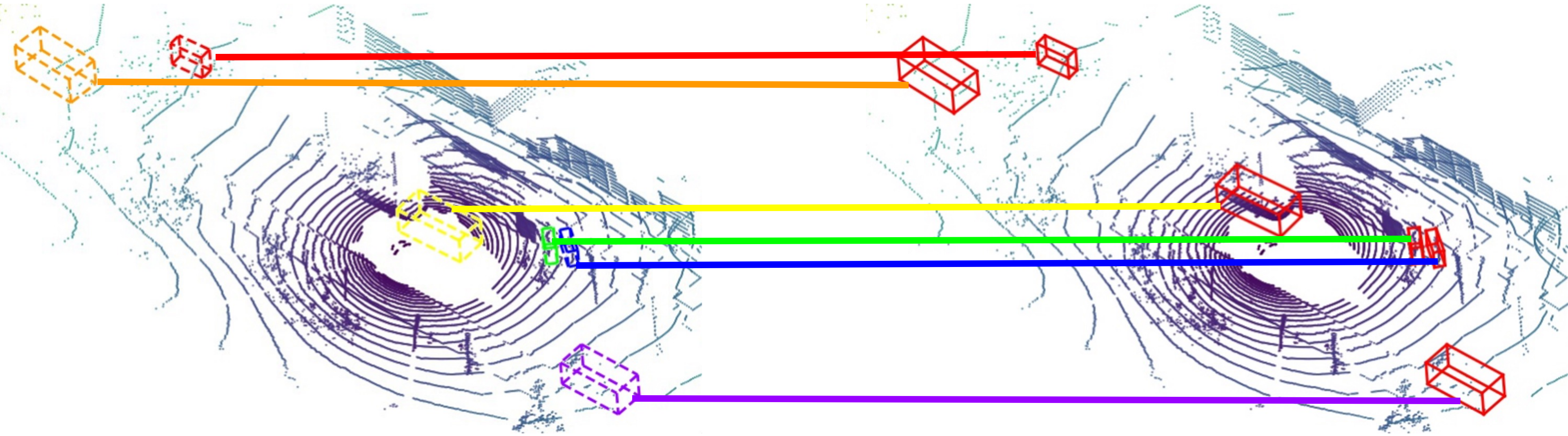
KalmanFilter Update
...
Tracking ID 1
Tracking ID 2
Tracking ID 3
Tracking ID 4
Tracking ID 5
Tracking ID 6
Tracking Result
Tracking Result
Tracking Result

Data Association

Mahalanobis Distance $m = \sqrt{(z_t - C\bar{\mu}_t)^T S_t^{-1} (z_t - C\bar{\mu}_t)}$

S = Innovation Covariance
$z_t - C\mu_t$ = innovation



Kalman Filter Predictions

Object Detections

Kalman Filter

1: **Algorithm Kalman_filter(**$\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$**):**

2: $\bar{\mu}_t = A_t\, \mu_{t-1} + B_t\, u_t$

3: $\bar{\Sigma}_t = A_t\, \Sigma_{t-1}\, A_t^T + R_t$

4: $K_t = \bar{\Sigma}_t\, C_t^T\, (C_t\, \bar{\Sigma}_t\, C_t^T + Q_t)^{-1}$ $= S_t^{-1}$

5: $\mu_t = \bar{\mu}_t + K_t(z_t - C_t\, \bar{\mu}_t)$

6: $\Sigma_t = (I - K_t\, C_t)\, \bar{\Sigma}_t$

7: return μ_t, Σ_t

Data Association

Mahalanobis Distance $m = \sqrt{(z_t - C\bar{\mu}_t)^T S_t^{-1} (z_t -} C\bar{\mu}_t)$

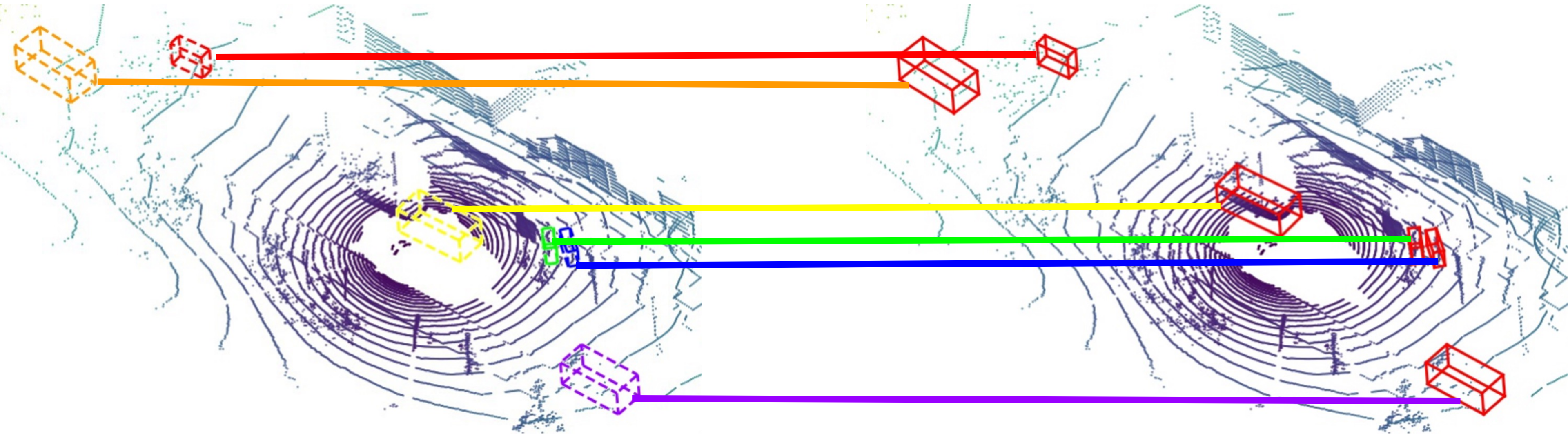
If $\boldsymbol{m > 3 * \sigma}$ **then reject as outlier. 99.7% of values lie within 3*standard deviation.**

Measuring the distance between the observation and the distribution of the predicted state.

