RANSAC: RANdom Sampling And Consensus

Roland Angst
rangst@stanford.edu
www.stanford.edu/~rangst
The Need for RANSAC

- Why do I need RANSAC? I know robust statistics!
  - “Robust Statistics” Huber [1981]
    - M-estimator, L-estimator, R-estimators, …
  - Least Median of Squares (LMedS), …

- Breakdown point of an estimator
  - “Proportion of incorrect observations … an estimator can handle before giving an incorrect … result” [Wikipedia]

- Robust estimators can achieve breakdown point of 50%
  - For example: median

- Usually a non-linear, non-convex optimization problem needs to be solved
The Need for RANSAC

- **Problems**
  - Estimators for more complex entities (e.g., homographies, essential matrices, …)?
  - Inlier ratio of computer vision data can be lower than 50%

- **Hough Transform**
  - Excellent candidate for handling high-outlier regimes
  - Can only handle models with very few parameters (roughly 3)

- **RANSAC** is a good solution for models with slightly larger number of parameters
  - Roughly up to 10 parameters (depending on inlier ratio)
RANSAC [Fischler & Bolles 81]

- Hypothesize-and-verify framework
  - Sample hypothesis and verify with data

- Assumptions
  - Outliers provide inconsistent (i.e. random) votes for models
  - There are sufficiently many inliers to detect a correct model

- Hypothesis generation
  - Sample subset of data points and fit model parameters to this subset
    - Plain RANSAC: sample points uniformly at random

- Verification on all remaining data points
Algorithm Outline

1. Select random sample of minimum required size to fit model parameters
2. Compute a putative model from sample set
3. Verification stage: Compute the set of inliers to this model from whole data set
4. Check if current hypothesis is better than any other of the previously verified
5. Repeat 1-4
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Number of Iterations

- Probability of selecting an inlier given by inlier ratio $p_{\text{inlier}}$
- Sample size $s$
- Confidence value for having sampled at least one all-inlier sample $P$
- Number of iterations $k$

Let’s put all of this together: $1 - P \geq (1 - p_{\text{inlier}}^s)^k$

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$k \geq \frac{\log(1 - P)}{\log(1 - p_{\text{inlier}}^s)}$

Probability of having selected at least one outlier in each of the $k$ trials
RANSAC Parameters

- How to find inlier ratio?
  - Provide lower bound for initialization and recompute when new best hypothesis has been found

- Scale of inlier noise

- Confidence for having sampled at least one all-inlier sample
Shortcomings of ‘Plain’ RANSAC

- Scale of inlier noise (for inlier-outlier threshold) needs to be specified
- Correct model is not generated with user-defined confidence
- Estimated model might be inaccurate
- Degenerate cases not handled
- Can be sped up considerably
  - Better hypothesis generation
  - Faster verification schemes
- Multiple models
  - Model selection
  - Interesting problem, but not covered in remainder
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Noisy Inliers

- Problem: not every all-inlier-sample provides a good solution
  - Sampling more than one all-inlier-set might be necessary!
  - In practice, solution often found only after roughly $k = \left( \frac{1}{p_{\text{inlier}}} \right)^{\text{sample size}}$ iterations
    - Simple calculation $k \geq \frac{\log(1 - P)}{\log(1 - p_{\text{inlier}}^s)}$ is inaccurate
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Increase Accuracy of Estimated Models

- **Lo-RANSAC**
  - Run inner RANSAC loop with non-minimal sample size to refine hypothesis of minimal sample size
  - “Locally Optimized RANSAC” Chum, Matas, Kittler [DAGM03]

- **MLESAC**
  - Fit model by max likelihood rather than max inlier count
  - “MLESAC: A new robust estimator with application to estimating image geometry” Torr & Zisserman [1996]
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Handling Degenerate Cases

- “Two-view geometry estimation unaffected by a dominant plane” Chum et.al. [CVPR05]
  - Estimate fundamental
  - If successful try to fit homography to triplet of 7-cardinality MSS
    - If homography can be found run plane-and-parallax fundamental estimation
      - 2 points off the plane need to get fundamental from known homography
      - 2-pt RANSAC over outliers of homography
    - else non-planar case

- Other approaches for making RANSAC robust w.r.t. degeneracies
  - “RANSAC for (quasi-)degenerate data (QDEGSAC)” Frahm & Pollefeys [CVPR06]
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Hypothesis Generation

- Trade-off between exploration and exploitation
  - Previously verified hypothesis tell us something about inlier set
  - Still, we should avoid narrowing our search too quickly
- Especially important for multi-model case
  - Eg. estimation of multiple planes in a scene
  - Points on other planes act as outliers to plane under consideration
PROSAC

- “Matching with PROSAC – progressive sample consensus” Chum & Matas [CVPR05]
- Use of a-priori knowledge
  - Confidence of a matching pair (e.g., based on descriptor matching distance)
- PROSAC: Favor high-quality matches while sampling points for minimal sample
  - Sort correspondences according to matching score
  - Consider progressively larger subsets of putative correspondences
  - Note: draws the same samples as RANSAC would, just in different order
- Pro
  - Can decrease the number of required hypothesis considerably
- Contra
  - Performance gain depends on data
  - Practical observation: high-confidence matches appear often appear in clusters on same spatial structure
    - Degenerate configurations…
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Verification

- Phrase hypothesis verification in sequential testing framework
  - Subsample remaining data and verify on this subset
  - If inlier ratio is sufficiently low: terminate verification

- Several papers have been published
  - Threshold determined based on $T_d,d$ tests
    - “Randomized RANSAC with $T_d,d$ test” Matas, Chum [IVC04]
  - Bail-Out test based on hyper-geometric distribution
    - “An effective bail-out test for RANSAC consensus scoring” Capel [BMVC05]
  - Wald’s Sequential Probability Ratio Test (WaldSAC)
    - “Optimal randomized RANSAC” Chum & Matas [PAMI07]

Speedup of 2-7 times compared to standard RANSAC according to:
“A Comparative Analysis of RANSAC Techniques Leading to Adaptive Real-Time Random Sample Consensus” Raguram et.al. [ECCV08]
Preemptive RANSAC

“Preemptive RANSAC for live structure and motion estimation” Nister [ICCV03]

Find a good estimate within a fixed time budget (eg. in a vSLAM system)

Idea
- Generate fixed number of hypothesis
- Verify all of them in parallel
  - Breadth-first verification scheme
  - Verify all hypothesis on a subset of the data
    - Prune unpromising hypothesis and retain promising ones
  - Verify on increasingly larger subsets, followed by pruning step
ARRSAC

- Adaptive Real-Time RANSAC
- Carefully designed combination of previous RANSAC approaches
- Achieves considerable speed-ups while still providing correct solution
- “A Comparative Analysis of RANSAC Techniques Leading to Adaptive Real-Time Random Sample Consensus” Raguram et.al. [ECCV08]
Further Evaluation and Comparisons

- “Performance Evaluation of RANSAC Family” Choi et.al. [BMVC09]

<table>
<thead>
<tr>
<th>Inlier Ratio</th>
<th>LMedS</th>
<th>RANSAC</th>
<th>MSAC</th>
<th>MLESAC</th>
<th>LO-RANSAC</th>
<th>R-RANSAC.T</th>
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Many different ‘flavours’

Still an active research area