CS231A Computer Vision: From 3D Reconstruction to Recognition



Gaussian Splatting for Novel View Synthesis

Silvio Savarese & Jeannette Bohg

Lecture 17



The problem of novel view synthesis



Inputs: sparsely sampled images of scene

Outputs: *new* views of same scene (rendered by our method)

2

Mildenhall et al. ECCV 2020. https://www.matthewtancik.com/nerf

Silvio Savarese & Jeannette Bohg

Lecture 17



2

Rendering (Graphics): Given 3D Scene + Camera parameters, yield images



3D Scene

Camera Poses

Images

Slide adopted from 6.5980 - ML for Inverse Graphics - Vincent Sitzmann

Silvio Savarese & Jeannette Bohg

Lecture 17

3

Inverse Graphics: Given Images, Infer Camera Poses & 3D Scene!



Images

3D Scene

Camera Poses

2-Jun-24

4

Slide adopted from 6.5980 - ML for Inverse Graphics - Vincent Sitzmann

Silvio Savarese & Jeannette Bohg

Lecture 17

How to get camera poses?



Images

3D Scene

Camera Poses

Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Silvio Savarese & Jeannette Bohg

Lecture 17

5

Can assume we know the camera poses.



Images

Camera Poses

3D Scene

Slide adopted from 6.S980 - ML for Inverse Graphics - Vincent Sitzmann

Silvio Savarese & Jeannette Bohg

Lecture 17

6



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Silvio Savarese & Jeannette Bohg

Lecture 17

7





Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Silvio Savarese & Jeannette Bohg

Lecture 17





Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Silvio Savarese & Jeannette Bohg

Lecture 17





Given an *observable* variable (pixel colors), we will build a differentiable forward model that we then use to estimate *unobserved (latent) variables* (geometry, appearance)!

Lecture 17

2-Jun-24

10

Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Silvio Savarese & Jeannette Bohg

Ways to Render



Surface rendering

Volume rendering

11

Silvio Savarese & Jeannette Bohg

Lecture 17

Volume rendering equation

$$I(D) = I_0 T(0) + \int_0^D c(s)\rho(s)T(s)ds$$
pixel color at coordinates D
$$T(s) = \exp\left(-\int_s^D \rho(t)dt\right) \quad \text{transparency}$$

$$I(s) = \exp\left(-\int_s^D \rho(t)dt\right) \quad \text{transparency}$$

LECLUIE

Represent a scene as a continuous 5D function



Generate views with traditional volume rendering



Mildenhall et al. ECCV 2020. https://www.matthewtancik.com/nerf

Silvio Savarese & Jeannette Bohg

Lecture 17

2-Jun-24

14

Generate views with traditional volume rendering



Lecture 17

2-Jun-24

15

$$\alpha_i = 1 - e^{-\sigma_i \delta t_i}$$

Silvio Savarese & Jeannette Bohg

Function of segment length δt_i and volume density σ

From Presentation by Matthew Tancik: Neural Radiance Fields for View Synthesis. 2020.

Optimize with gradient descent on rendering loss



Silvio Savarese & Jeannette Bohg

Lecture 17

Training network to reproduce all input views of the scene



From Presentation by Matthew Tancik: Neural Radiance Fields for View Synthesis. 2020.

Silvio Savarese & Jeannette Bohg

Lecture 17



The problem of novel view synthesis



Inputs: sparsely sampled images of scene

Outputs: *new* views of same scene (rendered by our method)

2-Jun-24

2

Mildenhall et al. ECCV 2020. https://www.matthewtancik.com/nerf

Silvio Savarese & Jeannette Bohg

Lecture 17

18

Vision-Only Navigation











(b) Point-NeRF Representation with Volume

location



Plenoxels: Radiance Fields ... [Yu et al. 2022] Direct Voxel Grid Optimization [Sun et al. 2021]

InstantNGP: Instant Neural ... [Müller et al. 2022]

PointNeRF: Point-based Neural ... [Xu et al. 2022]

Efficient Geometry-aware 3D... [Chan et al. 2022] TensorRF: Tensor Radiance Fields [Chen & Xu et al. 2022]

Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann



3D Gaussian Splatting for Real-Time Radiance Field Rendering

BERNHARD KERBL*, Inria, Université Côte d'Azur, France GEORGIOS KOPANAS*, Inria, Université Côte d'Azur, France THOMAS LEIMKÜHLER, Max-Planck-Institut für Informatik, Germany GEORGE DRETTAKIS, Inria, Université Côte d'Azur, France