Providing distance measurement when there is no overlap between the prediction and detection.

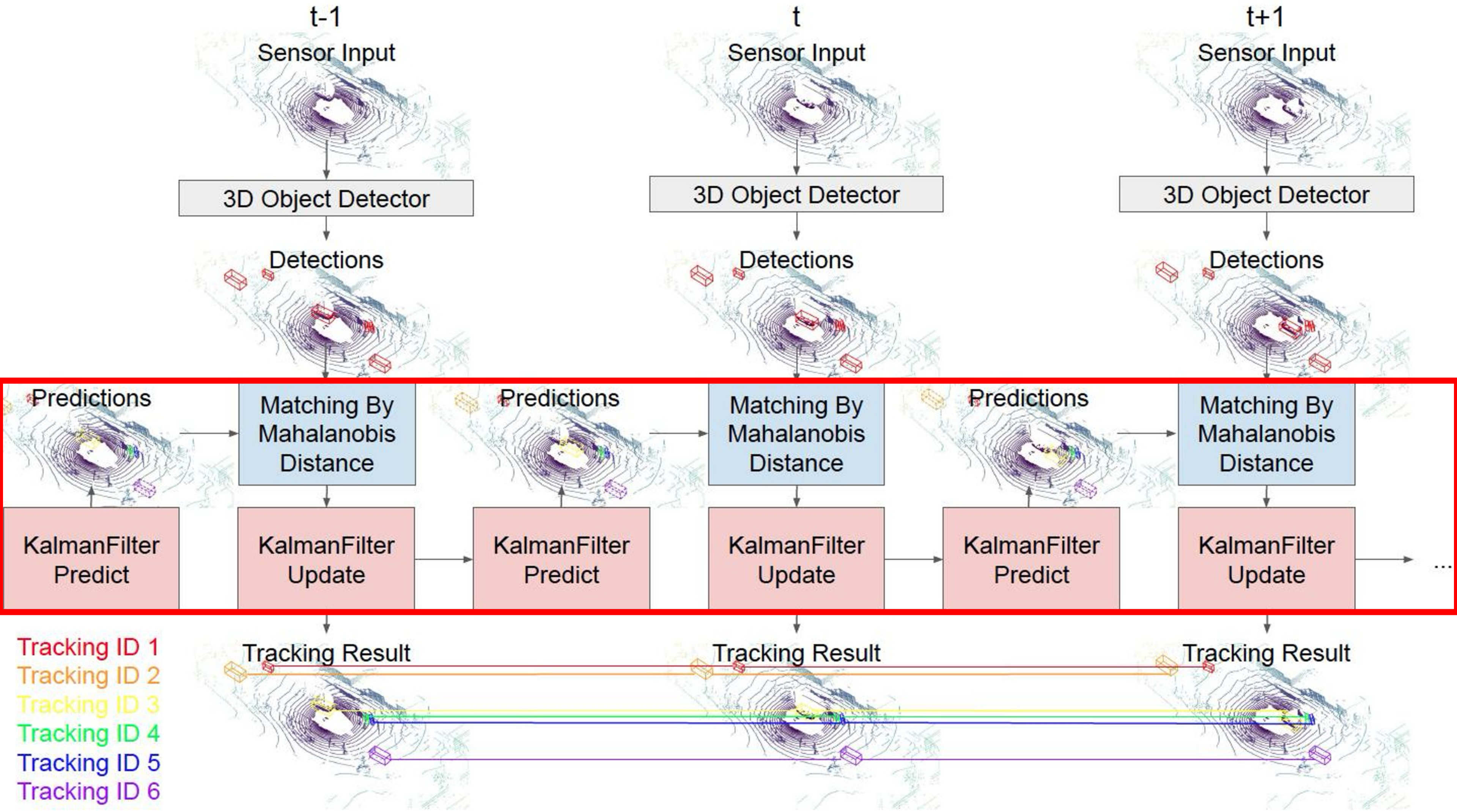
Taking the uncertainty information from the prediction into account.

Data Association - Greedy



Kalman Filter Predictions

Detections



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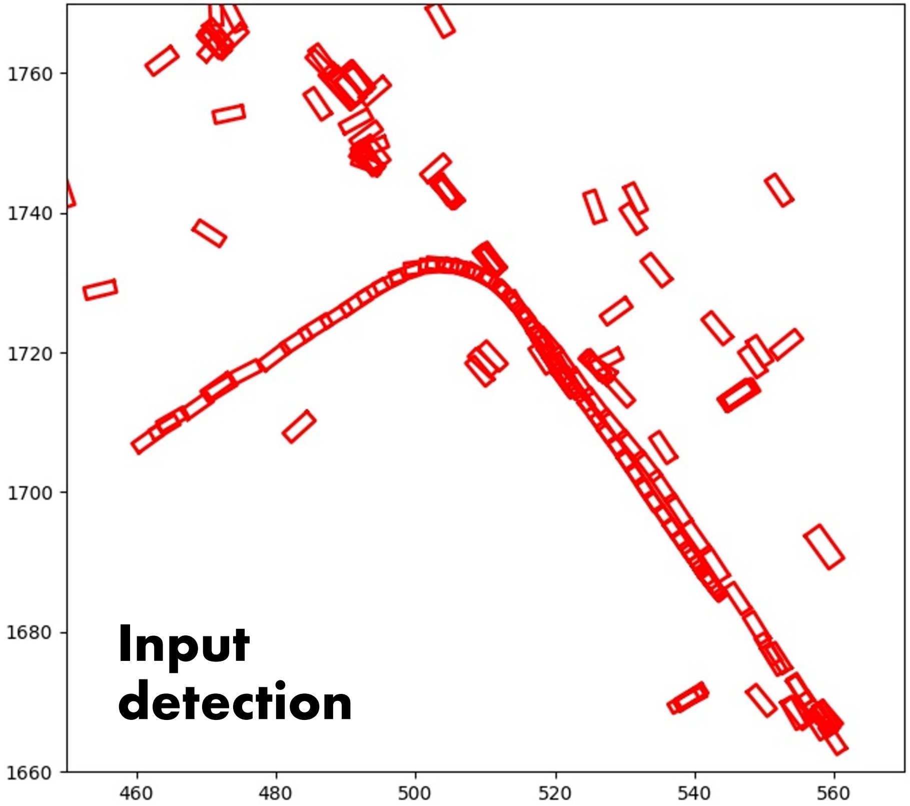
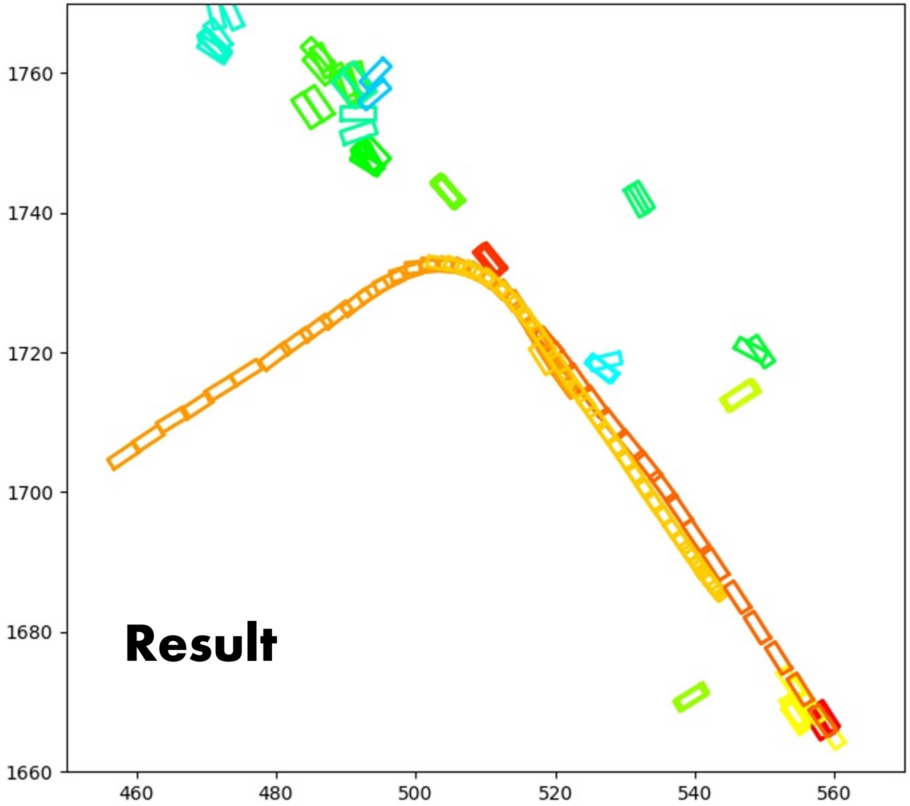
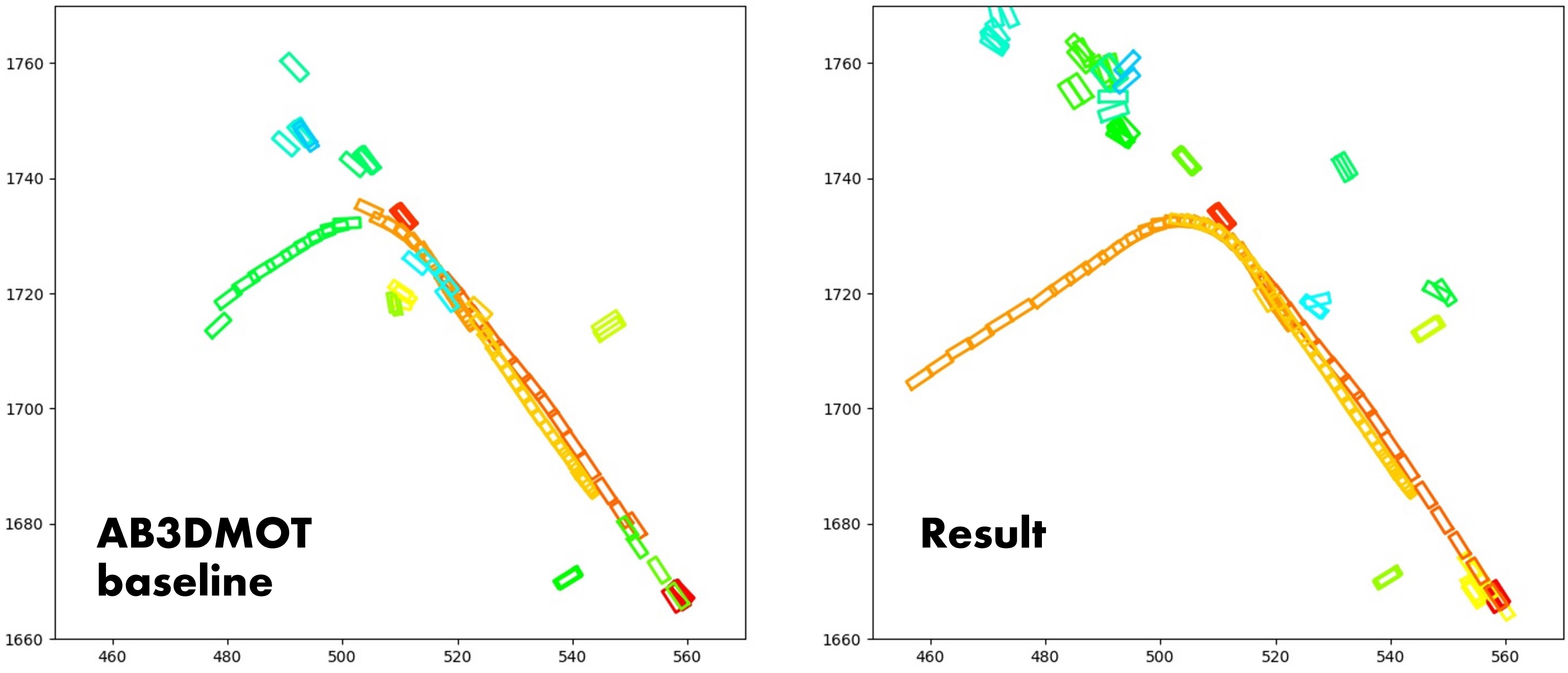
Tracking ID 3
Tracking ID 4
Tracking ID 5
Tracking ID 6
Tracking Result
Tracking Result
Tracking Result