Fig. 1. Our method achieves real-time rendering of radiance fields with quality that equals the previous method with the best quality [Barron et al. 2022], while only requiring optimization times competitive with the fastest previous methods [Fridovich-Keil and Yu et al. 2022; Müller et al. 2022]. Key to this performance is a novel 3D Gaussian scene representation coupled with a real-time differentiable renderer, which offers significant speedup to both scene optimization and novel view synthesis. Note that for comparable training times to InstantNGP [Müller et al. 2022], we achieve similar quality to theirs; while this is the maximum quality they reach, by training for 51min we achieve state-of-the-art quality, even slightly better than Mip-NeRF360 [Barron et al. 2022].



Neural Radiance Field: Parameterize Radiance Field densely, at *every* point in space



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

3D Gaussian Blobs

floating in Space



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

3D Gaussian Blobs

floating in Space



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

3D Gaussian Blobs

floating in Space



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

3D Gaussian Blobs

floating in Space



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Anisotropic Volumetric 3D Gaussians



Final Rendering

3D Gaussian Visualization









Gaussians are closed under affine transforms, integration







Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Gaussians are closed under affine transforms, integration







Slide adopted from 6.S980 – ML for Inverse Graphics – Vincent Sitzmann

Gaussians are closed under affine transforms, integration



Integrate along axis:

$$\int_{\mathrm{IR}} \mathcal{G}_{\mathbf{V}}^3(\mathbf{x} - \mathbf{p}) \, dx_2 = \mathcal{G}_{\hat{\mathbf{V}}}^2(\hat{\mathbf{x}} - \hat{\mathbf{p}})$$
$$\mathbf{V} = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \Leftrightarrow \begin{pmatrix} a & b \\ b & d \end{pmatrix} = \hat{\mathbf{V}}$$



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Throwback: The Kalman Filter Algorithm

1:	Algorithm Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):
2:	$\bar{\mu}_t = A_t \ \mu_{t-1} + B_t \ u_t$
3:	$\bar{\Sigma}_t = A_t \ \Sigma_{t-1} \ A_t^T + R_t$
4:	$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
5:	$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \ \bar{\mu}_t)$
6:	$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$
7:	return μ_t, Σ_t

1:Algorithm Bayes_filter($bel(x_{t-1}), u_t, z_t$):2:for all x_t do3: $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}) \ bel(x_{t-1}) \ dx$ Predict Step4: $bel(x_t) = \eta \ p(z_t \mid x_t) \ \overline{bel}(x_t)$ Update Step5:endfor6:return $bel(x_t)$

Silvio Savarese & Jeannette Bohg

Lecture 17

2-Jun-24

40

Instead: Rasterization



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Instead: Rasterization



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

"Cull" Gaussians with less than 99% confidence relative to view frustum



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann



Cam2world is affine mapping $\phi(x) = \mathbf{W}\mathbf{x} + \mathbf{p}$: $\mathcal{G}_{\mathbf{V}_{k}^{\prime\prime}}(\varphi^{-1}(\mathbf{u}) - \mathbf{t}_{k}) = \frac{1}{|\mathbf{W}^{-1}|}\mathcal{G}_{\mathbf{V}_{k}^{\prime}}(\mathbf{u} - \mathbf{u}_{k}) = r_{k}^{\prime}(\mathbf{u})$





Slide adopted from 6.S980 – ML for Inverse Graphics – Vincent Sitzmann

Cam2world is affine mapping $\phi(x) = Wx + p$: $\mathcal{G}_{\mathbf{V}_k^{\prime\prime}}(arphi^{-1}(\mathbf{u})-\mathbf{t}_k)=rac{1}{|\mathbf{W}^{-1}|}\mathcal{G}_{\mathbf{V}_k^\prime}(\mathbf{u}-\mathbf{u}_k)=r_k^\prime(\mathbf{u})$ · Projection m(u) is not an affine mapping :/





Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Cam2world is affine mapping $\phi(x) = \mathbf{W}\mathbf{x} + \mathbf{p}$: $\mathcal{G}_{\mathbf{V}_{k}^{\prime\prime}}(\varphi^{-1}(\mathbf{u}) - \mathbf{t}_{k}) = \frac{1}{|\mathbf{W}^{-1}|} \mathcal{G}_{\mathbf{V}_{k}^{\prime}}(\mathbf{u} - \mathbf{u}_{k}) = r_{k}^{\prime}(\mathbf{u})$ Projection m(u) is not an affine mapping :/ $\begin{pmatrix} x_{0} \\ x_{1} \\ x_{2} \end{pmatrix} = \mathbf{m}(\mathbf{u}) = \begin{pmatrix} u_{0}/u_{2} \\ u_{1}/u_{2} \\ \|(u_{0}, u_{1}, u_{2})^{T}\| \end{pmatrix}$ $\begin{pmatrix} u_{0} \\ u_{1} \\ u_{2} \end{pmatrix} = \mathbf{m}^{-1}(\mathbf{x}) = \begin{pmatrix} x_{0}/l \cdot x_{2} \\ x_{1}/l \cdot x_{2} \\ 1/l \cdot x_{2} \end{pmatrix}$, But can approximate with first-order Taylor Expansion as:

$$\mathbf{m}_{\mathbf{u}_k}(\mathbf{u}) = \mathbf{x}_k + \mathbf{J}_{\mathbf{u}_k} \cdot (\mathbf{u} - \mathbf{u}_k) \qquad \mathbf{J}_{\mathbf{u}_k} = rac{\partial \mathbf{m}}{\partial \mathbf{u}} (\mathbf{u}_k)$$

Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Camera

•

Propagating a Gaussian through a Linear Model



Silvio Savarese & Jeannette Bohg

Lecture 17



Propagating a Gaussian through a Non-Linear Model



Silvio Savarese & Jeannette Bohg

Lecture 17



Linearizing the Non-Linear Model



Silvio Savarese & Jeannette Bohg

Lecture 17



Throwback: The Extended Kalman Filter Algorithm

1: Algorithm Extended_Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):				
2: 3:	$ \bar{\mu}_t = g(u_t, \mu_{t-1}) \\ \bar{\Sigma}_t = G_t \ \Sigma_{t-1} \ G_t^T + R_t $	Predict		
4: 5: 6: 7:	$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$ $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$ $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$ return μ_t, Σ_t	Update		

	Kalman filter	EKF
state prediction (Line 2)	$A_t \ \mu_{t-1} + B_t \ u_t$	$g(u_t,\mu_{t-1})$
measurement prediction (Line 5)	$C_t \; ar{\mu}_t$	$h(ar{\mu}_t)$

Silvio Savarese & Jeannette Bohg

Lecture 17

50

Cam2world is affine mapping $\phi(x) = \mathbf{W}\mathbf{x} + \mathbf{p}$: $\mathcal{G}_{\mathbf{V}_{k}^{\prime\prime}}(\varphi^{-1}(\mathbf{u}) - \mathbf{t}_{k}) = \frac{1}{|\mathbf{W}^{-1}|} \mathcal{G}_{\mathbf{V}_{k}^{\prime}}(\mathbf{u} - \mathbf{u}_{k}) = r_{k}^{\prime}(\mathbf{u})$ Projection m(u) is not an affine mapping :/ $\begin{pmatrix} x_{0} \\ x_{1} \\ x_{2} \end{pmatrix} = \mathbf{m}(\mathbf{u}) = \begin{pmatrix} u_{0}/u_{2} \\ u_{1}/u_{2} \\ \|(u_{0}, u_{1}, u_{2})^{T}\| \end{pmatrix}$ $\begin{pmatrix} u_{0} \\ u_{1} \\ u_{2} \end{pmatrix} = \mathbf{m}^{-1}(\mathbf{x}) = \begin{pmatrix} x_{0}/l \cdot x_{2} \\ x_{1}/l \cdot x_{2} \\ 1/l \cdot x_{2} \end{pmatrix}$, But can approximate with first-order Taylor Expansion as:

$$\mathbf{m}_{\mathbf{u}_k}(\mathbf{u}) = \mathbf{x}_k + \mathbf{J}_{\mathbf{u}_k} \cdot (\mathbf{u} - \mathbf{u}_k) \qquad \mathbf{J}_{\mathbf{u}_k} = rac{\partial \mathbf{m}}{\partial \mathbf{u}} (\mathbf{u}_k)$$

Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Camera

•



But can approximate with first-order Taylor Expansion as:

$$\mathbf{M}_{\mathbf{u}_k}(\mathbf{u}) = \mathbf{x}_k + \mathbf{J}_{\mathbf{u}_k} \cdot (\mathbf{u} - \mathbf{u}_k) \qquad \mathbf{J}_{\mathbf{u}_k} = \frac{\partial \mathbf{m}}{\partial \mathbf{u}} (\mathbf{u}_k)$$