Qualitative Results

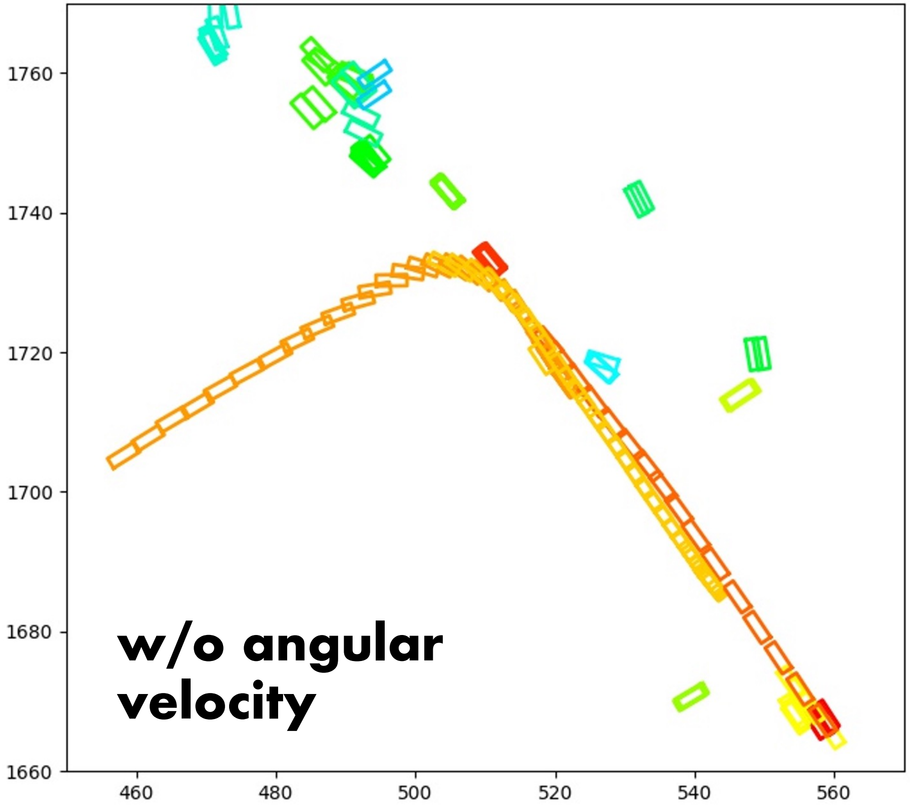


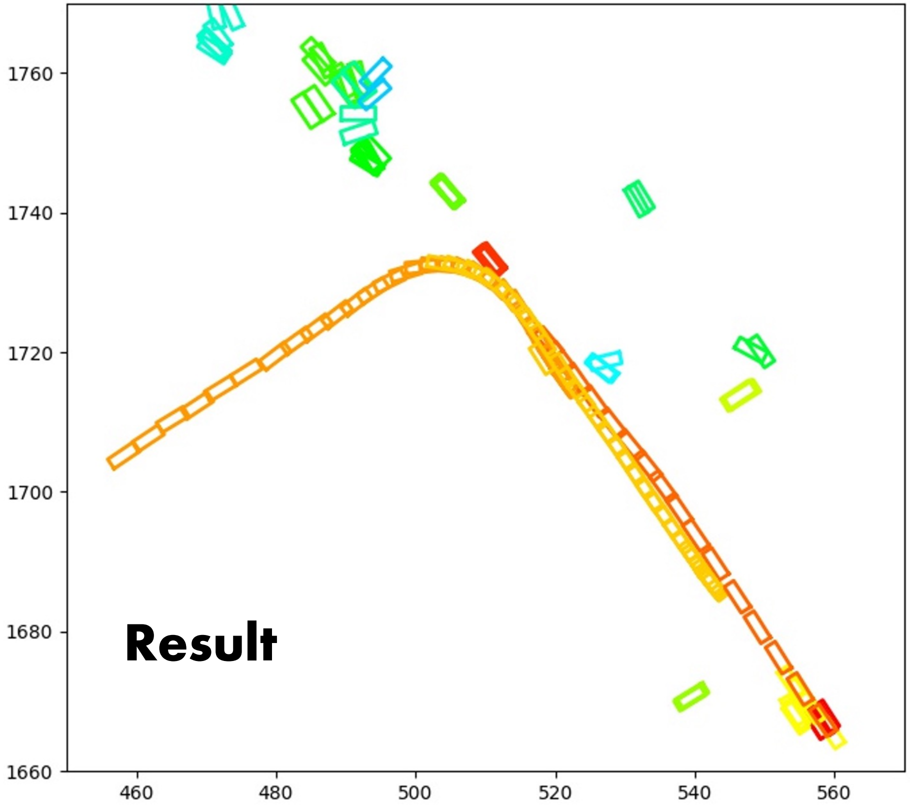


Qualitative Results

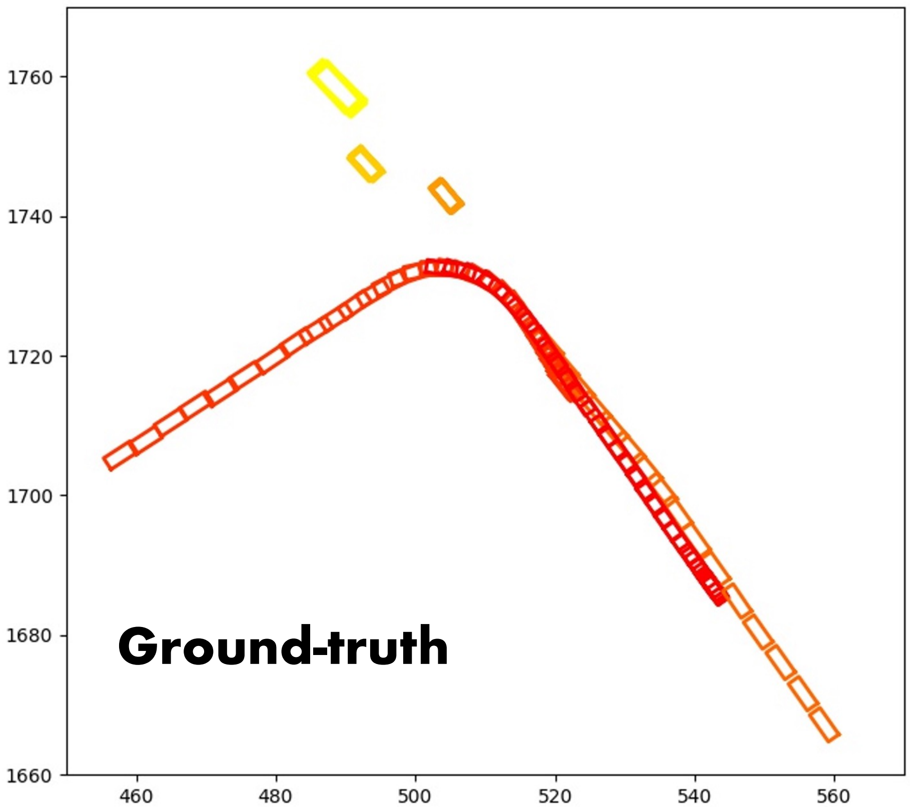


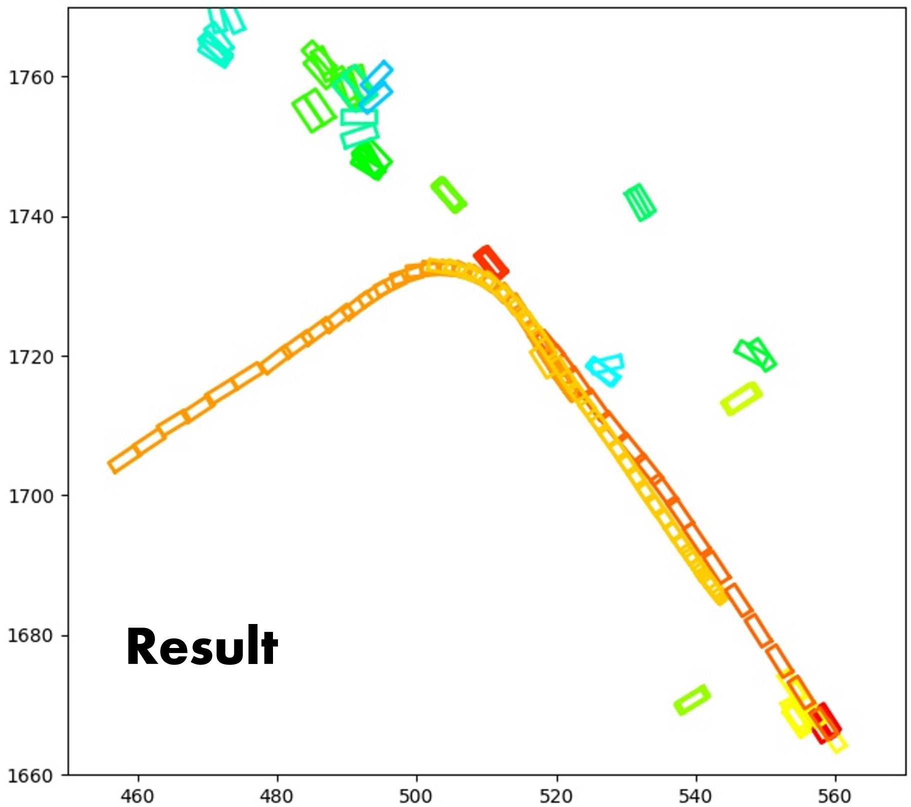
Qualitative Results





Qualitative Results





Priors and Hyperparameters

A lot of hardcoded knowledge!

- State Representation
- Models
  - Forward Model
    - State to next state
    - Action to next state
  - Measurement Model
- Probabilistic Properties
  - Process Noise
  - Measurement Noise

Differentiable filters

Can we learn models and hyperparameters from data?

Approach: Embed algorithmic structure of Bayesian Filtering into a recurrent neural network.

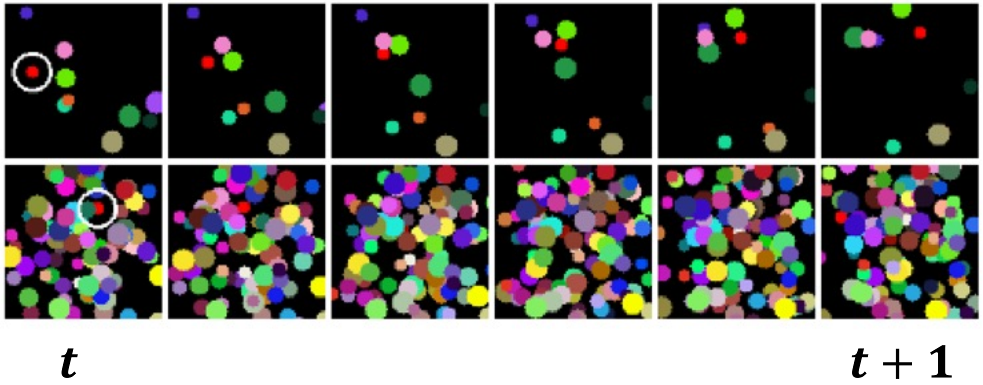
- prevents overfitting through regularization
- Avoids manual tuning and modeling

Estimators. Haarnoja et al. NeurIPS 2016

- Differentiable version of the Kalman Filter
- Uses Images as observations; learns a sensors that outputs state directly