Slide adopted from 6.S980 – ML for Inverse Graphics – Vincent Sitzmann



But can approximate with first-order Taylor Expansion as:

$$\mathbf{m}_{\mathbf{u}_k}(\mathbf{u}) = \mathbf{x}_k + \mathbf{J}_{\mathbf{u}_k} \cdot (\mathbf{u} - \mathbf{u}_k) \qquad \mathbf{J}_{\mathbf{u}_k} = \frac{\partial \mathbf{m}}{\partial \mathbf{u}} (\mathbf{u}_k)$$

Projected, 2D Gaussians are then:

$$\frac{1}{|\mathbf{W}^{-1}||\mathbf{J}^{-1}|}\mathcal{G}_{\mathbf{V}_k}(\mathbf{x}-\mathbf{x}_k)$$

$$\begin{aligned} \mathbf{V}_k &= \mathbf{J}\mathbf{V}'_k \, \mathbf{J}^T \\ &= \mathbf{J}\mathbf{W}\mathbf{V}''_k \mathbf{W}^T \mathbf{J}^T. \end{aligned}$$

Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann



But can approximate with first-order Taylor Expansion as:

$$\mathbf{m}_{\mathbf{u}_k}(\mathbf{u}) = \mathbf{x}_k + \mathbf{J}_{\mathbf{u}_k} \cdot (\mathbf{u} - \mathbf{u}_k) \qquad \mathbf{J}_{\mathbf{u}_k} = \frac{\partial \mathbf{m}}{\partial \mathbf{u}} (\mathbf{u}_k)$$

Projected, 2D Gaussians are then:

$$\frac{1}{|\mathbf{W}^{-1}||\mathbf{J}^{-1}|}\mathcal{G}_{\mathbf{V}_k}(\mathbf{x}-\mathbf{x}_k)$$

$$\mathbf{V}_k = \mathbf{J}\mathbf{V}'_k \mathbf{J}^T = \mathbf{J}\mathbf{W}\mathbf{V}''_k \mathbf{W}^T \mathbf{J}^T.$$

Finally, can integrate along rays:

$$q_k(\hat{\mathbf{x}}) = \int_{\mathbb{R}} \frac{1}{|\mathbf{J}^{-1}| |\mathbf{W}^{-1}|} \mathcal{G}_{\mathbf{V}_k}(\hat{\mathbf{x}} - \hat{\mathbf{x}}_k, x_2 - x_{k2}) dx_2$$

$$= \frac{1}{|\mathbf{J}^{-1}| |\mathbf{W}^{-1}|} \mathcal{G}_{\hat{\mathbf{V}}_k}(\hat{\mathbf{x}} - \hat{\mathbf{x}}_k)$$

Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Can compute volume rendering integral without ever sampling a single 3D point in space!



$$C = \sum_{i \in \mathcal{N}} c_i \alpha_i \prod_{j=1}^{i-1} (1 - \alpha_j), \tag{3}$$

where \mathbf{c}_i is the color of each point and α_i is given by evaluating a 2D Gaussian with covariance Σ [Yifan et al. 2019] multiplied with a learned per-point opacity.

Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Projected 3D Gaussian makes 2D Gaussian!





Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Any problems for inverse graphics, though...?



$$C = \sum_{i \in \mathcal{N}} c_i \alpha_i \prod_{j=1}^{i-1} (1 - \alpha_j),$$
(3)

where \mathbf{c}_i is the color of each point and α_i is given by evaluating a 2D Gaussian with covariance Σ [Yifan et al. 2019] multiplied with a learned per-point opacity.

Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Problem: Local minima...



Slide adopted from 6.5980 – ML for Inverse Graphics – Vincent Sitzmann

Fix 1: Start from SFM point cloud.



Slide adopted from 6.5980 - ML for Inverse Graphics - Vincent Sitzmann

Fix 2: Heuristic *pruning* and *spawning* operations



Slide adopted from 6.S980 - ML for Inverse Graphics - Vincent Sitzmann

Timelapse of the Optimization (NeRF-Synthetic Dataset)



CS231A Computer Vision: From 3D Reconstruction to Recognition



Next lecture: Guest Lecture by Adam Harley on Visual Tracking

Silvio Savarese & Jeannette Bohg Lecture 17