$$g(I_t) = z_t \approx x_t$$

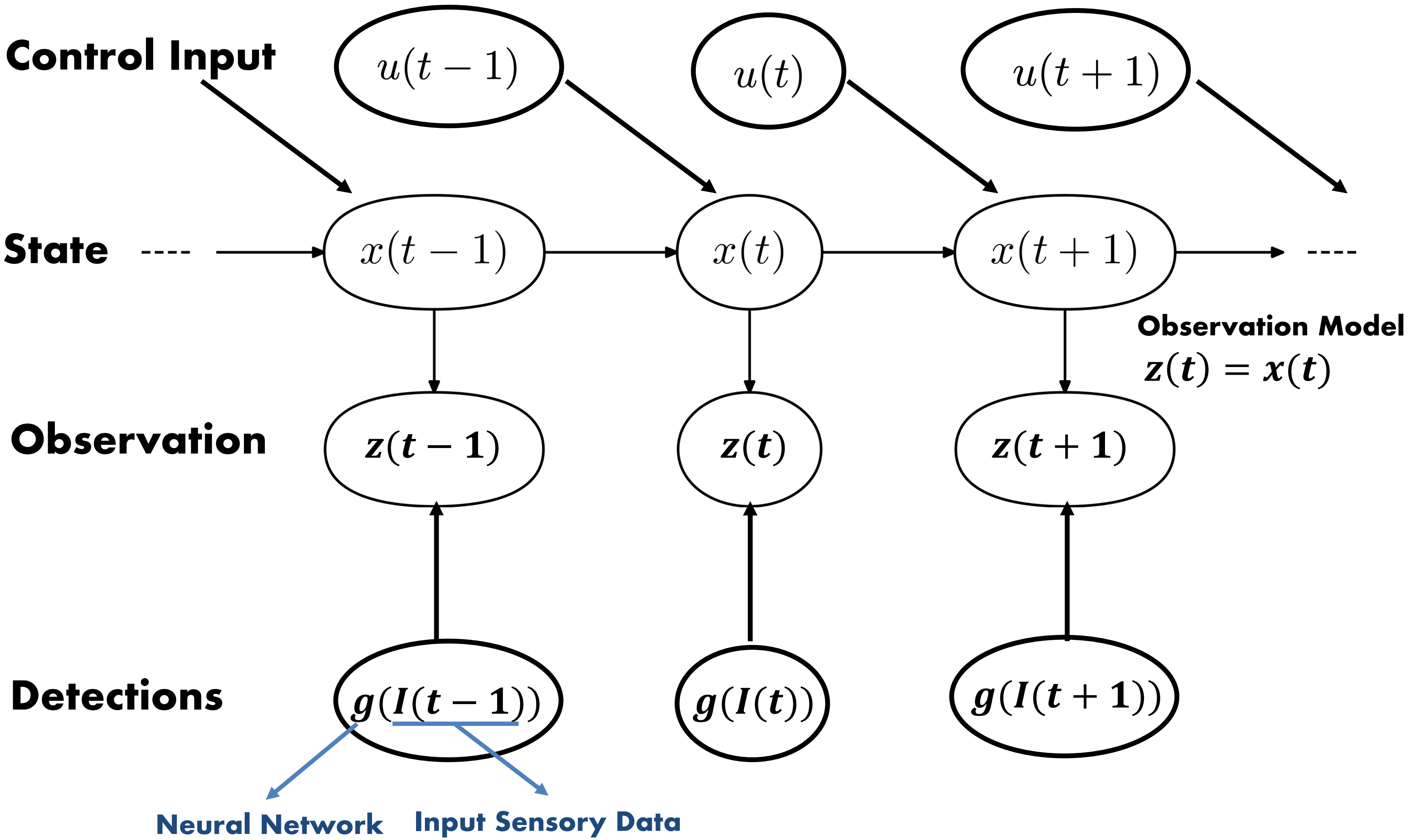
Track red disk position



Example Sequence w/ few occlusions

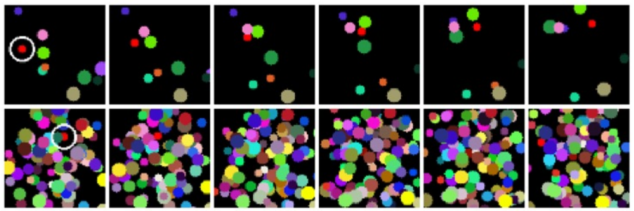
Example Sequence w/ many occlusions

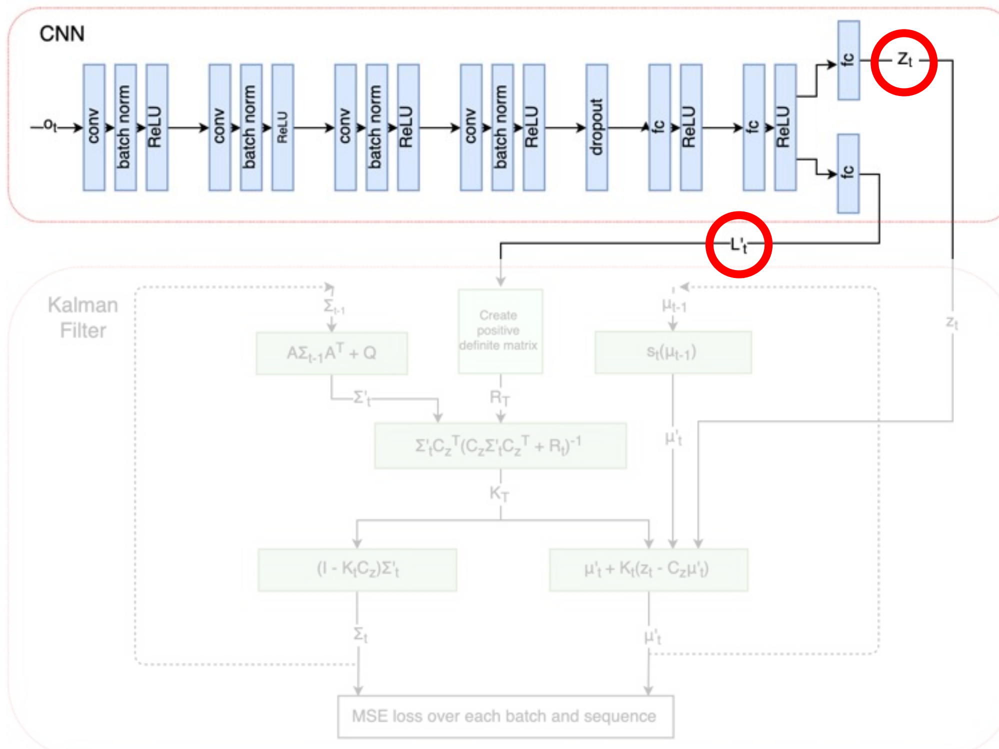
Tracking by Detection



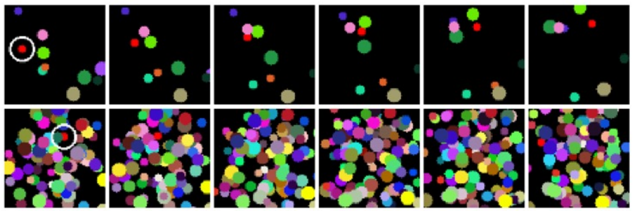
Differentiable Kalman Filter - Structure

$g(I_t) = z_t \approx x_t$



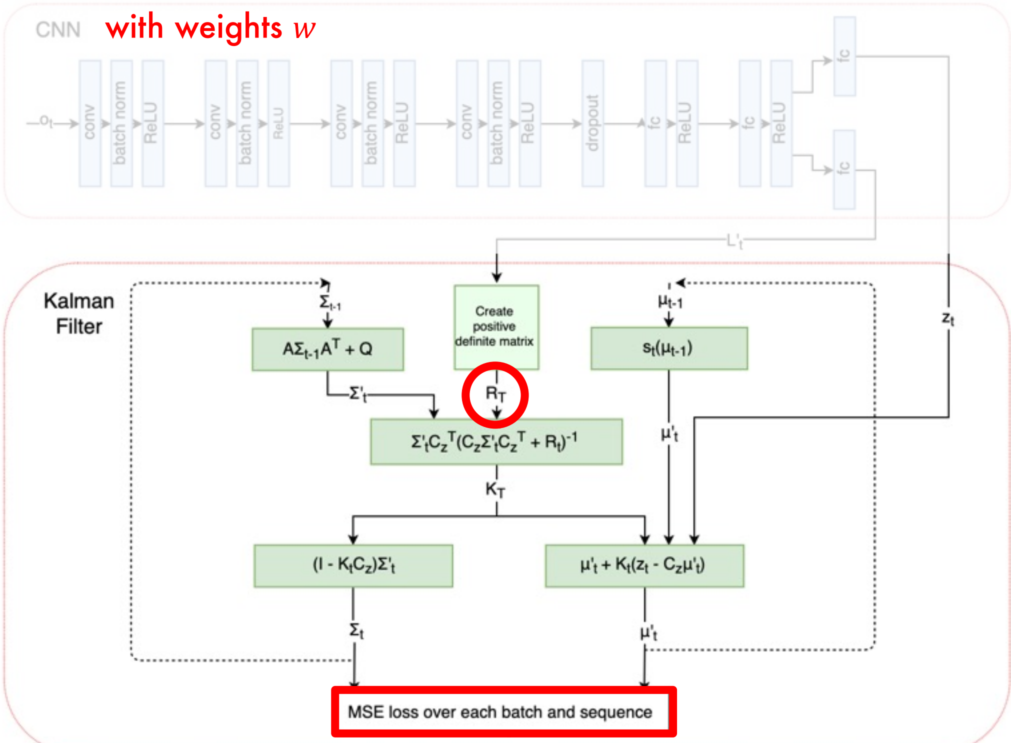


Differentiable Kalman Filter - Structure



R is high if red disk is occluded

$$L' L^T = R$$



$$\frac{\delta Loss}{\delta w}$$

Differentiable Kalman Filter – Loss Function

Ground truth state

Network weights

Belief

$$L(\mathbf{l}_{0\ldots T}, \mu_{0\ldots T}, \mathbf{\Sigma}_{0\ldots T}, \mathbf{w}) =$$

$$\lambda_1 \sum_{t=0}^{T} \frac{1}{2}((\mathbf{l}_t - \mu_t)^T \Sigma_t^{-1} (l_t - \mu_t) + \log(|\Sigma_t|)) + \lambda_2 \sum_{t=0}^{T} \| (l_t - \mu_t) \|_2 + \lambda_3 \| \mathbf{w} \|_2$$

Negative log likelihood of ground truth given current belief

Mean-Squared Error

Regularization

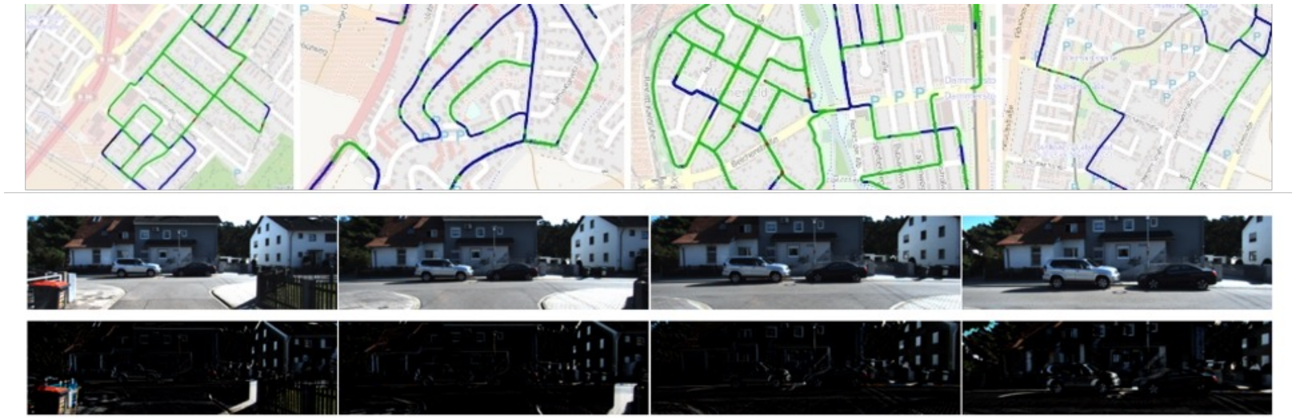
Differentiable Kalman Filter – Experiments and Baselines

Table 1: Benchmark Results

State Estimation Model	# Parameters	RMS test error $\pm\sigma$
feedforward model	7394	0.2322 ± 0.1316
piecewise KF	7397	0.1160 ± 0.0330
LSTM model (64 units)	33506	0.1407 ± 0.1154
LSTM model (128 units)	92450	0.1423 ± 0.1352
BKF (ours)	**7493**	$\mathbf{0.0537} \pm \mathbf{0.1235}$

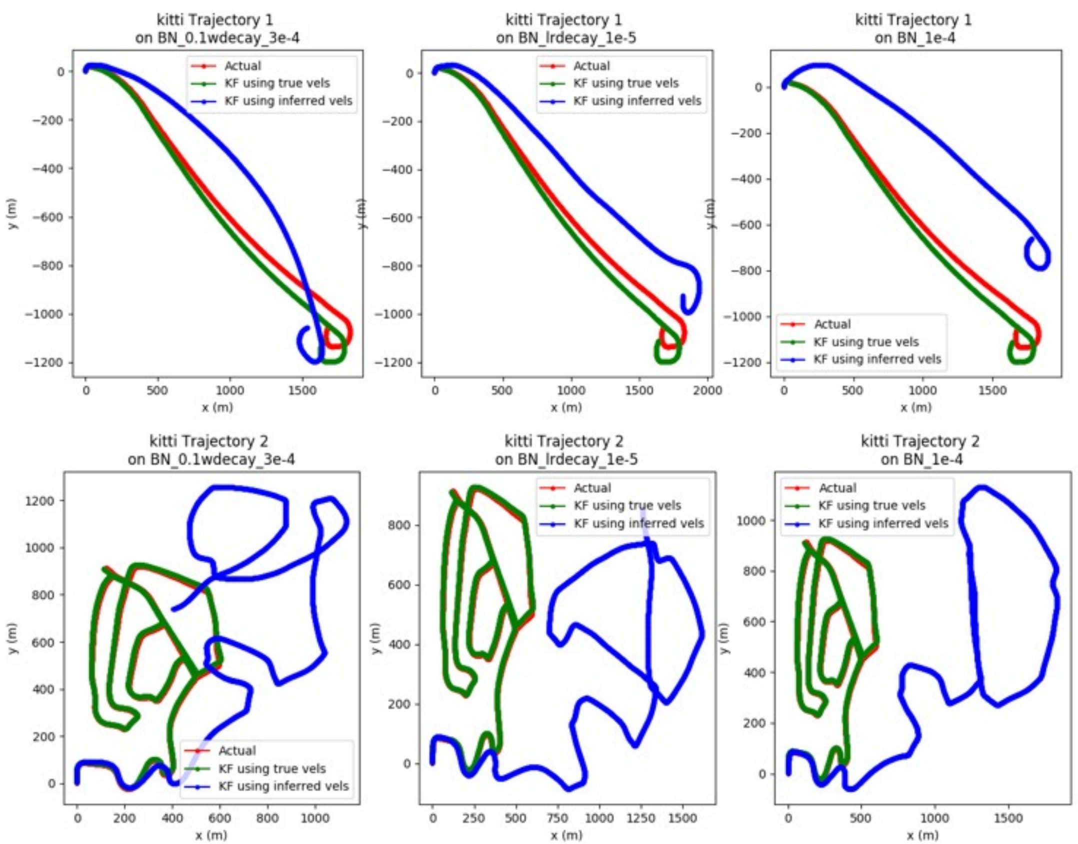
Differentiable Kalman Filter – Experiments and Baselines

- **Kitti – Visual Odometry Datatset**
- **22 stereo sequences with LIDAR**
  - **11 sequences with ground truth (GPS/IMU data)**
  - **11 sequences without ground truth (for evaluation)**



Differentiable Kalman Filter – Experiments and Baselines

Results reproduced by Claire Chen



Blue – Result of BackpropKF
Red – Ground truth
Green – w/ Ground truth velocities

CS231

Introduction to Computer Vision

Next lecture:

Neural Radiance Fields for Novel View Synthesis